The Island-Mainland Transition identified by Euler Number

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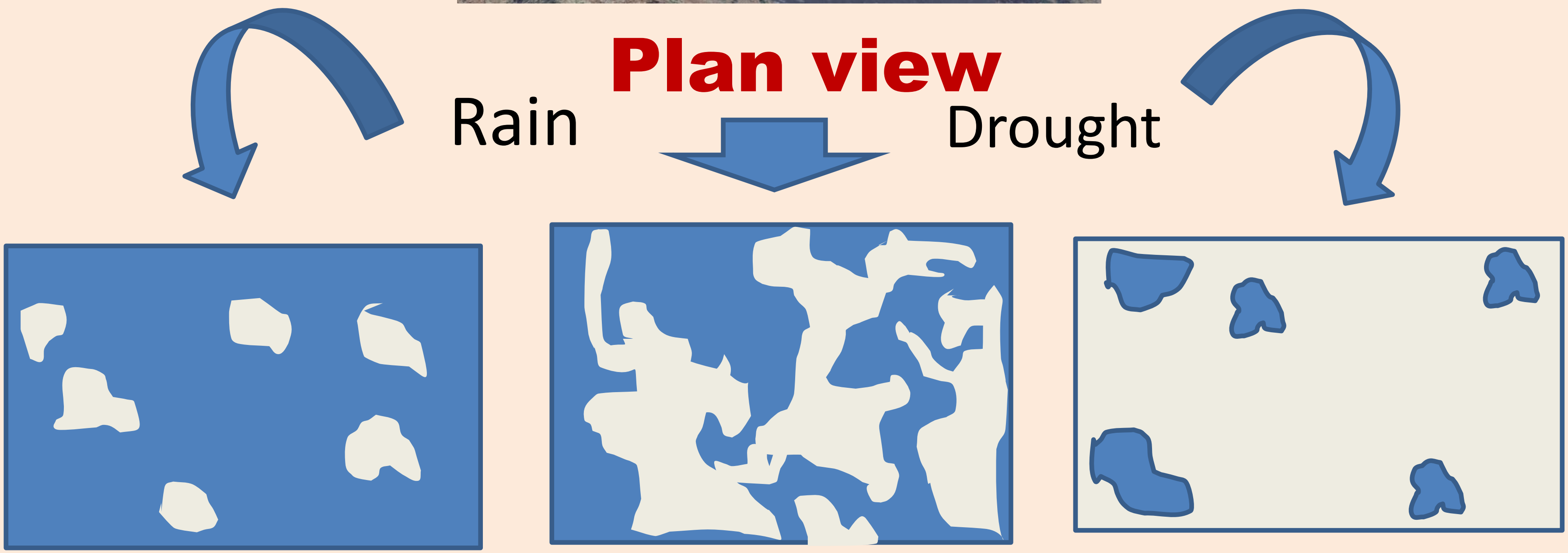


(I) Islands in sea

???
(II) Mixed Phase

(III) Lakes in mainland

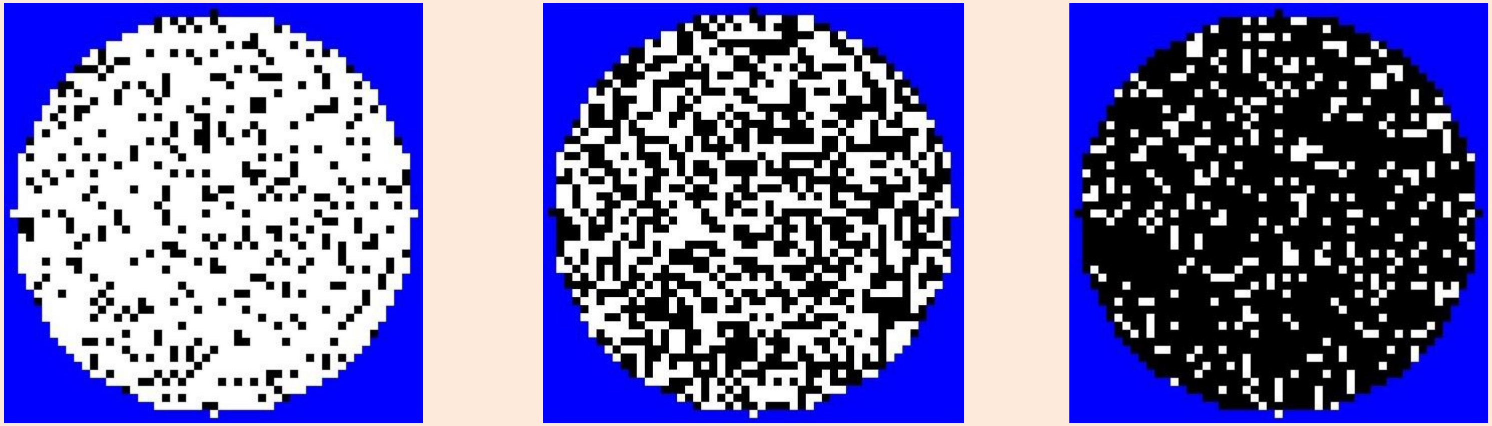
Our objective: To identify the transition where continuous Sea in (I) gets disconnected to give the mixed Phase in (II) and when mixed phase transforms to the Continuous mainland with lakes in (III)

Consider random deposition on square lattice with increasing –probability of occupation of site ‘p’.

We are NOT Looking for normal ‘Percolation threshold’

p = 0.17 **p = 0.50** **p = 0.83**

Similar results valid for system with circular boundary
Where standard percolation concept is not meaningful



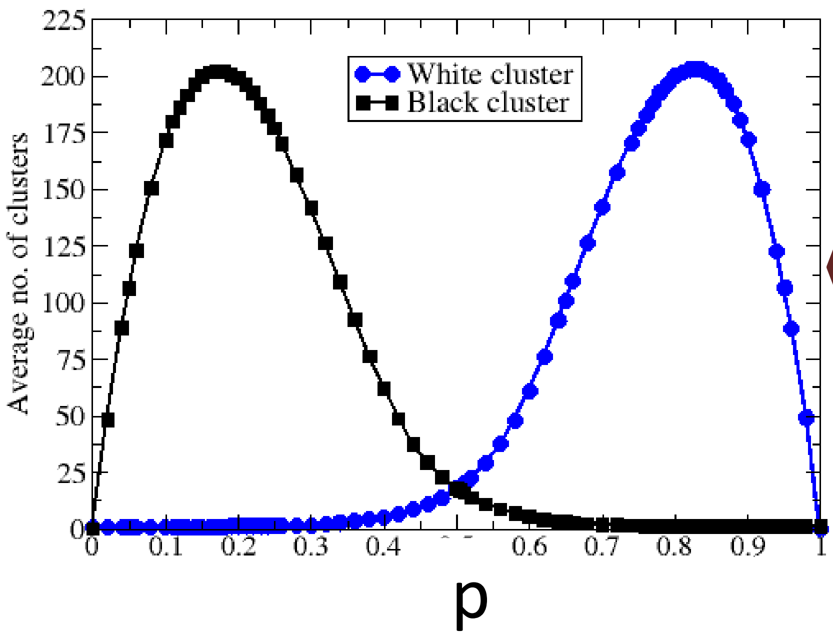
We define B(W) clusters as groups of B(W) sites connected by edges/corners

$N_B(p)$→ Number of *clusters* of black sites
$N_W(p)$→ Number of *clusters* of White sites

We borrow a concept from TOPOLOGY

The EULER NUMBER $\chi(p) = N_B(p) - N_W(p)$

(Ref: Vogel et al. Geoderma 125 (2005) 203– 211)

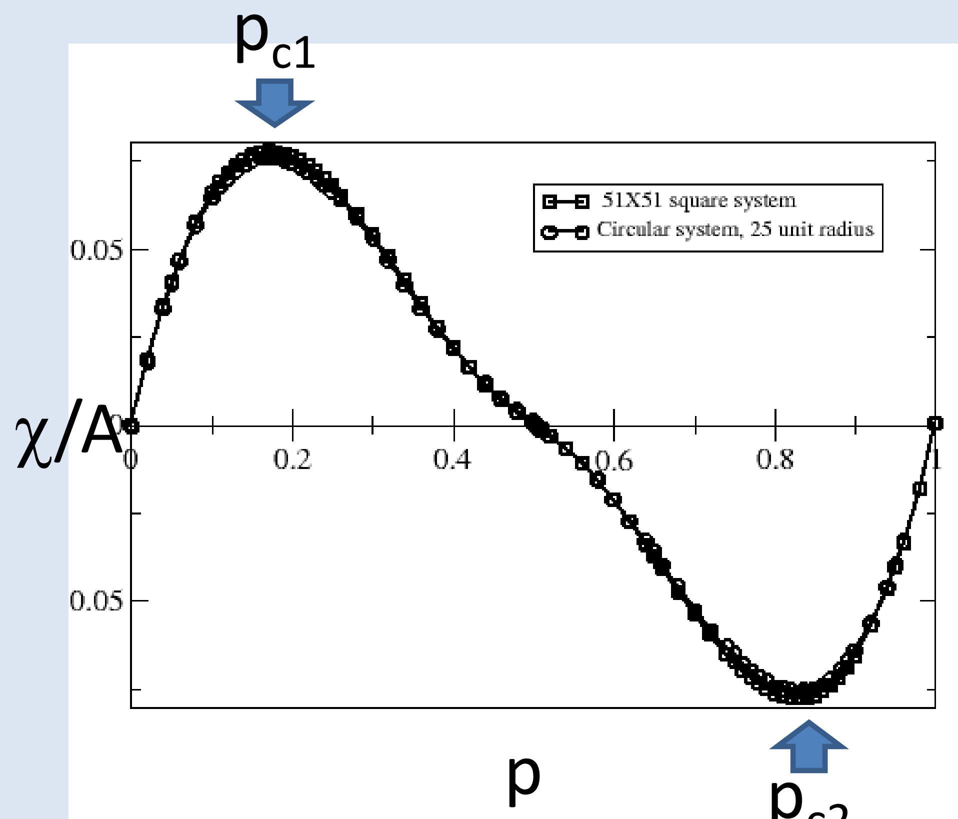


N_B(p) and **N_W(p)** plotted against **p**
Simulation performed over 51x51
Square lattice, averaged over 100
Configurations. A is total area of
The system.

The maximum p_{c1} and minimum p_{c2} of the χ(p) curve correspond to the

Island in sea → MP
and
MP → lakes in mainland

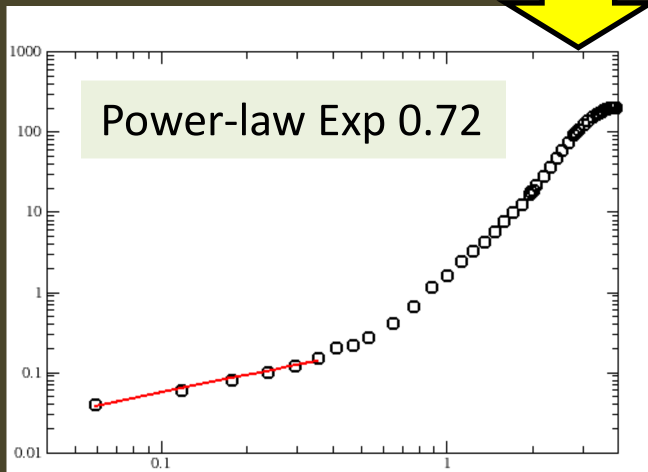
P_{ci} 's are different from standard percolation threshold (for black)
At p = 0.391



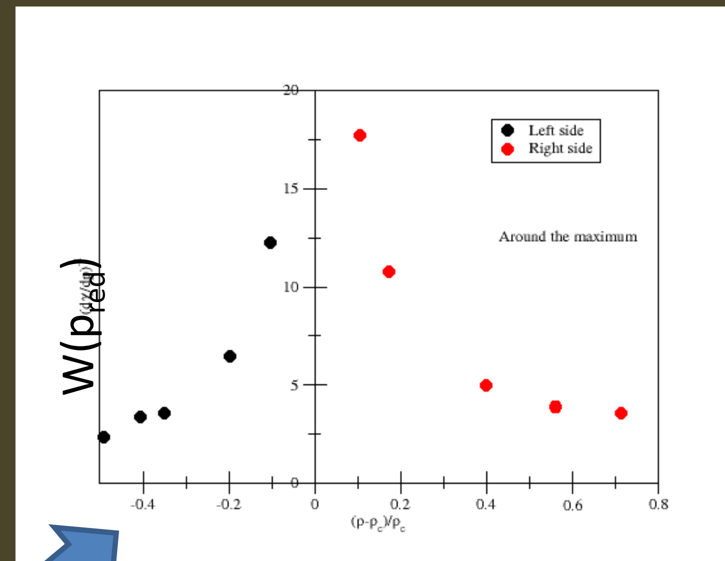
We define **Order Parameter** **$Q(p) \rightarrow (N_W(p) - 1)$**

How does it approach 0 with **p(reduced) = $(p - p_{ci})/p_{ci}$**

Q(p)

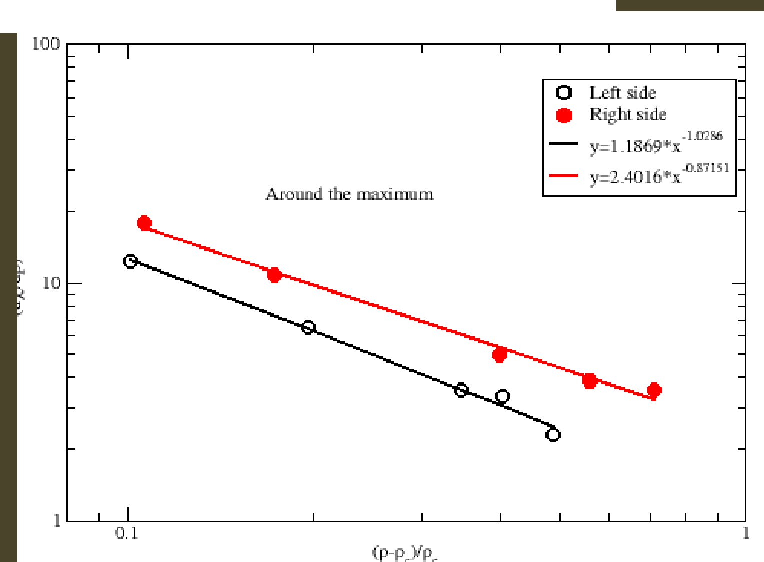


p(red)



$$W(p_{red}) \rightarrow \left| 1/\frac{\partial \chi}{\partial p_{red}} \right|$$ blows up

At p_{ci}. It also follows a power law
With exp -1.02 on left and -0.87
On right of p_{c1}



How it all started:
Experimental observations of IM Transition

Example I

A droplet of a sticky gel containing gelatin, water and NaCl is drying – water evaporates and NaCl crystallizes

Experiment shows a crossover between *faceted growth* of large NaCl Crystals to ***dendritic growth***,

we suggest
That the minimum in Euler number

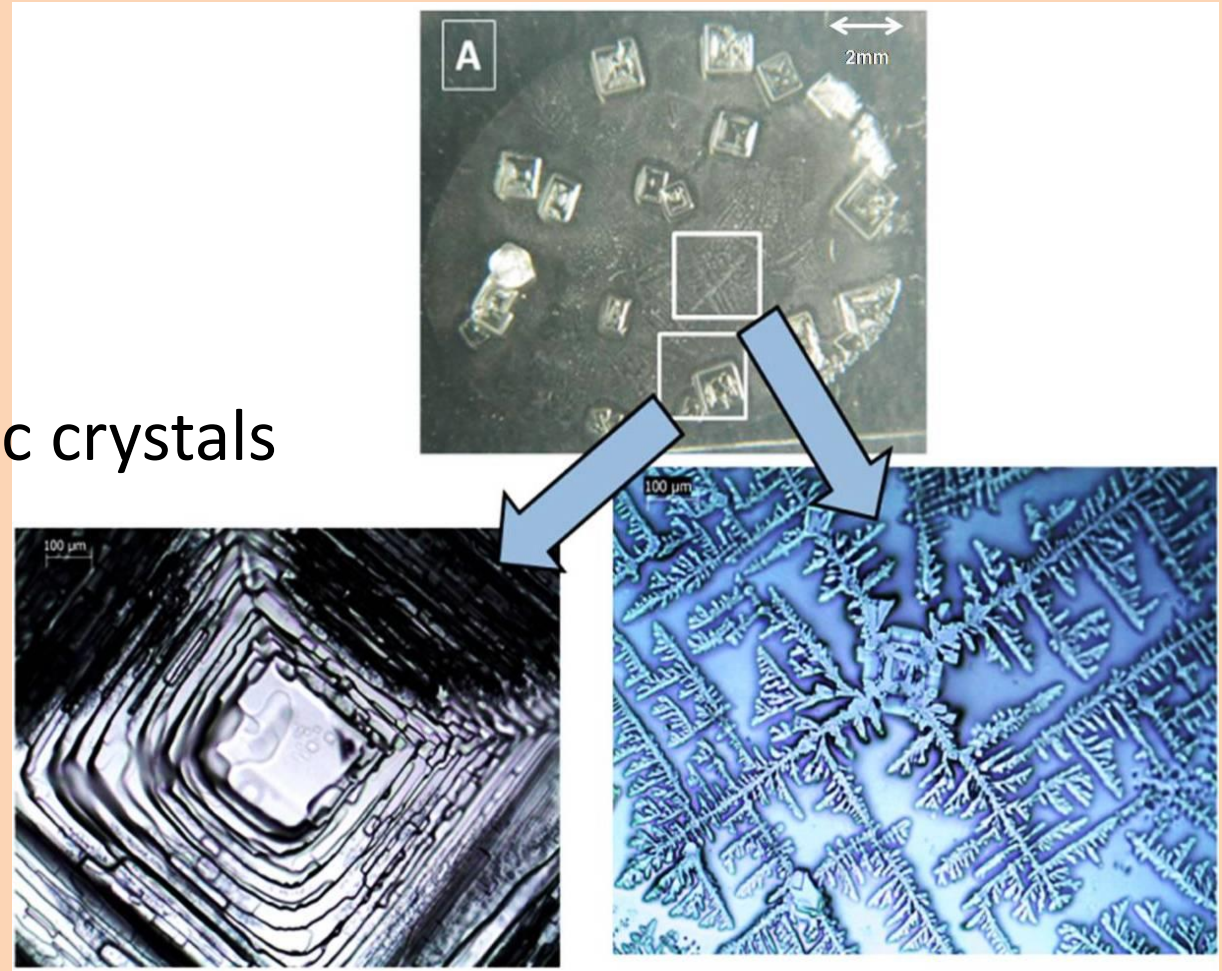
$$\chi(t) = N_v(t) - N_g(t)$$

Marks the **Mixed phase→ FLUID in VOID** transition

And crossover between two modes of crystallization
Occurs here

Gelatin + water + NaCl

Dutta Choudhury et al.Coll. Surf. A: Physicochem.Eng.Aspects 432 (2013) 110–118



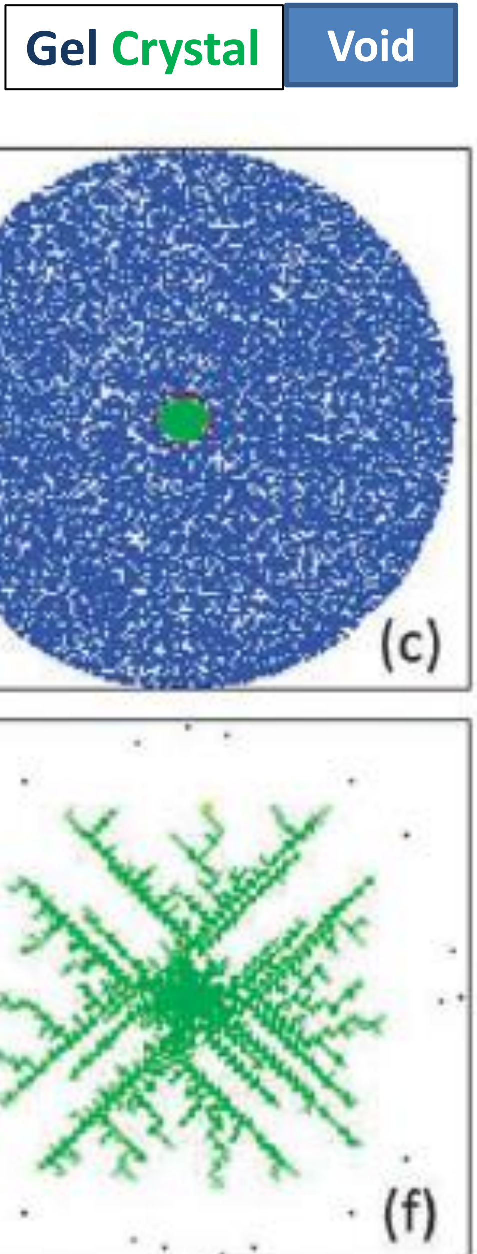
Cubic crystals

Dendrites are Multi-fractal

Giri et al. Cryst. Growth Des. 2013,13,341

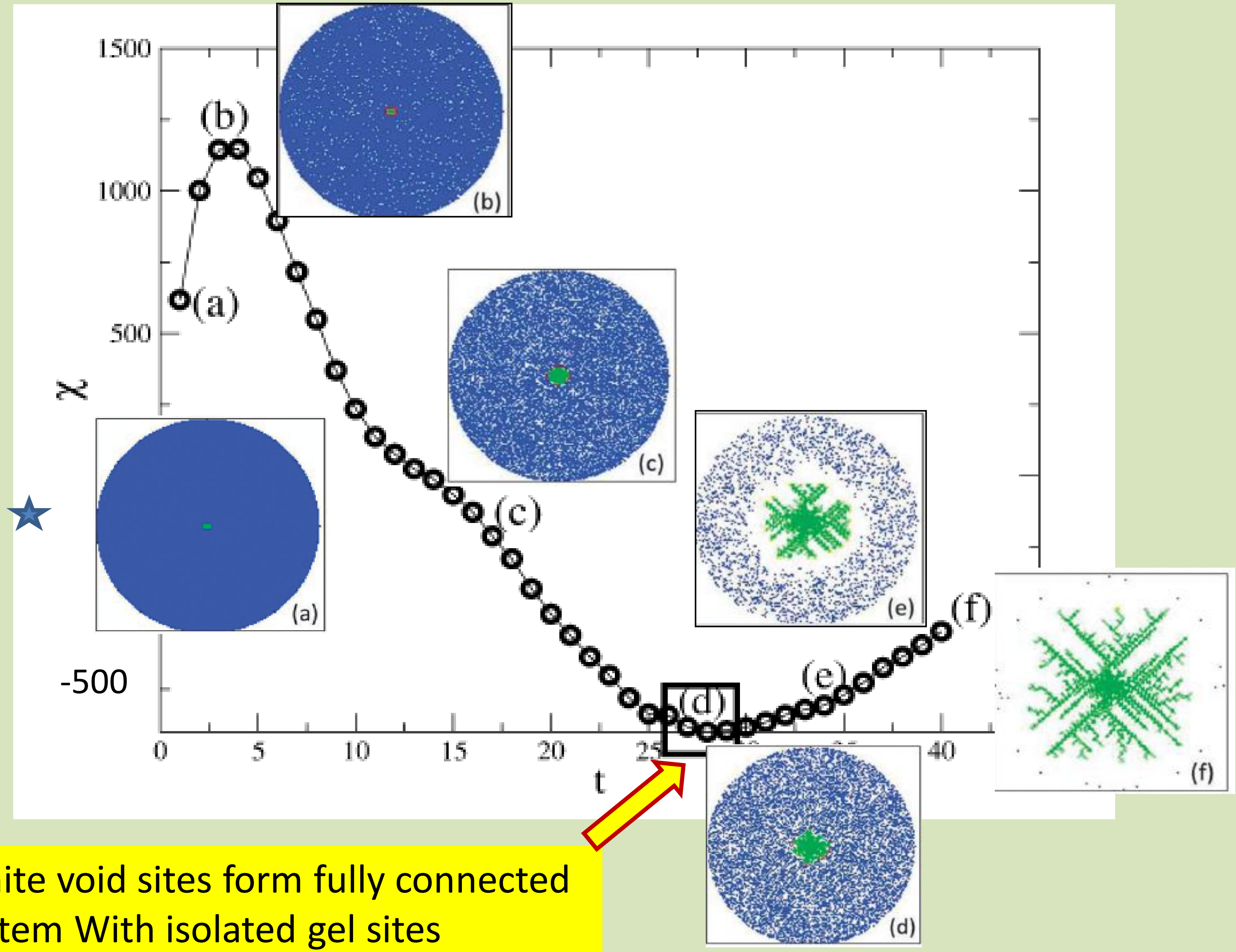
See video on youtube
www.youtube.com/watch?v=ovYjDXcEtxk

SIMULATING TRANSITION IN GROWTH PROCESS



M. Dutta Choudhury et al. Soft Matter, 2015

EVAPORATION makes blue GEL sites → white VOID sites



White void sites form fully connected System With isolated gel sites

Example II

Cracks form in a layer of bentonite-water-clay slurry, during drying.
The **connectivity** of the crack pattern depends
On the thickness (h) of the layer

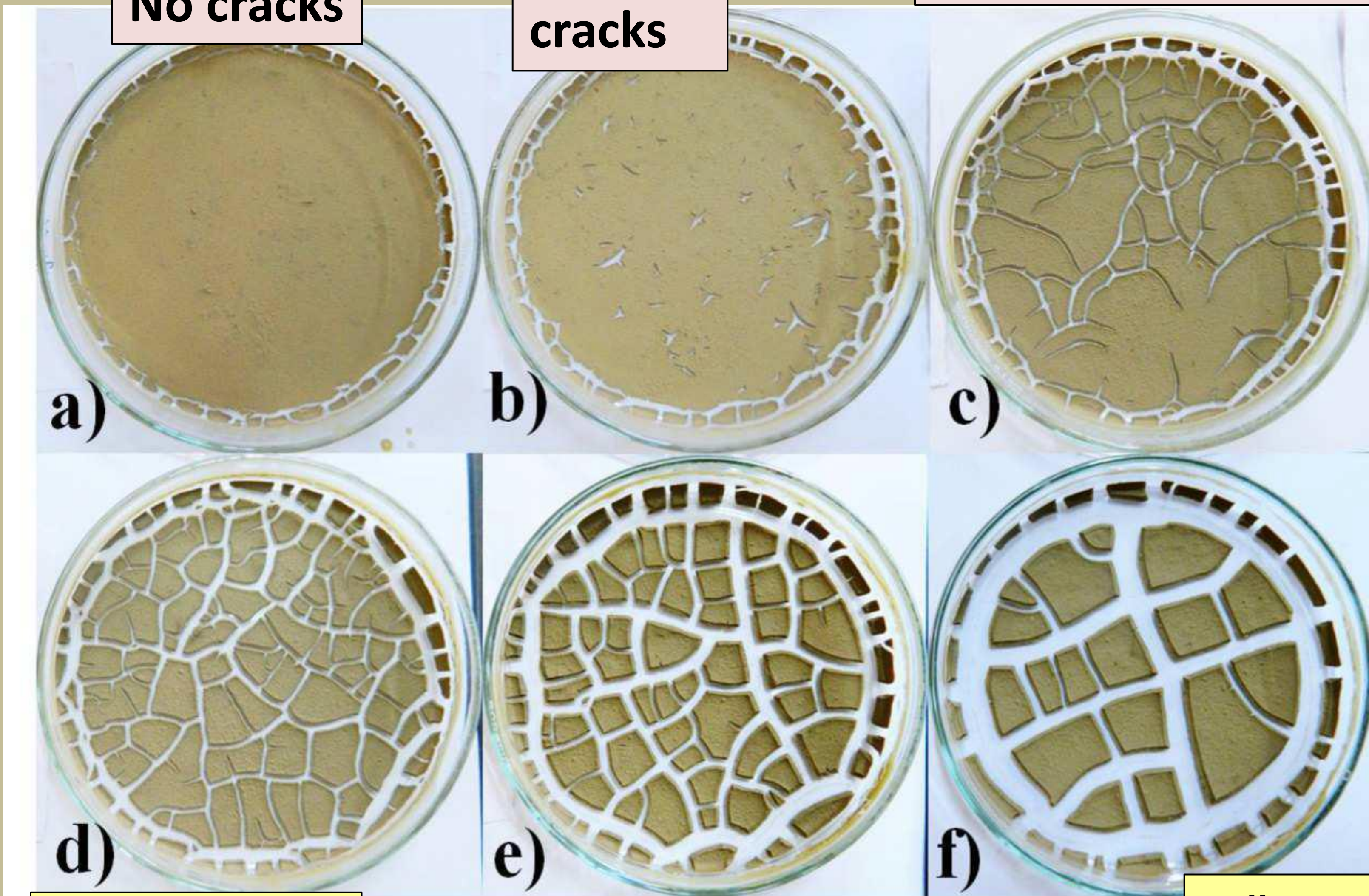
When do the cracks form a **fully connected network ?**
Let us see how the
Euler number varies with thickness (h).

h in mm: a) 0.29 cm, b) 0.34 cm, c) 0.38, d) 0.40, e) 0.56, f) 0. 89

No cracks

Isolated cracks

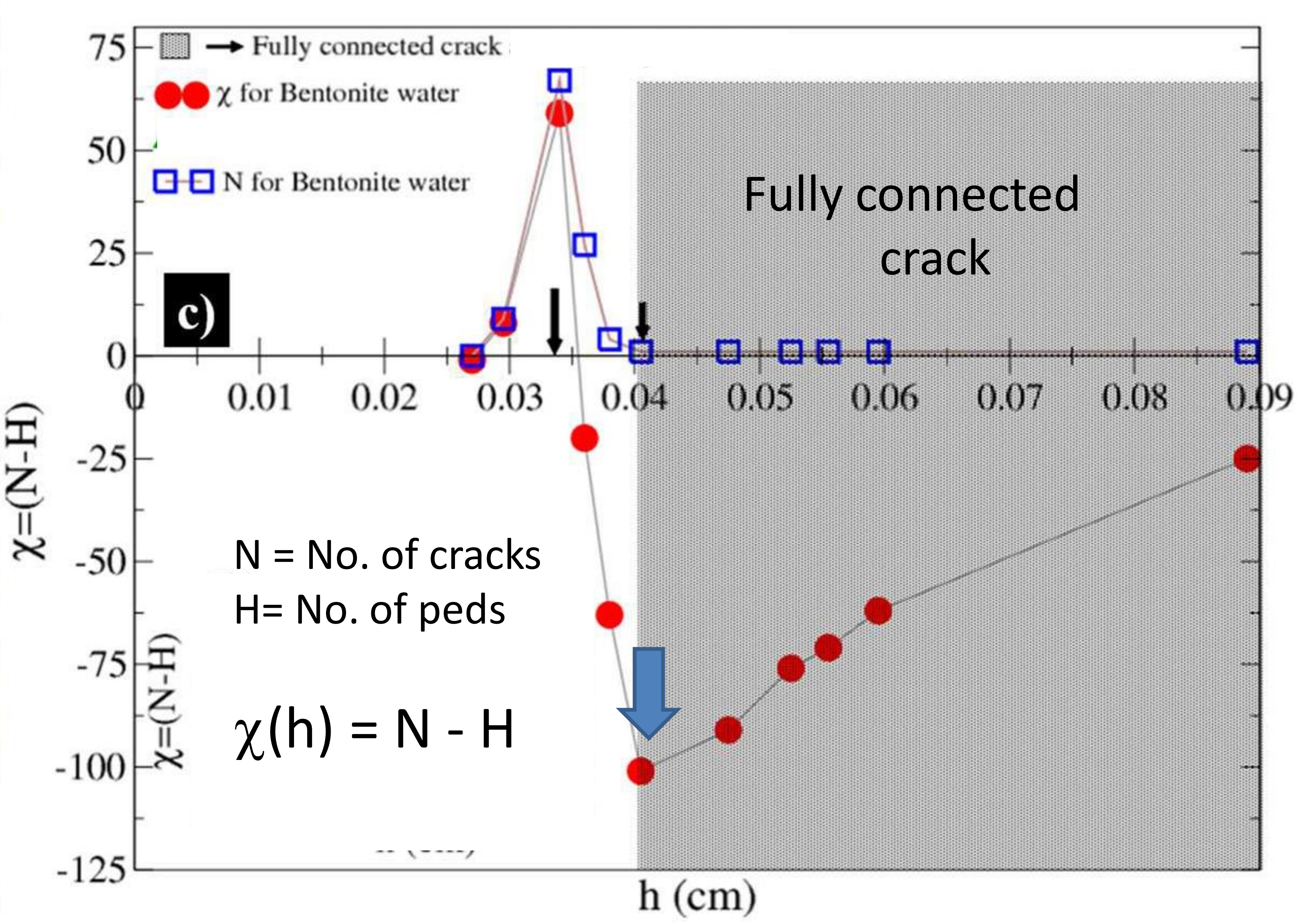
Partially connected cracks



Fully connected narrow cracks

*Khatun et al. EPJE, 2015,**38**,83*

Fully connected wide cracks



Fully connected crack
χ for Bentonite water
N for Bentonite water
c)
Fully connected crack
N = No. of cracks
H= No. of peds
χ(h) = N - H
χ=(N-H)
h (cm)
75
50
25
0
-25
-50
-75
-100
-125
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09

Conclusion: It seems that
The **Euler number** is a good
Indicator to identify a
new transition sequence.
We call this the
Island-mainland Transition.
The transition is shown to be
Significant In 2 real experimental systems

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Thank you for your Attention