Discrete Symmetry Violations

Lecture 2 :

Revision of previous lecture, EDM of non-relativistic/relativistic systems, Barr's chart, status of the Electron EDM experiment, theoretical determination of electron EDM

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Revision

- Non-zero electric dipole moments on non-dege-
-nerate systems imply P, T violation
- Atoms are rich sources of CP (equivalently P,T) violation.
- Closed-shell atoms get EDM from the nuclear sector
- Open-shell atoms get EDM due to electron EDM
- EDMs are enhanced in heavy atoms
- Comparison of atomic theory and experiments give information about the P,T violating coupling constants

EDM interaction (Lepton)

Interaction of the EDM of a spin-1/2 fermion with an electromagnetic field. Since EDM involves P, T - odd operators,

$$L_{\rm EDM} = -i\frac{d}{2}\bar{\Psi}\gamma^5\sigma^{\mu\nu}\Psi F_{\mu\nu}$$

where Ψ and $\bar{\Psi}$ are the Dirac field and the Dirac conjugate field, $\sigma^{\mu\nu} = (i/2)\left[\gamma^\mu, \gamma^\nu\right]$ and $F_{\mu\nu}$ is the field tensor. Using $\bar{\Psi} = \Psi^\dagger\gamma^0$ and expanding $\sigma^{\mu\nu}$,

$$L_{\rm EDM} = -i\frac{d}{2}\Psi^\dagger\gamma^0\gamma^5\sigma^{\mu\nu}\Psi\ F_{\mu\nu}$$

Use the following identities :

- $\sigma^{\mu\nu} = i\gamma^\mu\gamma^\nu$

  Proof :

$$\sigma^{\mu\nu} = (i/2)\left[\gamma^\mu, \gamma^\nu\right]$$

. But,

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

where $g^{\mu\nu}$ is the metric tensor. Now, since $\mu \neq \nu$,

$$\sigma^{\mu\nu} = (i/2)(2\gamma^\mu\gamma^\nu)$$

- $\{\gamma^5, \gamma^\mu\} = 0$

to show that

$$L_{\text{EDM}} = i\frac{d}{2}\Psi^\dagger\gamma^5\gamma^0\gamma^\mu\gamma^\nu\Psi\ F_{\mu\nu}$$

Using $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ to show that

$$L_{\text{EDM}} = d\bar{\Psi}\left[\Sigma.\mathbf{E} - i\alpha.\mathbf{B}\right]\Psi$$

where

$$\Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \text{ and } \alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}$$

E and B are the electric and magnetic fields respectively.

From L_{EDM}, it is easy to show that the single-particle Dirac Hamiltonian is

$$H_{\text{EDM}} = -d\gamma^0\Sigma.E + id\gamma.B$$

In the non-relativistic limit,

$$H_{\text{EDM}} = -d\sigma.E + id\gamma.B$$

Relativistic effects

The EDM of the atom is zero non-relativistically even if the electron is assumed to to have an intrinsic EDM.

Start from the atomic Hamiltonian

$$H_0 = \sum_i \left(p_i^2/2m_i + V_{\text{nuc}}(r_i)\right) + \sum_{i \neq j} \frac{e^2}{r_{ij}}$$

The EDM Hamiltonian (non-relativistic), as derived earlier is

$$H_1 = -d_e \sum_i \sigma_i \cdot E_i^{\text{int}}$$

where E_i^{int} is the internal electric field and is given by where E_i^{int} is the internal electric field and is given by

$$eE_i^{\text{int}} = -\nabla_i \left(V_{\text{nuc}}(r_i) + \sum_{i \neq j} \frac{e^2}{r_{ij}} \right)$$

Call the term in the bracket as $e\Phi$, a potential term. The perturbed atomic state would be

$$|\Psi\rangle = |\Psi_0^{(0)}\rangle + |\Psi^{(1)}\rangle$$

where

$$|\Psi^{(1)}\rangle = \sum_n \frac{|\Psi_n^{(0)}\rangle\langle\Psi_n^{(0)}|H_1||\Psi_0^{(0)}\rangle}{E_0 - E_n}$$

Consider an external field E in the z-direction,

$$H_{\text{ext}} = -eE \sum_i \left(z_i + \frac{d_e}{e}\sigma_{zi} \right)$$

This gives the definition of an atomic EDM.

$$D_{\text{atom}} = \langle \Psi | \sum_i d_e \sigma_{zi} + e z_i | \Psi \rangle$$

Let

$$D^{(0)} = d_e \langle \Psi_0^{(0)} | \sum_i \sigma_{zi} | \Psi_0^{(0)} \rangle$$

and

$$D^{(1)} = \langle \Psi_0^{(0)} | \sum_i e z_i | \Psi^{(1)} \rangle + \langle \Psi_0^{(1)} | \sum_i e z_i | \Psi_0^{(0)} \rangle$$

$$= \sum_n \frac{\langle \Psi_0^{(0)} | \sum_i e z_i | \Psi_n^{(0)} \rangle \langle \Psi_n^{(0)} | - d_e \sum_i \sigma_i \cdot E_i^{\text{int}} | \Psi_0^{(0)} \rangle}{E_0 - E_n}$$

$$+ \frac{\langle \Psi_0^{(0)} | - d_e \sum_i \sigma_i \cdot E_i^{\text{int}} | \Psi_n^{(0)} \rangle \langle \Psi_n^{(0)} | \sum_i e z_i | \Psi_0^{(0)} \rangle}{E_0 - E_n}$$

Or,

$$D = D^{(0)} + D^{(1)}$$

Consider the term $\sigma_i \cdot E_i^{\rm int}$ can be written as

$$\sigma_i \cdot E_i^{\rm int} = (1/e)\sigma_i \cdot \nabla_i(e\Phi)$$
$$= (-1/e)\left[\sigma_i \cdot \nabla_i, H_0\right]$$

Substitute this in $D^{(1)}$ and get

$$\sum_n \frac{\langle \Psi_0^{(0)} | \sum_i e z_i | \Psi_n^{(0)} \rangle \langle \Psi_n^{(0)} | (d_e/e)\left[\sigma_i \cdot \nabla_i, H_0\right] | \Psi_0^{(0)} \rangle}{E_0 - E_n}$$

$$+ \sum_n \frac{\langle \Psi_0^{(0)} | (d_e/e)\left[\sigma_i \cdot \nabla_i, H_0\right] | \Psi_0^{(n)} \rangle \langle \Psi_n^{(0)} | \sum_i e z_i | \Psi_0^{(0)} \rangle}{E_0 - E_n}$$

Drop the subscripts for the time-being

$$= (d_e/e)\frac{\langle\Psi_0^{(0)}|ez|\Psi_n^{(0)}\rangle\langle\Psi_n^{(0)}|(\sigma_i\cdot\nabla)H_0 - H_0(\sigma_i\cdot\nabla)|\Psi_0^{(0)}\rangle}{E_0 - E_n}$$

$$+(d_e/e)\frac{\langle\Psi_0^{(0)}|(\sigma_i\cdot\nabla)H_0 - H_0(\sigma_i\cdot\nabla)|\Psi_0^{(n)}\rangle\langle\Psi_n^{(0)}|ez|\Psi_0^{(0)}\rangle}{E_0 - E_n}$$

Simplifying,

$$D^{(1)} = -d_e\langle\Psi_0^{(0)}|\left[\sum_i \sigma_i\cdot\nabla_i, \sum_i z_i\right]|\Psi_0^{(0)}\rangle$$

Hence,

$$D^{(1)} = -d_e\langle\Psi_0^{(0)}|\sum_i \sigma_{zi}|\Psi_0^{(0)}\rangle$$

Finally,

$$D = D^{(0)} + D^{(1)} = 0$$

Electron edm in a non-relativistic atom

For a non-relativistic atom, though the intrinsic EDM of an electron is non-zero, the total EDM of the atom turns out to be zero.

Electron edm in a relativistic atom

- For the relativistic case, the same energy shift due to EDM interaction turns out to be non-zero!

Consider the situation where we use the relativistic form of the Hamiltonian H_0 and the non-relativistic form of the EDM interaction, $H_1 = -d_e \sum_i \sigma_i \cdot E_i^{\rm int}$. Check whether

$$\sigma_i \cdot E_i^{\rm int} = (-1/e)\left[\sigma_i \cdot \nabla_i, H_0\right]$$

is still valid.

$$H_0 = \sum_j \left(c\alpha_j \cdot p_j + \beta_j mc^2 + V_{\text{nuc}}(r_j)\right] + \sum_{j \neq l} \frac{e^2}{r_{jl}}$$

Consider the commutator, $(T = \sum_j \left(c\alpha_j \cdot p_j + \beta_j mc^2\right])$

$$(-1/e)\left[\sigma_i \cdot \nabla_i, T\right]$$

$$= -ic \sum_j \left[\sigma_i \cdot p_i, (\alpha_j \cdot p_j + \beta_j mc)\right]$$

$$= -ic\left[\sigma \cdot p, \alpha \cdot p\right] + mc\left[\sigma \cdot p, \beta\right]$$

$$= -ic \left(\sum_{kl} p_k p_l \left[\sigma_k, \alpha_l\right] + mc \sum_k p_k \left[\sigma_k, \beta\right] \right)$$

Consider,

$$\left[\sigma_k, \beta\right] = \left[-\gamma_5 \alpha_k, \beta\right] = 0$$

Since, $\{\gamma_5, \beta\}$ anti-commute.

Consider,

$$\sum_{kl} p_k p_l \left[\sigma_k, \alpha_l\right] = \sum_k p_k^2 \left[\sigma_k, \alpha_k\right] + \sum_{k,l;k\neq l} p_k p_l \left[\sigma_k, \alpha_l\right]$$

Since, $\left[\sigma_k, \alpha_k\right] = \left[\gamma_5 \alpha_k, \alpha_k\right] = 0$. And,

$$\left[\sigma_k, \alpha_l\right] = \left[-\gamma_5 \alpha_k, \alpha_l\right]$$

$$= -\gamma_5 \left[\alpha_k, \alpha_l\right]$$

$$\sum_{k,l;k\neq l} \left[\sigma_k, \alpha_l\right] = -\gamma_5 \sum_{k,l;k\neq l} \left[\alpha_k, \alpha_l\right] = 0$$

Therefore,

$$\left[\sigma_i \cdot \nabla_i, T\right] = 0$$

We get the same result as before i.e., EDM of an atom is zero.

We obtain a non-zero EDM of an atom only if the atomic Hamiltonian is relativistic and we use the correct relativistic form for the EDM interaction Hamiltonian

We already know,

$$-e\sigma_i \cdot E_i^{\text{int}} = \left[\sigma_i \cdot \nabla_i, \sum_j \left(V_{\text{nuc}}(r_j) + \sum_{j/nel} \frac{e^2}{r_{jl}} \right) \right]$$

$$= [\sigma_i \cdot \nabla_i, (H_0 - T)] = [\sigma_i \cdot \nabla_i, H_0] - [\sigma_i \cdot \nabla_i, T]$$

So,

$$-\beta e \sigma_i \cdot E_i^{\text{int}} = [\beta \sigma_i \cdot \nabla_i, H_0] - [\beta \sigma_i \cdot \nabla_i, T]$$

t can be proved that

$$[\beta \sigma_i \cdot \nabla_i, T] = 2ic\beta_i \gamma_{5i} p_i^2$$

Substituting in D and simplifying, after cancellations

$$D = 2icd_e\{\frac{\sum_n \langle \Psi_0^{(0)} | \sum_i z_i | \Psi_n \rangle \langle | \sum_i \beta_i \gamma_{5i} p_i^2 | \Psi_0^{(0)} \rangle}{E_0 - E_n}$$

$$+ \frac{\sum_n \langle \Psi_0^{(0)} | \sum_i \beta_i \gamma_{5i} p_i^2 | \Psi_n \rangle \langle | \sum_i z_i | \Psi_0^{(0)} \rangle}{E_0 - E_n}\}$$

We can identify the effective Hamiltonian,

$$H_{\mathrm{EDM}}^{\mathrm{eff}} = 2icd_e \beta \gamma_5 p^2$$

The Hamiltonian for P, T - odd interactions
:

$\bar{N}N \cdot \bar{e}\gamma^5 e$ (S-PS - scalar-pseudoscalar)

$\bar{N}\gamma^\mu N \cdot \bar{e}\gamma_\mu\gamma^5 e$ (V-A - vector-pseudovector)

$\bar{N}\sigma^{\mu\nu}N \cdot \bar{e}\sigma_{\mu\nu}\gamma^5 e$ (T-PT - tensor-pseudotensor)

$\bar{N}\gamma^\mu\gamma^5 N \cdot \bar{e}\gamma_\mu e$ (A-V - pseudovector-vector)

$\bar{N}\gamma^5 N \cdot \bar{e}e$ (PS-S - pseudoscalar-scalar)

Tensor-pseudotensor EDM interaction

$H_{\text{T-PT}} = \bar{N}\sigma^{\mu\nu}N \cdot \bar{e}\sigma_{\mu\nu}\gamma^5 e$ (T-PT - tensor-pseudotensor)

$$H_{\text{EDM}} = iC_T G_F \sqrt{(2)} \sum_i (\gamma_i \cdot I)\, \rho_N(r)$$

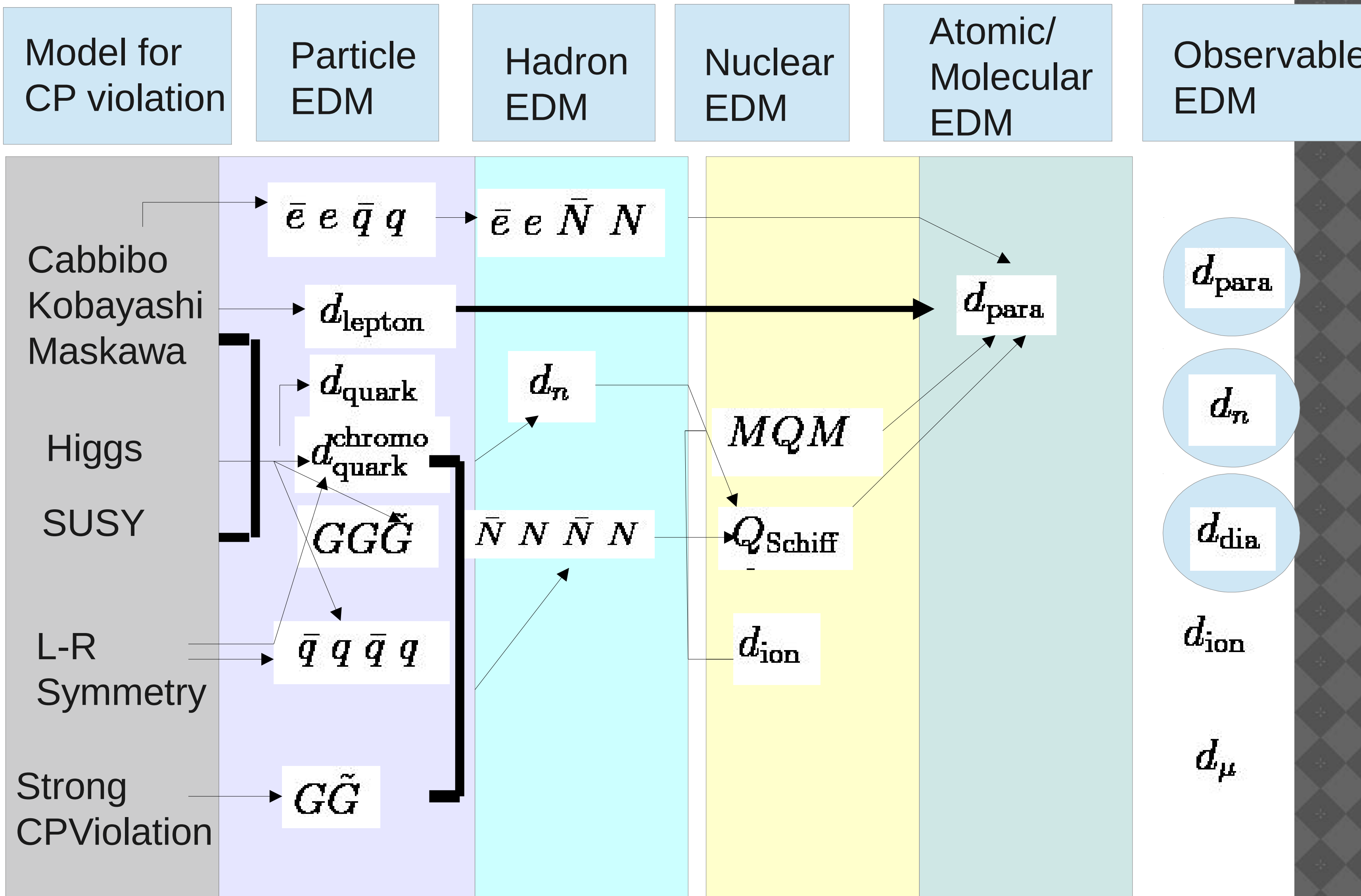
where $\rho_N(r)$ is the nuclear density, I is the nuclear spin, C_T is the T-PT coupling constant and G_F is the Fermi coupling constant.

Scalar-pseudoscalar EDM interaction

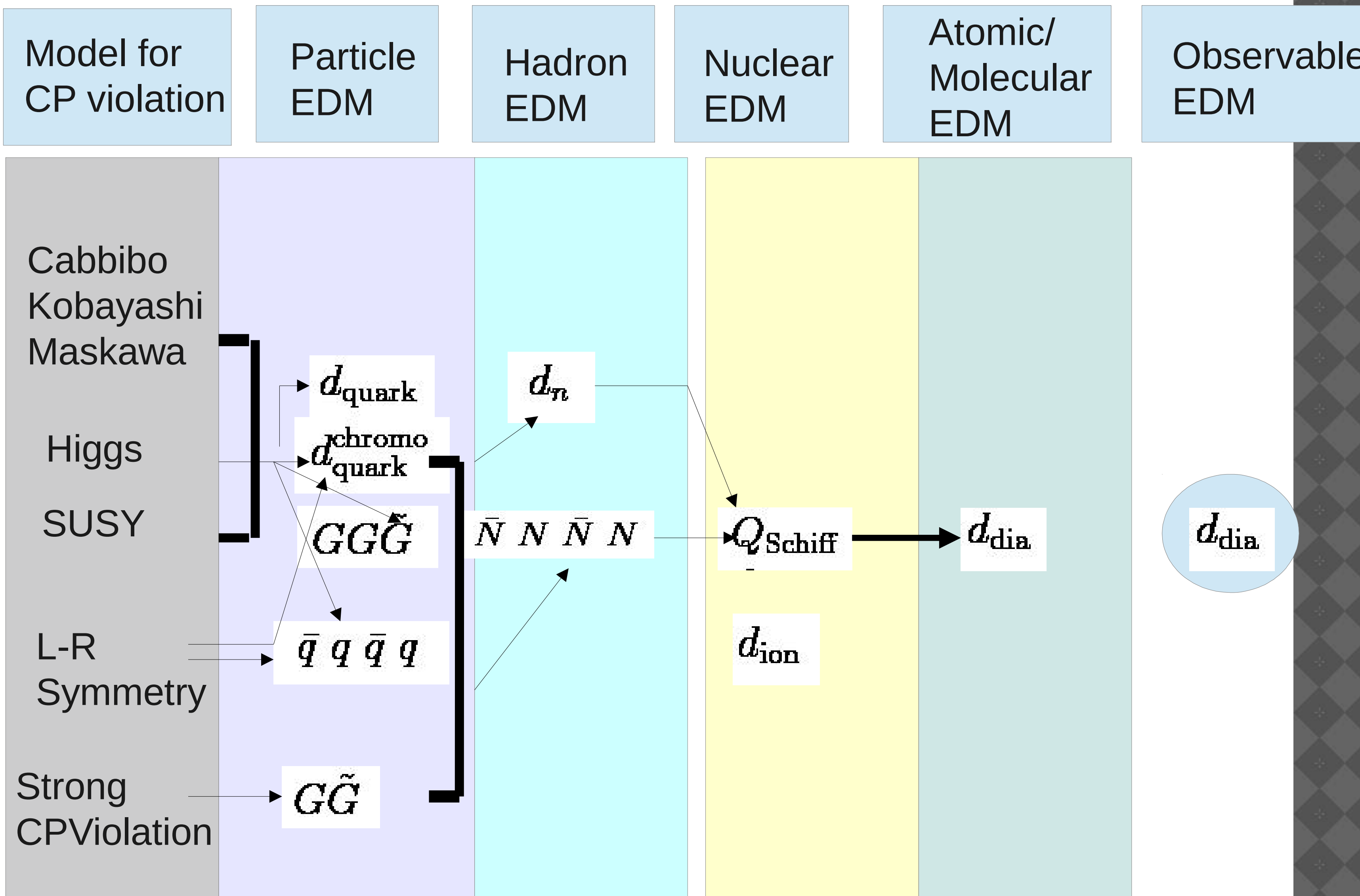
$H_{T\text{-}PT}$ = $\bar{N}N \cdot \bar{e}\gamma^5 e$ (S-PS - scalar-pseudoscalar)

$$H_{\text{EDM}} = iG_F/\sqrt{2}C_S\beta\gamma_5\rho_N(r)$$

Open-shell atoms



P, T-violating interactions in closed-shell atoms



EDM experiments – electron EDM

$$d_e = (-2.4 \pm 5.7\mathrm{stat} \pm 1.5\mathrm{syst}) \times 10^{-28} e\ \mathrm{cm}$$

Nature, 473, 493–496, (26 May 2011),J. J. Hudson,D. M. Kara,I. J. Smallman,B. E. Sauer,M. R. Tarbutt & E. A. Hinds

Mercury EDM - updates

$|d(199Hg)| < 3.1\times10^{-29}$ e cm (95% C.L.)

PRL. 102, 101601 (2009), W. C. Griffith, M. D. Swallows, T. H. Loftus, M. V. Romalis, B. R. Heckel, and E. N. Fortson

Thallium EDM - updates

$d_e \leq 1.6 \times 10^{(-27)}$ e cm

B. C. Regan, E. D. Commins, C. J. Schmidt, and D. DeMille, Phys. Rev. Lett. 88, 071805 (2002).

Neutron EDM - updates

$$d_n = -(3 \pm 5) \times 10^{-26}\ e\,\mathrm{cm}$$

K.F. Smith *et al.*, Phys. Lett. B **234**, 191 (1990).

TABLE 1. Some EDM experiments underway or planned.

Spin	System	Method	Location
Nuclear	^{199}Hg	4-cell vapor	Seattle
	^{129}Xe	Liquid cell	Princeton
	Ra	Optical trap	Argonne
	Neutron	Superfluid He bath	Los Alamos, SNS
		Neutron cell	Grenoble, ILL, PSI
Electron	YbF	Beam	Imperial College
	PbO	Cell	Yale
	Polar molecule	Optical and ion traps	Oklahoma, Boulder
	^{133}Cs	Optical lattice traps	Penn St, Austin
	Magnetic Crystal	Macroscopic B or E	Amherst, Yale, Indiana

Search for a Permanent Electric Dipole Moment of the Mercury Atom

E. N. Fortson

Citation: AIP Conf. Proc. **1182**, 629 (2009); doi: 10.1063/1.3293887