Particle Physics Models of Inflation in Modified Gravity

Jose Mathew

Indian Institute of Science Education and Research Thiruvananthapuram

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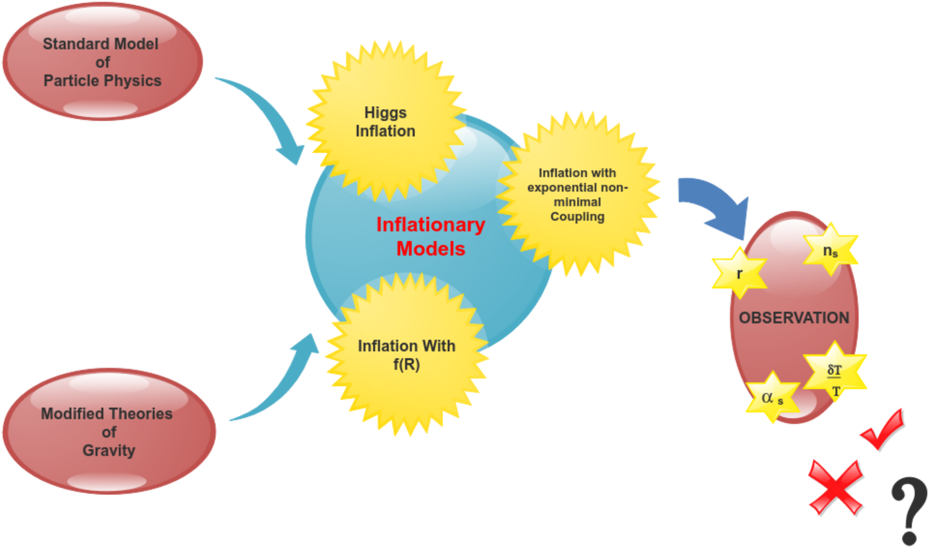
Theme of the work



Talk is based on the following works:

1. J. Mathew and S. Shankaranarayanan,**"Low scale Higgs inflation with Gauss–Bonnet coupling,"** Astropart. Phys. **84**, 1 (2016) [arXiv:1602.00411[astro-ph.CO]].

2. J. Mathew, J. P. Johnson, and S. Shankaranarayanan, **"Inflation with $f(R, \phi)$ in Jordan frame,"** [arXiv:1705.07945 [gr-qc]] (under review in Gen. Rel. Grav.)

3. J. P. Johnson, J. Mathew, and S. Shankaranarayanan, **"Inflation driven by exponential non-minimal coupling of inflaton with gravity,"** [arXiv:1706.10150 [gr-qc]](under review in Phy. Rev. D)

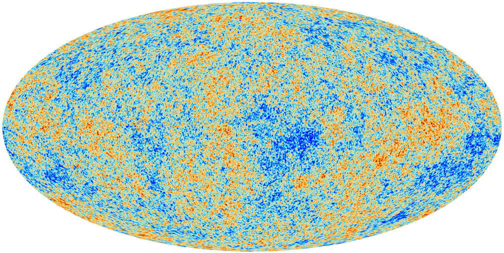
Outline

- Introduction
- Motivation
- Inflationary model realised in $Gauss - Bonnet$ gravity
- Inflationary model realised in $f(R)$ gravity that resembles scalar-tensor theories of gravity
- Inflationary model realised within $f(R)$ gravity, with an exponential non-minimal coupling
- Discussions and conclusions
- Further research interests and prospects

Cosmological Principle

- Cosmological principle states that, viewed on a sufficiently large scale, the properties of the universe are same for all the observers.
- That is, the Universe must be homogeneous and isotropic at large scales.
- What do observations suggest?

Figure : Fluctuations in Cosmic Microwave Background Radiation (CMB).



Credit: *ESA*

- CMB fluctuations are of the order of $\frac{\delta T}{T} \approx 10^{-5}$.
- Cosmological principle is consistent with observations.

Friedmann-Robertson-Walker Cosmology

- The mathematical frame work in which cosmological principle is incorporated are metric theories of gravity.
- The line element is:

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-Kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right)$$

where K= -1, 0, 1

- Friedmann Equations:

$$\frac{\dot{a}^2 + Kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

- We have $\rho \propto a^{-3(1+\omega)}$ where $\omega = \frac{P}{\rho}$.
- Within Friedmann-Robertson-Walker cosmology, the evolution of Universe follows:
  - Radiation dominated: $P = \frac{1}{3}\rho$ and $a \propto t^{\frac{2}{3}}$.
  - Matter dominated: $P = 0$ and $a \propto t^{\frac{1}{2}}$.
  - Λ dominated: $P = -\rho$ and $a \propto e^{Ht}$.

Problems with Standard Cosmology

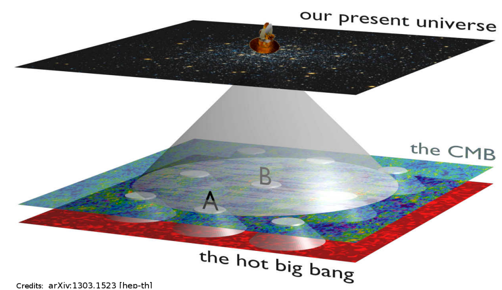
- Standard model of cosmology successfully trace the history of Universe from as early as 1 ns after Big Bang. However, it suffers from problems such as:

- Horizon Problem: The observed CMB radiation confirmed our belief of isotropic and homogeneous Universe. But why at this large scale?
- Fine tuning Problem: $K = 0$ is an unstable point which requires extreme fine tuning of the Universe at early time.
- Structure Formation: Density perturbations could generate structure. But what caused primordial density perturbations.

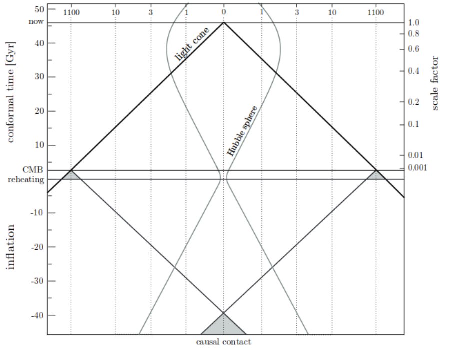
Cosmological Puzzles and Inflation as a solution

- It can be seen that cosmological puzzles are associated with an increasing comoving horizon.
- Inflation an early phase of Universe where comoving horizon decreases solves these problems

The Horizon Problem



Credits: arXiv:1303.1523 [hep-th]



http://www.damtp.cam.ac.uk/user/db275/Cosmology/Chapter2.pdf

Cosmological Puzzles and Inflation as a solution

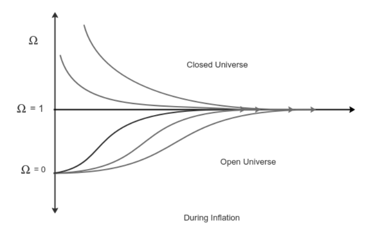
- It can be seen that cosmological puzzles are associated with an increasing comoving horizon.
- Inflation an early phase of Universe where comoving horizon decreases solves these problems

Fine Tuning Problem

- While $\Omega(a) = 1$ is an unstable point during radiation dominated expansion and matter dominated expansion. However, it is an attractor during Inflation.

$$1 - \Omega(a) = \frac{-K}{(aH)^2}$$

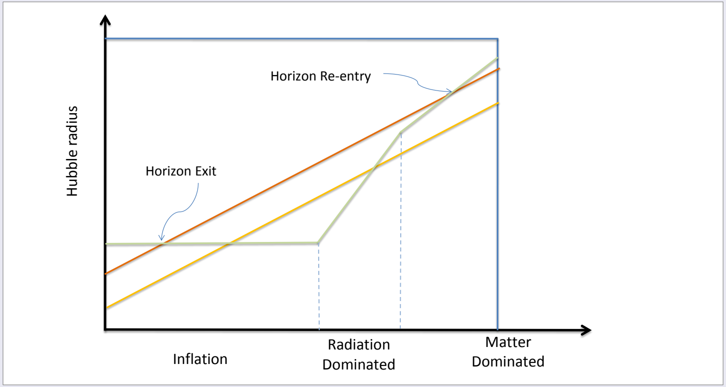
where $\Omega(a) = \frac{\rho(a)}{\rho_{crit}(a)}$.



Cosmological Puzzles and Inflation as a solution

- It can be seen that cosmological puzzles are associated with an increasing comoving horizon.
- Inflation an early phase of Universe where comoving horizon decreases solves these problems

Solution to the Problem of Density Perturbation



Dynamical Mechanism of Inflation

- The condition for Inflation is $\ddot{a} > 0$.
- Friedmann Equations:

$$\frac{\dot{a}^2 + Kc^2}{a^2} = \frac{8\pi G\rho}{3} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

- From Friedman Equations we can see that inflation demands $P < -\frac{1}{3}\rho$.
- For a scalar field we have:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- Models of Inflation
  - de-Sitter $a = a_0 e^{Ht}$. Driven by cosmoloical constant.
  - Power-law $a = a_0 t^p$. Driven by scalar with potential of form:

$$V(\phi) = \frac{(3p-1)}{2}(\sigma/t_i)^2 e^{-\frac{\phi-\phi_i}{\sigma}} \qquad \text{where} \sigma = \left(\frac{p}{4\pi}\right)^{1/2} M_p$$

- However, these exact solutions does not have an exit mechanism, and hence are not viable.

Slow-Roll Infation

- The widely used frame work of Inflation is of slow-roll, where a scalar field slowly rolls down the potential hill inflating the universe.

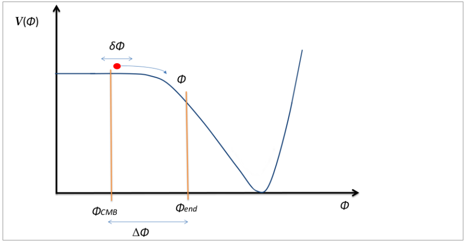
- During slow-roll the potential remains almost a constant.

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- An example- Higgs Inflation, with potential:

$$V(\phi) = M^4\left(1 - e^{-\sqrt{2/3}\phi/M_{pl}}\right)$$

- The quantum fluctuations of the scalar field act as the seeds of structure formation.



Problems of Inflation

- Inflation is the most successful paradigm that explains early Universe. However Inflation suffers from problems such as:
  - There is no unique mechanism for Inflation.
  - Inflation requires scalar fields with non-standard potentials. Based on canonical scalar field, Inflationary models require potentials of the form

$$V(\phi) = \sum_{n=0}^{N} c_{2n}\phi^{2n}$$

where c_{2n}'s are real numbers and $N > 2$.

  - Inflaton has to be light, the eta problem.

Problems of GR

- General relativity is successful. However people consider alternatives where GR is obtained as an approximation.
- The need for modified gravity: Observational aspects:
  1. The unsolved problems such as dark energy, dark matter etc.
  2. Absence of an unique mechanism for Inflation.
  3. ...
- The need for modified gravity: Theoretical aspects:
  1. General relativity permits singularities.
  2. General relativity cannot be conventionally quantized.
  3. ...
- Possible modifications to general relativity
  1. Scalar field theories.
  2. Tensor-theories.
  3. Scalar-tensor theories
  4. ...

Focus of the work

- What is the scalar field that drives inflation (inflaton)?
  - Scalar fields compatible with Standard Model of Particle Physics. In 4-D, we consider scalar field potentials of form.

$$V = \Lambda + m^2\phi^2 + \lambda\phi^4$$

- Is General relativity the right theory of gravity at Inflationary scales?
  - General relativity could be a low energy limit of a more fundamental theory such as $f(Gauss - Bonnet)$, $f(R)$, Scalar-tensor theories etc.
- How is Inflaton coupled to gravity?
  - We consider non-minimal Inflaton gravity coupling.

Inflation within Gauss-Bonnet gravity

[JM and SS]

- The Inflaton is Higgs Boson.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

$$\begin{aligned} \lambda &= 0.1291 \\ m &= M_H/\sqrt{2} \\ M_H &= 125\ GeV \quad [LHC\ Data] \end{aligned}$$

- Higgs field is non-minimally coupled to gravity through Gauss-Bonnet term.

$$\mathcal{S} = \int d^4x\sqrt{-g}\ \left[\frac{R}{2\kappa} + f(\phi)L_{GB} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\delta}R^{\mu\nu\rho\delta}$$

Background Equations

- For FRW metric, $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$ the field equations are

$$0 = -24H^2(\dot{H} + H^2)\dot{f}(\phi) + \dot{\phi}\ddot{\phi} + \dot{V}(\phi) + 3H\dot{\phi}^2$$

$$eom\ of\ scalar\ field$$

$$0 = -\frac{3H^2}{\kappa} + \frac{1}{2}\dot{\phi}^2 + V(\phi) - 24H^3\dot{f}(\phi)$$

$$0 - 0\ component\ of\ MEE$$

$$0 = -\frac{3H^2}{\kappa} - \frac{2\dot{H}}{\kappa} + V(\phi) - \frac{1}{2}\dot{\phi}^2 - 16H(\dot{H} + H^2)\dot{f}(\phi) - 8H^2\ddot{f}(\phi)$$

$$i - i\ component\ of\ MEE$$

Exact Power-law Solution

- Main Equation:

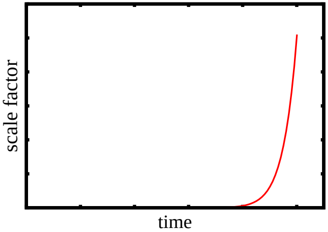
$$-2H^2 + \kappa\dot{\phi}^2 - 24\,\kappa\, H^3 \dot{f}\,(\phi) + 2\frac{\ddot{a}}{a} + 16\,\kappa H \frac{\ddot{a}}{a}\dot{f}\,(\phi) + 8\kappa H^2 \ddot{f}\,(\phi) = 0$$

- We have an exact Solution: Power-law

$a(t) = a_0 \left(\frac{t}{t_0} + \Upsilon\right)^p$, with $\phi(t) = \phi_0 \left(\frac{t}{t_0} + \Upsilon\right)^n$ for

$$V(\phi) = A_1\phi^{-\frac{2}{n}} + A_2\phi^{2-\frac{2}{n}} + A_3\phi^{\frac{p-1}{n}}$$

$$f(\phi) = B_1\phi^{\frac{2}{n}} + B_2\phi^{2+\frac{2}{n}} + B_3\phi^{\frac{p+3}{n}}$$



where '$p>1$' and 'n' can be any real number and the co-efficients:

$$A_1 = \frac{3\,(p-1)\,p^2}{\kappa\,(p+1)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^2 \qquad A_2 = \frac{(5n^2p - n^2 + 2n^3)}{2(1-2n+p)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^2 \qquad A_3 = 24p^3C\left(\frac{\phi_0^{1/n}}{t_0}\right)^{1-p}$$

$$B_1 = \frac{-1}{8\kappa p(1+p)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^{-2} \qquad B_2 = \frac{n^2}{16p^2(1+n)(1-2n+p)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^{-2} \qquad B_3 = \frac{C}{p+3}\left(\frac{\phi_0^{1/n}}{t_0}\right)^{-(p+3)}$$

Power-law - Special case

- $f(\phi) = \alpha\phi^{-4}$ and $V(\phi) = \frac{1}{4}\lambda\phi^4$ is a special case.
- Our analysis yielded the solution:

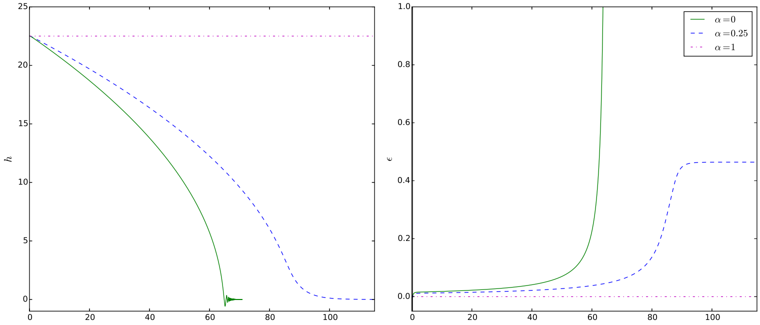
$$\begin{aligned} a(t) &= a_0 \left(\frac{t}{t_0} + \left(\frac{\phi(t_0)}{\phi_0} \right)^{-2} - 1 \right)^{p} \\ \phi(t) &= \phi_0 \left(\frac{t}{t_0} + \left(\frac{\phi(t_0)}{\phi_0} \right)^{-2} - 1 \right)^{-1/2} \end{aligned}$$

- Where the parameters are related by

$$\begin{aligned} \lambda\alpha &= -\frac{3p(p-1)}{2(p+1)^2\kappa^2} \\ \phi_0 &= \left(\frac{3p^2(p-1)}{t_0^2\kappa(p+1)\lambda} \right)^{\frac{1}{4}} \end{aligned}$$

An earlier study: Slow roll analysis

- $f(\phi) = \alpha\phi^{-4} = -\frac{1}{2}\xi_0\phi^{-4}$ and $V(\phi) = \frac{1}{4}\lambda\phi^4 = V_0\phi^4$.
- A coupling and potential of this form is well studied, but using the slow-roll frame work [Guo and Schwarz, PRD 2009,2010, Jiang and Guo, PRD 2013]
- It was shown that Higgs scalar can't act as the Inflaton with just the pure Gauss-Bonnet coupling under slow-roll. [Bruck and Longden, PRD 2016 March]; They define $\alpha = 4V_0\xi_0/3$

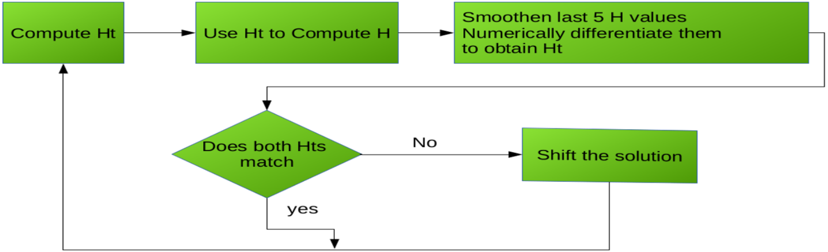


Details of Numerical Analysis

- What happens with $-\frac{1}{2}m^2\phi^2$ term in the potential?
- The field equations are:

$$0 = -24H^2(\dot{H} + H^2)\dot{f}(\phi) + \dot{\phi}\ddot{\phi} + \dot{V}(\phi) + 3H\dot{\phi}^2 \qquad eom\ of\ scalar\ field$$

$$0 = -\frac{3H^2}{\kappa} + \frac{1}{2}\dot{\phi}^2 + V(\phi) - 24H^3\dot{f}(\phi) \qquad 0-0\ component\ of\ MEE$$

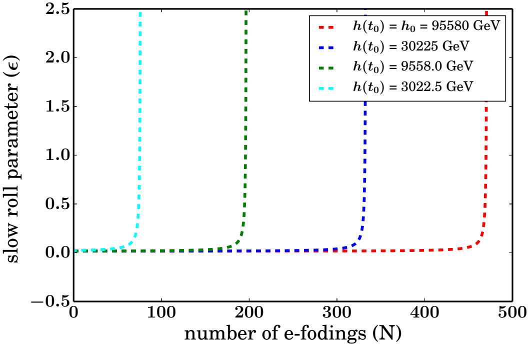


Inflation from Higgs field

- What happens with $-\frac{1}{2}m^2\phi^2$ term in the potential?

$$V\left(h\right) = \frac{\lambda}{4}\left(h^2 - \nu^2\right)^2$$

- Evoution of slow roll parameter $\epsilon = -\frac{\dot{H}}{H^2}$ with number of e-foldings for different initial values
- $\epsilon > 1$ corresponds to exit from Inflation.



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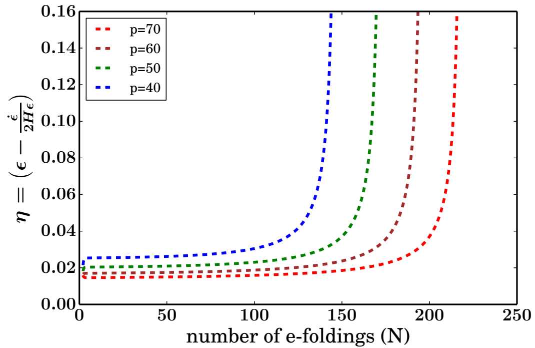
- Evoution of slow roll parameter η with number of e-foldings



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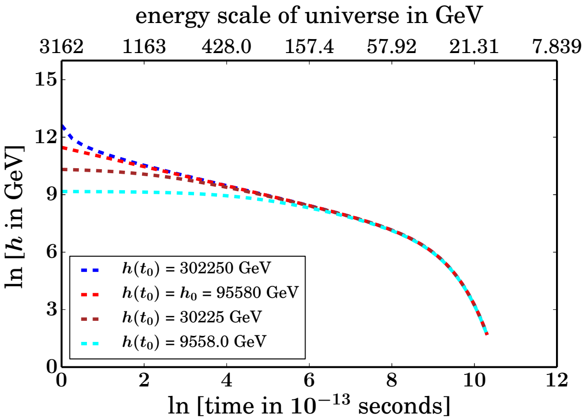
- What happens with $-\frac{1}{2}m^2\phi^2$ term in the potential?

$$V(h) = \frac{\lambda}{4}\left(h^2 - \nu^2\right)^2$$

- $\ln(h)$ vs $\ln(t)$

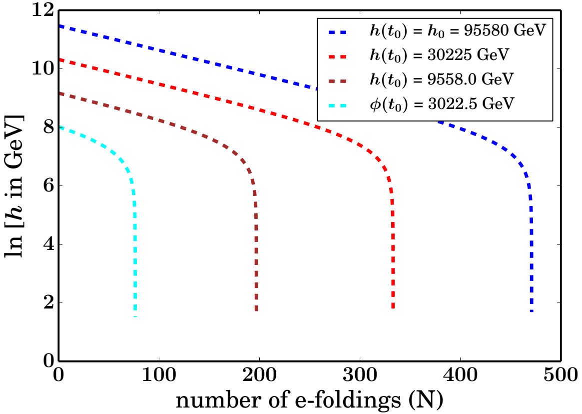


Inflation from Higgs field

- What happens with $-\frac{1}{2}m^2\phi^2$ term in the potential?

$$V(h) = \frac{\lambda}{4}\left(h^2 - \nu^2\right)^2$$

- $\ln(h)$ vs N

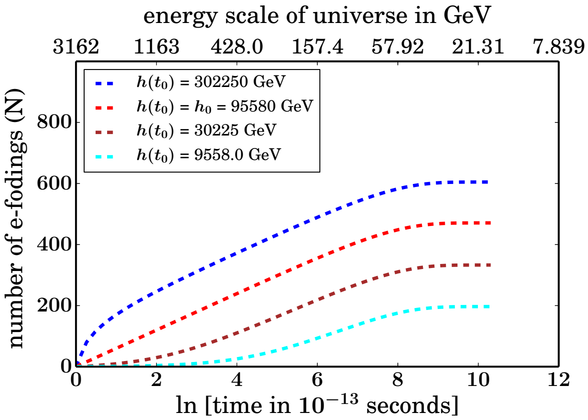


Inflation from Higgs field

- What happens with $-\frac{1}{2}m^2\phi^2$ term in the potential?

$$V(h) = \frac{\lambda}{4}\left(h^2 - \nu^2\right)^2$$

- e-foldings for different $h(t_0)$

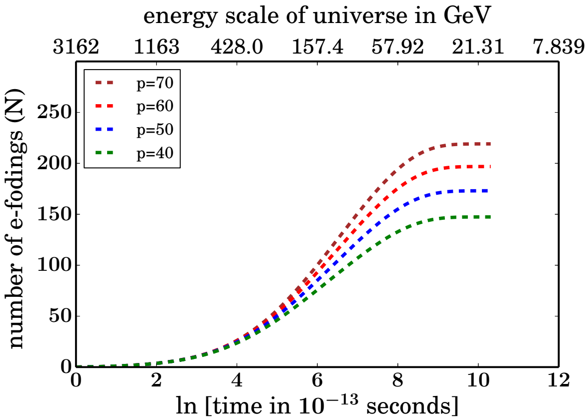


Inflation from Higgs field

- What happens with $-\frac{1}{2}m^2\phi^2$ term in the potential?

$$V(h) = \frac{\lambda}{4}\left(h^2 - \nu^2\right)^2$$

- e-foldings vs $\ln(t)$ for different values of p



Key features of our model

- The potential is consistent with standard model
- Exit can happen only with a mass term.
- We cannot have exit at any energy scale, it can only happen close to electroweak scale.
- No fine tuning.

Power spectrum - For approximated power-law soution

- The Fourier modes of scalar perturbations obey

$$\nu'' + \left(c_{\mathcal{R}}^2 k^2 - \frac{z_{\mathcal{R}}''}{z_{\mathcal{R}}} \right) \nu = 0$$

where $z_{\mathcal{R}} = a\sqrt{Q}$, Adapting the formulas from [Hwang and Noh, PRD 2000]

$$Q = \frac{\dot{\phi}^2 + \frac{3}{2}\frac{64H^4\dot{f}^2}{1/\kappa + 8H\dot{f}}}{\left(H + \frac{1}{2}\frac{8H^2\dot{f}}{1/\kappa + 8H\dot{f}}\right)^2} \qquad c_{\mathcal{R}}^2 = 1 - 8\dot{f}\,\frac{\frac{1}{2}\left(\frac{64H^2\dot{f}}{1/\kappa + 8H\dot{f}}\right)^2 \left(\ddot{f}/\dot{f} - H - \dot{H}\frac{1/\kappa + 8H\dot{f}}{2H\dot{f}}\right)}{\dot{\phi}^2 + \frac{3}{2}\frac{64H^4\dot{f}^2}{1/\kappa + 8H\dot{f}}}$$

- Scalar power spectrum given by $P_{\mathcal{R}} = \frac{k^3}{2\pi^2}|\nu/z_{\mathcal{R}}|^2$ is

$$P_{\mathcal{R}} = k^{3-2\nu_R} 2^{2\nu_{\mathcal{R}}-3} c_{\mathcal{R}}^{-2\nu_{\mathcal{R}}} \left(\frac{\Gamma(\nu_{\mathcal{R}})}{\Gamma(3/2)}\right)^2 \frac{1}{4\pi^2} \left(\frac{a_0(p-1)}{t_0}\right)^{\frac{2p}{p-1}} \frac{1}{a_0^2 Q}$$

where $\nu_{\mathcal{R}} = \frac{3p-1}{2(p-1)}$

Power spectrum - For approximated power-law soution

- The Fourier modes of tensor perturbations obey

$$u'' + \left(c_{\mathcal{T}}^2 k^2 - \frac{z_{\mathcal{T}}''}{z_{\mathcal{T}}} \right) u = 0$$

where $z_{\mathcal{T}} = a\sqrt{Q_g}$ and

$$Q_g = \frac{1}{\kappa} + 8H\dot{f}, \qquad c_{\mathcal{T}}^2 = \frac{\frac{1}{\kappa} + 8\ddot{f}}{\frac{1}{\kappa} + 8H\dot{f}}$$

- Tensor power spectrum, given by $P_{\mathcal{T}} = \frac{8k^3|u/z_{\mathcal{T}}|^2}{2\pi^2}$ is

$$P_{\mathcal{T}} = 8k^{3-2\nu_{\mathcal{T}}} 2^{2\nu_{\mathcal{T}}-3} c_{\mathcal{T}}^{-2\nu_{\mathcal{T}}} \left(\frac{\Gamma(\nu_{\mathcal{T}})}{\Gamma(3/2)} \right)^2 \frac{1}{4\pi^2} \left(\frac{a_0(p-1)}{t_0} \right)^{\frac{2p}{p-1}} \frac{1}{a_0^2 Q_g}$$

where $\nu_{\mathcal{T}} = \frac{3p-1}{2(p-1)}$

Constraints from Observations

- Scalar spectral index n_s, $P_{\mathcal{R}} \propto k^{n_s - 1}$

$$n_s = 3 - \frac{2p}{p-1}$$

- A value of $n_s \approx 0.968$ means $p \approx 60$, which constraints $\alpha = (1.823 M_p)^4$.

- Tensor spectral index n_t, $P_{\mathcal{T}} \propto k^{n_t}$, $n_t = n_s - 1$
- Tensor to scalar ratio, r

$$r \equiv \frac{P_{\mathcal{T}}}{P_{\mathcal{R}}} \approx 8 \times \left(\frac{c_{\mathcal{R}}}{c_{\mathcal{T}}}\right)^{2\nu_{\mathcal{R}}} \frac{Q}{Q_g}$$

- Tensor to scalar ratio is 0.012.
- $H_* = 10^{12}\ GeV$, $\phi_* = 10^{16}\ GeV$, $t_* = 8.013 \times 10^{-12} \text{GeV}^{-1} + t_0 - 60\sqrt{8|f\left(h(t_0)\right)|\kappa}$

Conclusion-I

Results

- Higgs field non-minimally coupled to Gauss-Bonnet term can drive Inflation.
- All parameters in the model are fixed.
- No fine tuning.
- Exit of Inflation happens close to electroweak scale.
- It is consistant with Planck data.

Inflation within f(R) gravity

[JM,Johnson and SS, 2017]

- We consider the action of form:

$$\int \sqrt{-g} d^4x \left[\frac{1}{2} f(R, \phi) - \frac{1}{2} g^{ab} \nabla_a \phi \nabla_b \phi - V(\phi) \right],$$

- We are interested in $f(R, \phi)$ of form:

$$f\left(R, \phi\right) = h(\phi) \left(R + \alpha R^2\right)$$

- We look for scalar fields that are compatible with standard model, i.e.,

$$V\left(\phi\right) = \Lambda + m^2 \phi^2 + \lambda \phi^4$$

Background Equations

- For FRW metric, $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$ the field equations are

$$0 = 6\dot{h}H^2 + 72\dot{h}H^4\alpha + 72\dot{h}H^2\alpha\dot{H} + 3\dot{h}\dot{H} + 18\dot{h}\dot{H}^2\alpha - \dot{V} - \omega\,\dot{\phi}\ddot{\phi} - 3\omega H\dot{\phi}^2$$

equation of motion of scalar

$$0 = -\frac{1}{2}\omega\dot{\phi}^2 + 3hH^2 + 108\alpha hH^2\dot{H} - 18h\dot{H}^2\alpha - V + 3H\dot{h} + 72\dot{h}\alpha H^3 + 36H\dot{h}\alpha\dot{H} + 36Hh\alpha\ddot{H}$$

0-0 component of MEEs

$$0 = 2h\dot{H} + 108\alpha hH^2\dot{H} + 48\dot{h}H^3\alpha + 54h\dot{H}^2\alpha + 3hH^2 + \ddot{h} + \frac{1}{2}\omega\dot{\phi}^2 120H\dot{h}\dot{H}\alpha + 72h\ddot{H}\alpha + 2H\dot{h} - V + 24\alpha H^2\ddot{h} + 12\alpha\dot{H}\ddot{h} + 24\alpha\ddot{H}\dot{h} + 12h\alpha\dddot{H}$$

i-i component of MEEs

Exact de-Sitter solution

- Main Equation:

$$-2h\dot{H} - 72h\dot{H}^2\alpha - \omega\dot{\phi}^2 - \ddot{h} - 84H\dot{h}\dot{H}\alpha - 36Hh\ddot{H}\alpha + 24\alpha\dot{h}H^3 + H\dot{h} - 24\ddot{h}H^2\alpha - 12\alpha\ddot{h}\dot{H} - 24\alpha\dot{h}\ddot{H} - 12\alpha h\dddot{H} = 0$$

- We have an exact solution, de-Sitter: $a = a_0 e^{H_D t}$ with $\phi = \phi_0 e^{-pH_D t}$, for

$$\begin{aligned} V(\phi) &= \lambda_0 + m^2\,\phi^2 + \lambda_p\,\phi^{-p} \\ h(\phi) &= \mu_0 + \mu_2\,\phi^2 + \mu_p\,\phi^{-p} \end{aligned}$$

where

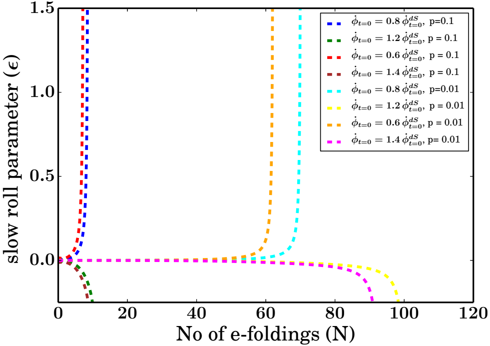
$$\mu_0 = \frac{1}{3\,H_D^2}\lambda_0 \qquad \mu_2 = -\frac{\omega\,p}{\left(1 + 24\,\alpha\,H_D^2\right)(2 + 4\,p)}$$

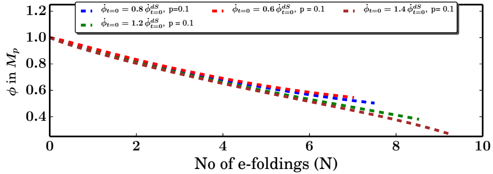
$$m^2 = \left(3 + p(2p-5)(1 + 24\,\alpha\,H_D^2)\right)\,H_D^2\,\mu_2 \qquad \mu_p = \frac{1}{6\,H_D^2\left(12\,\alpha\,H_D^2 + 1\right)}\lambda_p\,,$$

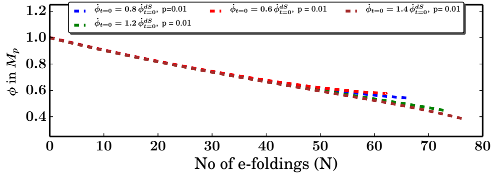
- λ_0 and λ_p are arbitrary, we use this freedom to set $\lambda_0 = \mu_0 = \lambda_p = \mu_p = 0$.

Natural Exit

- The numeric computation shows that the de-Sitter solution obtained is not an attractor.







$$\alpha = -10^8 M_p^2,\ \alpha_2 = 10^{-4} M_p^{-2}$$
and
$$m(H_D = 4.17167 \times 10^{-4} M_p, p = 0.1) = 5.9437 \times 10^{-5};$$
$$m(H_D = 1.4437 \times 10^{-4}, p = 0.01) = 3.368 \times 10^{-6} M_p$$

Complete analytical solution

- The de-Sitter solution is an unstable equillibrium point.
- We define :

$$\boldsymbol{v} = \begin{pmatrix} H \\ \dot{H} \\ \Delta \end{pmatrix} \quad \text{where} \quad \Delta = \dot{\phi}/\phi \; ; \quad \text{Also we have} \quad \{\boldsymbol{v}\}_{eq} = \begin{pmatrix} H_D \\ 0 \\ -pH_D \end{pmatrix}$$

$$\dot{\boldsymbol{v}} = f(\boldsymbol{v}) = \begin{pmatrix} \dot{H} \\ \ddot{H} \\ \dot{\Delta} \end{pmatrix} = \left\{ \begin{matrix} \dot{H} \\ -4H^2\Delta - 3H\dot{H} - 2\Delta\dot{H} + \frac{1}{72}\frac{\Delta^2}{\alpha\mu_2 H} + \frac{1}{2}\frac{\dot{H}^2}{H} + \frac{1}{36}\frac{m^4}{\alpha\mu_2 H} - \frac{1}{12}\frac{H}{\alpha} - \frac{1}{6}\frac{\Delta}{\alpha} \\ 144\alpha\mu_2 H^4 + 144\alpha\mu_2\dot{H}H^2 + 36\alpha\mu_2\dot{H}^2 - 3H\Delta - \Delta^2 + 12\mu_2 H^2 + 6\mu_2\dot{H} - 2m^2 \end{matrix} \right\}$$

- The trajectories with initial conditions close to the equilibrium can be written as $\boldsymbol{v} = \boldsymbol{v}_{eq} + \delta\boldsymbol{v}$ taylor expanding we have $\dot{\delta v_i} = \{\partial_j f_i\}_{eq}\delta v_j = J_{ij}\delta v_j$.

$$J_{ij} = \begin{bmatrix} 0 & 1 & 0 \\ 1/6\,\frac{72\,p\alpha\,{H_D}^2 + p - 1}{\alpha} & 2\,pH_D - 3\,H_D & 1/9\,\frac{-1 - 24\,\alpha\,{H_D}^2 + p + 24\,p\alpha\,{H_D}^2}{\alpha} \\ 3\,\frac{(-3+2\,p)pH_D}{1+2\,p} & -3\,\frac{p}{1+2\,p} & 2\,pH_D - 3\,H_D \end{bmatrix}$$

- The complete solution is:

$$\boldsymbol{v_i} = \{\boldsymbol{v}_i\}_{eq} + \delta\boldsymbol{v}_i = \{\boldsymbol{v}_i\}_{eq} + \sum_{i=1}^{i=3} c_i u_i e^{(\lambda_i t)}$$

Number of e-foldings

- The approximate expression for the number of e-foldings is:

$$N \approx \frac{H_D}{\lambda} ln \left(\frac{H_D^2}{\lambda(H_D - H_i)} \right).$$

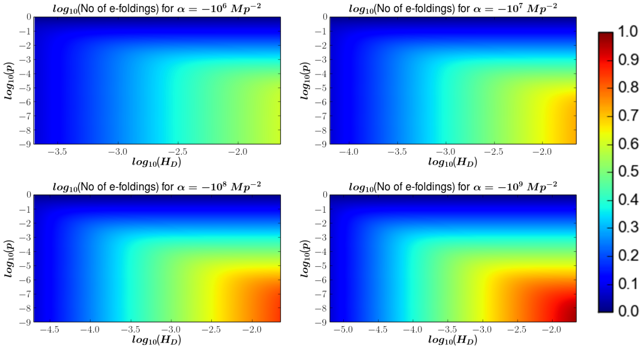


Figure : Contour plot showing the dependence of the number of e-folding on H_D, p and α.

Key features of our model

- The potential is consistent with standard model
- The exact solution is an unstable fixed point solution (saddle point solution).
- Exit depends on the initial conditions
- We have a complete analytical expression for the Inflationary evolution.

Scalar Power Spectrum

- For the metric with most general perturbations:

$$ds^2 = -(1+2\theta)dt^2 - a(\beta_{,\alpha} + B_\alpha)dtdx^\alpha + a^2[g^{(3)}_{\alpha\beta}(1-2\psi) + 2\gamma_{,\alpha|\beta} + 2C_{\alpha|\beta} + 2C_{\alpha\beta}].$$

- The quantity we need to evaluate inorder to compare with the observations is 3-Curvature perturbation ($\mathcal{R}$) which is given by:

$$\mathcal{R} = \psi + \frac{H}{\dot\phi}\delta\phi$$

- **For models where $F = \frac{\partial f(R,\phi)}{\partial R} \equiv F(R,\phi)$, we can't derive the conventional Mukhanov-sasaki equation.**
- However, we can derive the following equation using the perturbed field equations using Newtonian gauge, for $\Theta = \theta + \psi$ (Bardeen Potential in Einstein frame):

$$F\ddot\Theta + \left(3\dot F + H\,F - -\frac{2F\ddot\phi}{\dot\phi}\right)\dot\Theta + \left(\frac{k^2}{a^2}F - \ddot F - \frac{2FH\ddot\phi}{\dot\phi} + \frac{2\dot F\ddot\phi}{\dot\phi} + H\dot F + 4\dot H F\right)\Theta = \left(\dot\phi^2 + 6F\dot H - 3\dot F H - 3\ddot F + \frac{6\dot F\ddot\phi}{\dot\phi}\right)\theta$$

- For the exact analytical solution in the previous section, above equation becomes:

$$\ddot\Theta + H_D(1-4p)\dot\Theta + \frac{k^2}{a^2}\Theta - 4\,p\,H_D^2\,(1-p)\theta = 0$$

- For $k \gg 1$, we have

$$\ddot\Theta + H_D(1-4p)\dot\Theta + \frac{k^2}{a^2}\Theta \simeq 0 \qquad (2)$$

Scalar Power Spectrum

- For $p \ll 1$, We can obtain

$$\delta\phi = \frac{\phi_0}{2H_D}\, e^{-p\,H_D\,t}\,(\dot{\Theta} + H_D\Theta)$$

- We can derive:

$$\begin{aligned}
\psi &= \frac{1}{3}\Theta - \frac{2}{3}\frac{1}{\frac{k^2}{a^2}}\ddot{\Theta} - \frac{1}{\frac{k^2}{a^2}}\dot{\Theta}\left(H_D - \frac{1}{12\alpha H_D}\right) \\
\theta &= \frac{2}{3}\frac{1}{\frac{k^2}{a^2}}\ddot{\Theta} + \frac{1}{\frac{k^2}{a^2}}\dot{\Theta}\left(H_D - \frac{1}{12\alpha H_D}\right) + \frac{2}{3}\Theta \\
\delta F &= n\phi_0^2 e^{-2pH_D t}\left(\frac{1}{6}\Theta + \frac{2}{3}\frac{1}{\frac{k^2}{a^2}}\ddot{\Theta} + \frac{1}{\frac{k^2}{a^2}}\dot{\Theta}\left(H_D - \frac{1}{12\alpha H_D}\right)\right) \\
\psi &= \mathcal{R} - \frac{H}{\dot{\phi}}\delta\phi
\end{aligned}$$

$$\Theta = e^{(4p-1)H_D t/2} U_1 \qquad \text{where} \quad U_1 = C_1 H^{(1)}_{\frac{1}{2}-2p}\left(\frac{k e^{-H_D t}}{a_0 H_D}\right) + C_2 H^{(2)}_{\frac{1}{2}-2p}\left(\frac{k e^{-H_D t}}{a_0 H_D}\right)$$

Scalar Power Spectrum

- The perturbed equations can be rewritten in terms of the new variables $\Theta,\ \dot{\Theta},$ and $\mathcal{R}$ or equivalently $U_1,\ \dot{U}_1$ and $\mathcal{R}$.
- Obtain a second order differential eqn for $\mathcal{R}$.

$$\ddot{\mathcal{R}}_< + 3H_D\mathcal{R}_< + \frac{k^2}{a^2}\mathcal{R}_< = 0$$

- Solve it and use the $Bunch - Davis$ vacuum solution at early epoch, to get:

$$\mathcal{R}_< \ = \frac{H_D}{2a\sqrt{k}}e^{-ik\eta}$$

- for $\frac{k}{a} \ll 1$ i.e., for large wavelength modes, we have:

$$\mathcal{R}_> = C\,.$$

- Matching the large wavelength and small wavelength solution at $k\eta = 2\pi$, the scalar power spectrum is given by

$$\mathcal{P}_{\mathcal{R}} = H_D^2\,.$$

Tensor Power Spectrum

- Following [Hwang and Noh, PRD 2000]

$$\ddot{C}^{\alpha}_{\beta} + (3 - 2p)\, H_D \dot{C}^{\alpha}_{\beta} + \frac{k^2}{a^2} C^{\alpha}_{\beta} = 0.$$

- We can simplify this equation by rewriting $C^{\alpha}_{\beta} = \nu_g / z_g$, where $z_g = a e^{-pH_D t}$ to get

$$\nu_g'' + \left(k^2 - \frac{z_g''}{z_g}\right) \nu_g = 0.$$

- Then solution to the above equation is given by

$$\nu_g = \sqrt{-\eta} \left(C_1 H^{(1)}_{3/2-p}(-k\eta) + C_2 H^{(2)}_{3/2-p}(-k\eta) \right).$$

- At the initial epoch of Inflation, setting the field to be in the Bunch-Davies vacuum, we have

$$\nu_g = \sqrt{\frac{\pi}{4}} \sqrt{-\eta} H^{(1)}_{3/2-p}(-k\eta),$$

- Tensor power spectrum is given by

$$\mathcal{P}_g = 8 \left(\frac{k}{k_*}\right)^{2p} \frac{2^{-2p}}{4\pi^2} H_D^2 \left(\frac{\Gamma(3/2 - p)}{\Gamma(3/2)}\right)^2 e^{2pH_D t_*}$$

Conclusion-II

Results

- Inflationary model with $f(R,\phi)$ gravity driven by a massive scalar field is constructed.
- Calculated the scalar power spectrum for $p \ll 1$ and was able to show that the spectrum is nearly scale invariant.
- Calculated the tensor power spectrum and obtained a blue tilt.
- Exit depends on the initial velocity of the scalar field and initial value of Hubble parameter.

Inflation from exponential non-minimal coupling

[Johnson, JM, and SS, 2017]

- We consider the action of form:

$$S_J = \int d^4x\sqrt{-g}\left[\frac{1}{2}f(R,\phi) - \frac{\omega}{2}g^{ab}\nabla_a\phi\nabla_b\phi - V(\phi)\right],$$

- We are interested in $f(R,\phi)$ of form:

$$f(R,\phi) = \frac{1}{\kappa}Re^{h(\phi)R} \simeq \frac{1}{\kappa}\left[R + h(\phi)R^2\right].$$

- The physical motivation for such a scenario comes from the fact that the quantum corrections to the gravity and scalar field can have a scale dependent corrections.
- We look for scalar fields that are compatible with standard model, i.e.,

$$V(\phi) = \Lambda + m^2\phi^2 + \lambda\phi^4$$

Einstein frame calculations

- $f(R)$ gravity is a higher derivative theory, hence field equations are 4th order in Jordan frame.
- However, Physics can be described equivalently in both Jordan frame and in Einstein frame where the field equations are second order, related through a conformal transformation.
- For our model, using the conformal transformation $\tilde{g}^{ab} \to \Omega^2 g^{ab}$, we rewrite the action in the form.

$$S_E = \int \sqrt{-\tilde{g}} dx^4 \left[\frac{1}{2\kappa}\tilde{R} - \frac{\tilde{g}^{ab}}{2e^{\sqrt{\frac{2\kappa}{3}}\zeta}} \partial_a\phi\partial_b\phi - \frac{1}{2}\tilde{g}^{ab}\partial_a\zeta\partial_b\zeta - W \right]$$

where $\Omega^2 = F = \frac{\partial f(R,\phi)}{\partial R}$, $\zeta = \sqrt{\frac{3}{2\kappa}} \ln F$ and $W = \frac{FR-f}{F^2} + \frac{V}{F^2}$

- We verified that the equations in Einstein frame are satisfied by the transformed form of the solution obtained in Jordan frame.
- However, we find it difficult to proceed with our technique in Einstein frame.

Background Equations

- In Jordan frame, for FRW metric, $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$, the field equations are

$$0 = \frac{1}{\kappa}72H^4\dot{h} + \frac{1}{\kappa}\dot{H}\dot{h}H^2 + \frac{1}{\kappa}18\dot{h}\dot{H}^2 - \dot{V} - \omega\dot{\phi}\ddot{\phi} - 3\omega H\dot{\phi}^2$$

equation of motion of scalar

$$\begin{aligned} 0 = {} & \frac{1}{\kappa}108h\dot{H}H^2 - \frac{1}{\kappa}18h\dot{H}^2 + \frac{1}{\kappa}3H^2 - V - \frac{1}{2}\omega\dot{\phi}^2 + \frac{1}{\kappa}72\dot{h}H^3 + \\ & \frac{1}{\kappa}36H\dot{h}\dot{H} + \frac{1}{\kappa}36Hh\ddot{H} \end{aligned}$$

0-0 component of MEEs

$$\begin{aligned} 0 = {} & \frac{1}{\kappa}48\dot{h}H^3 - V + \frac{1}{\kappa}72Hh\ddot{H} + \frac{1}{\kappa}120H\dot{h}\dot{H} + \frac{1}{\kappa}12\dot{H}\ddot{h} + \frac{1}{\kappa}24\ddot{h}H^2 + \\ & \frac{1}{\kappa}24\dot{h}\ddot{H} + \frac{1}{\kappa}12h\dddot{H} + \frac{1}{2}\omega\dot{\phi}^2 + \frac{1}{\kappa}2\dot{H} + \frac{1}{\kappa}3H^2 + \frac{1}{\kappa}108h\dot{H}H^2 + \frac{1}{\kappa}54h\dot{H}^2 \end{aligned}$$

i-i component of MEEs

Exact de-Sitter solution

- Main Equation:

$$0 = -\omega\kappa\dot{\phi}^2 - 72h\dot{H}^2 + 24\dot{h}H^3 - 84H\dot{h}\dot{H} - 36Hh\ddot{H} - 12\dot{H}\ddot{h} - 24\ddot{h}H^2 - 24\dot{h}\ddot{H} - 12h\dddot{H} - 2\dot{H}$$

- We have an exact solution, de-Sitter: $a = a_0 e^{H_D t}$ with $\phi = \phi_0 e^{-nH_D t}$, for

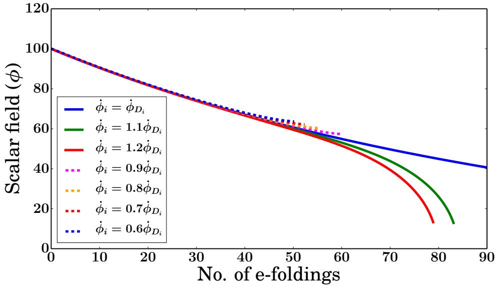
$$h(\phi) = -\lambda\phi^2, \qquad V(\phi) = m^2\phi^2 + V_0$$

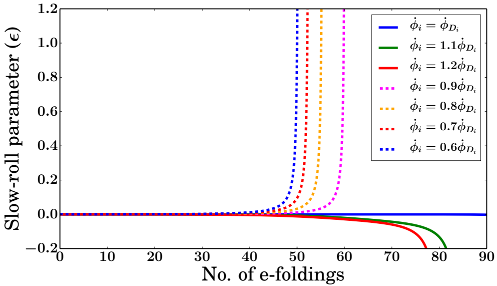
where

$$\lambda = \frac{1}{48}\frac{\omega n\kappa}{(2n+1)H_D^2}. \qquad m^2 = \frac{\omega n^2 H_D^2}{(2n+1)}\left(\frac{5}{2} - n\right), \quad V_0 = \frac{3H_D^2}{\kappa}.$$

- Here we don't consider the integration constants, which we set to zero.

- The numeric computation shows that the de-Sitter solution obtained is not an attractor.
- Hence for a wide range of initial conditions there exist an inflationary solution with graceful exit.





$$H_D = 4\times 10^{-4}M_p, \quad n = 0.01, \quad \phi_0 = 100M_p, \quad m^2 = 3.90588\times 10^{-11}M_p^2, \quad \lambda = 1276.5522, \quad V_0 = 4.81\times 10^{-7}M_p^{-4}, \quad \kappa = 1.$$

Key features of our model

- The potential is consistent with standard model
- The exact solution is an unstable fixed point solution (saddle point solution).
- Exit depends on the initial conditions

Power spectrum

- We obtained the scalar perturbation following a similar procedure that we used in our earlier work.
- For $n \ll 1$ the scalar power spectrum was obtained to be:

$$\mathcal{P}_{\mathcal{R}} = H_D^2\,.$$

- For tensor perturbations, we follow [Hwang and Noh, PRD 2000]
- Here the differential equation is complicated. Hence to simplify we choose the approximation $2h(\phi)R \gg 1$, then the evolution equation is:

$$\ddot{C}^{\alpha}_{\beta} + (3-2n)\,H_D\dot{C}^{\alpha}_{\beta} + \frac{k^2}{a^2}C^{\alpha}_{\beta} = 0.$$

- The tensor power spectrum is

$$\mathcal{P}_g = 8\left(\frac{k}{k_*}\right)^{2n}\frac{2^{-2n}}{4\pi^2}H_D^2\left(\frac{\Gamma(3/2-n)}{\Gamma(3/2)}\right)^2 e^{2nH_Dt}.$$

Conclusion-III

Results

- Inflationary model, within $f(R, \phi)$ gravity with an exponential non-minimal coupling, driven by a massive scalar field is constructed.
- Showed that the exact solution obtained is an unstable solution.
- Showed that exit depends on the initial velocity of the scalar field and initial value of Hubble parameter.
- We showed that the scalar power spectrum obtained is scale invariant for $n \ll 1$.
- We showed that the tensor power spectrum have a blue tilt.

Concluding Remarks

- We were able to show that Higgs scalar can act as the Inflaton when non-minimally coupled with Gauss-Bonnet term leading to exit at electro-weak scale. The tensor to scalar ratio will be lowered by such a coupling. The exit is happening at electro-weak scale also suggests possible implications at LHC.
- We were able to build a successful inflationary model driven by a massive scalar field in $f(R, \phi)$ gravity. We were able to show that the model predicts a blue tilt for tensor power spectrum. We were also able to show that the scalar power spectrum is nearly scale invariant.
- We were able to build an inflationary model in $f(R)$ gravity with an exponential non-minimal coupling with gravity. We were able to show that the model predicts a blue tilt for tensor power spectrum. We were also able to show that the scalar power spectrum is nearly scale invariant.

Future Research

- There are several possibilities for the application of the techniques we have used in our works, also there are scope for detailed analysis of our works and results.
  - I would like to model early universe (focusing on bounce) and late time acceleration in different theories of gravity, using the technique we have used.
  - I would like to see whether the technique we used could be used to obtain the approximate solutions, once we have fixed the model.
  - I would like to see the possibility to obtain the power-spectrum in f(R,ϕ) models of gravity,for slow-roll. Where $F \equiv F(R, \phi)$
  - I am interested in the detailed investigation of Gauss-Bonnet Higgs non-minimal coupling from the particle physics aspect.

THANK YOU!