Topological Quantum Phase Trasition in Interacting Helical Liquid

Sujit Sarkar
Poornaprajna Institute of Scientific Research
4 Sadashivanagar, Bangalore-5600 80

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Outline

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Nobel Prize in Physics 2016

Strange phenomena in matter's flatlands

This year's Laureates opened the door on an unknown world where matter exists in strange states. The Nobel Prize in Physics 2016 is awarded with one half to ***David J. Thouless****, University of Washington, Seattle, and the other half to* ***F. Duncan M. Haldane****, Princeton University, and* ***J. Michael Kosterlitz****, Brown University, Providence. Their discoveries have brought about breakthroughs in the theoretical understanding of matter's mysteries and created new perspectives on the development of innovative materials.*

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Physics of Majorana modes in interacting helical liquid

Sujit Sarkar

Difference Between Quantum Phase Transition and Topological Quantum Phase Transition

Landau's theory of Phase Transition provide the phenomenological footing on which symmetry broken states can be explained.

In past decade it has become clear that a series of phases of matter with so called topological order do not have a local order.

Difference between topological quantum phase transition and quantum phase transition

Topological quantum phase transition describes with the topologically invariant number while the quantum phase transition describes by order parameter. Topological number changes by an integer number during the topological quantum phase transition from topological state to the non-topological state, which is related to the appearance of Majorana zero modes localized at the edge of the system. This detailed study based on exact calcula-

Properties of Topological Phases

- The bulk of the system is gapped, namely, there is a finite energy gap between the ground state and the excited states. Hence the bulk is an insulator at low temperatures

- The band structure of the bulk of the system is characterized by a topological invariant which is a non-zero integer

- There are gapless states at the boundaries of the system; these contribute to electronic transport

- Bulk-boundary correspondence: The number of boundary states is equal to the topological invariant; it does not change if the parameters in the Hamiltonian are changed a bit or if a small amount of disorder is present

What is Helical Spin Liquid?

Helical spin liquid originates from the quantum spin Hall effect in a system with or without Landau levels. In the quantum spin Hall effect, the left movers in the edge are connected with the down spin and the right movers with the up spin and the transport process is quantized. This physics is generally termed as a helical liquid which describes the connection between the spin and momentum.

Spin Hall Effect?

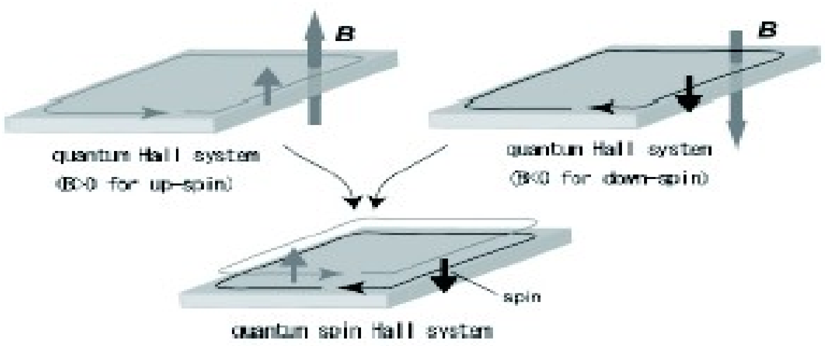
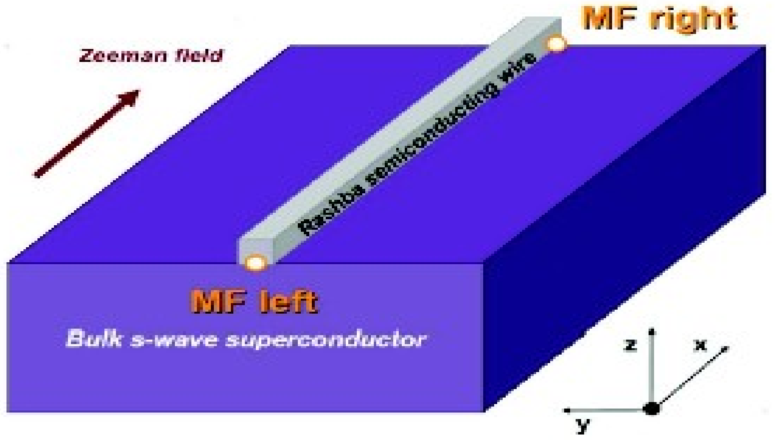


Figure 1:



A 2D system displays the quantum spin Hall effect if,

(1) its Hamiltonian is invariant under time-reversal and all states are twofold Kramers degenerate.

(2) The bulk system has a gap and the topological quantumnumber Z_2 is odd.

(3) When the system is opened, there is an odd number of Kramers doublets on each edge at the Fermi energy.

Model Hamiltonian

One can write the Hamiltonian for the low energy collective excitation in one dimensional system as

$$H_0 = \int \frac{dk}{2\pi} v_F [(\psi_{R\uparrow}{}^{\dagger}(x)(i\partial_x)\psi_{R\uparrow}(x) - \psi_{L\downarrow}(x)^{\dagger}(i\partial_x)\psi_{L\downarrow}(x)) + (\psi_{R\downarrow}{}^{\dagger}(x)(i\partial_x)\psi_{R\downarrow}(x) - \psi_{L\uparrow}{}^{\dagger}(x)(i\partial_x)\psi_{L\uparrow}(x))], \quad (1)$$

where $\psi_{R\uparrow}(x)$ and $\psi_{L\downarrow}(x)$ are the field operators for spin up right moving and spin down left moving electrons

Here we consider a low-dimensional quantum many body system of topological insulator in the proximity of s-wave superconductor and an external magnetic field along the edge of this system. The additional part in the Hamiltonian is

$$\delta H = \Delta\psi_{L\downarrow}(x)\psi_{R\uparrow}(x) + B\psi_{L\downarrow}{}^{\dagger}(x)\psi_{R\uparrow}(x) + h.c., \quad (2)$$

where Δ is the proximity induced superconducting gap and B is the applied magnetic field along the edge of the

Now we consider the generic interaction, considering the two particles having forward and umklapp scattering as

$$H_{fw} = g_2 \psi_{L\downarrow}^{\dagger}(x)\psi_{L\downarrow}(x)\psi_{R\uparrow}^{\dagger}(x)\psi_{R\uparrow}(x). \tag{3}$$

The analytical expression for umklapp in a conventional form can be expressed as[23].

$$H_{um} = g_u \psi_{L\downarrow}^{\dagger}(x)\partial_x \psi_{L\downarrow}^{\dagger}(x)\psi_{R\uparrow}(x)\partial_x \psi_{R\uparrow}(x) + h.c. \tag{4}$$

Therefore, the total Hamiltonian of the system is $H = H_0 + H_{fw} + H_{um} + \delta H$. Now we can write the above Hamiltonian as $H_{XYZ} = \sum_l H_l$ (up to a constant)[16], where

$$H_l = \sum_{\alpha} J_\alpha S_l^{\alpha} S_{l+1}^{\alpha} - [\mu + B(-1)^l] S_l^z. \tag{5}$$

and $J_{x,y} = J \pm \Delta > 0$, $J = v_F$ and $J_z > 0$.

The bosonized form of this model Hamiltonian is (for detail, please see method).

$$H = \frac{v}{2}\left(\frac{1}{K}(\partial_x\phi(x))^2 + K(\partial_x\theta(x))^2\right) - \left(\frac{\mu}{\pi}\right)\partial_x\phi(x) + \frac{B}{\pi}\cos(\sqrt{4\pi}\phi(x)) - \frac{\Delta}{\pi}\cos(\sqrt{4\pi}\theta(x)) + \frac{g_u}{2\pi^2}\cos(4\sqrt{\pi}\phi(x)), \tag{6}$$

where $\theta(x)$ and $\phi(x)$ are the dual fields and K is the Luttinger liquid parameter of the system. The first and second

Majorana-Ising Transition

Majorana-Ising transition and nature of phase transition. Our starting Hamiltonian is $H_2 = H_0 + \delta H$. We recast the fermionic field in terms of the Majorana fields as, $\psi_{L\downarrow}(x) = \frac{1}{2}(i\chi_1(x) + \chi_2(x))$ and $\psi_{R\uparrow}(x) = \frac{1}{2}(\tilde{\chi}_1(x) + i\tilde{\chi}_2(x))$. The total Hamiltonian, $H = H_0 + \delta H$, becomes

$$H = \sum_{l=1,2}\left(i\chi_l(x)\frac{v_F}{2}\partial_x\chi_l(x) - i\tilde{\chi}_l(x)\frac{v_F}{2}\partial_x\tilde{\chi}_l(x) + im_l\chi_l(x)\tilde{\chi}_l(x)\right), \tag{7}$$

where $m_{1,2} = \Delta \mp B$ (here $\Delta > 0$). At $\Delta = B$, one of the two Majorana fermion modes becomes gapless which is the signature of bulk Majorana-Ising quantum phase transition.

It is well known that the critical theory is invariant under the rescaling. Then the singular part of the free energy density satisfies the following scaling relations.

$f_s[\Delta, B] = e^{-2l} f_s[e^{(2-1/K)l}\Delta, e^{(2-K)l}B]$. The scale l can be fixed from the following analytical relation, $e^{(2-1/K)l^*}\Delta = 1$. Finally, after few steps of calculation, we arrive at the following relation: $f_s[\Delta, B] = \Delta^{2/(2-1/K)} f_s[1, \Delta^{-(2-K)/(2-1/K)}B]$ and the equation for phase transition is $\Delta^{-(2-K)/(2-1/K)} B \sim 1$. The phase boundary between these two quantum phases can be obtained by using the above relation. When $K = 1$(non-interacting case), the phase

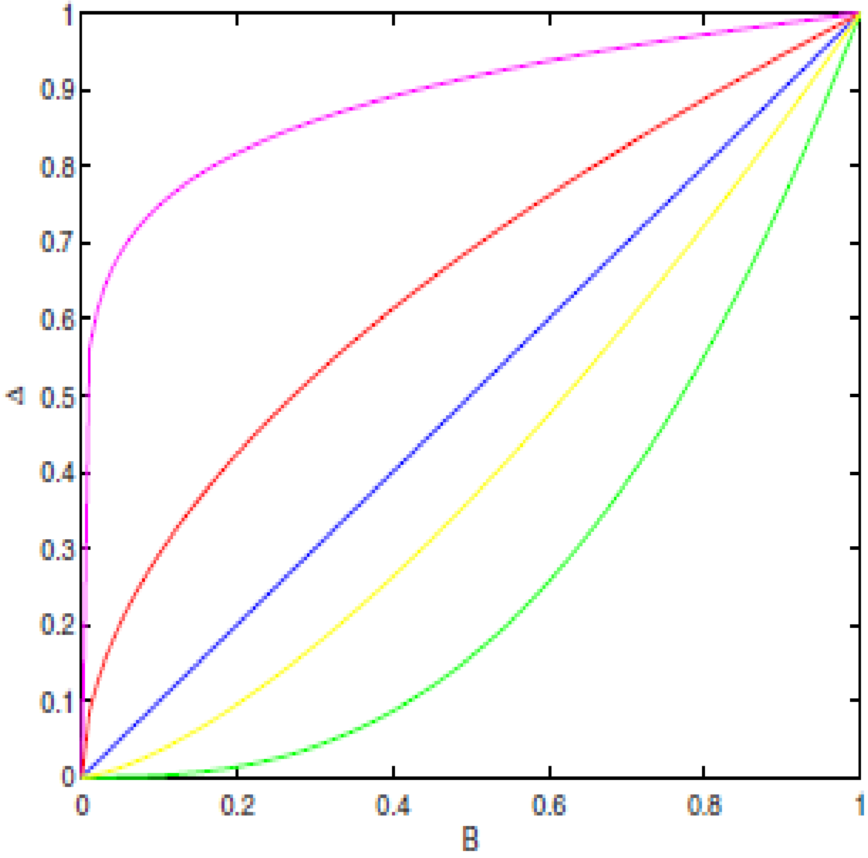


Figure 1. Phase boundary between the two different phases Δ and *B*. The color blue, magenta, red, green and yellow are for $K = 1, 0.55, 0.75, 1.5, 1.2$ respectively.

This is the procedure to derive these RG equations of the present problem, which are the following:

$$
\begin{aligned}
\frac{dB}{dl} &= (2-K)B + 4Kg_u B, \\
\frac{d\Delta}{dl} &= \left(2-\frac{1}{K}\right)\Delta, \\
\frac{dg_u}{dl} &= (2-4K)g_u + 2KB^2 \\
\frac{dK}{dl} &= \frac{\Delta^2}{4} - K^2B^2.
\end{aligned} \tag{13}
$$

Where $l = ln\left[\frac{\Lambda}{\Lambda_0}\right]$, is the flow parameter and Λ_0 is the initial value of the momentum cut-off parameter. In the RG process, one can write the RG equations themselves in a perturbative expansion in coupling constant ($g(l)$). They cease to be valid beyond a certain length scale, where $g(l) \sim 1$[34]. It is very clear from the above RG equations that in the absence of umklapp scattering, these equations reflect the duality in our helical liquid model system. The duality is the following: $\phi \leftrightarrow \theta$, $K \leftrightarrow K^{-1}$ and $\Delta \leftrightarrow B$. These RG equations have trivial ($\Delta^* = 0 = B^*$) fixed points for any arbitrary K. Apart from that, these RG equations have also two non-trivial fixed lines, $\Delta = B$ and $\Delta = -B$ for $K = 1$.

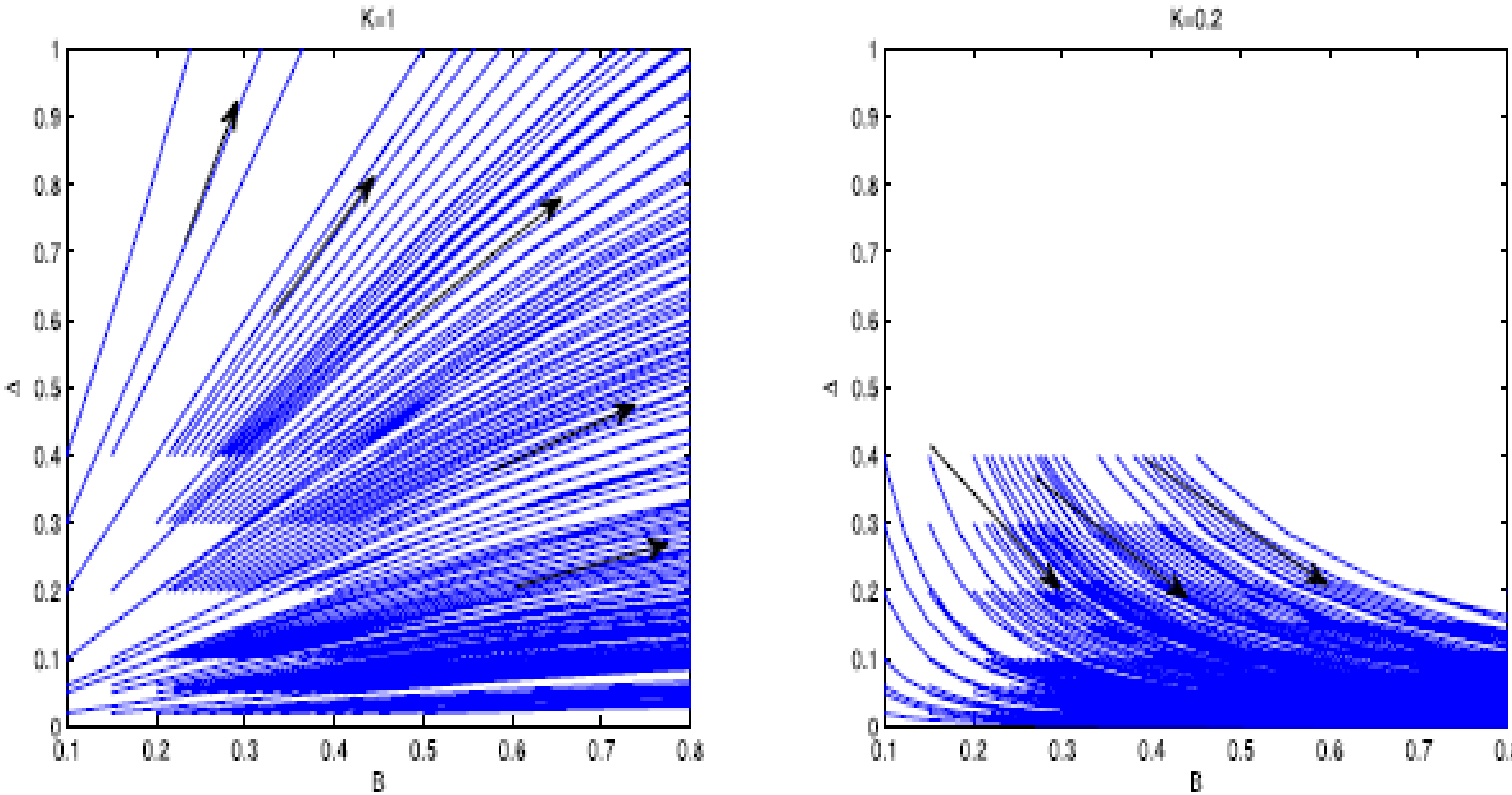


Figure 2. RG flow of Δ with B, in the absence of umklapp scattering. The left panel (Fig. 2A) is for $K = 1$ and the right panel (Fig. 2B) is for $K = 0.2$.

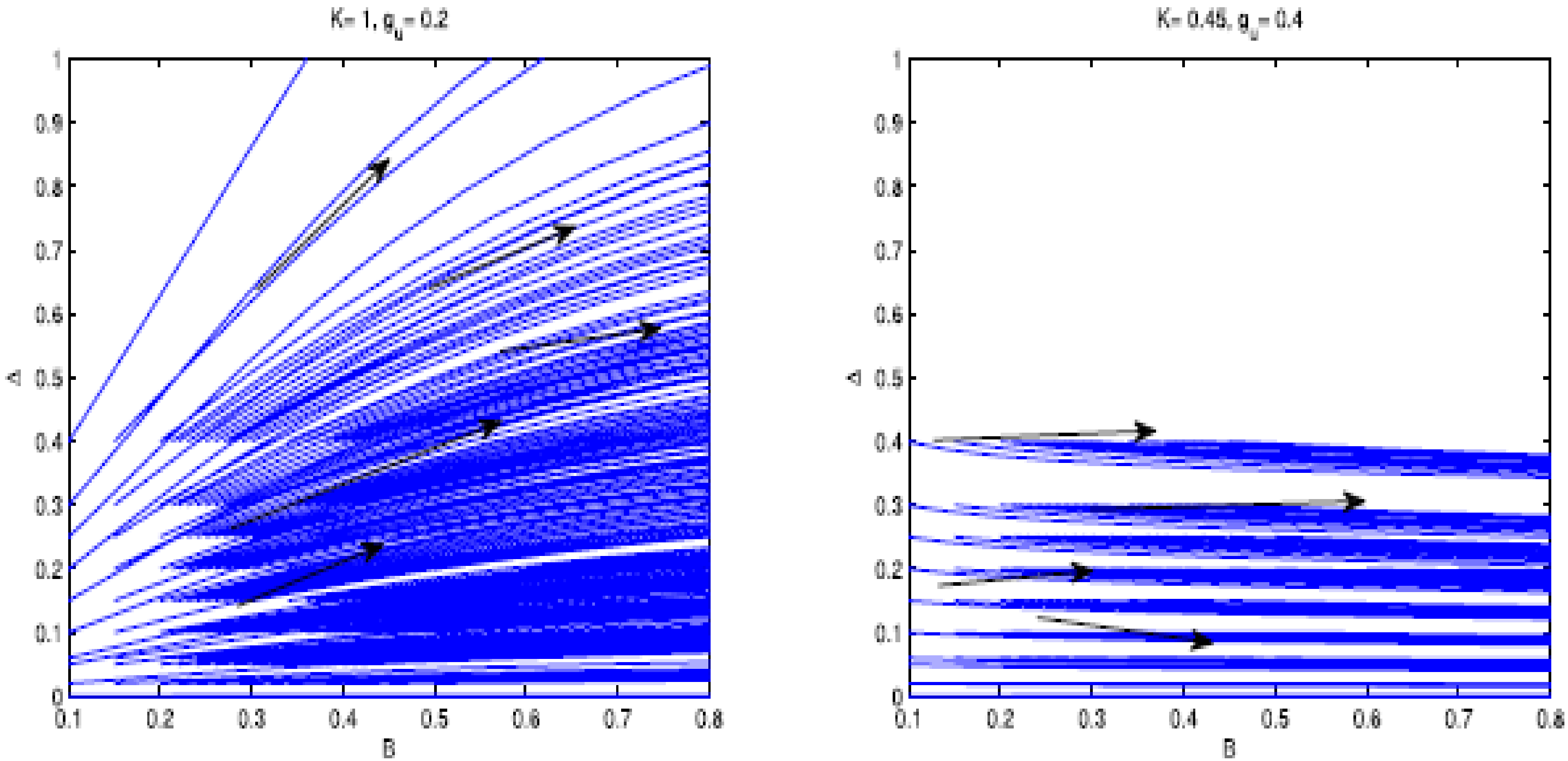


Figure 3. RG flow lines of Δ with B. The left panel (Fig. 3A) is for $K=1$, $g_u=0.2$ and the right (Fig. 3B) panel is for $K=0.45$, $g_u=0.4$.

Conclusions

(1). We have presented Majorana-Ising topological transition from the scaling analysis of sine-Gordan coupling term.

(2) We have derived and presented the results based on Renormalization Group study to predict the topological and non-topological state.