# Defect-Driven Structures for Self-Assembly 



MJB and L. Giomi, Adv. Phys. 58, 449-563 (2009) (arXiv:0812.3064)

## Introduction

Among soft and biological materials there are many compelling examples of objects with curvature and intrinsic order. These systems span a very broad range of length and energy scales and the physical mechanisms that lead to their assembly and mechanical stability can be very different.


Lengths (in meters)

| + | 1 | 1 |  | 1 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1.42 \times 10^{-10}$ | $5 \times 10^{-9}$ | $1.2 \times 10^{-8}$ | $10^{-7}$ | $10^{-6}$ | $10^{-3}$ |

Energy ranges between hundreds of $k_{B} T$ ( $\mathrm{sp}^{2}$ bonds in graphene) to $1 k_{B} T$ (Van der Waals interactions).

Suppose we want to design nano to meso scale building blocks (super-atoms) for creating super-molecules and subsequent 3D structures without the restrictions of quantum mechanics

Requirements:

## - Distinguished regions for the attachment of ligands - High degree of specificity

For small scale objects the surface to volume ratio is large so surface topological defects can provide the distinguished regions

The effective valence will then be determined by the number of defective regions and the type of directional bonding will be determined by the relative arrangement of the defects.

Lubensky \& Prost (1992)

The number of defective regions is an energetic question with underlying topological constraints

## Strategy:

- Take a set of microscopic objects interacting on some fixed surface
- Map to an effective interacting defect problem by treating everything but the defects as a continuum
- Find the ground state of the defect Hamiltonian


## Example: Divalent Metal Nanoparticles



SAM consisting of 2 types of thiol-terminated ligands (1nonanethiol and 4methybenzenethiol) on Au

DeVries et al (Stellacci):
Science (2007)

 Activation



Functionalization by MUA plus linking by DAH


Redo all of polymer physics with nanoparticles!

Membranes

J.F. Hainfeld and T.L. Steck, J. Supramol. Struct. 6, 301 (1977) (courtesy Leo Van Hemmen) 10,000X

## Membranes

Crystalline (elastic) membranes

$$
\mathrm{E}=\mathrm{E}_{e l}+E_{b e n d}
$$

2d Elasticity of Sheet + Shape Change from Height Fluctuations

$$
\mathrm{E}_{e l}=\frac{1}{2} \int d^{2} x\left[2 \mu u_{i j}^{2}+\lambda u_{k k}^{2}\right]
$$

where $u_{i j}=\frac{1}{2}\left(\partial_{i} u_{j}+\partial_{j} u_{i}+\partial_{i} h \partial_{j} h\right)$ (strain tensor)

$$
\mathrm{E}_{b e n d}=\frac{\kappa}{2} \int d^{2} x\left(\nabla^{2} h\right)^{2}
$$

$$
\begin{aligned}
& \quad \text { Energy minimized when } u_{i j} \approx 0 \\
& \Longrightarrow \partial_{j} u_{i}+\partial_{i} u_{j}=-\partial_{i} h \partial_{j} h
\end{aligned}
$$

## BUT

This is impossible for single-valued phonon fields $u_{i}\left(x_{1}, x_{2}\right)$
Defects, e.g; disclinations and dislocations, are an essential part of the ground state!

In fact,

$$
\frac{1}{2} \epsilon_{i m} \epsilon_{j n} \partial_{m} \partial_{n}\left(\partial_{j} u_{i}+\partial_{i} u_{j}\right)=s\left(x_{1}, x_{2}\right)=\operatorname{det}\left(\partial_{i} \partial_{j} h\right)=K\left(x_{1}, x_{2}\right)
$$

What are the defects?

1. Disclinations= Bond-Orientation Defects

$5=+1$


$$
7=-1
$$

$$
E \sim R^{2} \quad \text { (planar) }
$$

## 2. Dislocations (translational defects) $\equiv$ 5-7 dimers



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Integrating out the phonons one finds

$$
E=\frac{Y}{2} \iint d^{2} x d^{2} y\left[K(x) \frac{1}{\Delta^{2}(x, y)} K(y)\right]+\kappa E_{\text {bending }}
$$

$K(x)=$ Gaussian curvature $\quad Y=\frac{4 \mu(\mu+\lambda)}{2 \mu+\lambda} \quad$ (2D Young's Modulus)

$$
\begin{gathered}
E=\frac{y}{2} \iint d^{2} x d^{2} y[K(x)-s(x)] \frac{1}{\Delta^{2}(x, y)}[K(y)-s(y)]+N E_{c}+\kappa E_{b} \\
s=\frac{\pi}{3} \sum_{i=1}^{N} q_{i} \delta\left(x, x_{i}\right) \\
G(x, y)=\frac{1}{\Delta^{2}(x, y)} \sim|x-y|^{2} \ln |x-y|
\end{gathered}
$$

MJB, D.R. Nelson and A. Travesset (ISU) : PRB 62 (2000) 8738
MJB, A. Cacciuto (Columbia), D.R. Nelson and A. Travesset, PRB 73 (2006) 024115

From planar packing to spherical packing: topological constraints

$$
V-E+F=\chi=2
$$

For triangulations $(2 E=3 F)$ this implies


## $\sum_{i}\left(1-\frac{c_{i}}{6}\right)=2$

$c_{i}=$ degree of vertex $i$
$q_{i}=6-c_{i}=$ disclination charge

$$
Q_{v o t}=\sum q_{i}=12
$$

$i$

## More specimens from sphereland



SV40 Viral Capsid


Bilayer vesicle of Ph5C60K molecules: Zhou et al


Adenovirus


MP AuNP: DeVries et al (Stellacci)


C60


Irvine \& Chaikin

## Shape of defective regions

> Map particle interaction to (universal) defect Hamiltonian in a continuum elastic background with 2 parameters: bulk modulus $Y$ and core energy $E_{c}$

Constant positive Gaussian curvature $\quad K(x)=1 / R^{2}$

Disclination elastic energies grow like $R^{2}$
For small R/a (<5) the disclinations are
localized - isolated point-like disclinations as in 2d melting from the hexatic to the fluid

For large R/a the disclinations are delocalized - leads to formation of freely terminating linear defect arrays

## SCARS



## Large Scale Simulations



$$
\mathrm{N}=752(\mathrm{~V}=1 / \mathrm{r}) \quad \text { D.J. Wales, H. Mackay and E.L. Altshuler, PRB (2009) }
$$

http://www-wales.ch.cam.ac.uk/~wales/CCD/Thomson2/table.html
Thomson Applet: http://thomson.phy.syr.edu/thomsonapplet.htm MJB, C. Cecka (Harvard) and A.A. Middleton (Syracuse)

# PMMA particles on curved surfaces, imaging and reconstruction (Charge stabilized: Chaikin \& Irvine: NYU) 

Oil
(Cyclohexyl bromide)

PMMA particles

Water droplet
Glass cover slip




[^0]
## Changing the Order

For $p$-fold order the total number of distinct defective regions is $2 p$

| $p$ | Local order | Valence |
| :--- | :--- | :--- |
| 6 | Crystalline | 12 |
| 2 | Nematic | 4 |
| 1 | Vector | 2 |
| 4 | Tetradic | 8 |


$\mathrm{p}=2$

## Spherical nematics ( $\mathrm{p}=2$ ) give rise to 4

 elementary $+1 / 2$ defects.

Lubensky \& Prost (1992)
D.R. Nelson, Nano Lett. 2 (2002) 1125

In the isotropic (one Frank constant) approximation the $+1 / 2$ defects would sit at the vertices of a tetrahedron

H. Shin, MJB and X. Xing: Phys. Rev. Lett. 101 (2008) 037802

Hard and soft rod MC fluid simulation produces jammed splay-dominated nematic state with four defects lying on a great circle!


$$
F=\frac{K_{1}}{2} \int(D \cdot n)^{2}+\frac{K_{3}}{2} \int(D \times n)^{2}
$$

For hard rods splay is preferred over bend
Take the extreme limit of pure splay deformations

$$
D \times n=0 \Longrightarrow(\vec{n} \cdot \vec{D}) \vec{n}=0
$$

Director is a completely determined integral curve (follows geodesics)
+1 disclinations are degenerate with $2+1 / 2 s$

Why are all defects on a great circle?
There is a special bending-free ground state


Cut and rotate by an arbitrary angle
Director field is continuous after surgery, except at defect cores
All four $1 / 2$ defects form a rectangle of arbitrary aspect ratio One parameter family of degenerate ground states

## Making nematic shells: double emulsions

Fernandez-Nieves et al; PRL (2007); Vitelli and Nelson, PRE (2006)


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## Toroidal Crystals

Toroidal crystals are two-dimensional crystalline assemblages of monodisperse objects on a torus. Example: toroidal micelles from self-assembly of amphiphilic dumbbell molecules (Kim et al 2006)


Amphiphiles form double-dumbbells of 16 nm in length and self-assemble in spherical and uncapped cylindrical micelles. These turn into toroidal micelles in the course of a week. Diameters: $D_{1}=70-300 \mathrm{~nm}, D_{2}=16 \mathrm{~nm}$.


## Changing the Manifold: $T^{2}$

Although defects not required topologically ( $\chi=0$ ) they appear as minimizers


## Toroidal Scars



E. Pairam and A. Fernandez-Nieves PRL 102, 234501 (2009)

# Crystalline Order on CMC Delaunay surfaces 



Irvine,Vitelli and Chaikin, Nature (2010) MJB, Z. Yao, EPL (201I)

## Catenoids

Glass cover slip

## PMMA particles

## Water

Glass cover slip




## Euler number 0



Negative and varying gaussian curvature!




## Self-interstitials (MJB, Irivine \& Chaikin)



## Curvature driven fractionalization

- MJB, H. Shin and A. Travesset, PRE 75 (2007) [cond-mat/0610819]
- MJB, D. R. Nelson and H. Shin, Phys. Chem. Chem. Phys. (2008) [arxiv:0707.1909]


## http://thomson.phy.syr.edu/thomsonapplet.htm



Glass slide


# Scan objective up and down 

 for confocal operation

Glass slide



Glass slide


## Manipulate independently



## Scan objective up and down for confocal operation

## 3D Imaging

## And manipulation



$\cdots \cdots, 8 \%$

## Where did the particle go?



## Interstitial fractionalization on a catenoid

$\downarrow$

## Interstitial fractionalization on a catenoid



## Interstitial fractionalization on a catenoid



## Conclusions

- Topology determines broad structure of ground state
- Energetics determines the detailed structure = shape of defective regions


## Curvature-driven effects

- Disclination delocalization
- Novel structures (disclinations) in ground state

Defects allow design of superatoms
\# Defective regions = valence
Global Geometry of Defective regions determines type of directional bonding available (controllable via elastic moduli)

## Structure controls shape in some cases: faceted liquid crystalline vesicles from block copolymers


[^0]:    Thursday, February 2, 12

