Quantum Field Theory as the Language of Physics

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CELEBRATING THE UNITY OF SCIENCE
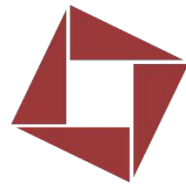
Happy Birthday

Classical Physics
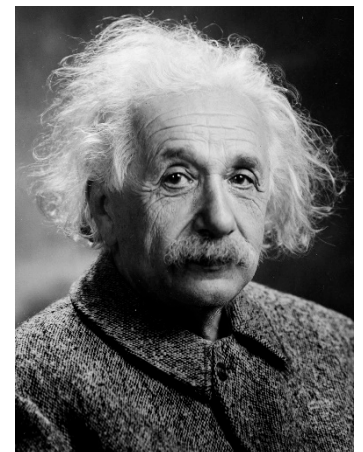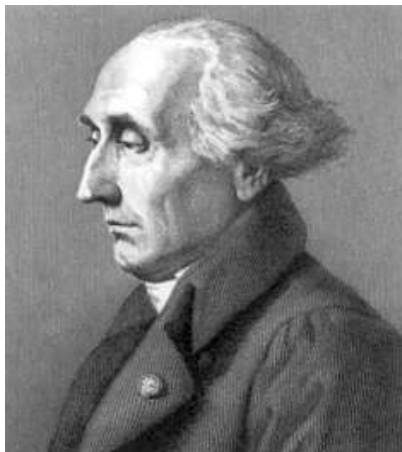
Classical Mechanics

Time evolution of a finite number of particles

Ordinary differential equations

Classical Field Theory

Time evolution of an infinite number of degrees of freedom, e.g. electromagnetic field, velocity of fluid, metric

Partial differential equations

Calculus as the Language of Classical Physics (and more)

- New mathematics
- Motivated by physics
- Many applications in mathematics, physics, and other branches of science, and engineering
- Sign that this is a deep idea
- Calculus is a mature field. Streamlined – most books and courses are more or less the same
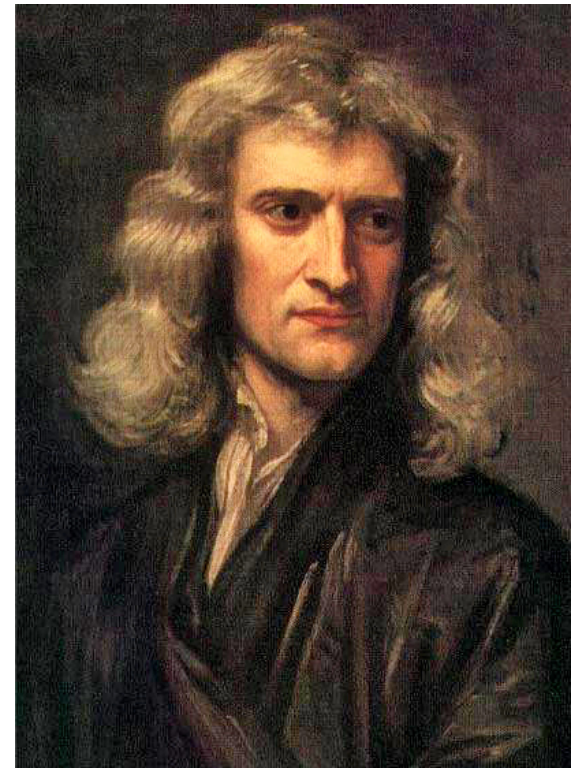
Quantum Physics

Quantum Mechanics

Time evolution of a finite number of quantum particles

Operators in a Hilbert space

Functional integral

Quantum Field Theory

Time evolution of an infinite number of quantum degrees of freedom, e.g. electromagnetic field

A lot is known. Still very exciting progress.

My personal view: a new intellectual structure is needed – QFT

Quantum Field Theory is Everywhere

- Particle physics: the language of the Standard Model
  - Enormous success, e.g. the electron magnetic dipole moment is theoretically   *1.001 159 652 18 …*
    experimentally   *1.001 159 652 180…*
- Condensed matter
  - Description of the long distance properties of materials: phases and the transitions between them
- Cosmology
  - Early Universe, inflation
- …

Quantum Field Theory is Everywhere

- String theory/quantum gravity
  - On the string world-sheet
  - In the low energy approximation
  - The whole theory (gauge/gravity duality)
- Applications in mathematics especially in geometry and topology

Quantum Field Theory is Everywhere

- Quantum field theory is the modern calculus
  - Natural language for describing diverse phenomena
- Enormous progress over the past decades, still continuing
- Indications that our understanding is incomplete.
  - Not that it is wrong.
  - Perhaps it should be reformulated

Calculus vs. Quantum Field Theory

- New mathematics (in fact, not yet rigorous)
- Motivated by physics (particle physics, condensed matter)
- Many applications in mathematics and physics
- Sign that this is a deep idea
- QFT is not yet mature – books and courses are very different (different perspective, order of presentation)
- Indications that we are still missing big things – perhaps QFT should be reformulated

Presentations of quantum field theory

- Traditional. Use a Lagrangian…
- More abstractly, operators and their correlation functions
  - In the traditional Lagrangian approach this is the outcome
- Others?

Abstract presentation of QFT

Collection of operators and their correlation functions

- The correlation functions are local and unitary
- Can place in various backgrounds including on various manifolds

Many consistency conditions – highly over-constrained. It is miraculous that a solution of the constraints exists.

Seems unsatisfactory – no clear starting point.

Lagrangian

- Natural starting point – quantize a classical system
  - Canonical quantization
  - Functional integral
  - Others
- Need to regularize (e.g. a lattice) to make it meaningful. (In the condensed matter applications the lattice is real.) Then need to prove the existence of
  - the continuum limit
  - the large volume limit

Lagrangian

Questions:

- Do we know all possible Lagrangians?

- Do we know all consistency conditions?

- When do different Lagrangians lead to the same theory? Duality

- More below

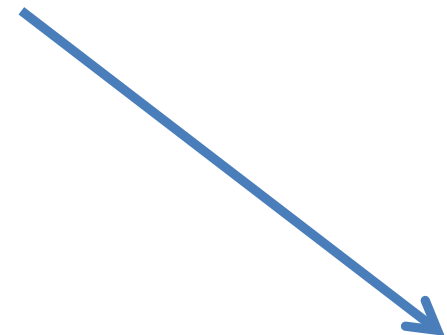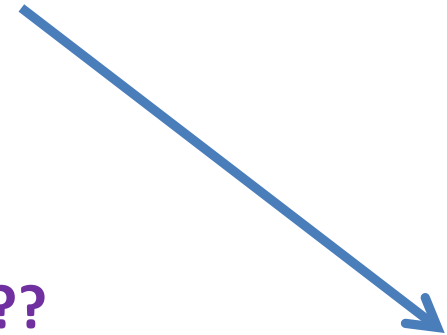# Lagrangians are meaningful and useful when they are weakly coupled

- Free UV theory perturbed by a relevant operator, e.g.
  - Asymptotically free *3+1d* gauge theory (e.g. QCD)
  - . . .
- Free IR theory perturbed by an irrelevant operator, e.g.
  - *3+1d* QED
  - Chiral theory of pions
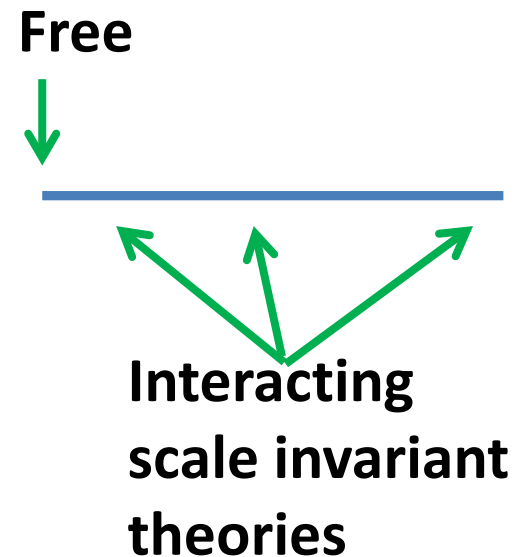 - . . .

Free UV theory

???   **???**

Free IR theory

Lagrangians are meaningful and useful when they are weakly coupled

- Family of scale invariant theories connected to a free theory, e.g.

    - *3+1d* $\mathcal{N} = 4$ super-Yang-Milles

    - *2+1d* Chern-Simons theory (large level)

    - *1+1d* sigma model with Calabi-Yau target space

    - . . .

Free

Interacting scale invariant theories

Lagrangians are not good enough

- Strong coupling
  - Usually hard to determine the behavior at strong coupling
- Not used in exact solutions
  - In fact the abstract presentation with its consistency conditions is more useful
- Duality
  - Extremely surprising
- Theories without Lagrangian
  - E.g. in 4, 5, and 6 dimensions

Suggest that QFT should be reformulated

- Lagrangians are not good enough
- Not mathematically rigorous
- Extensions of traditional local QFT
  - Field theory on a non-commutative space
  - Little string theory
  - Others?
- Because of the gauge/gravity duality QFT is the same as gravity. This calls for a deeper understanding.
- QFT constantly surprises us with new phenomena. If we are surprised, it means that we do not fully understand it.

Thank you for your attention

Happy Birthday