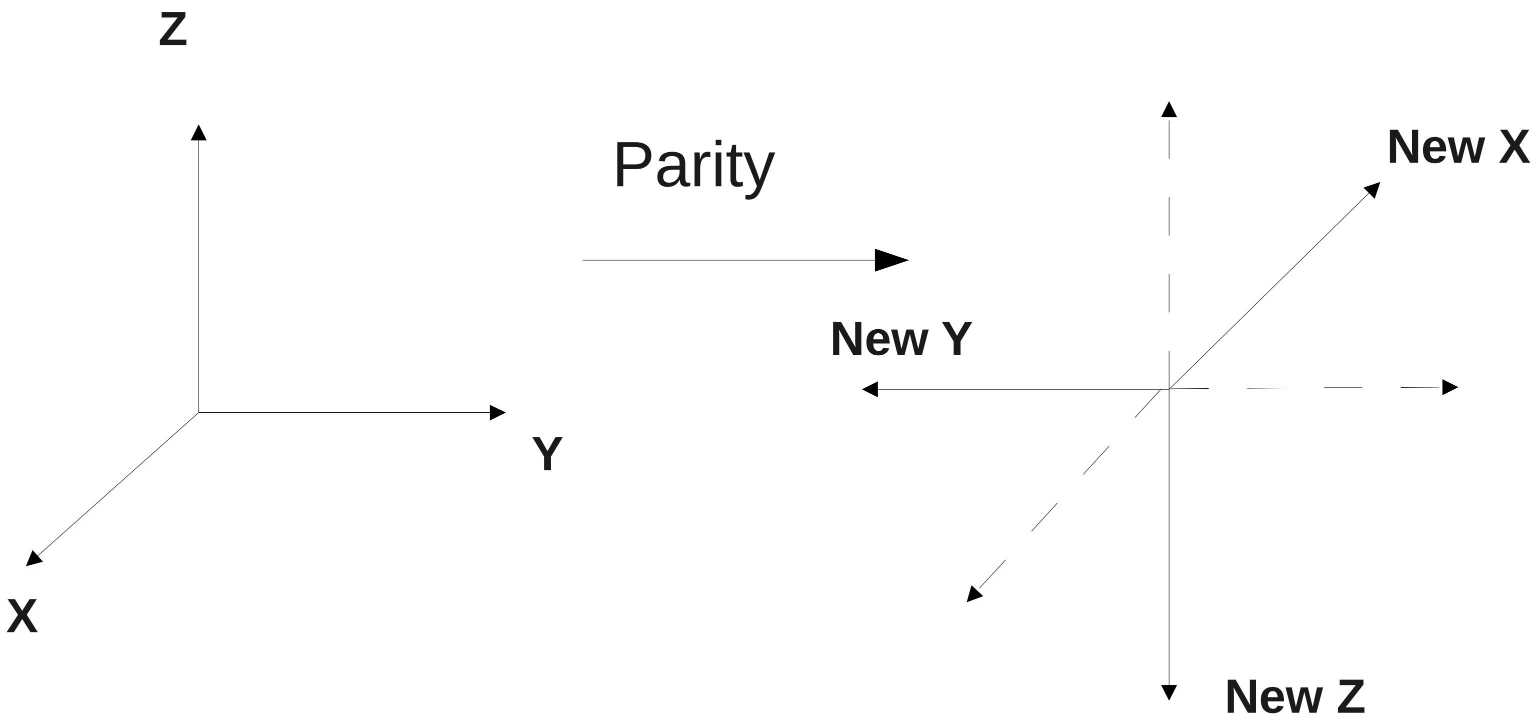
Discrete Symmetry Violations

Lecture 1 :
Parity, Time-reversal, violation of parity & time-reversal, experimental search for (P, T) violation, (P,T) – violation in atoms, permanent electric dipole moments.

K.V.P. Latha,
Department of Physics,
Pondicherry University,
Puducherry – 605014
PCPV 2013, Mahabaleshwar

Discrete symmetries

- Parity / space inversion – Geometric meaning



X → – X
Y → – Y
Z → – Z

Let the parity operator act on vectors of Hilbert space while the coordinate system remains fixed.

1. Associate a Unitary operator for a symmetry operation, like in the case of translations, rotations, etc.

2. Wigner's theorem : States that any symmetry acts as a Unitary or anti-Unitary transformation in the Hilbert space.

$$\pi^\dagger \pi = 1 \text{ or } \pi^\dagger = \pi^{-1}$$

Parity as a discrete symmetry

Parity transformation on state kets :

Given $|\alpha\rangle$, consider a space inverted state obtained by applying a unitary operator (π) on $|\alpha\rangle$ as :

$$|\alpha\rangle \to \pi|\alpha\rangle$$

The expectation value of a length $\mathbf{x}$ taken with respect to the space-inverted state is

$$\langle\alpha|\pi^{\dagger}\mathbf{x}\pi|\alpha\rangle$$

This quantity should give opposite sign,

$$\langle\alpha|\pi^{\dagger}\mathbf{x}\pi|\alpha\rangle = -\langle\alpha|\mathbf{x}|\alpha\rangle$$

This is accomplished if $\pi^\dagger \mathbf{x} \pi = -\mathbf{x}$. That is,

$$\mathbf{x}\pi = -\pi\mathbf{x}$$

(since π is unitary as it is associated with a symmetry operation).

The quantities **X,** momentum **p** are **odd** under parity

Wave functions under parity :

The parity transformation of the wavefunction of a quantal system is

$$|\Psi\rangle \rightarrow |\Psi'\rangle = \pi|\Psi\rangle$$

Consider the wavefunction of a quantal system, $\Psi(\mathbf{x}') = \langle \mathbf{x}'|\alpha\rangle$. The wavefunction of the space-inverted state $\pi|\alpha\rangle$ is

$$\langle \mathbf{x}'|\pi|\alpha\rangle = \langle -\mathbf{x}'|\alpha\rangle = |\Psi(-\mathbf{x}')\rangle$$

Consider an eigenket of the parity operator,

$$\pi|\alpha\rangle = p_\alpha|\alpha\rangle$$

Then

$$\pi^2|\alpha\rangle = p_\alpha^2|\alpha\rangle = |\alpha\rangle$$

Therefore $p_\alpha^2 = \pm 1$. If eigenvalue $= +1$, then

$$\Psi(-x) = \Psi(x) \text{ (even parity)}$$

else

$$\Psi(-x) = -\Psi(x) \text{ (odd parity)}$$

Also, π is linear,

$$\pi\left[\Psi_1(x) + \Psi_2(x)\right] = \pi\Psi_1(x) + \pi\Psi_2(x)$$

and

$$\pi\left[c\Psi_1(x)\right] = c\pi\Psi_1(x)$$

Suppose $|\alpha\rangle$ is an eigen ket of parity. Then,

$\pi|\alpha\rangle = \pm|\alpha\rangle$ since eigen values of $\pi = \pm 1$ → Prove this using $\Pi^2 = 1$

The corresponding wavefunction,

$$\langle \mathbf{x}'|\pi|\alpha\rangle = \pm\langle \mathbf{x}'|\alpha\rangle$$

But,

$$\langle \mathbf{x}'|\pi|\alpha\rangle = \pm\langle -\mathbf{x}'|\alpha\rangle$$

Hence, $|\alpha\rangle$ is even or odd underparity depending on whether $\Psi(-\mathbf{x}') = \pm\Psi(\mathbf{x}')$ (i.e., even/odd parity).

Parity operator is (a) Hermitian (b) Linear (c) Unitary

Question : How does an eigenket of position vector transform under parity ?

That is,

$$\pi|x'\rangle = ?$$

Consider

$$\begin{aligned} x\pi|x'\rangle &= -\pi x|x'\rangle \\ &= -\pi x'|x'\rangle \\ &= (-x')\pi|x'\rangle \end{aligned} \qquad (1)$$

Implies that $\pi|x'\rangle$ is an eigenket of x with the eigen value $-x'$. Therefore,

$$\pi|x'\rangle = e^{i\delta}|-x'\rangle$$

Not all wavefunctions of physical interest have definite parities.

- Ex. Momentum does not commute with π --- momentum eigen ket is not a parity eigen ket.

- Orbital angular momentum commutes with π ---- eigenket of L is also a parity eigenket.

SUMMARY :

- $p\pi = -\pi p$, implies

$$\left\{\pi, p\right\} = 0$$

$$\left\{\pi, x\right\} = 0$$

-

$$[\pi, \boldsymbol{J}] = 0 = [\pi, \boldsymbol{L}]$$

THEOREM :

Suppose $[H, \pi] = 0$ *and* $|n\rangle$ *is a non-degenerate eigen ket of* H *with eigen value* E_n*, that is,*

$$H|n\rangle = E_n|n\rangle$$

Then, $|n\rangle$ *is also a parity eigen ket.*

Proof : We have,

$$H|n\rangle = E_n|n\rangle$$

This implies

$$\pi H|n\rangle = E_n\pi|n\rangle \text{ and } H\pi|n\rangle = E_n\pi|n\rangle$$

Therefore, $\pi|n\rangle$ is also an eigenket of H with the same eigen value, E_n.

- They should be linearly dependent, that is, $\pi|n\rangle = c|n\rangle$
- Since $\pi^2 = 1$ this gives $c^2 = 1$ or

$$\pi|n\rangle = \pm|n\rangle$$

Suppose $|\alpha\rangle$ and $|\beta\rangle$ are parity eigenstates :

$$\pi|\alpha\rangle = \epsilon_\alpha|\alpha\rangle \text{ and}$$

$$\pi|\beta\rangle = \epsilon_\beta|\beta\rangle$$

Consider

$$\langle\beta|\mathbf{x}|\alpha\rangle = \langle\beta|\pi^{-1}\pi\mathbf{x}\pi^{-1}\pi|\alpha\rangle$$

$$= \epsilon_\alpha\epsilon_\beta\left(-\langle\beta|\mathbf{x}|\alpha\rangle\right)$$

For a finite non-zero $\langle\beta|\mathbf{x}|\alpha\rangle$, the constants ϵ_α and ϵ_β must be opposite in sign. The parity odd operator $\mathbf{x}$ connects states of opposite parity, or

$$\langle n|\mathbf{x}|n\rangle = 0$$

CONCLUSION : If $[H, \pi] = 0$ and $|n\rangle$ is a non-degenerate eigenket of H with eigenvalue E_n :

$$H|n\rangle = E_n|n\rangle$$

then $|n\rangle$ is also a parity eigenket.

Notes :

- For a degenerate state, it is perfectly alright to have an electric dipole moment.
- Operators that are odd under parity like $\boldsymbol{p}$ or $\boldsymbol{S} \cdot \boldsymbol{x}$, have non-vanishing matrix elements only between states of opposite parity
  AND
  Operators that are even under parity connect states of the same parity.

Time-reversal

- Time-reversal is more appropriately understood as motion reversal.

Consider the motion of a charged particle in an electric field described by the Schroedinger equation,

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left[\frac{-\hbar^2}{2m}\nabla^2 + q\phi(\mathbf{r})\right]\Psi(\mathbf{r},t)$$

Under the operation $\Psi(\mathbf{r},t) \to \Psi(\mathbf{r},-t)$, the Schroedinger equation changes sign. The transformation that preserves the form should be $\Psi(\mathbf{r},t) \to \Psi^*(\mathbf{r},-t)$. In QM, time-reversal operator is anti-linear.

Consider the expectation value of the momentum operator, $\mathbf{p}$. We expect

$$\langle\alpha|\mathbf{p}|\alpha\rangle = -\langle\tilde{\alpha}|\mathbf{p}|\tilde{\alpha}\rangle$$

where $|\tilde{\alpha}\rangle$ is the time-reversed state. So,

$$\Theta\mathbf{p}\Theta^{-1} = -\mathbf{p}$$

Likewise,

$$\Theta\mathbf{x}\Theta^{-1} = \mathbf{x}$$

From the relation

$$[x_i, p_j]|\ \rangle = i\hbar\delta_{ij}|\ \rangle$$

Applying Θ on both sides,

$$\Theta[x_i, p_j]\Theta^{-1}\Theta = \Theta i\hbar\delta_{ij}|\ \rangle$$

This leads to

$$[x_i, (-p_j)]\Theta|\ \rangle = -i\hbar\delta_{ij}|\ \rangle$$

Θ has to be anti-unitary to preserve the commutation relation. Similarly, to preserve

$$[J_i, J_j] = i\hbar\epsilon_{ijk}J_k$$

the angular momentum operator must be odd under time-reversal,

$$\Theta \mathbf{J} \Theta^{-1} = -\mathbf{J}$$

Violation of P and T symmetries

- If a symmetry is not violated, the result of any experi- -ment, is invariant under the corresponding symmetry operation.
- In Classical Mechanics, Noether's theorem gives the direct relationship between symmetries and conserved Quantities.
- Variation of physical laws under the transformation $r \rightarrow -r$ leads to Parity violation
- Differences in the laws of Physics under the transfor- mation, $t \rightarrow -t$ leads to violation of time-reversal symme -try

Experimental search for PTV

- CP violation was observed in the neutral kaon system
- If CPT was an exact symmetry, CP violation would mean
- T violation.
- P violation was observed in the decay of Co nuclei
- T violation was detected in the neutral B-meson decays. BaBar has made the first direct measurement of T violation.

TIME – REVERSALSYMMETRYVIOLATION

- BaBar detector : At PEP-II facility at SLAC, California. Capable of studying electron-positron collisions, CP violation and determination of differences between matter and anti-matter.
- BaBar made the first direct measurement of time-reversal violation.
- CP violation was observed in 1964 in neutral kaon decays. It took 50 yrs to measure T-violation
- In 1998, claims to have found T-violation by CPLEAR experiment existed, but results were controversial.

- Application of CPT symmetry transformation on a system shows no difference from the original system.
- CP and P violations were observed
- A T-violation is expected. That is, a transformation from one state to another would occur at different rates when the process is reversed in time, showing T violation.
- The $e^{-}-e^{+}$ collisions inside BaBar are made to produce the Y mesons, composed of the bottom quark and its anti-quark. The Y particles decay into B-mesons, which are used for this study.
- Since the B-meson pairs are created from the same Y meson, they inherit their quantum numbers from the parent Y.

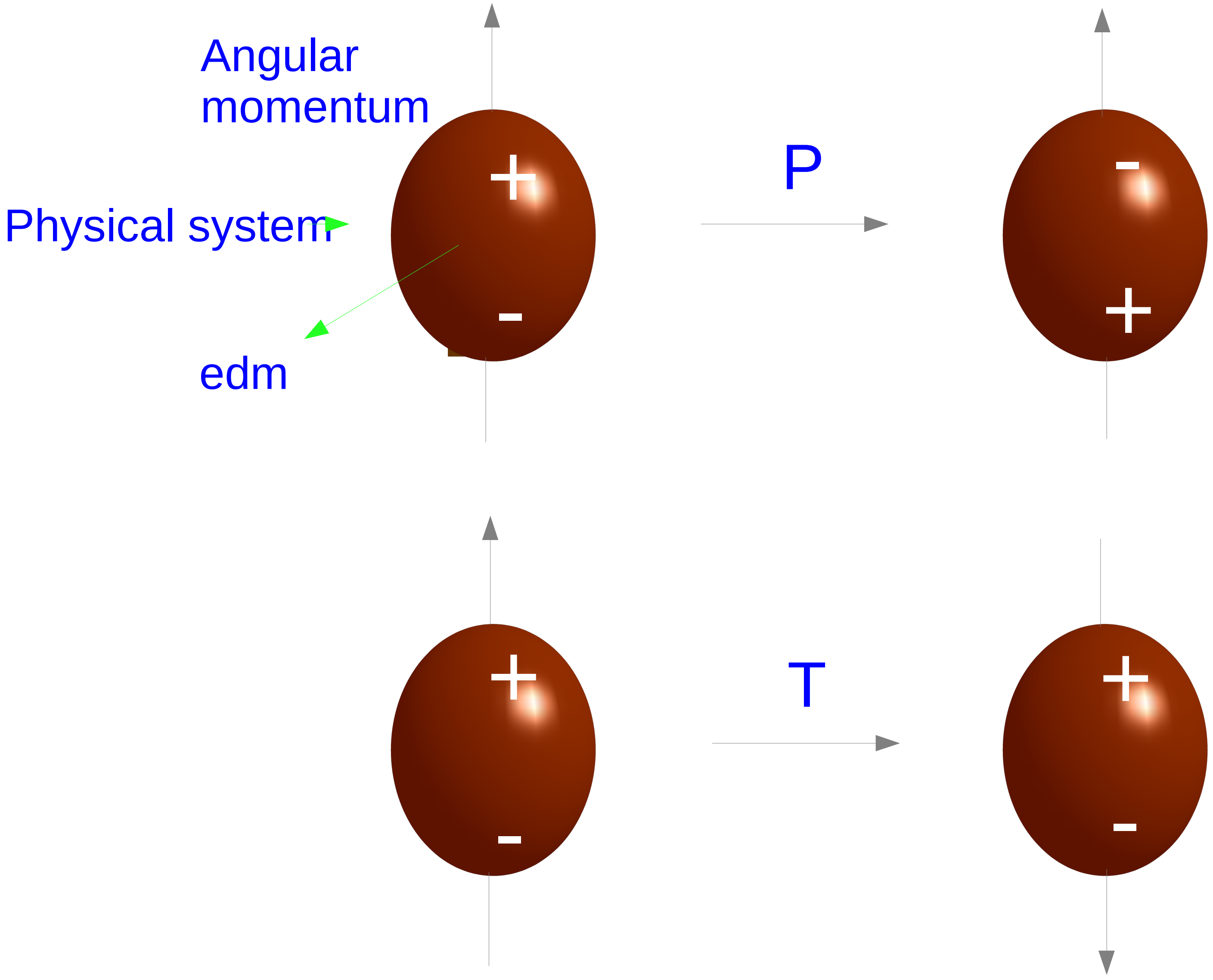
- Comparison of the decay modes of the B-mesons in the original and time-reversed states was done. The transformations of the mesons from their initial to final states and then back to initial states, showed differences in the decay rates.

Atomic Electric Dipole Moments

Quantity	Parity	Time-reversal	Original Frame	Transformed Frame
D	-D	D	D = d J	D = - d J
J	J	-J		

For non-zero EDM of an atom, P and T have to be violated

P & T violation for a non-zero EDM



Non-zero EDMs are direct evidence for Parity and Time-reversal violations

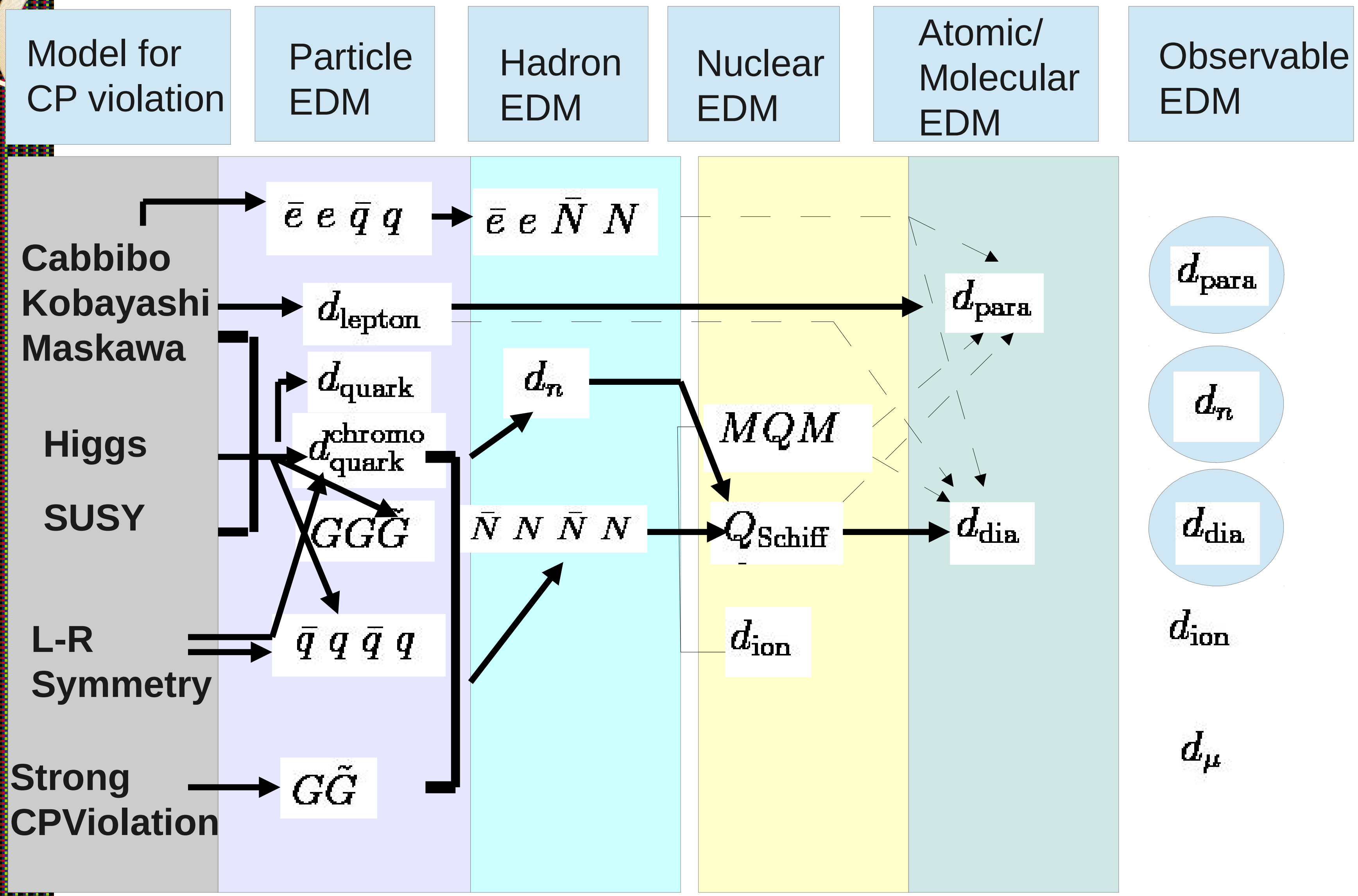
EDMs and PTV

There is a violation, because the dipole must lie either parallel or anti-parallel to the spin, but the choice of one over the other violates parity and time reversal symmetry.

We can think of the electron as a little ball of electrical charge surrounded by a fluctuating halo of particles and anti-particles. What we do is measure whether this ball is round or not. We know that if the ball is not perfectly round then the matter and antimatter must be behaving differently. So by making a very careful measurement of the electron's shape we can infer something about the nature of antimatter.
---- Professor Ed Copeland , Univ. of Nottingham.

More on :
http://www.sixtysymbols.com/videos/electron_sphere.htm

Barr’s chart



Model for CP violation
Particle EDM
Hadron EDM
Nuclear EDM
Atomic/ Molecular EDM
Observable EDM
Cabbibo Kobayashi Maskawa
Higgs
SUSY
L-R Symmetry
Strong CPViolation
$\bar{e}\ e\ \bar{q}\ q$
$\bar{e}\ e\ \bar{N}\ N$
d_{lepton}
d_{quark}
$d_{\text{quark}}^{\text{chromo}}$
$GG\tilde{G}$
$\bar{q}\ q\ \bar{q}\ q$
$G\tilde{G}$
d_n
$\bar{N}\ N\ \bar{N}\ N$
MQM
Q_{Schiff}
d_{ion}
d_{para}
d_{dia}
d_{para}
d_n
d_{dia}
d_{ion}
d_μ

EDMs of Closed & Open shell atomic systems

The dominating effects or processes that produce EDMs in closed shell atoms may be different from those in open shell atoms

For closed-shell atoms

- Nuclear Schiff moment
- Electron-nuclear interactions
- EDM of an electron
- Magnetic quadrupole moment

For open shell atoms

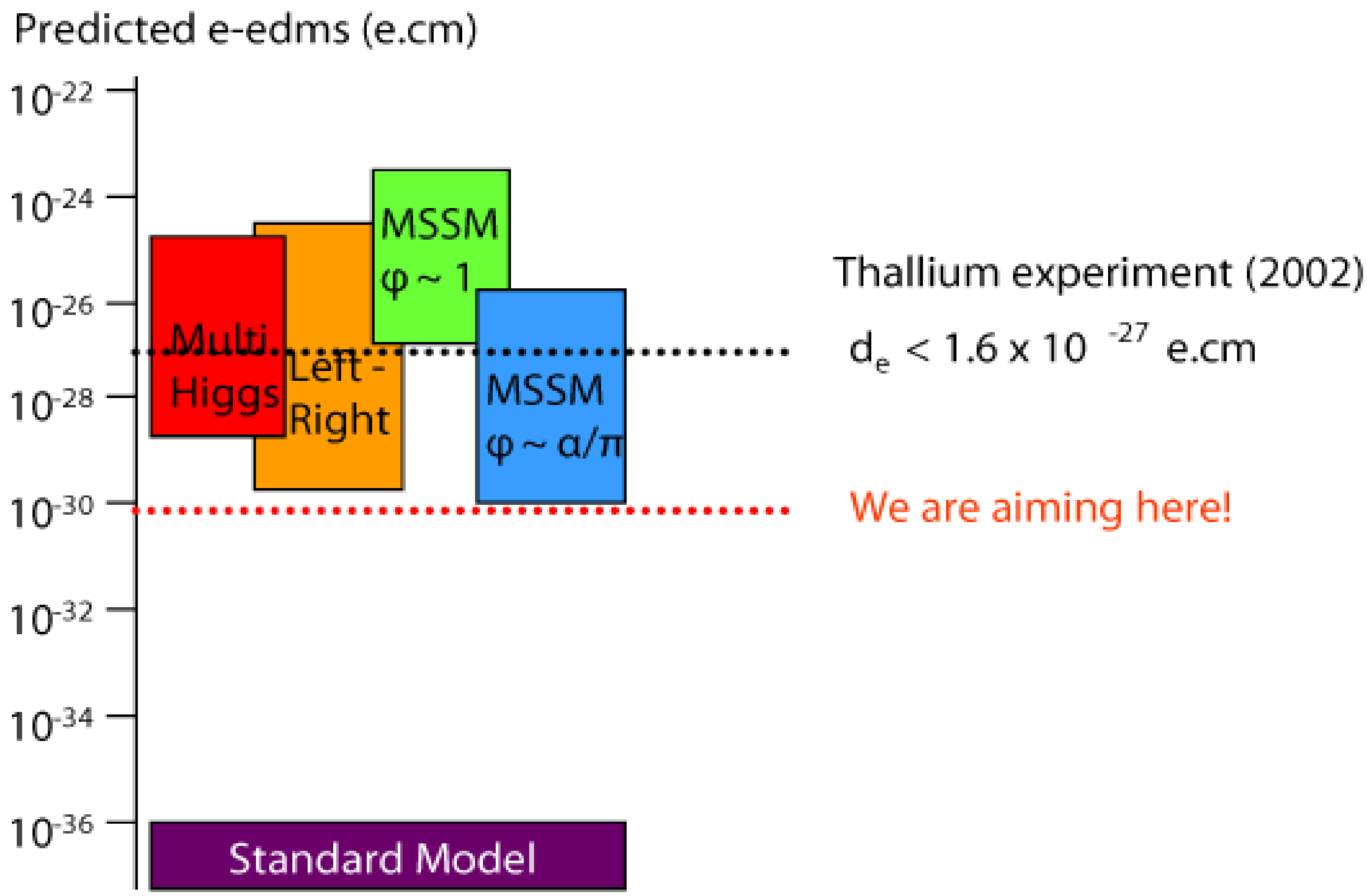
- EDM of an electron
- Electron-nuclear interactions
- Magnetic quadrupole moment
- Nuclear Schiff moment

EDM of an electron

Standard Model Predictions

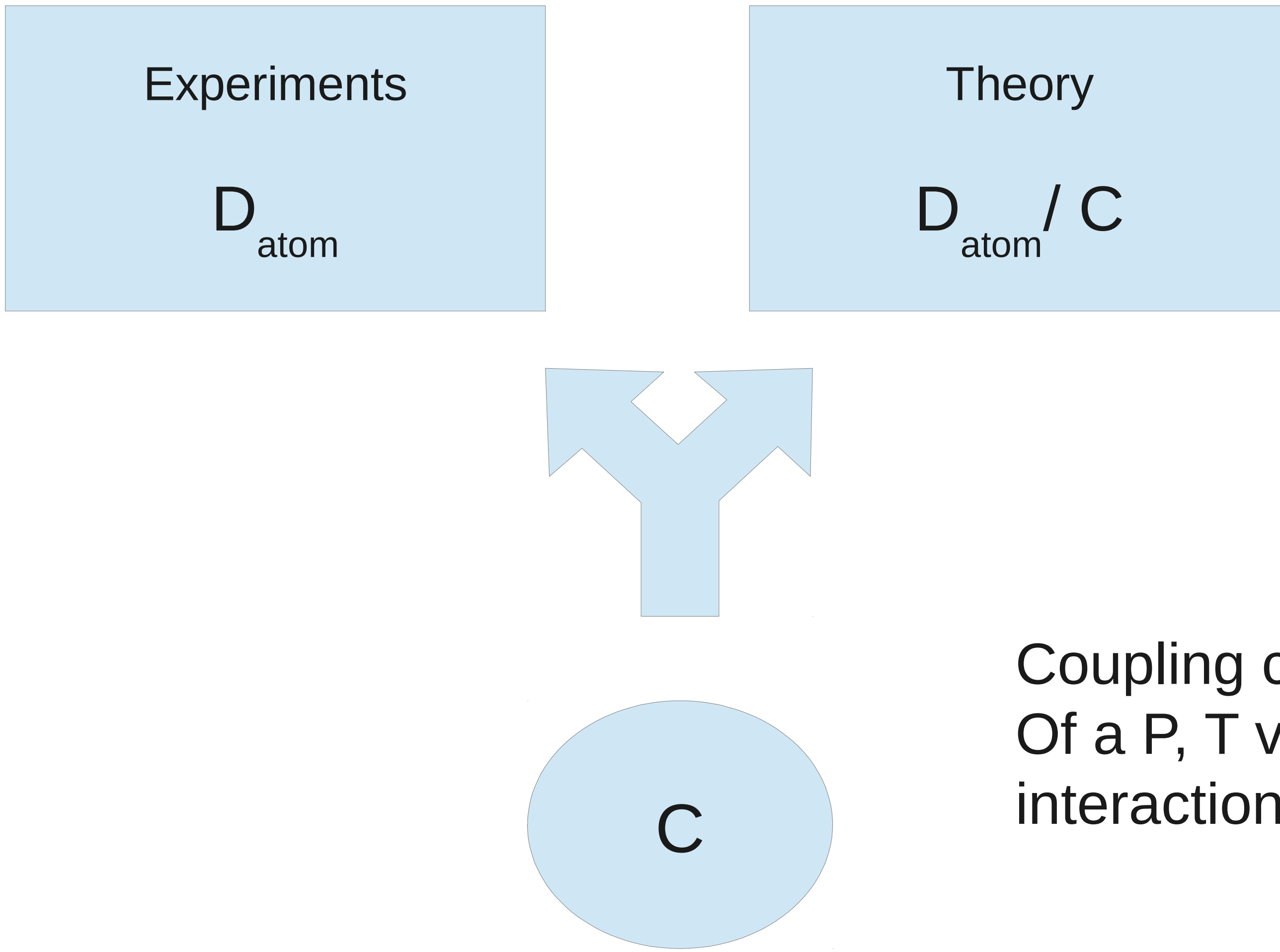
Quantity	Standard Model	Experimental Limit
EDM of an electron	~ 10^{-36} e cm	~ 10^{-28} e cm

J. J. Hudson, D. M. Kara, I. J. Smallman, B. E. Sauer, M. R. Tarbutt & E. A. Hinds, Nature 473, 493–496 (26 May 2011)



(Taken from http://www3.imperial.ac.uk/ccm/research/edm/overview, Centre for Cold Atoms, Imperial College, London)

Atomic Calculations – Theoretical Challenges



Coupling constant
Of a P, T violating
interaction

Heavy atoms for EDM measurements

- The EDM matrix elements scale as Z^2 or Z^3, where Z is the atomic number.
- Implies that EDM is enhanced in heavy atoms
- Interesting candidates for EDM measurements : Hg^{80}, Xe^{54}, Yb^{70}, Ra^{88}, Tl^{81}, Cs^{55}, etc