From chords to series:
The evolution of sine function in India

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Introduction

- When people talk of contribution of India, generally it centers around religion, philosophy, art forms, caste system, etc. Their contributions towards science and technology is almost completely ignored.
- Here is a representative list of different branches S & T that were practised in Ancient India.

Sciences

1. Astronomical
2. Chemical
3. Mathematical
4. Agricultural
5. Psychological
6. Medicinal
7. Veterinary

Technology

1. Water harvesting
2. Iron and Steel
3. Casting statues
4. Ship building
5. Constructing temples
6. Textile manufacturing
7. Town planning

- Right at the beginning of his famous drama *Mālavikāgnimitram* (1.2) (through the words of *sūtradhāra*) Kālidāsa observes:

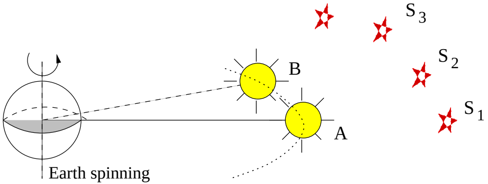
  पुराणमित्येव न साधु सर्वं न चापि काव्यं नवमित्यवद्यम्।
  सन्तः परीक्ष्यान्यतरद्भजन्ते मूढः परप्रत्ययनेयबुद्धिः॥

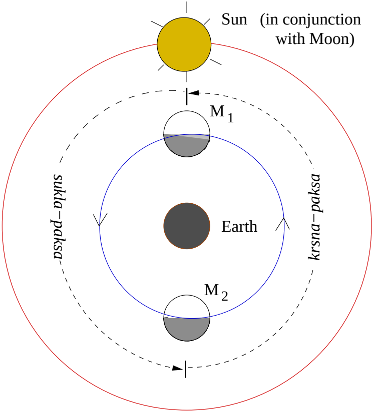
  By virtue of being ancient, not all is good. And [similarly], by virtue of being new, modern literature should not be condemned. The noble ones having examined, choose either of them; the unintelligent is carried away by the views of others.

- Mindless glorification of the past $\rightsquigarrow$ counterproductive.
- Equally counterproductive would be downright condemnation.
- Indians have not shied to break the moulds of the past! As and when required they had indeed done so!

Motivation for finding accurate sine values

- The positions of the planets in the background of stars forms the basis for reckoning time.
- Determination of their positions crucially depends upon accurate knowledge of sine function.





Determination of time from shadow measurement

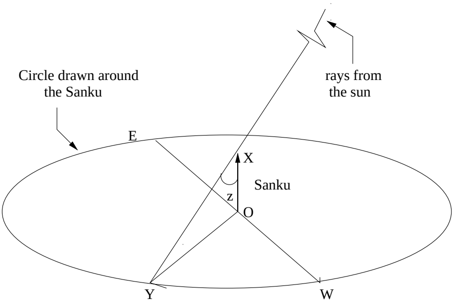


Figure: Zenith distance and the length of the shadow.

$$t = (R\sin)^{-1}\left[\frac{R\cos z}{\cos\phi\cos\delta} \pm R\sin\Delta\alpha\right] \mp \Delta\alpha.$$

If ϕ and δ are known ($\Delta\alpha = f(\phi, \delta)$), then t is known.

The significance of sine function in astronomy

Muniśvara at the beginning of the second chapter of *Siddhānta-sārvabhauma* observes:

> स्वस्पष्टभोगैः फलदा ग्रहेन्द्राः यतोऽखिले कर्मणि तेन तेषाम् ।
> स्फुटक्रियामागमयुक्तिमूलां अपास्तनिर्युक्तिमतां प्रवच्मि ॥ ॥ उपजातिः ॥
>
> Based on the true positions, ... The correction procedure which is based on *āgama* and *yukti* and which has discarded schools that are devoid of rationales, is being stated by me.

Then outlining the significance of sine function, he notes:

> देहं यथा स्नायुनिबद्धमस्ति तथा ग्रहस्पष्टमुखक्रिया हि ।
> ज्याभिस्तथा ज्याः प्रथमं वदामि प्रत्येकभागं विवरेऽनुपातात् ॥ ॥ उपजातिः ॥
>
> As the body is tied together [i.e., completely criss-crossed| with veins, so too are the procedures for to computation of the true position of the planets with the sines. [Hence,] I present the values of sines for every degree; ...

The sine table presented by Muniśvara

सत्र्यंशरामाः त्रिलवोनभागाः दिशः सरमांशगुणेन्दवः स्युः ।
वित्र्यंशमेघाः खयमाः स्वपादगुणाश्विनः तत्त्वकलोनभानि ॥ ॥ उपजातिः ॥

Arc in deg.	Verbal value of the Sine	Symbolic form	Sexagesimal form	From his 4-fig Sine table
1	सत्र्यंशरामाः	$3+\frac{1}{3}$	3; 20	3; 20,00,17
2	त्रिलवोनितागाः	$7-\frac{1}{3}$	6; 40	6; 39,56,54
3	दिशः	10	10; 0	9; 59,46,12
4	सरमांशगुणेन्दवः	$13+\frac{1}{3}$	13; 20	13; 19,24,33
5	वित्र्यंशमेघाः	$17-\frac{1}{3}$	16; 40	16; 38,48,17
6	खयमाः	20	20; 0	19; 57, 53,46
7	स्वपादगुणाश्विनः	$23+\frac{1}{4}$	23; 15	23; 16,37,22
8	तत्त्वकलोनभानि	$27-25'$	26; 35	26; 34,55,25

The sine of sum and difference of two arcs presented by Nityānanda

अन्योन्यकोटिमौर्व्या गुणिते ये चेष्टचापयोर्दोर्ज्ये ।
त्रिज्योद्धृते तयोर्यो योगः सा चापयोगज्या ॥ ४१ ॥ ॥ आर्या ॥

The Sines of the two desired arcs are mutually multiplied by the Cosines of the other two; when divided by the Radius, whatever is sum of the two, that is the Rsine of the sum of the [two] arcs.

$$R\sin(\theta+\phi) = \frac{R\sin\theta R\cos\phi}{R} + \frac{R\cos\theta R\sin\phi}{R}. \qquad (1)$$

अन्योन्यकोटिमौर्व्या गुणिते ये चेष्टचापयोर्दोर्ज्ये ।
त्रिज्योद्धृते तयोर्या वियुतिः सा चापविवरज्या ॥ ४१ ॥ ॥ आर्या ॥

$$R\sin(\theta-\phi) = \frac{R\sin\theta R\cos\phi}{R} - \frac{R\cos\theta R\sin\phi}{R}. \qquad (2)$$

Computing Rsines: Approach in earlier *siddhāntas*

- Varāhamihira has given the following Rsine values and relations in his Pañcasiddhāntikā (c. 505 CE):

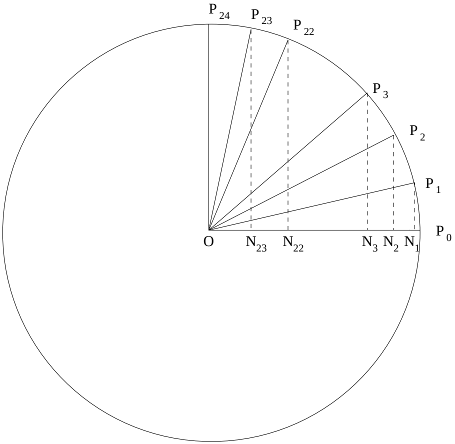
$$
\begin{aligned}
R\sin(30^\circ) &= \frac{R}{2} && (3a)\\
R\sin(45^\circ) &= \frac{R}{\sqrt{2}} && (3b)\\
R\sin(60^\circ) &= \frac{\sqrt{3}}{2}R && (3c)\\
R\sin(90^\circ) &= R && (3d)
\end{aligned}
$$

$$
\begin{aligned}
R\sin(\theta) &= R\cos(90-\theta) && (4a)\\
R\sin^2(\theta) + R\cos^2(\theta) &= R^2 && (4b)\\
R\sin\left(\frac{\theta}{2}\right) &= \left(\frac{1}{2}\right)\left[R\sin^2(\theta) + R\,\mathrm{vers}^2(\theta)\right]^{\frac{1}{2}} && (4c)
\end{aligned}
$$

- The above Rsine values given by (3) and relations (4) can be derived using the *bhujā-koṭi-karṇa-nyāya* and *trairāśika* (rule of three for similar triangles). Equations (4) can be used to compute all the 24 tabular Rsine values.

Construction of the sine-table

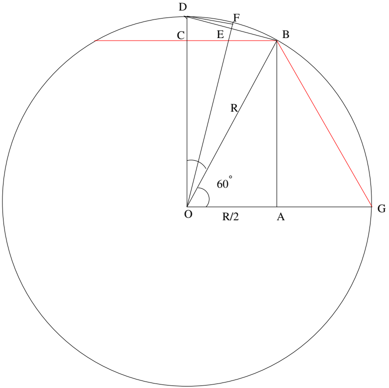
A quadrant is divided into 24 equal parts, so that each arc bit $\alpha = \frac{90}{24} = 3^\circ 45' = 225'$.



- Āryabhaṭa presents two different methods for finding $R\sin i\theta, (P_i N_i)$ $i = 1, 2, \ldots 24$.
- The Rsines of the intermediate angles are determined by interpolation (I order or II order).

Finding tabular sines: Geometrical approach

समवृत्तपरिधिपादं छिन्द्यात् त्रिभुजाच्चतुर्भुजाच्चैव ।
समचापज्यार्धानि तु विष्कम्भार्धे यथेष्टानि ।।



- We know, chord $60^\circ = R = 3438$
- From the triangle OBC,

$$R \sin 30 = BC = R/2 = 1719$$

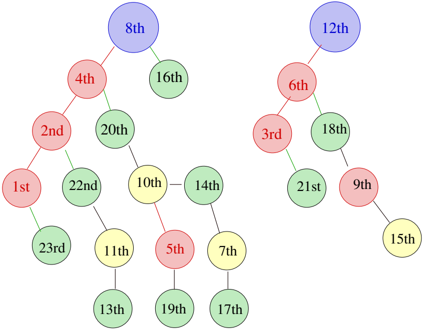
- Using *bhujā-koṭi-karṇa-nyāya* from 8th Rsine (30°) we can get 16th Rsine (60°).
- In other words, $R \sin \theta \rightsquigarrow R \sin(90 - \theta)$.
- Further, noting that

$$R \sin \theta \rightsquigarrow R \cos \theta \rightsquigarrow R \operatorname{vers} \theta$$
$$R \sin \theta \;\&\; R \operatorname{vers} \theta \rightsquigarrow R \sin \frac{\theta}{2},$$

it can be seen that all the 24 Rsines can be calculated.

Finding tabular sines: Geometrical approach (contd.)

- Most of the Indian astronomers have presented their sine tables by dividing the quadrant (90°) into 24 parts.
- By the principle outlined above, it can be easily shown that all the 24 Rsines can be obtained provided the 24th, 12th and 8th Rsines are known.



- The circumference of the circle was taken by Āryabhaṭa to be 21600 units.
- From that using the approximation for π given by him, we get $R = 24th$ Rsine ≈ 3438.
- Once this it known, it is noteworthy that in the proposed scheme of constructing the table, all that is required is extraction of square root, for which Āryabhaṭa had clearly evolved an efficient algorithm.

Finding tabular sines: Analytic approach

प्रथमाच्चापज्यार्धात् यैरूनं खण्डितं द्वितीयार्धम् ।
तत्प्रथमज्यार्धांशैः तैस्तैरूनानि शेषाणि ।।

Let $B_k = R\sin(k \times 225'),\ (k = i, 2, \ldots, 24)$, be the twenty-four Rsines, and let $\Delta_k = B_k - B_{k-1},\ (k = i, 2, \ldots, 24)\ldots$ be the Rsine-differences. Then, the formula coded in the above *sūtra* may be expressed as[1]

$$\Delta_2 = B_1 - \frac{B_1}{B_1} \tag{5}$$

$$\Delta_{k+1} = B_1 - \frac{(B_1 + B_2 + \ldots + B_k)}{B_1} \qquad (k = 1, 2, \ldots, 23). \tag{6}$$

This second relation is also sometimes expressed in the equivalent form

$$\Delta_{k+1} = \Delta_k - \frac{(\Delta_1 + \Delta_2 + \ldots + \Delta_k)}{B_1} \qquad (k = 1, 2, \ldots, 23). \tag{7}$$

From the above, the discrete version of the harmonic equation follows

$$\Delta_{k+1} - \Delta_k = \frac{-B_k}{B_1} \qquad (k = 1, 2, \ldots, 23). \tag{8}$$

[1] Āryabhaṭa is using the approximation $\Delta_2 - \Delta_1 \approx 1'$.

- Using either/both the approaches, Āryabhaṭa having obtained Rsine values has presented a table in *Gītikā-pāda* of *Āryabhaṭīya* (verse 12).
- This verse[2] lists the 24 first order Rsine-differences (in arc-minutes):

  मखि भखि फखि धखि णखि ञखि
  ङखि हस्झ स्ककि किष्ग स्घकि किघ्व ।
  घ्लकि किग्र हक्य धकि किच
  स्ग श्झ ङ्व क्ल प्त फ छ कलार्धज्याः ॥

  225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, and 7—these are the Rsine-differences [at intervals of 225′ of arc] in terms of the minutes of arc.

- In Āryabhaṭa's notation: म $\rightarrow$ 25; & ख $\rightarrow$ 200;

[2] This verse is one of the most terse verses in the entire Sanskrit literature that I have ever come across. Only after several trials would it be ever possible to read the verse properly, let alone deciphering its content.

Bhāskara's approximation

- We know that sine function
  1. is symmetric about 90° point
  2. is concave over the range $0^\circ \to 180^\circ$

- Bhaskara's (7th cent.) formula clearly satisfies these properties.

Bhaskara's approximation

$$\sin\theta = \frac{4\theta(180 - \theta)}{40500 - \theta(180 - \theta)}$$

- It would indeed have been a mathematician's delight to arrive at an expression for sine function that at once captures the properties as well as serves as a very good approximation ($\approx 99\%$) for the entire range ($0 - 180^\circ$).
- Bhāskara II (12th century) discusses sine at great length by exclusively devoting a chapter to it.
- The 17th century astronomer Nityānanda devotes almost 65 verses!

The structure of the five sections on trigonometry

The organization of different sections is as follows:

verses 19–23		Preamble and definitions
verses 24–30	Section 1	The Sines of ninety, thirty, eighteen degrees
verses 31–36	Section 2	The Sine of half the arc
verses 37–40	Section 3	The Sine of double the arc
verses 41-48	Section 4	The Sine of the sum of two arcs
verses 49–54	Section 5	The Sine of the difference of two arcs
verses 55–59		Demonstration of equivalences by geometrical construction (*śilpa*)
verses 60–85	Section 6	Obtaining the Sine of one degree

Nityānanda introduces each section with a short statement in prose which describes which has a distinct structure:

- they announce the section number
- indicate the relation that is to be discussed (in locative absolute), and
- specify what relation is going to be presented.

Setting the background for defining Sines

अथ ज्याकथनम् ।

नवत्यंशजीवा भवेद्व्यासखण्डं
तदर्धं खरामांशजीवा निरुक्ता ।
तदर्धोनितं विस्तृतेरङ्घ्रिवर्गात्
सपादात् पदं ज्याष्टचन्द्रांशकानाम् ॥ २४ ॥ ॥ भुजङ्गप्रयातम् ॥

$$\text{Sin } 90 = R, \tag{9}$$

$$\text{Sin } 30 = \frac{R}{2}, \tag{10}$$

$$\text{Sin } 18 = \sqrt{\left(\frac{D}{4}\right)^2 + \frac{1}{4}\left(\frac{D}{4}\right)^2} - \frac{1}{2} \cdot \frac{D}{4}. \tag{11}$$

The relation given here is justified by means of a geometrical construction. A diagram depicting the construction has been presented in some of the manuscripts.

The multiple-angle formula that forms the basis of all the three algorithms presented by Nityānanda is:

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta. \tag{12}$$

Given that the modern sine and the non-unit-radius Sine are related by the expression $R\sin\theta = \mathrm{Sin}\,\theta$, the above formula reduces to:

$$\mathrm{Sin}\,3\theta = 3\,\mathrm{Sin}\,\theta - \frac{4\,\mathrm{Sin}^3\,\theta}{R^2} \tag{13}$$

Substituting $\theta = 1^\circ$ and denoting $\mathrm{Sin}\,1^\circ = y$, equation (13) becomes:

$$\mathrm{Sin}\,3^\circ = 3y - \frac{4y^3}{R^2}, \tag{14}$$

$$\text{or,} \qquad y = \frac{\mathrm{Sin}\,3^\circ}{3} + \frac{4y^3}{3R^2}. \tag{15}$$

Determination of the Sine of one degree

Method due to al-Kashi (15th cent)

- Since the Arabic sources employed chords, Nityānanda appears to be emulating this by multipliying equation (15) by two, so that we have:

$$2y = \frac{2 \cdot \text{Sin}\,3}{3} + \frac{(2y)^3}{3R^2}. \tag{16}$$

- The left hand side $2y$ can be considered to be made up of successive sexagesimal digits. Denoting them by $p_0, p_1, p_2, \ldots$ we have: [3]

$$2y = p_0, p_1, p_2, \ldots$$

- By a careful analysis of equation (16), al-Kashi establishes each of these components one by one, by an iterative process. In essence y is updated with each iteration by an additional sexagesimal place.
- At any stage of the operation, an approx. value of the Sine of one degree is obtained by dividing the current value of $2y$ by 2.

[3] The text calls each of these components: degrees (*lava*), minutes (*liptā*), seconds (*viliptā*), and so on, respectively.

Determination of the Sine of one degree

Method due to al-Kashi (15th cent)

- Since y is small, neglecting y^3, a good first approximation to y is
$$2y \approx \frac{2 \cdot \text{Sin}\, 3}{3} = p_0 + \frac{r_0}{3},$$
- Now, p_0 is then used to generate the next segasesimal digit p_1:
$$\frac{r_0 + \dfrac{(p_0)^3}{R^2}}{3} = p_1 + \frac{r_1}{3}.$$
- The third sexagesimal digit p_2 is to be found using the relation:
$$\frac{r_1 + \dfrac{(p_0, p_1)^3 - (p_0)^3}{R^2}}{3} = p_2 + \frac{r_2}{3}.$$
- In the n^{th} iteration, one produces p_n using the equation:
$$\frac{r_n + \dfrac{(p_0, \ldots, p_{n-1})^3 - (p_0, \ldots, p_{n-2})^3}{R^2}}{3} = p_n + \frac{r_n}{3}.$$

Numerical value of the Sine of three degrees

- Nityānanda simply describes the algorithm, and does not specify the numerical value of the Sine of one degree.
- However, we carried out a few iterations of the procedure to demonstrate how the numbers might have been generated.
- The one and only numerical parameter Nityānanda does give is the Sine of three degrees. Its value, recorded using the object-numeral system is:

$$\text{Sin}\,3^\circ = 3; 8, 24, 33, 59, 34, 28, 14, 50$$

- Interestingly, this value is not the same as the one given by al-Kashi. In their scholarly paper, it has been noted by Rosenfeld and Hogendijk[4] that the correct value of the 8th place is 50 and not 29.
- This clearly suggests that Nityānanda has independently evaluated it and not simply borrowed this parameter.

[4] Rosenfeld and Hogendijk 2003, fn. 57, p. 45.

Nityānanda's approach to the problem

- The second method Nityānanda describes to solve the cubic is again an iterative procedure. He commences with the approximation:
$$y_0 = \frac{2 \cdot \text{Sin}\, 3}{3}.$$
- This is to be increased by its own cube, and divided by the product of three and the square of the radius to get the next iterate:
$$y_1 = y_0 + \frac{y_0^3}{3R^2}.$$
- Now we use y_1 to get the next iterate. That is,
$$y_2 = y_0 + \frac{y_1^3}{3R^2}.$$
- Subsequent iterations are generated via:
$$y_n = y_0 + \frac{(y_{n-1})^3}{3R^2}.$$
- The iterative process is terminated when $y_{n+1} \approx y_n$.

Convergence of the iterates (Nityānanda's approach)

Itn. No. (n)	Sin $1^\circ = y_n/2$	difference = $\frac{y_n - y_{n-1}}{2}$
0	1 2 48 11 19 51 29 24 56 40	
1	1 2 49 43 04 32 00 53 37 37	0 00 01 31 44 40 31 28 40 57
2	1 2 49 43 11 14 14 50 04 39	0 00 00 00 06 42 13 56 27 02
3	1 2 49 43 11 14 44 14 17 10	0 00 00 00 00 00 29 24 12 31
4	1 2 49 43 11 14 44 16 26 08	0 00 00 00 00 00 00 02 08 58
5	1 2 49 43 11 14 44 16 26 17	0 00 00 00 00 00 00 00 00 09
6	1 2 49 43 11 14 44 16 26 17	0 00 00 00 00 00 00 00 00 00

- It may be noted from the table above, that the successive difference keep rapidly decreasing (in 5 steps you get 10 sexagesimal digits).
- This may have to do with the problem being akin to fixed point iteration technique applied to a cubic.
- It has been noted before[5] that such approaches were employed by Indian astronomers of earlier period to solve similar problems.

[5] See the recent studies by Plofker 2002, and Sriram, Ram and Pai 2012.

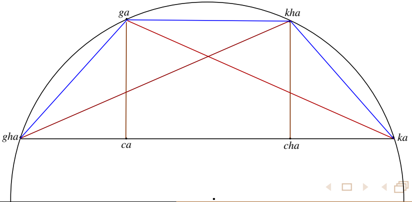
Arriving at the cubic through geometrical construction

- Nityānanda presents an interesting geometrical construction in order to arrive at the cubic.
- Having described the construction in detail he finally states:

  evaṃ jāto bhūmivaktrāvaghātaḥ
  dorvargāḍhyaḥ karṇavargāssa eva ||
  Thus, what is obtained is the product of the base and the face increased by the square of the chord. This is indeed the square of the diagonal.

- This amounts to the following equation:

$$(ka\text{-}ga)^2 = (\mathrm{Crd}\,\theta)^2 + \mathrm{Crd}\,\theta \cdot \mathrm{Crd}\,3\theta \qquad (17)$$



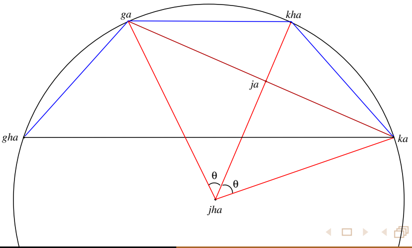
Arriving at the cubic through geometrical construction

- Nityānanda having presented an alternative expression for the diagonal states:

  evaṃ prakāradvayato 'pi kuryāt
  karṇasya vargau gaṇakapravīṇaḥ |
  Thus, the expert mathematician may compute the squares of the diagonal by both the approaches.

- The other equation for the diagonal given is:

$$(kha\text{-}gha)^2 = 4(\mathrm{Crd}\,\theta)^2 - \frac{(\mathrm{Crd}^2\,\theta)^2}{R^2}. \tag{18}$$



Arriving at the cubic through geometrical construction

- Equating the two expressions for the diagonals, and replacing all instances of Crd θ with an unknown, say y, we have:

$$y^2 + y \cdot \text{Crd}\,3\theta = 4y^2 - \frac{(y^2)^2}{R^2}.$$

- Reducing both sides by the unknown quantity produces:

$$y + \text{Crd}\,3\theta = 4y - \frac{y^3}{R^2}. \tag{19}$$

- Rearranging the above we have,

$$3y = \text{Crd}\,3\theta + \frac{y^3}{R^2}$$

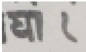
$$y = \frac{\text{Crd}\,3\theta}{3} + \frac{y^3}{3R^2}. \tag{20}$$

- This is the cubic to be solved, for which the various iterative methods were prescribed.

Representation of algebraic equations in the manuscripts

Simple expressions

$\mathrm{Crd}\,\theta = \textit{ka-kha} = y$



$\textit{yā}\ 1$

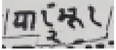
Now, the base (*bhū*) is to be diminished by that:

$b - y$

$\textit{yā}\ \dot{1}\ \textit{bhū}\ 1$

Half of that is the part (*sandhi*):

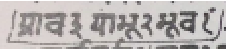
$\dfrac{b - y}{2}$



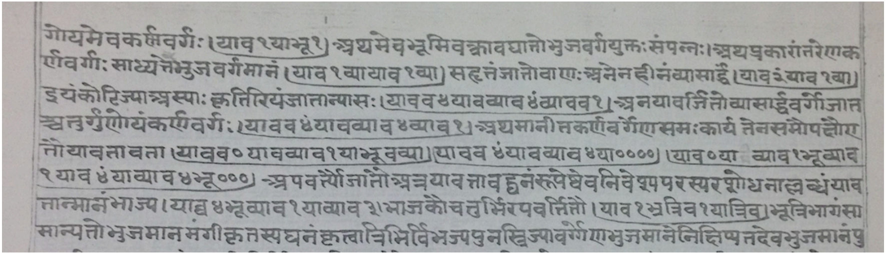
$\begin{array}{c} \textit{yā}\ \dot{1}\ \ \textit{bhū}\ 1 \\ 2 \end{array}$

Herein below we present an example of how more complex expressions are represented in Sanskrit manuscripts.[6]

$$y^2 - \left(\frac{b-y}{2}\right)^2 = \frac{3y^2 + 2by - b^2}{4}$$



yāva 3 *yābhū* 2 *bhūva* i̇
[4]



[6] Clemency and Ramasubramanian, SCIAMVS 19 (2018), 1–52

Signal achievements of Kerala mathematicians

- The "Newton" series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots, \tag{21}$$

- The "Gregory-Leibniz"[7] series

$$Paridhi = 4 \times \textit{Vyāsa} \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\right) \tag{22}$$

- The derivative of sine inverse function

$$\frac{d}{dt}\left[\sin^{-1}\left(\frac{r}{R}\sin M\right)\right] = \frac{\frac{r}{R}\cos M \ \frac{dM}{dt}}{\sqrt{1-\left(\frac{r}{R}\sin M\right)^2}} \tag{23}$$

and many more remarkable results are found in the works of Kerala mathematicians (14th–16th cent.)

[7] The quotation marks indicate the discrepancy between the commonly employed names to these series and their historical accuracy.

Why should we look into the past?

- In The Times of India (Kolkata) August 17 (p 10), there appeared an interview of Manjul Bhargava (Fields Medalist, 2014). When asked:

  > Why is India still a middle power in mathematics despite its famed legacy?

- The reply that Manjul gave was:

  > Students in India should be taught about the great ancient Indian mathematicians like Paṇini, Piṅgala, Hemacandra, Āryabhaṭa and Bhāskara. Their stories and works inspired me, and I think they would inspire students across India. Many of these works were written in Indian languages in beautiful poetry, and contain important breakthroughs[8] in the history of mathematics.

[8]For instance: zero, Pythagorean theorem, combinatorial rules, analytical expression for finding sine function, solution to indeterminate equations (I and II order), infinite series, their fast convergent approximations, etc.

Do the books on history present the truth?

Alex Bellos, a British journalist in his text authored in 2010 observes:

For two thousand years the only way to pinpoint pi was by using polygons. But, in the seventeenth century, Gottfried Leibniz and John Gregory ushered in a new age of pi appreciation with the formula:

$$\frac{\text{pi}}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \ldots$$

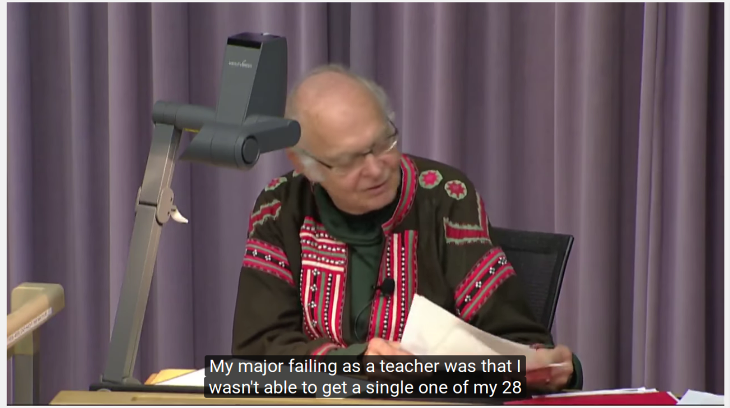
// Historically incorrect

In other words, a quarter of pi is equal to one minus a third plus a fifth minus a seventh plus a ninth and so on, alternating the addition and subtraction of unit fractions of the odd numbers as they head to infinity. Before this point scientists were aware only of the scattergun randomness of pi's decimal expansion. Yet here was one of the most elegant and

Making unverifiable claims or exaggerated claims is unscientific, but what is scientific about suppressing or making blind rejection?

The thrill in investigating historical material

An interesting observation by Donald Knuth – the author of *The Art of Computing*



My major failing as a teacher was that I wasn't able to get a single one of my 28 PhD students to realize what a thrill it is to work on source material!

Concluding Remarks

- In 1969 Sir C V Raman observes:

> *We have, I think, developed an inferiority complex. I think what is needed in India today is the destruction of that defeatist spirit. We need a spirit of victory, a spirit that will carry us to our rightful place under the sun; a spirit which will recognise that we, as inheritors of a proud civilisation, are entitled to our rightful place on the planet. If that indomitable spirit were to arise,*[9] *nothing can hold us back from achieving our rightful destiny.*[10]

- Raman brings out an extremely important point here. The inferiority complex, residing in us as a parasite, without our knowledge, has been inhibiting us for centuries!
- One way (if not the only way) to get rid of this problem is to make the citizens of our nation to shed way the cultivated ignorance by making them aware of their own scientific heritage.

[9] The Upanisadic passage – उत्तिष्ठत जाग्रत प्राप्य वरान्निबोधत – essentially does this.
[10] This, and other quotes shared by Amartya Dutta, ISI Kolkata.

Concluding Remarks
What is to be done?

- Efforts should be made to highlight important discoveries made in India[11] as a part of the school curriculum.
- History as taught is schools today is primarily political history, [12] and at best social history, talking of caste system, suppression, oppression, and so on — and that too as portrayed by others to us!
- The scientific and cultural history of India — which is currently either completely absent, or presented in a banal manner — must be introduced in an "accurate" and attractive manner!
- Such an introduction, if not anything else, will at least help the people of India shed away their ignorance/myth that Indians were only interested in spirituality, religion, philosophy, literature and arts.

[11] As a first step books must be prepared based on authentic facts presenting what, where, who, why and how great ideas emerged. Eminent scientists (not merely historians) who appreciate history, should be involved for they will be better equipped to fathom the depth and significance of Indian contributions.

[12] filled with lurid details of betrayals, machinations, etc.

Thanks!

THANK YOU !