Solvable model for a dynamical quantum phase transition from fast to slow scrambling

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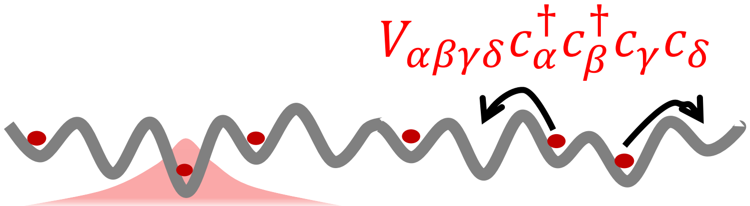
Work done with Ehud Altman (UC Berkeley)

SB & E. Altman, arXiv:1610.04619

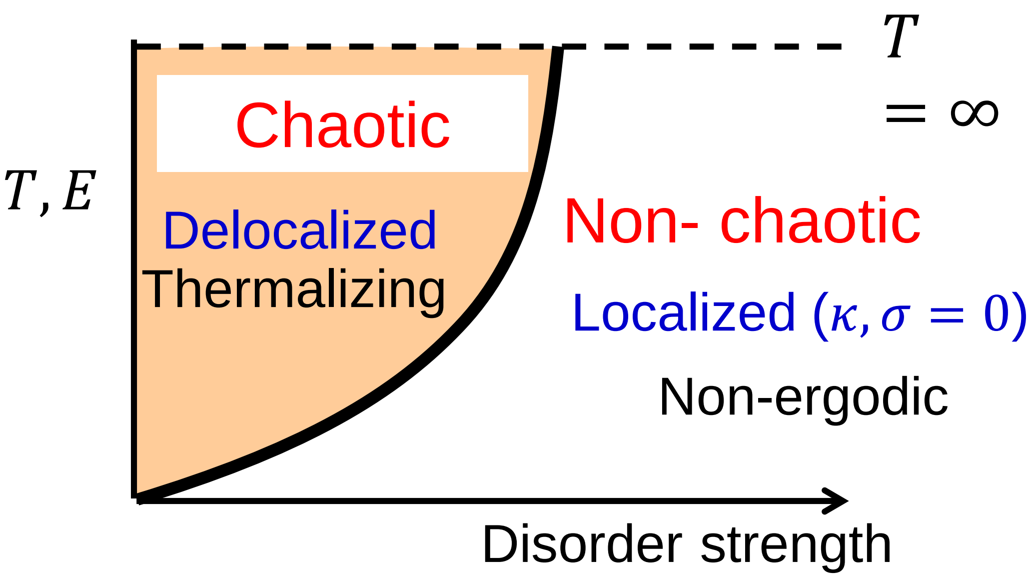
Chaotic to non-chaotic transition in many-body systems



Many-body localization (MBL) in interacting disorder system (isolated)

→ Failure to thermalize

→ Non-chaotic



Basko, Aleiner, Altshuler (2005);
Gornyi, Mirlin, Polyakov (2005)

Oganesyan and Huse (2007),
Pal and Huse (2010)
……….

→ Large-N Solvable model for MBL transition???

<u>This talk:</u> Only transition fast to slow thermalization

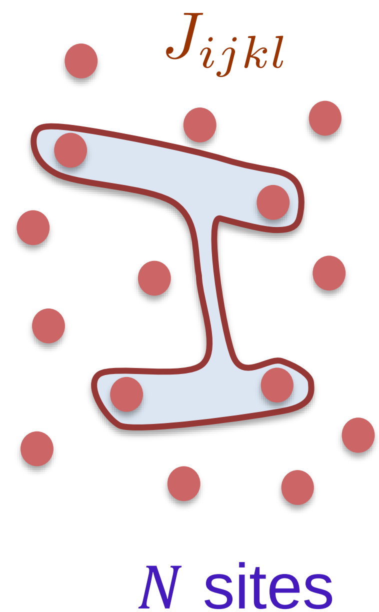
Large-N model for thermalization?

Sachdev-Ye-Kitaev (SYK) model

Sachdev & Ye, PRL (1993)
Kitaev, KITP (2015)
Sachdev, PRX (2015)

$$H_{SYK} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l - \mu \sum_{i}^{N} c_i^\dagger c_i$$

$$P(J_{ijkl}) \sim e^{-\frac{|J_{ijkl}|^2}{J^2}}$$



- Solvable in strong coupling for large N

- Emergent conformal symmetry at low energy.

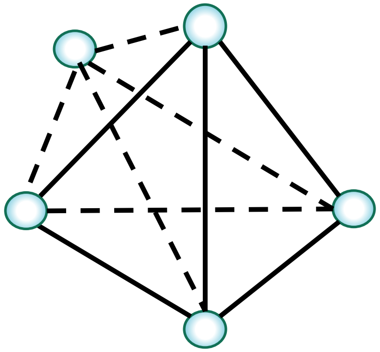
- 'Maximally chaotic'
Quantum chaos or 'scrambling' with Lyapunov exponent, $\lambda_L = 2\pi T$

'Upper bound' to quantum chaos as in a black hole
Maldacena, Shenker & Stanford (2016)

Kitaev → Solvable model for holography

Contrast with quadratic infinite range model (model for quantum dot)

$$H = \frac{1}{\sqrt{N}} \sum_{ij} t_{ij} c_i^\dagger c_j$$



$$P(t_{ij}) \sim e^{-\frac{|t_{ij}|^2}{t^2}}$$

- Fermions occupying states of a $N \times N$ random matrix.
- No thermalization or chaos in the many-body sense.

Add weak interaction →

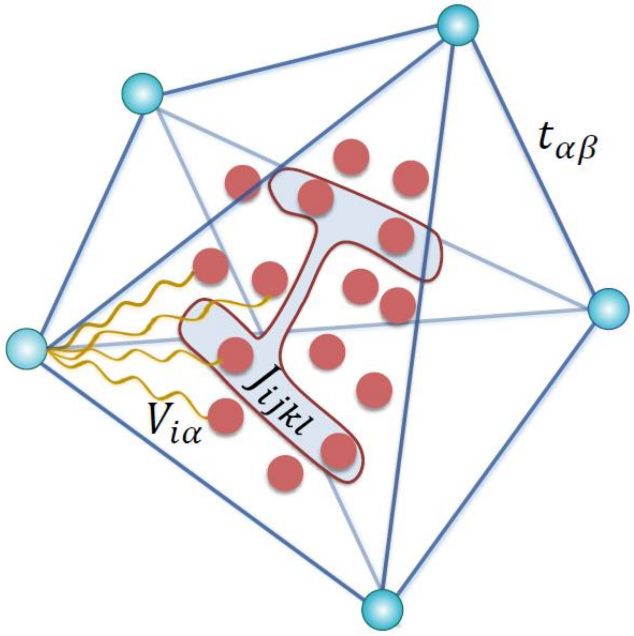
- Fermi liquid state at infinite N.

Quasi-particle lifetime → $\tau \sim 1/T^2$

Expectation, Lyapunov exponent $\lambda_L \sim \frac{1}{\tau} \sim T^2$

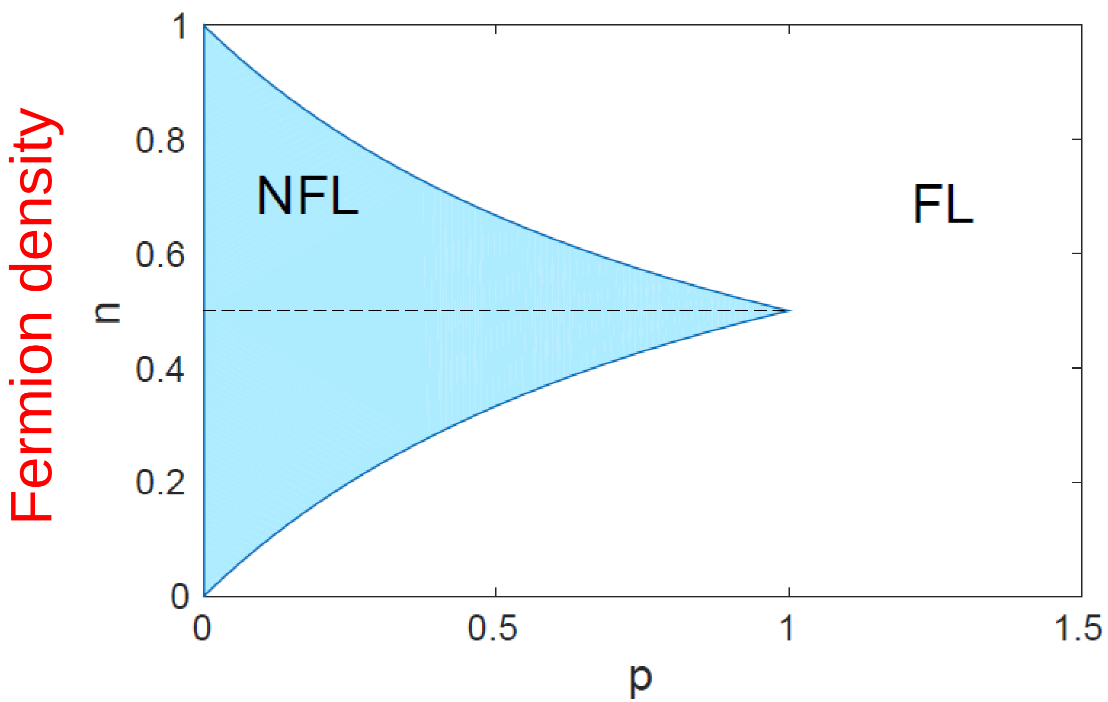
This talk: Solvable model with a quantum critical point two distinct quantum chaotic fixed points, SYK and Fermi-liquid.

Classifying phases and phase transitions in terms of quantum chaos?



Two-species fermion model

How spectrum and quantum chaos evolve?

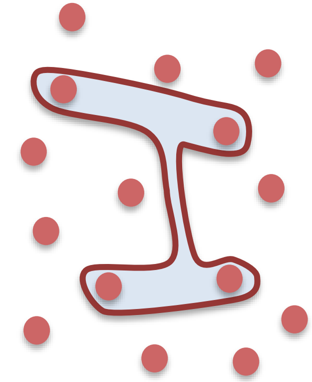


Ratio of number of sites of two species

Review of SYK model

$\langle J_{ijkl}^2 \rangle = J^2$

Sachdev, PRX (2015)



N sites

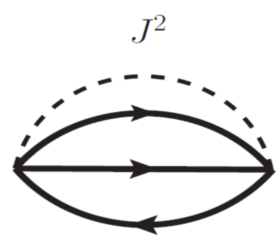
- Complex fermions

$$\mathcal{H}_{SYK} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l - \mu \sum_i c_i^\dagger c_i$$

Half filling, $\mu = 0$

- Disordered averaged saddle point for $N \to \infty$

$$G^{-1}(\omega) = \cancel{\omega} + \mu - \Sigma(\omega)$$

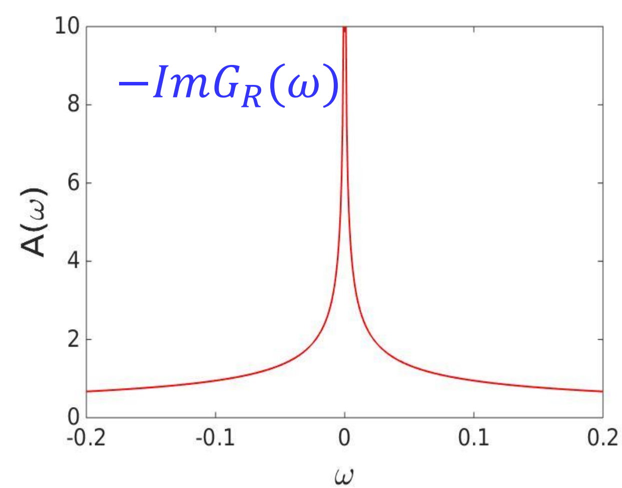


$$\Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

→ Conformal symmetry

- Non Fermi liquid fixed point

$$\Sigma_R(\omega) \sim -\Lambda^3 e^{\frac{i\pi}{4}} \sqrt{J\omega} \qquad G_R(\omega) = \Lambda e^{-\frac{i\pi}{4}} / \sqrt{\omega}$$



- Extensive zero-temperature entropy

Quantum chaos in SYK model

Kitaev, KITP (2015)
Polchinski & Rosenhaus (2016)
Maldacena & Stanford (2016)

Out-of-time-order correlation (OTOC)

$$\langle\, c_i^\dagger(t) c_j(0) c_i^\dagger(t) c_j(0) \rangle \sim 1 - \left(\frac{\beta J}{N}\right) e^{\lambda_L t}$$

$$\lambda_L = 2\pi T$$

Fastest scrambler! Maximally chaotic.

Upper bound to quantum chaos

Maldacena, Shenker & Stanford (2015)

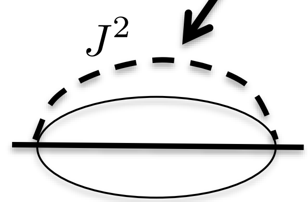
How to drive a phase transition out of this maximally chaotic non-Fermi liquid fixed point?

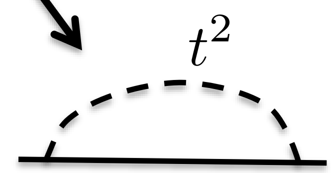
Naive attempt: add a quadratic term

$$\mathcal{H}_{SYK} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{\sqrt{N}} \sum_{ij} t_{ij} c_i^\dagger c_j$$

$$G^{-1}(\omega) = \omega - \Sigma_J(\omega) - t^2 G(\omega)$$

$$\Sigma_J(\tau) = -J^2 G^2(\tau) G(-\tau) =$$





The ansatz $G(\omega) \sim 1/\sqrt{\omega}$ is not self-consistent in the limit $\omega \to 0$

$$G^{-1}(\omega) \sim \omega - \sqrt{J\,\omega} - \frac{t^2}{\sqrt{\omega}}$$

The free fermion ansatz is self consistent: $G(\omega) \sim -i/t$

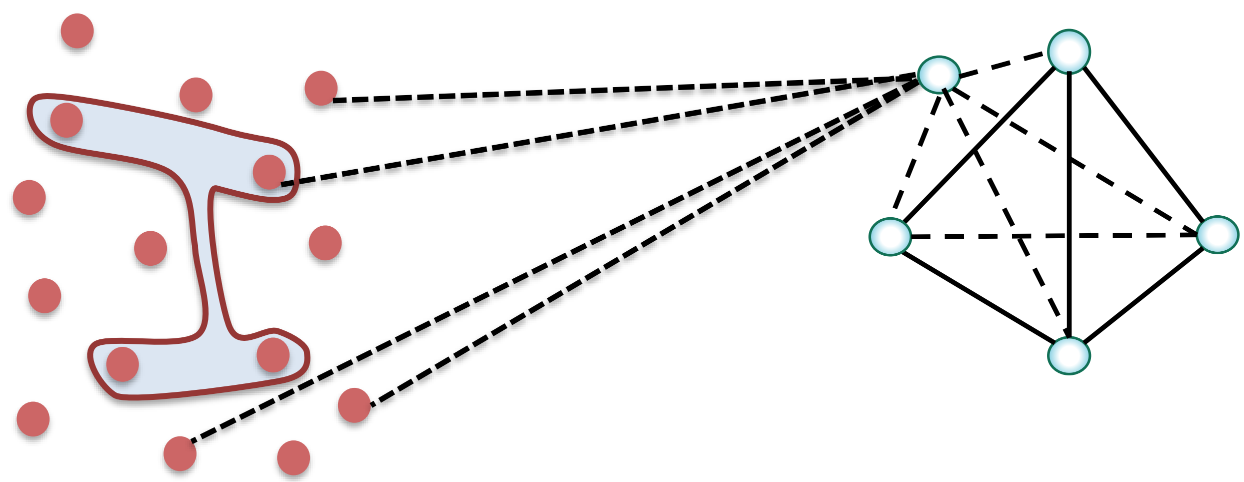
Quadratic interaction is relevant. Always a Fermi liquid. No transition!

Consider a model with two types of sites

N sites:
SYK coupling

M sites:
Random hopping

$$\overline{J_{ijkl}^2} = J^2 \qquad \overline{V_{i\alpha}^2} = V^2 \qquad \overline{t_{\alpha\beta}^2} = t^2$$



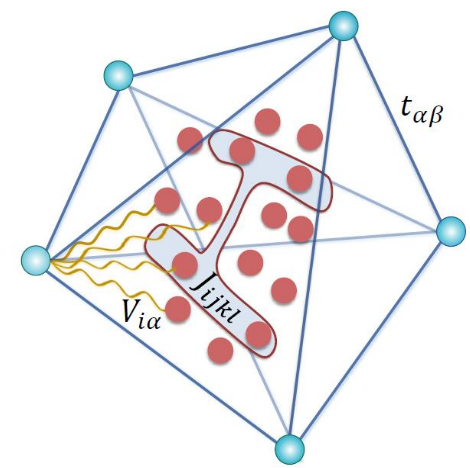
$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{\sqrt{M}} \sum_{\alpha\beta} t_{\alpha\beta} \psi_\alpha^\dagger \psi_\beta - \mu\Big(\sum_i c_i^\dagger c_i + \sum_\alpha \psi_\alpha^\dagger \psi_\alpha\Big)$$

$$+ \frac{1}{(MN)^{1/4}} \sum_{i\alpha} (V_{i\alpha} c_i^\dagger \psi_\alpha + h.c.)$$

Saddle point equations at large N

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{\sqrt{N}} \sum_{\alpha\beta} t_{\alpha\beta} \psi_\alpha^\dagger \psi_\beta + \frac{1}{(MN)^{1/4}} \sum_{i\alpha} (V_{i\alpha} c_i^\dagger \psi_\alpha + h.c.)$$

$$-\mu\left(\sum_i c_i^\dagger c_i + \sum_\alpha \psi_\alpha^\dagger \psi_\alpha\right)$$

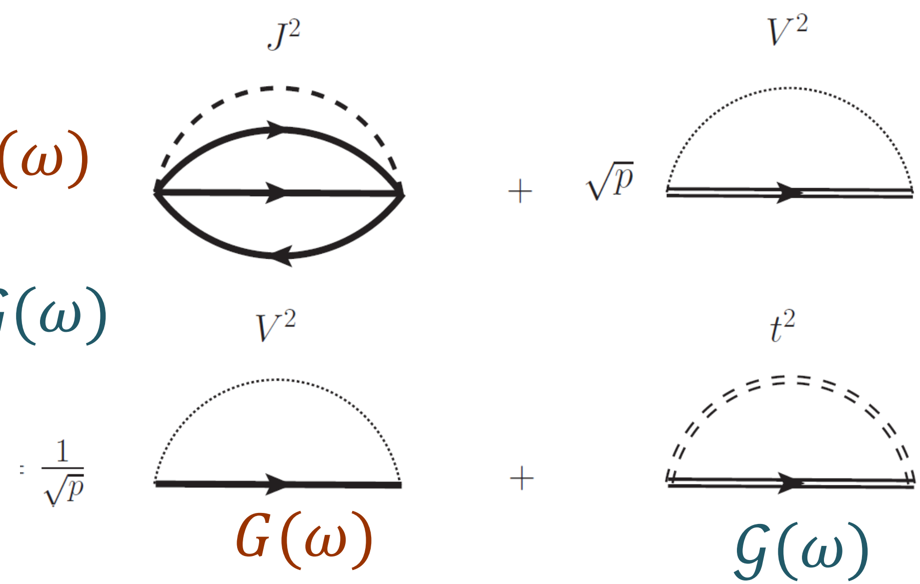


$$p = M/N \qquad N, M \to \infty$$

$$\mathcal{G}^{-1}(\omega) = \omega + \mu - t^2 \mathcal{G}(\omega) - \frac{V^2}{\sqrt{p}}\, G(\omega)$$

$$G^{-1}(\omega) = \omega + \mu - \Sigma_{\mathrm{J}}(\omega) - V^2 \sqrt{p}\, \mathcal{G}(\omega)$$

$$\Sigma_{\mathrm{J}}(\tau) = -J^2 G^2(\tau) G(-\tau)$$

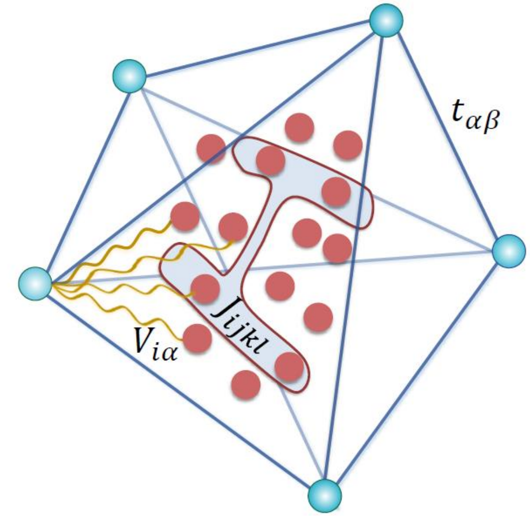


Non Fermi liquid fixed point

$$G^{-1}(\omega) = \cancel{\omega + \mu} - \Sigma_{\mathrm{J}}(\omega) - V^2\sqrt{p}\,\mathcal{G}(\omega)$$

$$\mathcal{G}^{-1}(\omega) = \cancel{\omega + \mu - t^2\mathcal{G}(\omega)} - \frac{V^2}{\sqrt{p}}\,G(\omega)$$

$$\Sigma_{\mathrm{J}}(\tau) = -J^2 G^2(\tau) G(-\tau)$$



→ Emergent Conformal symmetry

Scaling dimension

$$\tau = f(\sigma)$$

$$\tilde{G}(\sigma_1, \sigma_2) = [f'(\sigma_1) f'(\sigma_2)]^{\Delta_c}\, G(f(\sigma_1), f(\sigma_2))$$

$$\Delta_c = \frac{1}{4}$$

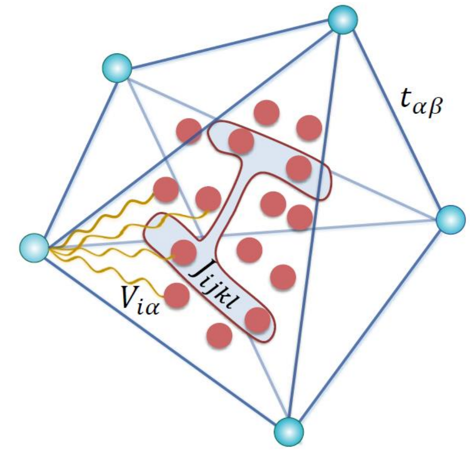
$$\tilde{\mathcal{G}}(\sigma_1, \sigma_2) = [f'(\sigma_1) f'(\sigma_2)]^{\Delta_\psi}\, \mathcal{G}\big(f(\sigma_1), f(\sigma_2)\big)$$

$$\sim \tilde{\Sigma}(\sigma_1, \sigma_2)$$

$$\Delta_\psi = \frac{3}{4}$$

→ Power-law solution

Half filling: Non-Fermi liquid



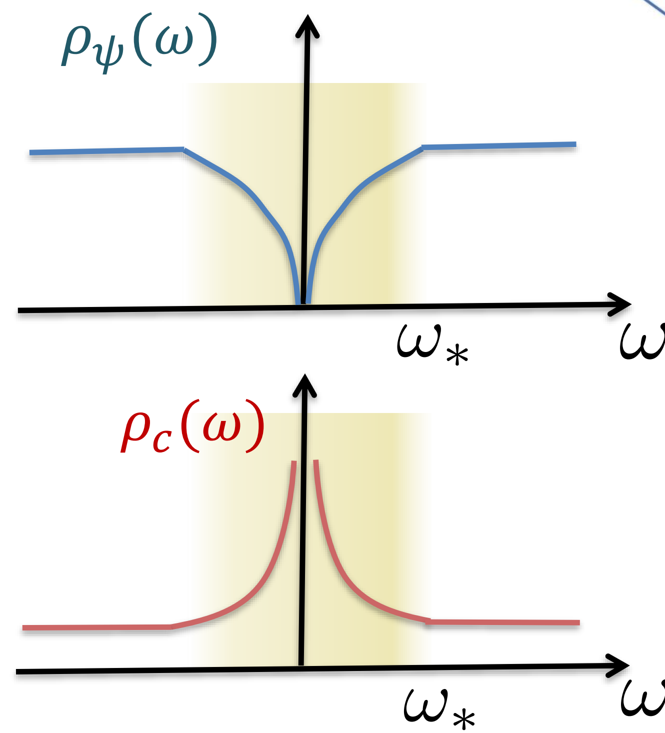
$\mu = 0$

→ Solution at $T = 0$, for $p = \frac{M}{N} < 1$

$$G_R(\omega) \sim \frac{(1-p)^{\frac{1}{4}}}{\sqrt{J\omega}} e^{-i\pi/4}$$

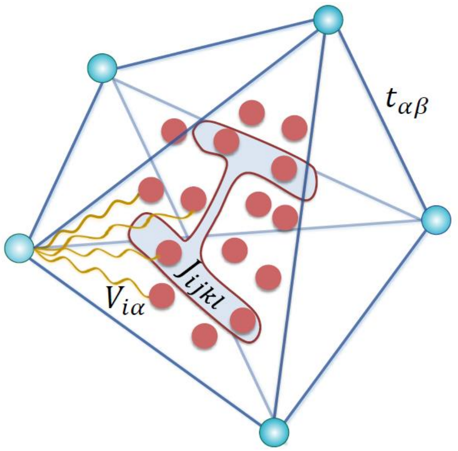
$$\mathcal{G}_R(\omega) \sim -\frac{\sqrt{p}}{(1-p)^{\frac{1}{4}}} \frac{\sqrt{J\omega}}{V^2} e^{i\pi/4}$$

$$\omega_* = \frac{V^4(1-p)^{1/2}}{t^2 J p}$$



Weight and bandwidth of the singularity in $\mathcal{G}_R(\omega)$ vanishes continuously as $p \to p_c = 1$

Half filling: Fermi-liquid



- Solution for $p = M/N > 1$

$$G_R(\omega) \simeq -i\frac{1}{\sqrt{p(p-1)}}\frac{t}{V^2}$$

$$\mathcal{G}_R(\omega) \simeq -i\sqrt{\frac{p-1}{p}}\frac{1}{t}$$

→ Constant DOS for $\omega < \omega_* \approx \left(\frac{V^2}{t}\right)\sqrt{p(p-1)}$

Self-energy, $\mathrm{Im}\Sigma_{\mathrm{J}}(\omega) \sim -\left(\frac{J^2 t^3}{V^6}\right)\frac{1}{(p(p-1))^{3/2}}\;\omega^2$

- Free fermion fixed point, emergent conformal symmetry

$$\tilde{G}(\sigma_1, \sigma_2) = [f'(\sigma_1)f'(\sigma_2)]^{1/2}\, G\big(f(\sigma_1), f(\sigma_2)\big) \sim \tilde{\mathcal{G}}(\sigma_1, \sigma_2)$$

→Critical point at $p = M/N = 1$ separates NFL and FL fixed points

Quantum critical point (QCP) between NFL and FL

Half filling

$$\omega^*_{NFL} \sim (1-p)^{\frac{1}{2}} \qquad \omega^*_{FL} \sim (p-1)^{\frac{1}{2}}$$

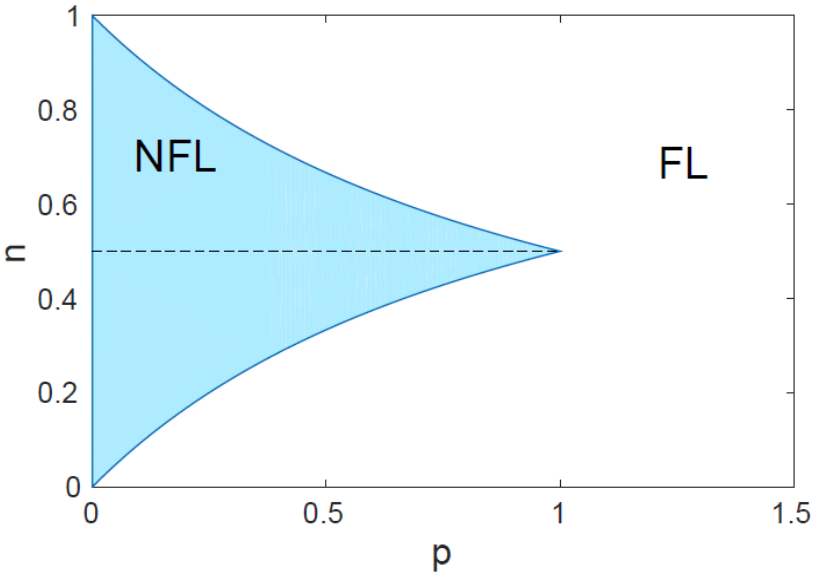
$$G_R(\omega) \sim (1-p)^{\frac{1}{4}}/\sqrt{\omega}$$

$$\mathcal{G}_R(\omega) \sim \sqrt{\omega}\,/(1-p)^{\frac{1}{4}}$$

$$G_R(\omega) \sim 1/(p-1)^{\frac{1}{2}}$$

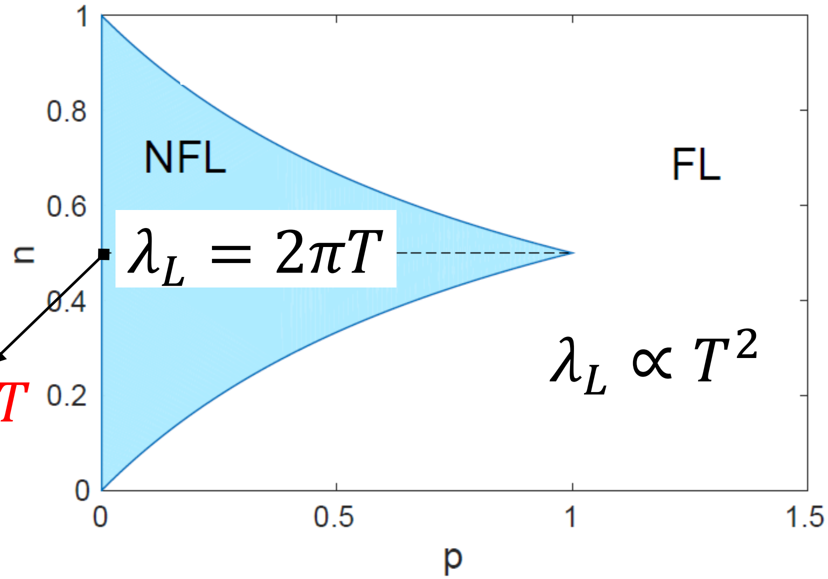
$$\mathcal{G}_R(\omega) \sim (p-1)^{\frac{1}{2}}$$

$p = 1$ $\qquad$ p



NFL-FL phase diagram

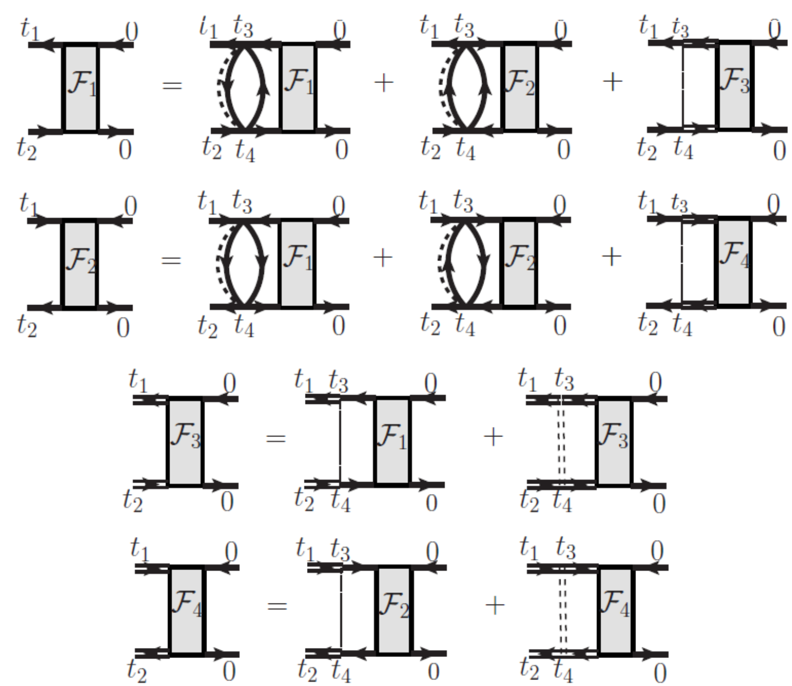
Quantum chaos across QCP?



$\lambda_L = 2\pi T$

Out-of-time-order correlation (OTOC)

$$\langle c_i^\dagger(t) c_j(0) c_i^\dagger(t) c_j(0) \rangle \sim 1 - \left(\frac{\beta J}{N}\right) e^{\lambda_L t}$$

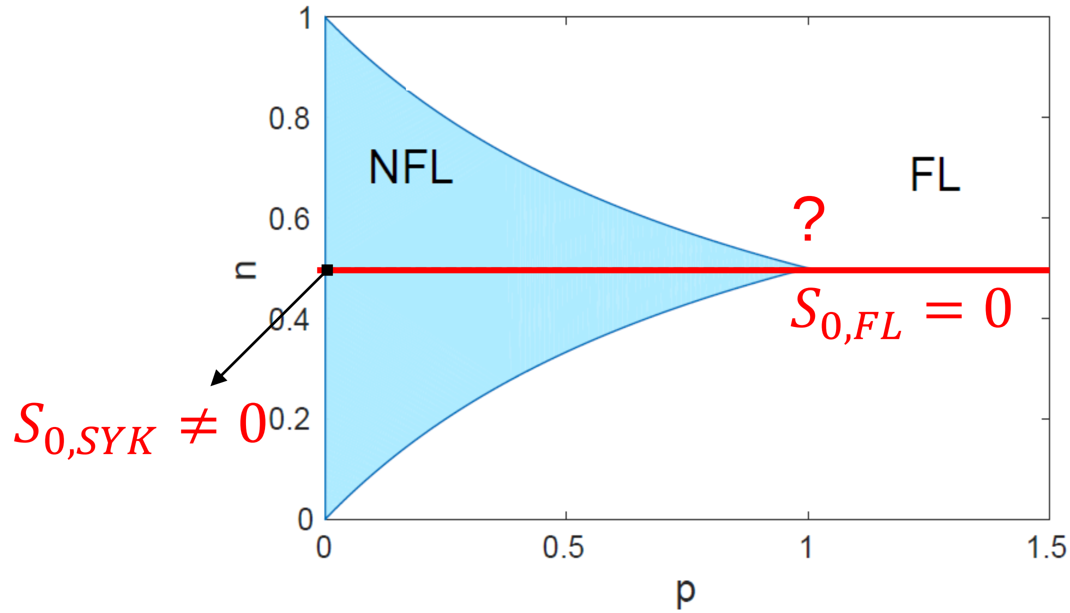


- Entire NFL phase is maximally chaotic

→ Dual to a black hole

- Transition from fast to slow scrambling

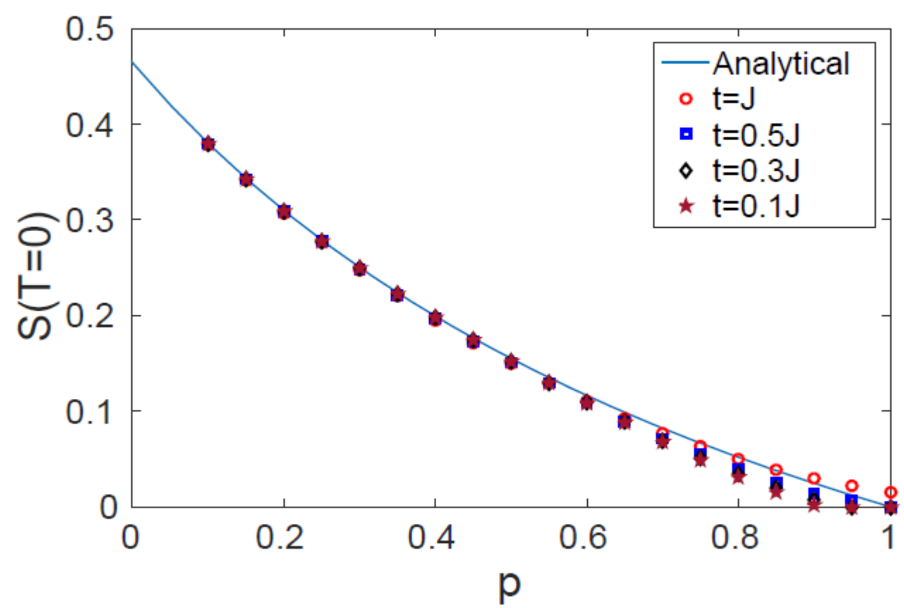
Zero-temperature entropy across QCP?



Luttinger theorem for NFL
+Maxwell's relation →

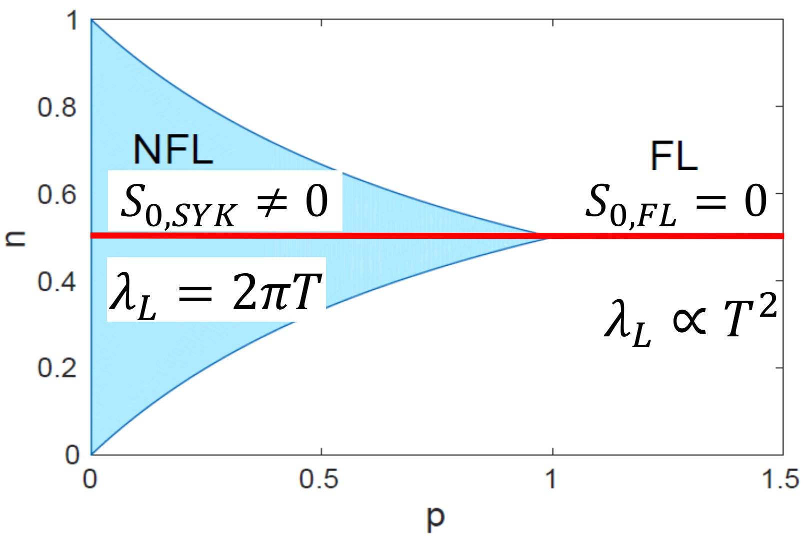
$$S_0(n = 1/2) = \frac{1-p}{1+p} S_{0,SYK}$$

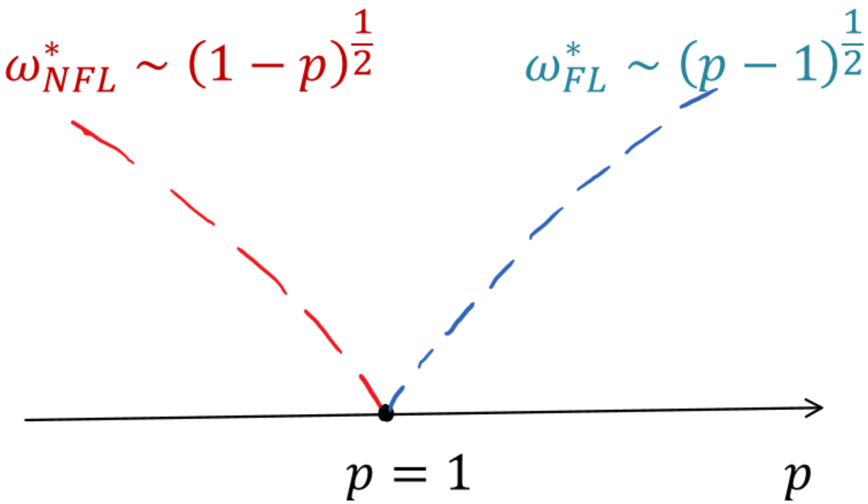
→ Change of geometry in dual gravity across QCP?



Conclusions and outlook

Exactly Solvable model for a non-Fermi liquid to Fermi liquid transition and fast to slow scrambling.





- Theory for the critical point? New chaotic fixed point?
- Extension to large-*N* description for MBL and MBL transition?
- Holographic interpretation? Phase transition involving elimination of black hole?

→Quench across QCP → Black hole evaporation.

Thank you!

Quantum chaos

- Classical chaos

$$\frac{\partial x(t)}{\partial x(0)} \sim e^{\lambda_L t}$$

λ_L, Lyapunov exponent

- Quantum chaos

→ 'Semiclassical billiards'

Larkin & Ovchinikov (1969)

- Chaos correlator $C(t) = -\langle [x(t), p(0)]^2 \rangle$

$$[x(t), p(0)] \quad \Rightarrow \quad i\hbar\{x(t), p\}$$

←Poisson bracket

$$= i\hbar \frac{\partial x(t)}{\partial x(0)}$$

$$C(t) \sim \hbar^2 \, e^{2\lambda_L t}$$

'Srambling time' $t_* \sim \frac{1}{\lambda_L} \ln\left(\frac{1}{\hbar}\right)$ $C(t_*) \sim 1$

Generalize to quantum chaotic (interacting) many-body systems

$$C(t) = -\langle [A(t), B(0)]^2 \rangle \qquad\qquad [A, B] = 0$$

$$A(t) = e^{i\mathcal{H}t} A e^{-i\mathcal{H}t} = A + it[\mathcal{H}, A] - \frac{t^2}{2!}[\mathcal{H}, [\mathcal{H}, A]] - \frac{it^3}{3!}\Big[\mathcal{H}, \big[\mathcal{H}, [\mathcal{H}, A]\big]\Big] + \cdots$$

Chaotic evolution →
Local operator will grow in size encompassing the whole system

$$C(t) \sim \epsilon\, e^{\lambda_L t} \qquad\qquad \epsilon \to \hbar^2$$

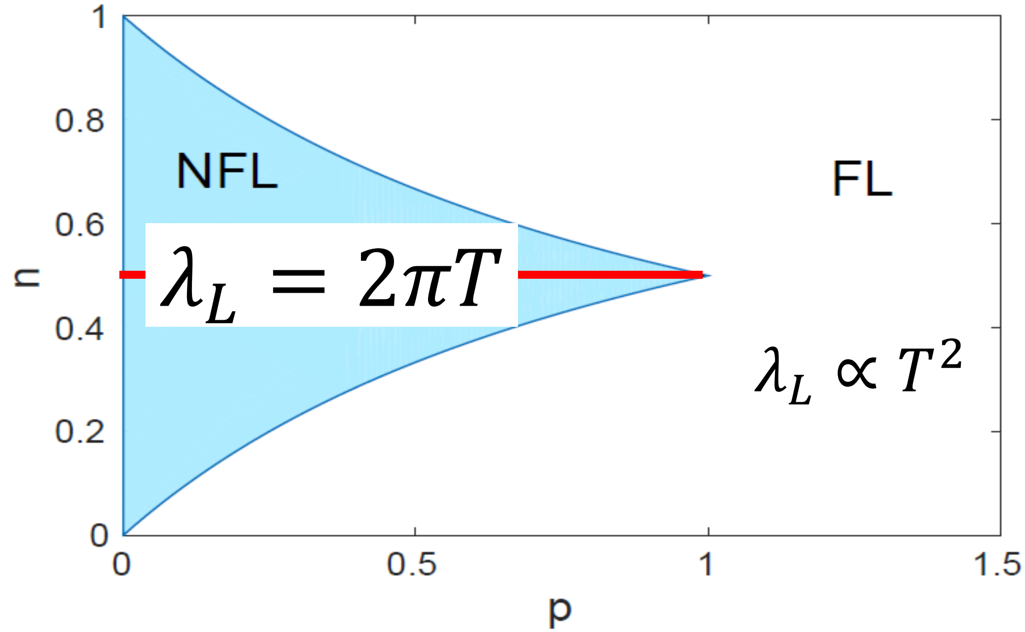
→Information scrambling

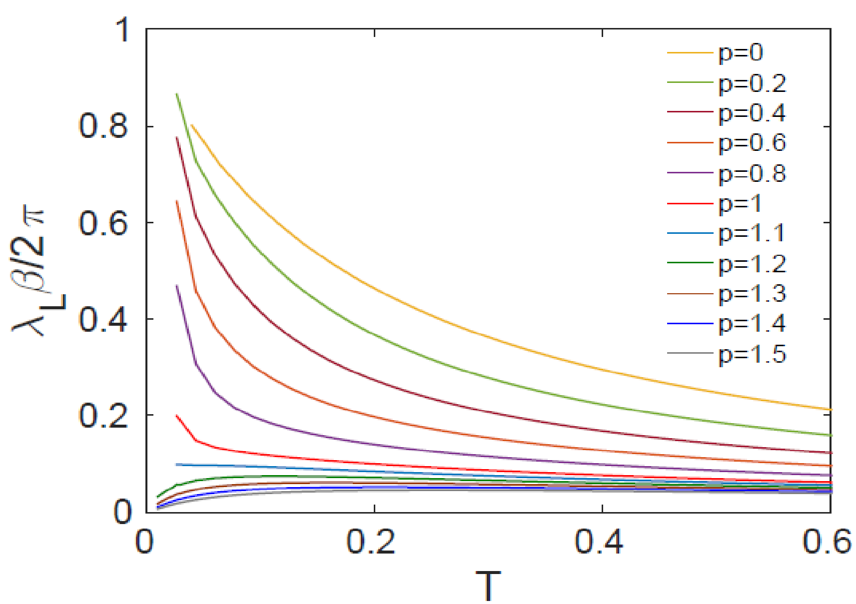
→Thermalization

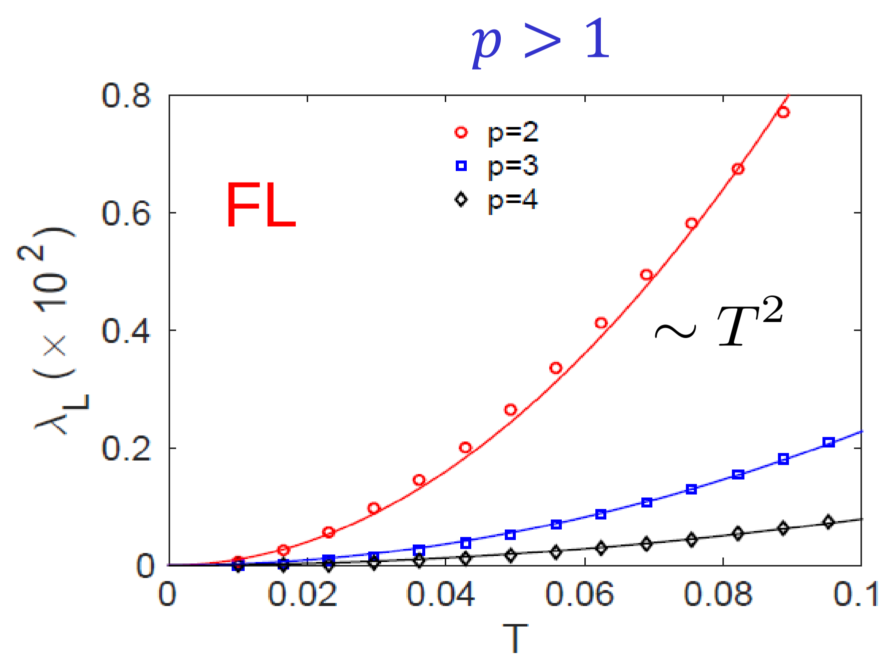
Out-of-time-order (OTO) correlator

$$F(t) = \langle A(t) B(0) A(t) B(0) \rangle \sim 1 - \epsilon e^{\lambda_L t}$$

Decays exponentially







Transition from fast to slow scrambling

Physical motivation: Is MBL unstable in dimension $D > 1$?

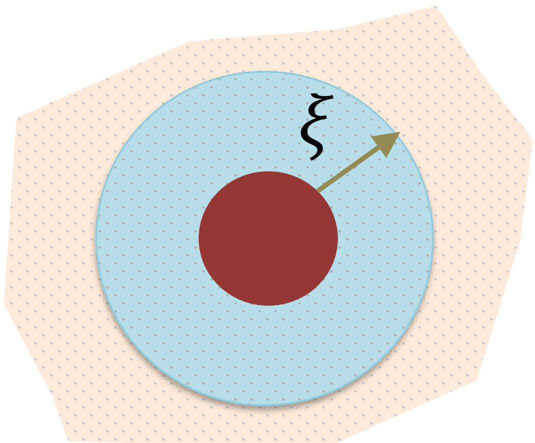
De Roeck and Huveneers (2016)

N sites:
SYK coupling

$\overline{J_{ijkl}^2} = J^2$

M sites:
Random hopping

$\overline{V_{i\alpha}^2} = V^2 \qquad \overline{t_{\alpha\beta}^2} = t^2$



Ergodic bubble
in an Anderson insulator

$\boxed{M \sim \xi^D}$

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{\sqrt{N}} \sum_{\alpha\beta} t_{\alpha\beta} \psi_\alpha^\dagger \psi_\beta - \mu\Big(\sum_i c_i^\dagger c_i + \sum_\alpha \psi_\alpha^\dagger \psi_\alpha\Big)$$
$$+ \frac{1}{(MN)^{1/4}} \sum_{i\alpha} (V_{i\alpha} c_i^\dagger \psi_\alpha + h.c.)$$

SB, Potirniche & Altman (unpublished)

Breaking of reparameterization symmetry

$$G^{-1}(i\omega_n) = \cancel{i\omega_n} - \Sigma(i\omega_n) \qquad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

- Low-energy conformal symmetry, $\tau = f(\tau), \quad 0 < \tau < \beta$

$$\tilde{G}(\sigma_1, \sigma_2) = [f'(\sigma_1) f'(\sigma_2)]^{1/4}\, G(f(\sigma_1), f(\sigma_2)), \qquad f \in \mathrm{Diff}(S^1)$$

- The saddle point spontaneously breaks the symmetry to $SL(2,R)$

$$G(\tau_1, \tau_2) \sim |\tau_1 - \tau_2|^{-\frac{1}{2}} \to [f'(\sigma_1) f'(\sigma_2)]^{\frac{1}{4}} |f(\sigma_1) - f(\sigma_2)|^{-\frac{1}{2}} = |\sigma_1 - \sigma_2|^{-\frac{1}{2}}$$

$$f(\tau) = \frac{a\tau + b}{c\tau + d}, \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,R)$$

- Symmetry is also broken explicitly by $i\omega_n$ term.

→Pseudo Goldstone mode

→ Quantum chaos and thermalization in SYK model.

Conformal Green's functions at finite temperature

From $T=0$, $G_R(\omega), \mathcal{G}_R(\omega) \Rightarrow \quad G(\tau) \sim -\frac{\mathrm{sgn}(\tau)}{\sqrt{J|\tau|}}, \qquad \mathcal{G}(\tau) \sim -\frac{\mathrm{sgn}(\tau)}{(J|\tau|)^{\frac{3}{2}}}$

→ Finite temperature Green's function obtained by conformal transformation

$$\tau = \left(\frac{\beta}{\pi}\right)\tan(\pi\sigma/\beta)$$

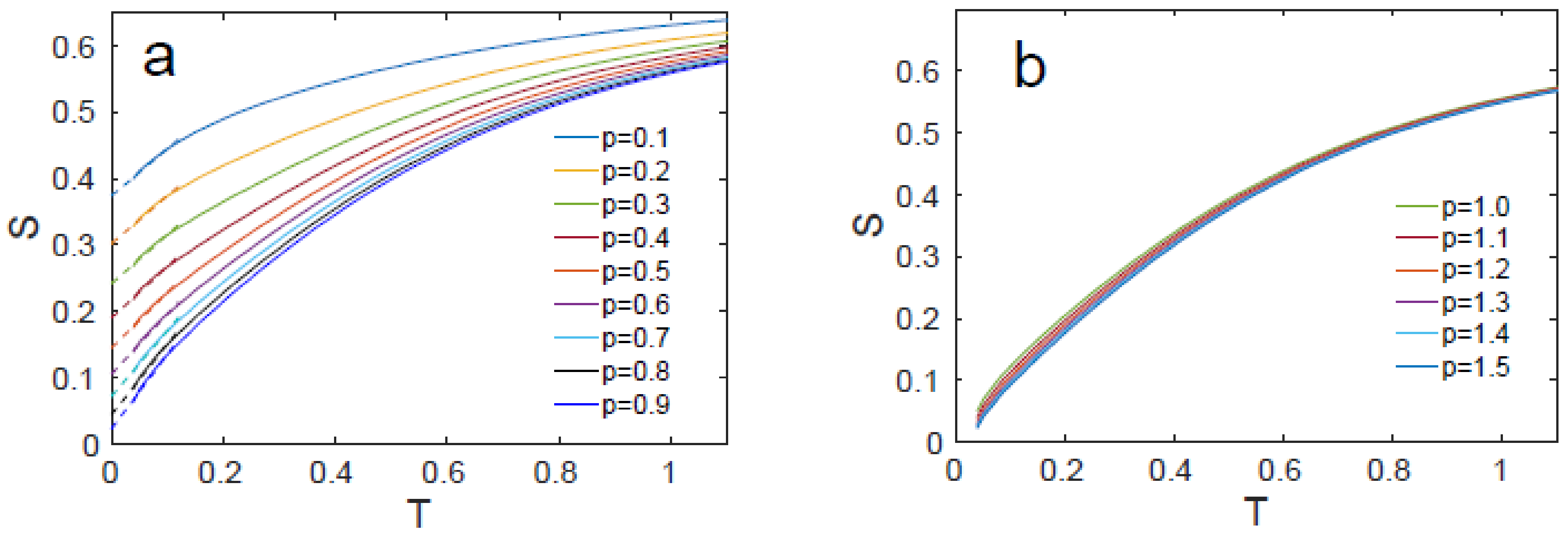
$$G(\tau) \sim -\frac{\mathrm{sgn}(\tau)}{\beta J \sin\left(\frac{\pi|\tau|}{\beta}\right)^{1/2}} \qquad \mathcal{G}(\tau) \sim -\frac{\mathrm{sgn}(\tau)}{\beta J \sin\left(\frac{\pi|\tau|}{\beta}\right)^{3/2}}$$

$$G_R^{\Delta}(\omega) \sim \frac{T^{2\Delta-1}}{\Gamma(2\Delta)\sin(2\pi\Delta)} \frac{\Gamma\left(\Delta - i\frac{\omega}{2\pi T}\right)}{\Gamma\left(1-\Delta-i\frac{\omega}{2\pi T}\right)}$$

$$\Delta = \frac{1}{4} \to G_R(\omega) \qquad \Delta = \frac{3}{4} \to \mathcal{G}_R(\omega)$$

Entropy

$$f = \frac{F}{N+M} = -\frac{T}{1+p}\sum_n \left[\ln(-\beta G^{-1}(i\omega_n)) + p\ln(-\beta \mathcal{G}^{-1}(i\omega_n))\right] e^{i\omega_n 0^+}$$
$$-\frac{1}{1+p}\int_0^\beta d\tau \left[\frac{3}{4}\Sigma_J(\tau)G(-\tau) + V^2\sqrt{p}\mathcal{G}(\tau)G(-\tau) + \frac{pt^2}{2}\mathcal{G}(\tau)\mathcal{G}(-\tau)\right].$$



$$F_1(t_1,t_2) = \frac{1}{N^2}\sum_{ij}\overline{Tr[yc_i^\dagger(t_1)yc_j^\dagger(0)yc_i(t_2)yc_j(0)]} \tag{32a}$$

$$F_2(t_1,t_2) = \frac{1}{N^2}\sum_{ij}\overline{Tr[yc_i(t_1)yc_j^\dagger(0)yc_i^\dagger(t_2)yc_j(0)]} \tag{32b}$$

$$F_3(t_1,t_2) = \frac{1}{NM}\sum_{i\alpha}\overline{Tr[y\psi_\alpha^\dagger(t_1)yc_i^\dagger(0)y\psi_\alpha(t_2)yc_i(0)]} \tag{32c}$$

$$F_4(t_1,t_2) = \frac{1}{NM}\sum_{i\alpha}\overline{Tr[y\psi_\alpha(t_1)yc_i^\dagger(0)y\psi_\alpha^\dagger(t_2)yc_i(0)]} \tag{32d}$$