Discrete Symmetry Violations

Lecture III :

Revision of previous lecture, Origin of EDMs in closed-shell systems, Barr’s chart, (P, T) violating interactions in closed-shell atoms, Schiff theorem, Violation of Schiff theorem, Schiff moment Hamiltonian, nuclear Schiff moment, nuclear spin and NSM

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Revision

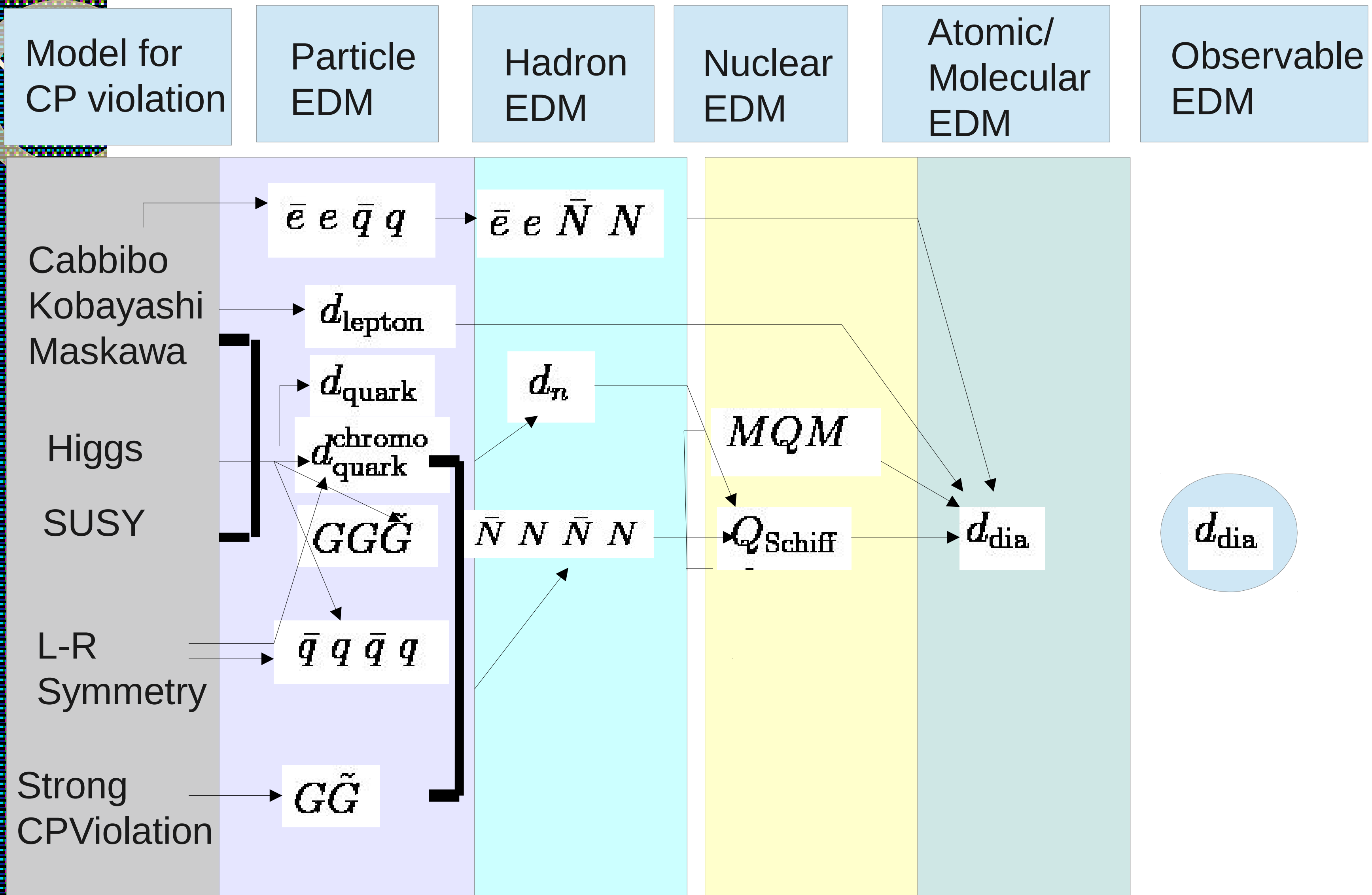
- Non-relativistically, an atomic EDM is always zero even if its constituents have non-zero EDM. This happens because there is an exact cancellation of external field with the internal field to give a zero net field in an atom.

- The atomic EDM is non-zero only when the relati- -vistic atomic Hamiltonian is used along with the relativistic form of the EDM interaction Hamiltonian

- The e-N (P,T)-odd interactions can be of S-PS, T-PT, V-A, A-V, PS-S types.

- Closed-shell and open-shell atomic EDMs arise from different sources. What are dominating sources for closed-shell case, need not be so for open-shell cases.

Closed-shell atoms, Barr's chart



PTV in closed-shell atoms

EDM of a diamagnetic atom arises predominantly from,

- Nuclear Schiff moment ← nucleon-nucleon interactions ← quark interactions and chromo EDMs.
- Electron-nucleus interactions ← electron-quark interactions .

EDM of closed-shell atoms dominantly arises from

- Nuclear Schiff moment (S)
- Tensor-pseudotensor electron-nuclear interactions.

Other sources include d_e , P, T – odd electron-nuclear interac- -tions and nuclear magnetic quadrupole moment.

Schiff theorem

Consider an assemblage of particles

1) That are non-relativistic
2) Interact only through electrostatic forces
3) The electric dipole distribution within each particle is same as its charge distribution

will have no permanent EDM.

- The energy shift for a system having an electric dipole moment D is Є = - D . E.
- The electric field for ith particle having dipole moment,

$$-\nabla_i V$$

Consider the unperturbed Hamiltonian,

$$H_0 = \sum_i \frac{p_i^2}{2m_i} + V_{\mathrm{nuc}}(r_i) + \sum_{i<j} \frac{e^2}{r_{ij}}$$

The EDM corrections to the Hamiltonian can be written as

$$H^{\mathrm{EDM}} = -\mathbf{d}_{\mathrm{i}} \cdot \mathbf{E}_i$$

where

$$\mathbf{d}_{\mathrm{i}} \cdot \mathbf{E}_i = d_e \sigma_i \cdot \mathbf{E}_i$$

It can be proved that

$$\sigma_i \cdot \mathbf{E}_i = -1/e\,[\sigma_i \cdot \nabla_i, H_0]$$

Using this,

$$H^{\mathrm{EDM}} = \sum_i (i/\hbar) \left[\frac{\mathbf{d}_{\mathrm{i}} \cdot \mathbf{p}_i}{q_i}, H_0 \right]$$

Denote $\frac{\mathbf{d}_{\mathrm{i}} \cdot \mathbf{p}_i}{q_i} = Q$. Therefore,

$$H^{\mathrm{EDM}} = i \left[Q, H_0 \right]$$

The Hamiltonian without the interaction of the EDM is H = T + V, where V includes contributions from external And internal electric fields.

EDM interaction is expressed as a commutator with the Hamiltonian. Calculate the expectation value of H^{EDM} :

$$\langle \Psi_0 | H^{\mathrm{EDM}} | \Psi_0 \rangle = ?$$

$$= \langle \Psi_0 | i \left[Q, H_0 \right] | \Psi_0 \rangle$$

$$= i \langle \Psi_0 | Q H_0 - H_0 Q | \Psi_0 \rangle = 0$$

$\Rightarrow$ Gives no contribution to an expectation value. Question Is how to observe the EDM of a charged particle ? EDMs of neutral atoms can be detected by several Mechanisms.

Violation of Schiff theorem

- This conclusion is true for particles interacting only through electrostatic forces. If only electrostatic forces are involved, when external field is applied, the particles rearrange in such a way that net field is zero. There should be non-electrostatic forces involved, so that the net field is non-zero.

- A nucleus is held together by strong interactions which give it a finite size. In this case, distribution of EDM and charge is different. Then, nucleus is said to have a Schiff moment.

- Relativistic effects cannot be ignored. The interaction of the EDM of a relativistic particle cannot be expanded in a simple commutator form and hence gives a non-zero expectation value.

These situations may lead to a non-zero dipole moment.

- The operator Q commutes with the kinetic energy, T only for the non-relativistic case, where $T = p_i^2/2\text{m}$
- For the relativistic case,

$$T = \Sigma_i c\,\alpha_i \bullet p_i + \beta_i m_i c^2$$

and with relativistic expression of Q, the cancellation is incomplete and leads to a residual effective interaction term.

- It can be rigorously shown that the commutator of H^{EDM} will vanish by explicitly considering the interaction of the system with external field :

In the presence of external field,

$$H = H_0 - \sum_i q_i \mathbf{r}_i \cdot \mathbf{E}_i = H_0 + H^E$$

To first order in the external field the wavefunction correction $\delta\Psi^E$ caused by H^E can be obtained from the results of perturbation theory

$$(E_0 - H_0)\,|\delta\Psi^E\rangle = H^E|\Psi_0\rangle$$

Now, expand the commutator,

$$i\langle\Psi|\,[Q,H]\,|\Psi\rangle$$

where H is the total Hamiltonian.

$$= i\left(\langle\Psi_0| + \langle\delta\Psi^E|\right)[Q,H]\left(|\Psi_0\rangle + |\delta\Psi^E\rangle\right)$$

Where E_0 is the unperturbed eigen value, $H_0\Psi_0 = E_0\Psi_0$. By expanding the commutators $[Q,H_0]$ and using the equation for $|\delta\Psi^E\rangle$ from perturbation theory to replace few terms by $H^E|\Psi_0\rangle$,

we find that after simplification, leads to an exact cancellation of the terms involving the internal and external electric fields.

Nuclear Schiff moment

There can be a residual EDM interaction when Schiff theorem is violated.

The electronic matrix elements of the potential $\Phi_0(R)$ are related to the nuclear Schiff moment. The first P, T – odd term in the multipole expansion can be written in terms of a vector **S** as

$$S = \frac{1}{10}\left(\int e\rho(r) r^2 d^3r - \frac{5}{3}\mathbf{d}(1/Z)\int \rho(r) r^2 d^3r\right)$$

The nuclear potential arising from the nuclear Schiff moment is

$$\Phi(\vec{R}) = -3\frac{\vec{S}\cdot\vec{R}}{B}\rho(\vec{R})$$

where $B = \int R^4\rho(\vec{R})dR$

Closed-shell Atomic EDMs

The P and T violating T-PT electron-nucleon interaction is

$$H_{e-N} = \sum_{j=1}^{n} i\frac{G_F}{\sqrt{2}} C_T \left[i\bar{\Psi}_N \sigma_{\mu\nu}\Psi_N\right]\left[\bar{\Psi}_e \gamma^5 \sigma_{\mu\nu}\Psi_e\right]$$

$$H_{e-N} = \sum_{j=1}^{n} iG_F(2\sqrt{2})C_T \beta \alpha \vec{I} \rho_N(\vec{r})$$

Where

C_T Is the T-PT coupling constant,

G_F is the Fermi constant = $2.22 \quad 10^{-14}$ a.u,

$\sigma_{\mu\nu}$, γ^5 are built from the Dirac matrices,

' I ' is the nuclear spin.

The matrix elements of the $H_{e\text{-}N}$ operator ~ Z^2, hence heavy elements are preferred.

C_T is zero in the framework of Standard Model

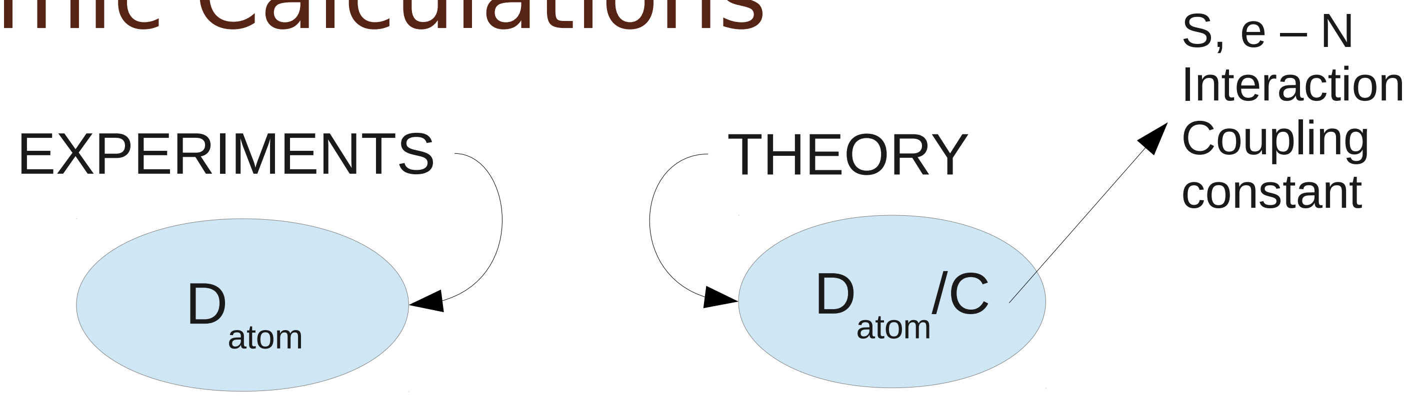
A non-zero value of C_T would imply new physics beyond the Standard Model. To set limits on specific models of CP violation, the experimental results must be related to EDMs of fundamental particles.

Open-shell Atomic EDMs

P, T violating effects in open-shell atoms dominantly arise From

- Electron EDM,
- P, T – odd electron-nuclear interactions,
- Schiff moment
- Magnetic quadrupole moment of the nucleus

Determination of NSM from Atomic Calculations



Atomic EDM is

$$D_a = \frac{\langle \tilde{\Psi} | \vec{D} | \tilde{\Psi} \rangle}{\langle \tilde{\Psi} | \tilde{\Psi} \rangle}$$

where $|\tilde{\Psi}\rangle$ are the perturbed atomic exact states, given by

$$|\tilde{\Psi}\rangle = |\Psi_0\rangle + \lambda|\Psi_1\rangle$$

Then,

$$D_a/\lambda = \frac{\langle\Psi_0|\vec{D}|\Psi_1\rangle\langle\Psi_1|\vec{D}|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle}$$

EDMs are enhanced in atoms having,

High nuclear charge (Z) $\Rightarrow$ P, T – odd effects are dominant in heavy atoms.

Close levels of opposite parity, $D_{atom} \sim 1 / \Delta E$

Interpretation of EDM results

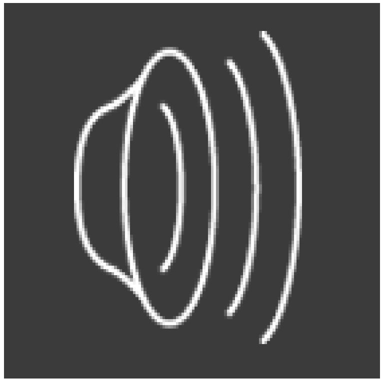
1. C_T is zero in the framework of Standard Model. C_T may be used to arrive at the electron-quark CP violating coupling constants.
2. The nuclear Schiff moment can arise from

- A nucleon EDM (for nuclei with unpaired nucleons)
- P, T violating nucleon-nucleon interactions.

The N-N interactions take place via a pion exchange. The Schiff moment can be related to the CP-violating pion-nucleon coupling constant.

- Inturn, the pion-nucleon coupling constant is related to the QCD vaccuum angle and to the chromo-EDMs of quarks and also to the EDM of a neutron.

Fantastic video by Brady Haran in which Ed Copeland, a theorist at the University of Nottingham, talks about their latest Result and the significance of measuring 'the shape of the electron'.



Determination of PTV π – N coupling constant

The nucleon-nucleon interactions are dominated by pion-exchanges. For ^{199}Hg,

$$-g_{\pi NN}\bar{g}_{\pi NN} = \eta_{np}\frac{G_F m_\pi^2}{\sqrt{2}}$$

where $g_{\pi NN}$ and $\bar{g}_{\pi NN}$ are the CP-conserving and the CP violating pion-nucleon couplings. The constant η_{np} is the (n, p) coupling constant, which is directly related to the Schiff moment as

$$S/(\mathrm{efm}^3) = -1.4 \times 10^{-8}\eta_{np}$$

Determination of QCD angle and chromo-EDMs of quarks

The constant $\bar{g}_{\pi NN}$ for ^{199}Hg can be used to set limits on the QCD vacuum angle using

$$\bar{g}_{\pi NN} \approx -0.027\theta_{\rm QCD}$$

Determination of nucleon EDMs

It is also possible to set limits on the neutron and the proton EDMs from ^{199}Hg EDM, using

$$d_N \approx \left(5.2 \times 10^{-16}\right) \theta ecm$$

Validity of Particle Physics Models

Electron EDM can also be deduced from closed shell atoms by Considering the hyperfine interaction as a perturbation.

The EDM of an electron, as predicted by the Standard model of Particle physics in comparison with other models :

Model	d_e e-cm
Standard Model	$< 10^{-38}$
SUSY Multi-Higgs Left-right asymmetric	$10^{-26} – 10^{-28}$