Inequalities on spin observables and application to hadronic reactions and structure functions

Jean-Marc Richard
j-m.richard@ipnl.in2p3.fr

Insitut de Physique Nucléaire de Lyon
Université Lyon 1 – IN2P3 -CNRS
4, rue Enrico Fermi, Villeurbanne, France

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Collaboration with

- Mohktar Elchikh (Oran, Algeria),
- J. Soffer (Temple U.), X. Artru (Lyon) and O. Teryaev (Dubna)

Outline

Introduction-1

- Spin observables are necessary to test the details of the dynamics,
- For instance, vector mesons anticipated (Breit) from the onset of spin-orbit forces in proton–proton scattering,
- Typical scenario: one or two observables are measured first,
- Question naturally arises: which new observable will provide the best improvement of knowledge,
- Corollary: If two or three observables are measured independently, is it possible to test whether they are compatible, without performing a full amplitude analysis,

Introduction-2

- If X, Y, Z, etc. are typical spin observables, with standard normalisation $-1 \leq X \leq +1$
- the domain for $\{X, Y\}$ is often smaller than the square $[-1, +1]^2$
- the domain for $\{X, Y, Z\}$ is often smaller than the cube $[-1, +1]^3$
- Explicit inequalities are obtained relating two or three spin observables, for instance $X^2 + Y^2 \leq 1$, $X^2 + Y^2 + Z^2 \leq 1$.
- Hence some observables might be very much constrained by previously measured observables,
- While the range of others can be still very much open
- Our study aims at reviewing the inequalities and their physical interpretation (derivation purely algebraic in some early works)

$\pi\mathrm{N}$ scattering (1)

From textbooks

$$\mathcal{M} = -2m\bar{u}(\tilde{p}')\left[-\mathcal{A} + i\gamma.\frac{\tilde{q}+\tilde{q}'}{2}\mathcal{B}\right]u(\tilde{p}) = 8\pi\sqrt{s}\,\chi_f^\dagger\,(f + ig\,\boldsymbol{\sigma}.\boldsymbol{n})\,\chi_i\ ,$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = I_0 = |f|^2 + |g|^2\ , \quad I_0 P_n = I_0 A_n = 2\,\Im\mathrm{m}(fg^*)\ .$$

$$I_0 A = (|f|^2 - |g|^2)\cos\vartheta + 2\,\Re\mathrm{e}(fg^*)\sin\vartheta\ ,$$

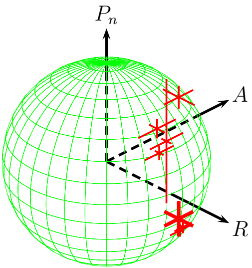
$$I_0 R = (|f|^2 - |g|^2)\sin\vartheta - 2\,\Re\mathrm{e}(fg^*)\cos\vartheta\ .$$

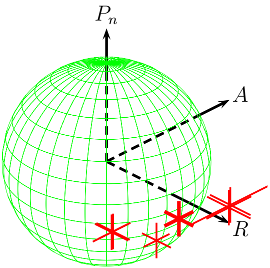
and hence

$$P_n^2 + A^2 + R^2 = 1\ .$$

either ignored in the analysis (and checked after) or used in the analysis.

πN scattering (2)

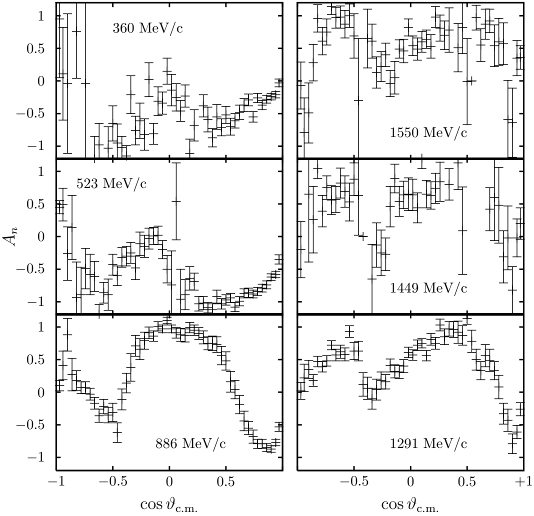




πN scattering (3)

$\bar{p}p \to \pi\pi$

Large spin effects observed in the crossed reaction (LEAR)



$\bar{p}p \to \pi\pi$

Similar identity
$A_n^2 + A_{mm}^2 + A_{ml}^2 = 1$

as for πN

Hence
$A_{mm} = A_{ml} = 0$ when
$|A_n| = 1$

Motivations for $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$

$\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$ measured at LEAR to study how strangeness is produced.

The most popular possibilities envisaged:

- Kaon exchange, similar to $\pi^{\pm}$ exchange for $\bar{\mathrm{p}}\mathrm{p} \to \bar{\mathrm{n}}\mathrm{n}$.
- Light quark pair annihilation and $s\bar{s}$ creation in different variants, such as ${}^3\mathrm{S}_1$ or ${}^3P_0$

Also proposed

- s-channel resonances (Roberts)

During the course of the experiment, a new scenario proposed:

- $s\bar{s}$ extracted from the nucleon (or antinucleon) sea.

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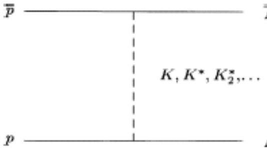
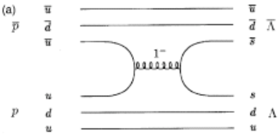
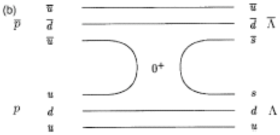
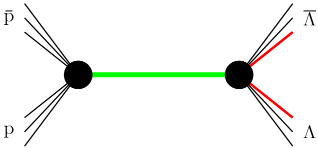
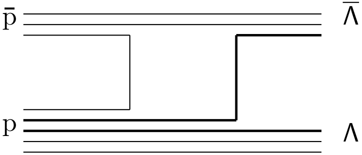
Mechanisms


$\bar{p}$
$\bar{\Lambda}$
$K, K^*, K_2^*, \ldots$
p
Λ

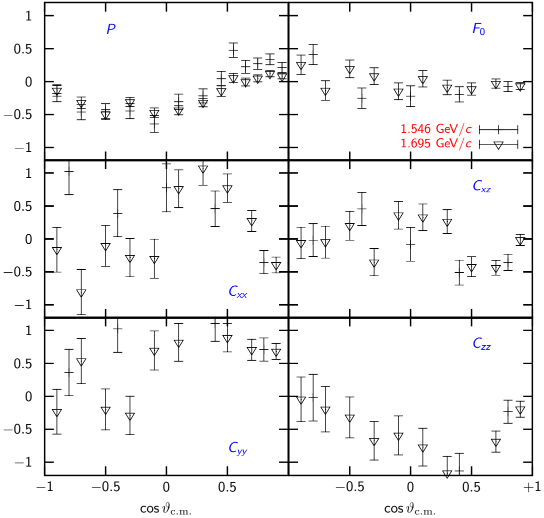

(a)
$\bar{p}$
$\bar{u}$ $\bar{d}$ $\bar{u}$
1^-
$\bar{u}$ $\bar{d}$ $\bar{s}$ $\bar{\Lambda}$
p
u d u
s d u Λ


(b)
$\bar{p}$
$\bar{u}$ $\bar{d}$ $\bar{u}$
0^+
$\bar{u}$ $\bar{d}$ $\bar{s}$ $\bar{\Lambda}$
p
u d u
s d u Λ

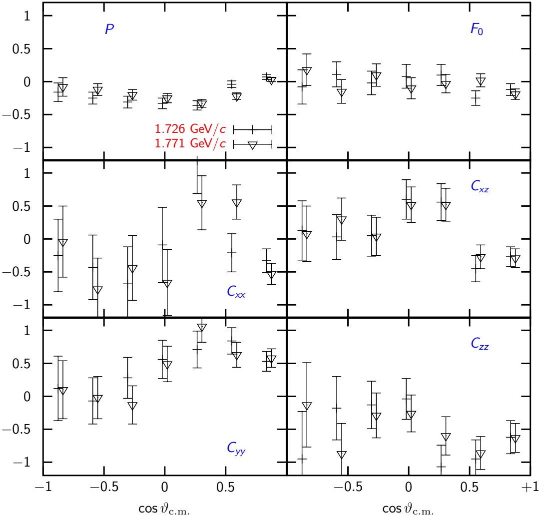

$\bar{p}$
p
$\bar{\Lambda}$
Λ


$\bar{p}$
p
$\bar{\Lambda}$
Λ

Spin observables without polarised target-1



Spin observables without polarised target-2



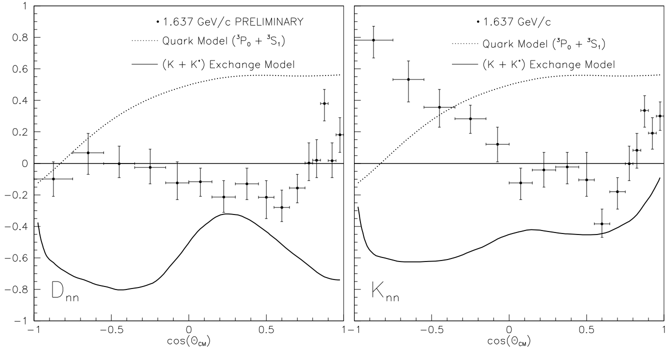
Towards measurements with a polarised target

- Suggestion to resume with a polarised target,
- Holinde et al., compare K-exchange and simple quark models: transverse depolarisation D_{nn} and transfer K_{nn} well suited to test the dynamics.
- Longitudinal polarisation tests tensor-force effects, as for $\bar{\mathrm{p}}\mathrm{p} \to \bar{\mathrm{n}}\mathrm{n}$ (Dover, R.), but gives less observables.
- Alberg, Ellis and Kharzeev proposed the model with extraction of a $s\bar{s}$ pair from the sea, which give a large effect on D_{nn}.
- However, it was pointed out that the existing data on final-state polarisation, and the data on final-state spin-correlation parameters already restrict the allowed domain for D_{nn}. (R., Elchikh+R.) This was the beginning of a systematic investigation of the correlations among observables.

Results with a polarised target at 1.637 GeV/c

Meanwhile, the experiment was carried out, and the analysis lead to D_{nn} and K_{nn} and several other observables.

Results of Paschke et al. at 1.637 GeV/c vs. some popular models



Results with a polarised target at 1.525 GeV/c

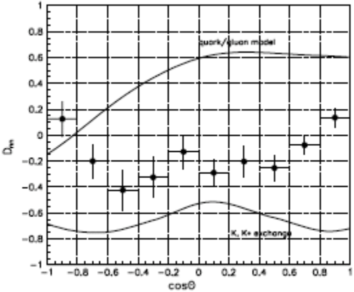


Fig. 4. Measured depolarization D_{nn} at 1525 MeV/c (preliminary), with the combined-vector/scalar quark-gluon calculation by Alberg et al. [14] at 1525 MeV/c, and the mixed-kaon-exchange calculation by Haidenbauer et al. [7] for 1546 MeV/c (curve approximated).

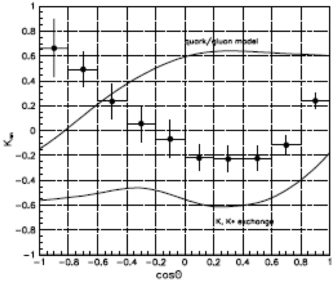


Fig. 5. Measured spin transfer K_{nn} at 1525 MeV/c (preliminary), with the combined-vector/scalar quark-gluon calculation by Alberg et al. [14] at 1525 MeV/c, and the mixed-kaon-exchange calculation by Haidenbauer et al. [7] for 1546 MeV/c (curve approximated).

Results at 1.525 GeV/c

Formalism of $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$

For $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$, there are six amplitudes,

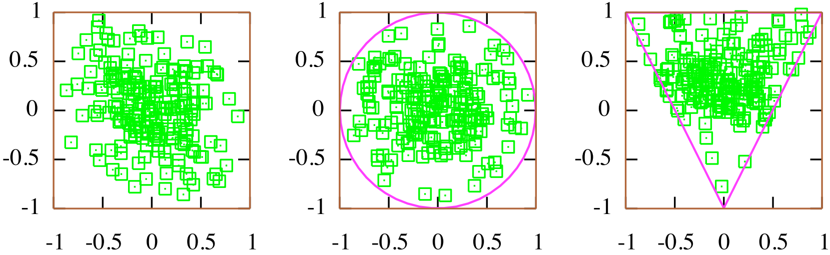
$$\begin{aligned}\mathcal{M} =&(a+b)I + (a-b)\,\boldsymbol{\sigma}_1.\boldsymbol{n}\,\boldsymbol{\sigma}_2.\boldsymbol{n} + (c+d)\,\boldsymbol{\sigma}_1.\boldsymbol{m}\,\boldsymbol{\sigma}_2.\boldsymbol{m}\\ &+ (c-d)\,\boldsymbol{\sigma}_1.\boldsymbol{l}\,\boldsymbol{\sigma}_2.\boldsymbol{l} + ie\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2).\boldsymbol{n} + g\,(\boldsymbol{\sigma}_1.\boldsymbol{l}\,\boldsymbol{\sigma}_2.\boldsymbol{m} + \boldsymbol{\sigma}_1.\boldsymbol{m}\,\boldsymbol{\sigma}_2.\boldsymbol{l})\ ,\end{aligned}$$

difficult to anticipate the relations among observables just by looking at their expression in terms of the amplitudes, which are

$$\begin{aligned}
I_0 &= |a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 + |g|^2\ , & P_n I_0 &= 2\,\Im\mathrm{m}(ae^* + dg^*)\ ,\\
C_{nn} I_0 &= |a|^2 - |b|^2 - |c|^2 + |d|^2 + |e|^2 + |g|^2\ , & A_n I_0 &= 2\,\Im\mathrm{m}(ae^* - dg^*)\ ,\\
D_{nn} I_0 &= |a|^2 + |b|^2 - |c|^2 - |d|^2 + |e|^2 - |g|^2\ , & C_{ml} I_0 &= 2\,\Re\mathrm{e}(ag^* - de^*)\ ,\\
K_{nn} I_0 &= |a|^2 - |b|^2 + |c|^2 - |d|^2 + |e|^2 - |g|^2\ , & D_{mm} I_0 &= 2\,\Re\mathrm{e}(ab^* + cd^*)\ ,\\
C_{mm} I_0 &= 2\,\Re\mathrm{e}(ad^* + bc^* - ge^*)\ , & D_{ml} I_0 &= 2\,\Re\mathrm{e}(cg^* + be^*)\ ,\\
C_{ll} I_0 &= 2\,\Re\mathrm{e}(-ad^* + bc^* + ge^*)\ , & K_{mm} I_0 &= 2\,\Re\mathrm{e}(ac^* + bd^*)\ ,\\
C_{nlm} I_0 &= 2\,\Im\mathrm{m}(ge^* - a^*d + b^*c)\ , & K_{ml} I_0 &= 2\,\Re\mathrm{e}(bg^* + ce^*)\ ,\\
C_{nml} I_0 &= 2\,\Im\mathrm{m}(ge^* - a^*d - b^*c)\ , & C_{nmm} I_0 &= 2\,\Im\mathrm{m}(de^* - ag^*)\ ,\\
C_{mnl} I_0 &= 2\,\Im\mathrm{m}(ab^* - cd^*)\ , & C_{mln} I_0 &= 2\,\Im\mathrm{m}(ac^* - bd^*)\ ,\\
C_{mnm} I_0 &= 2\,\Im\mathrm{m}(be^* + cg^*)\ , & C_{mmn} I_0 &= 2\,\Im\mathrm{m}(ce^* + bg^*)\ ,
\end{aligned}$$

$\bar{\mathrm{p}}\mathrm{p} \to \bar{\Lambda}\Lambda$: empirical method for pairs of spin obervables

The method adopted by Elchikh and R. consists of generating randomly 6 complex amplitudes, compute the observables, look at constraints and then derive the inequality that is observed from the explicit expression of the observables in terms of amplitudes.



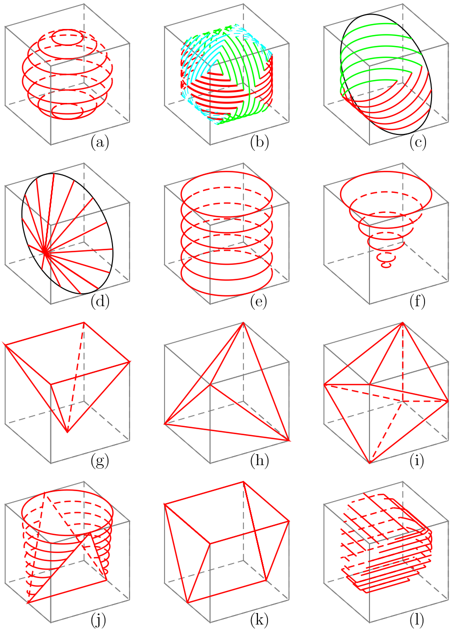
Random simulation of P_n vs. A_n (square),
A_n vs. D_{mm} (disk) and P_n vs. C_{nn} (triangle)

Case of three observables

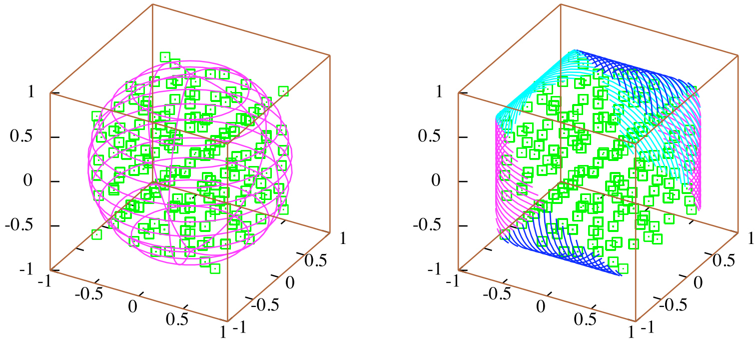
The method was extended by Artru, Elchikh and R., later helped by Soffer and Teryaev, to the case of three simultaneous observables.
Several shapes observed for the allowed domain: sphere, cone, pyramid, tetrahedron octahedron, intersection of two cylinders, intersection of three cylinders, some "coffee filter".
In particular:

- any triple is constrained,
- constraint even if all pairs are unconstrained,

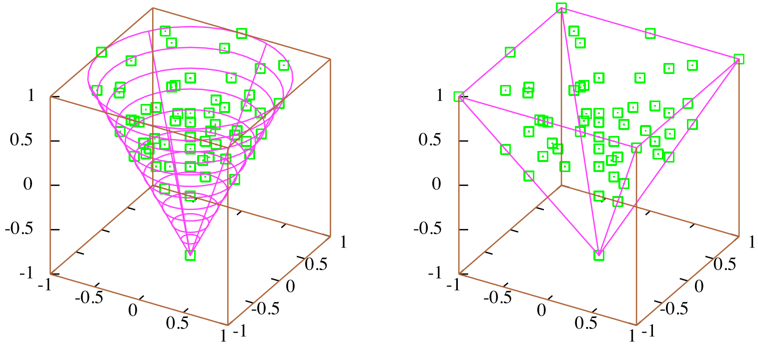
For instance the tetrahedrons (next slide) has $[-1,+1]^2$ projection on all faces.


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)

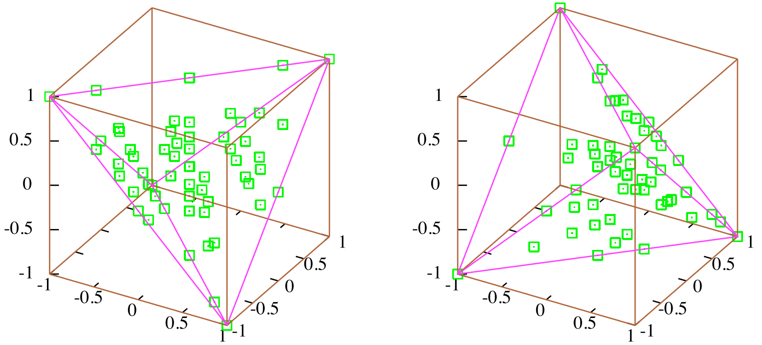
Some remarkable cases



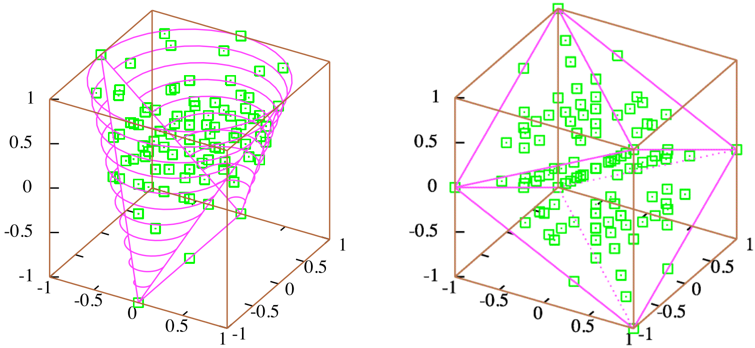
Fictitious observables $\{A_n, D_{mm}, K_{ml}\}$ obtained with amplitudes whose real and imaginary parts are chosen to be either 0 or ± 1, shown against the unit sphere (left) or the intersection of the three unit cylinders.



Domain for $\{P_n, C_{ml}, C_{nn}\}$ *(left) and* $\{P_n, A_n, C_{nn}\}$ *(right).*



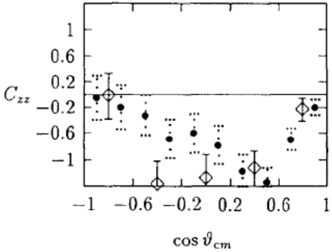
Domain for $\{P_n, A_n, D_{nn}\}$ (left) and $\{C_{nn}, C_{mm}, C_{ll}\}$ (right).



Observables $\{P_n, C_{mm}, C_{nn}\}$ *(left) and* $\{C_{mm}, C_{ll}, C_{nmm}\}$ *(right).*

Phenomenological applications

When PS185/3 (with polarised target) was proposed, much emphasis was put on D_{nn}.
But $C_{zz}^2 + D_{nn}^2 \leq 1$ and similar constraints, and



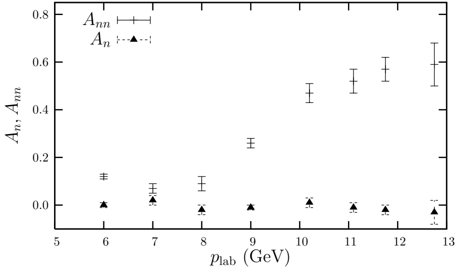
$|C_{zz}|$ already large from data without target polarisation.

Understanding the observed inequalities

- From the algebraic expressions of the observables in terms of a set of amplitudes
  - case by case (see the $\pi\mathrm{N}$ example),
  - Systematic (Fierz transformation, see Tabakin for $\gamma\mathrm{N} \to \mathrm{K}Y$
- Without requiring explicit amplitudes
  - Anticommutation: If $\{X, Y\} = 0$, then $X^2 + Y^2 \leq 1$ and similarly, if $\{X, Y\} = \{Y, Z\} = \{Z, X\} = 0$, then $X^2 + Y^2 + Z^2 \leq 1$ This implies that if $X^2 + Y^2 \leq 1$, and $Y^2 + Z^2 \leq 1$, and $Z^2 + X^2 \leq 1$, in most of the cases, this is due to anticommutation, and then $X^2 + Y^2 + Z^2 \leq 1$ is expected; But there are counterexamples. This means that some of the disk constraints are *not* due to anticommutation.
  - Positivity of the initial state density matrix and final-state density matrix,
  - Positivity of the density matrix describing the crossed reaction $\bar{\mathrm{p}} + \mathrm{p} + \Lambda + \bar{\Lambda} \to \emptyset$ necessary to write down all possible inequalities.

Nucleon-nucleon scattering

- Historical importance Prediction of vector mesons (Breit), determination of the NN potential
- Beautiful experiments at low energy with polarised beam and target
- Crucial to point out necessary refinements of 3-body forces
- High energy? Current wisdom, based on perturbative QCD: spin effects small at large s and $|t|$
- But dramatic effect found by Krisch et al.

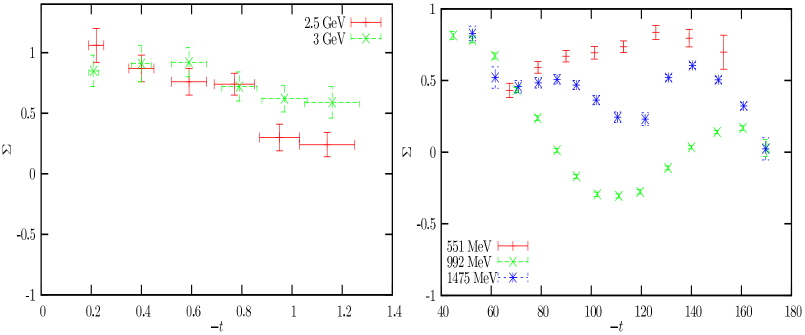


Nucleon-nucleon-2

- $$\alpha = M_{++,++}\ , \quad \beta = M_{+-,+-} = M_{-+,+-}\ , \quad \delta = M_{--,++}\ ,$$
- $I_0 = |\alpha|^2 + 2|\beta|^2 + |\delta|^2$
  $I_0 A_{nn} = 2\,\Re\mathrm{e}(\alpha^*\delta) + 2|\beta|^2$
  $I_0 A_{ll} = |\alpha|^2 + |\delta|^2 - 2|\beta|^2$
  $I_0 A_{mm} = -2\,\Re\mathrm{e}(\alpha^*\delta) + 2|\beta|^2$
- $$A_{nn}+A_{ll}+A_{mm} = 1\ , \quad A_{nn}+A_{ll} > 0\ , \quad A_{nn}+A_{mm} > 0\ , \quad A_{ll}+A_{mm} > 0\ .$$
- Unfortunately, $A_{nn} = 0.8$ leaves the interval $[-0.8, 1]$ available for both A_{ll} and A_{mm}.

Beam asymmetry for $\gamma\mathrm{p} \to \eta\mathrm{p}$ and $\gamma\mathrm{p} \to \pi^0\mathrm{p}$

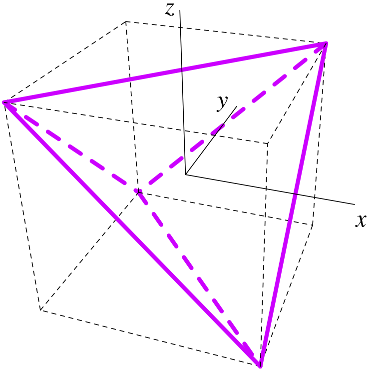
Bussey et al. (1976) for η, GRAAL (2005) for π^0



Spin observables in $\gamma\mathrm{N} \to \mathrm{K}\Lambda$

$\gamma + \mathrm{N} \to \mathrm{K} + Y$, and similar, have 4 amplitudes only (Chew et al., 1959).
All triples of observables are constrained in a domain smaller than the $[-1, +1]^3$ cube.

For instance, for
$A = T =$ target asymmetry
$P =$ polarisation of recoil baryon
$\Sigma =$ beam asymmetry

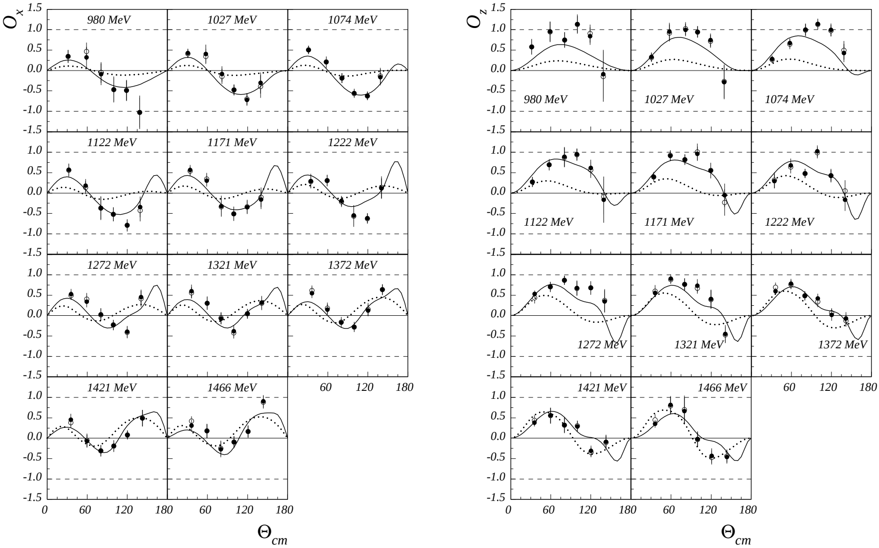


Tetrahedron domain limiting the observables $x = T$, $y = P$ and $z = \Sigma$.

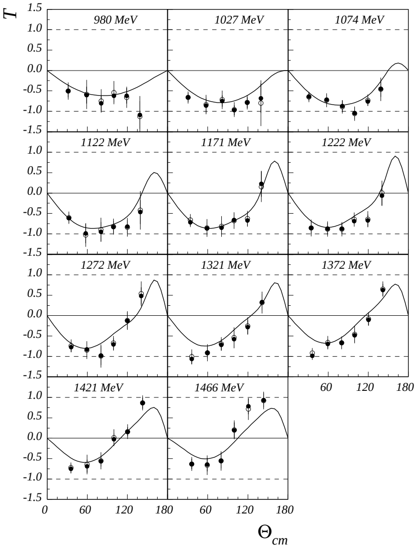
GRAAL-CLAS analysis

- The reaction $\gamma\mathrm{N} \to \mathrm{K}\Lambda$ recently measured by GRAAL (Grenoble) and CLAS (Jlab) at about the same energies, but with different spin observables.
- It is perhaps premature to attempt an unambiguous amplitude analysis combining both data sets, but our inequalities (Phys.Rev.C75:024002,2007, see also, Tabakin et al., Goldstein et al., etc.) can be used to check whether GRAAL and CLAS are compatible.

Beam recoil O_x and O_z



Target asymmetry T



Some inequalities for $\gamma\mathrm{N} \to \mathrm{K}\Lambda$

$A = T$ = target asymmetry
P = polarisation of recoil baryon
Σ = beam asymmetry
O_i = beam-recoil
C_i = target-recoil

$$C_x^2 + C_z^2 + O_x^2 + O_z^2 = 1 + T^2 - P^2 - \Sigma^2 \,,$$

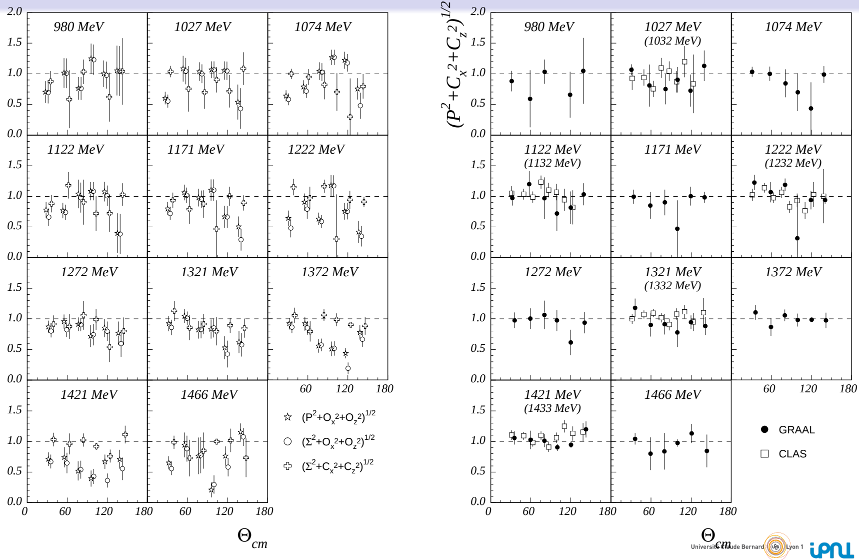
$$(P \text{ or } \Sigma)^2 + (O \text{ or } C)_x^2 + (O \text{ or } C)_z^2 \leq 1 \,,$$

$$|T \pm P| \leq 1 \mp \Sigma \,.$$

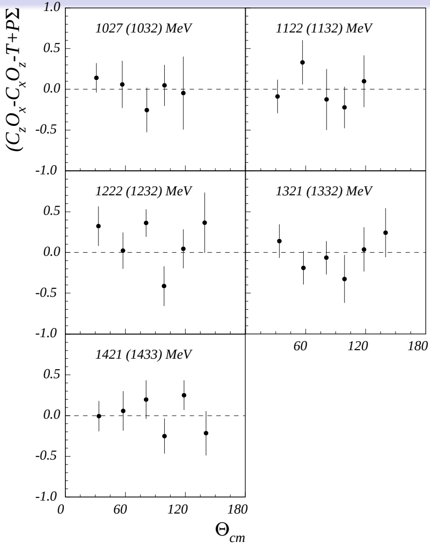
$$|O \text{ or } C|_{x,z}| \leq \min\{\sqrt{1-\Sigma^2}, \sqrt{1-P^2}\} \,,$$

etc.

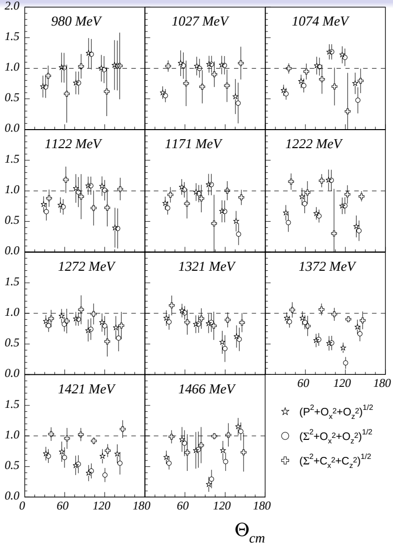
$(P^2 + O_x^2 + O_z^2)^{1/2} \leq 1$, $(\Sigma^2 + O_x^2 + O_z^2)^{1/2} \leq 1$, $(\Sigma^2 + C_x^2 + C_z^2)^{1/2} \leq 1$ and $(P^2 + C_x^2 + C_z^2)^{1/2} \leq 1$



Combined CLAS-GRAAL test of $C_z O_x - C_x O_z - T + P\Sigma = 0$

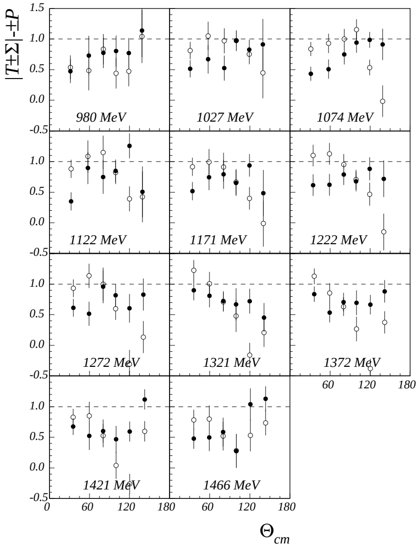


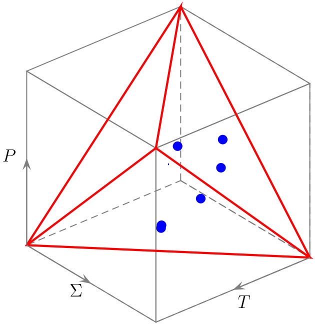
$(P^2 + O_x^2 + C_z^2)^{1/2} \leq 1$, $(\Sigma^2 + O_z^2 + C_x^2)^{1/2} \leq 1$, $(\Sigma^2 + O_x^2 + C_x^2)^{1/2} \leq 1$ and $(P^2 + O_z^2 + C_z^2)^{1/2} \leq 1$



From a combination of CLAS data (C_x, C_z) and GRAAL data (O_x, O_z)

Test of the tetrahedron inequality $|T \pm \Sigma| \mp P \leq 1$

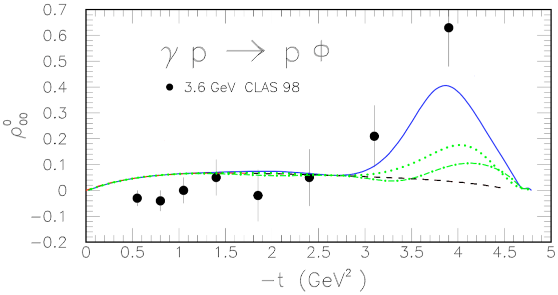




Case of 1222 MeV data.

Vector-meson photoproduction

$\gamma + \mathrm{N} \to \phi + \mathrm{N}$ and similar, with 12 amplitudes. Some triples of observables are unconstrained. Note that if the vector meson is identified through its decay into two pseudoscalars, such as $\rho \to \pi\pi$ or $\phi \to \mathrm{KK}$, only the *tensor* polarisation is accessed. To get the axial polarisation, one needs other decay modes.

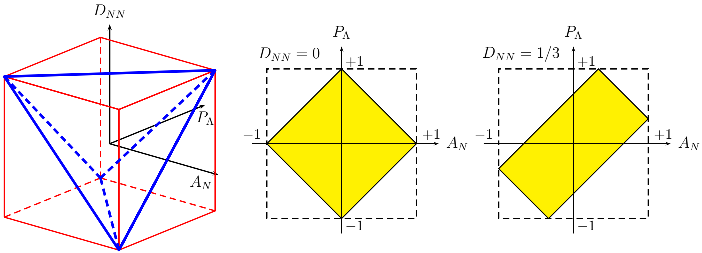


Inclusive hadronic reactions

For $a(\text{spin } 1/2) + b(\text{unpolarised}) \to c(\text{spin } 1/2) + X$, then

$$(1 \pm D_{NN})^2 \geq (A_{aN} \pm P_{cN})^2 + (D_{LL} \pm D_{SS})^2 + (D_{LS} \mp D_{SL})^2 \ .$$

in particular for $\mathrm{p}^{\uparrow}\mathrm{p} \to \Lambda^{\uparrow} X$, $1 \pm D_{NN} \geq |P_{\Lambda} \pm A_N|$,



The allowed domain corresponding to the constraints (*left*). The slice of the full domain for $D_{NN} = 0$ (*middle*) and for $D_{NN} = 1/3$ (*right*).

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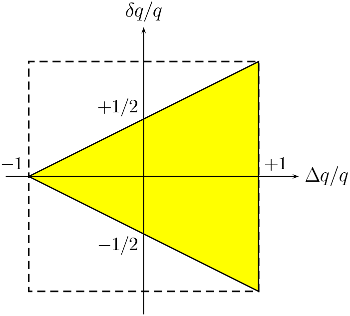
Quark distribution function, Soffer's inequality

Let $q(x)$ be a quark distribution function,
$q_\pm(x)$ the quark distributions of definite helicity,
with $q(x) = q_+(x) + q_-(x)$ and $\Delta q(x) = q_+(x) - q_-(x)$ the usual spin-dependent distribution.
The positivity of each $q_\pm$ implies $q(x) \geq \Delta q(x)$.
To construct the transversity distributions, one also needs the non-diagonal term in the helicity basis, δq. ($\delta q = q_\uparrow - q_\downarrow$ for a $N_\uparrow$)

The Soffer inequality $[q + \Delta q]/2 \geq \delta q$ can be viewed as in the figure, similar the the triangle inequality on some pairs of observables for $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$.



Outlook-1

- Rediscovery of the works by Michel, Minnaert, etc., and further development of limits on the domain allowed for spin observables (Artru, Elchikh, Soffer, etc.),
- Constraints useful when a few observables are measured
  - to see whether they are compatible
  - to determine which of the yet unknown observables has the widest range left
- Identities and inequalities on spin observables first derived by algebraic methods
- Better understood from the positivity of the density matrices describing the reaction and the crossed reactions
- Link with the theory of quantum information: a quantum state (initial spin) undergoes a quantum process (scattering), leading to a new state (final state). This is submitted to the usual restrictions, differences between pure and entangled spin states, the increase of entropy, etc.

Outlook-2

- Promising field of spin observables, particularly stimulated by recent experiments
  - $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$,
  - photoproduction,
  - parton distributions,
  - etc.
- Possibility of better polarised targets in the future
- Possibility of new polarised beams (POSIPOL workshops for positrons, some old and new ideas considered for antiprotons, etc.)
- Ongoing analysis of $\overrightarrow{N} + \overrightarrow{d}$