Quantum spin ice in external electric field

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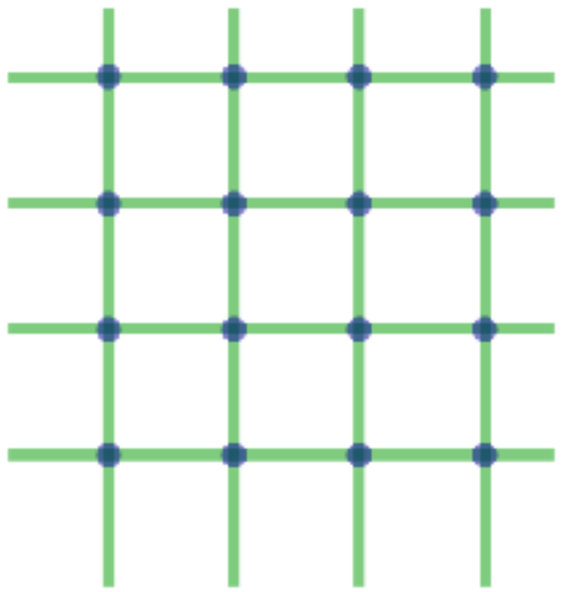
(With : Ètienne L. Hurtubise and R. Moessner)

Charge Response in Magnetic Insulators



Consider 1/2 filling (1 e/site)

$$\mathcal{H} = -t\sum_{ij} c^\dagger_{i\sigma}c_{j\sigma} \quad + \; U\sum_i n_{i\uparrow}n_{i\downarrow}$$

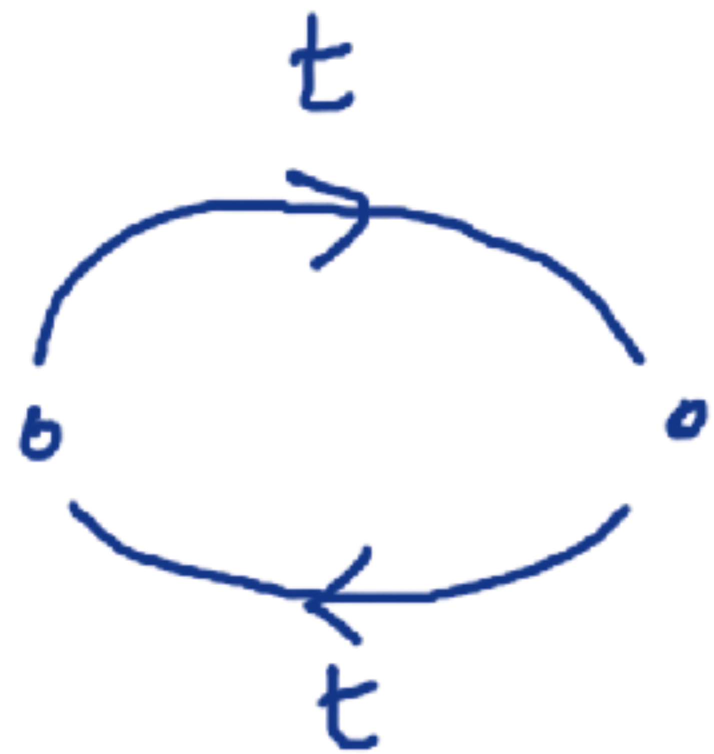
$$\frac{t}{U} > 1: \ \text{metal}$$

$$\frac{t}{U} < 1: \ \text{Insulator}$$

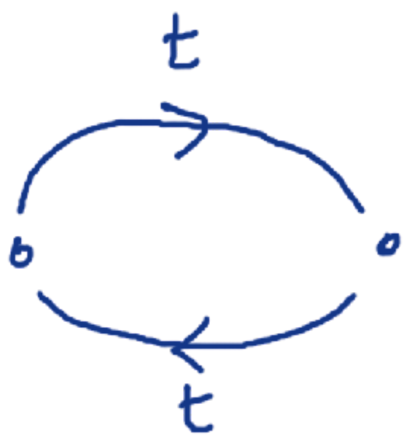
With S=1/2 magnetic moment/site

Virtual Charge fluctuations and effective low energy energetics

$$\mathcal{H}_{eff} = J \sum_{ij} \mathbf{s}_i \cdot \mathbf{s}_j + \cdots \qquad J \sim \frac{t^2}{U}$$



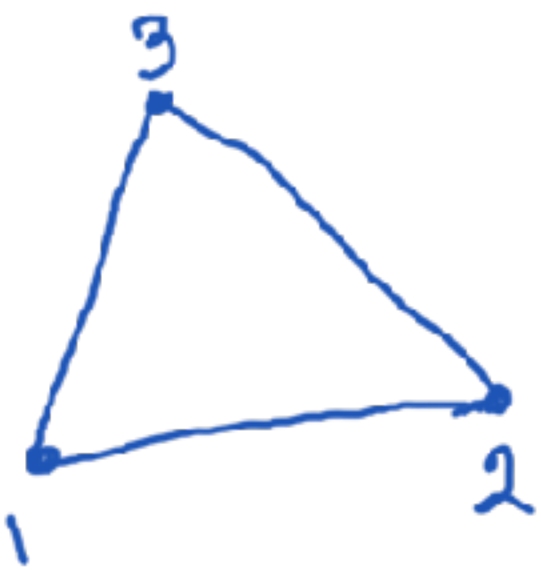
Virtual Charge fluctuations : electric polarisation



On-site charge operator

$$\hat{n}_i = \sum_{\sigma} c^{\dagger}_{i\sigma} c_{i\sigma}$$

$$\hat{n}_i^{eff} = 1 - \frac{t^3}{U^3} f(\mathbf{s})$$



$$\hat{n}_1^{eff} = 1 - 8\frac{t^3}{U^3} \left(\mathbf{s_1} \cdot (\mathbf{s_2} + \mathbf{s_3}) - \mathbf{2 s_2} \cdot \mathbf{s_3}\right)$$

[Bulaevskii et. al., 2008]

Such Charge fluctuations can lead to electric polarisation moment (also contribution from lattice distortion)

Electric polarisation : From symmetry considerations

- The form of such electric polarisation moment is essentially fixed by lattice symmetries (upto coupling constants)

- Anything that transforms as a 3-vector (in three dim.) and is

    - even under time-reversal

    - odd under inversion

Electric polarisation : magneto-electric effect

$$\mathcal{H}_{pert} = -\mathbf{p} \cdot \mathbf{E}$$

Electric polarisation : Multiferroics

(Spontaneously/explicitly Broken inversion symmetry)

Electric polarisation : Beyond Multiferroics

(NO Spontaneously/explicitly Broken inversion symmetry)

Electric field effects in magnetic insulators

(With inversion symmetry)

$$\mathcal{H}_{eff} = J \sum_{ij} \mathbf{s}_i \cdot \mathbf{s}_j + \cdots$$

$$\mathcal{H}_{pert} = -\mathbf{p} \cdot \mathbf{E}$$

What is the effect ?

If the magnetic systems order then such effects,

though interesting,

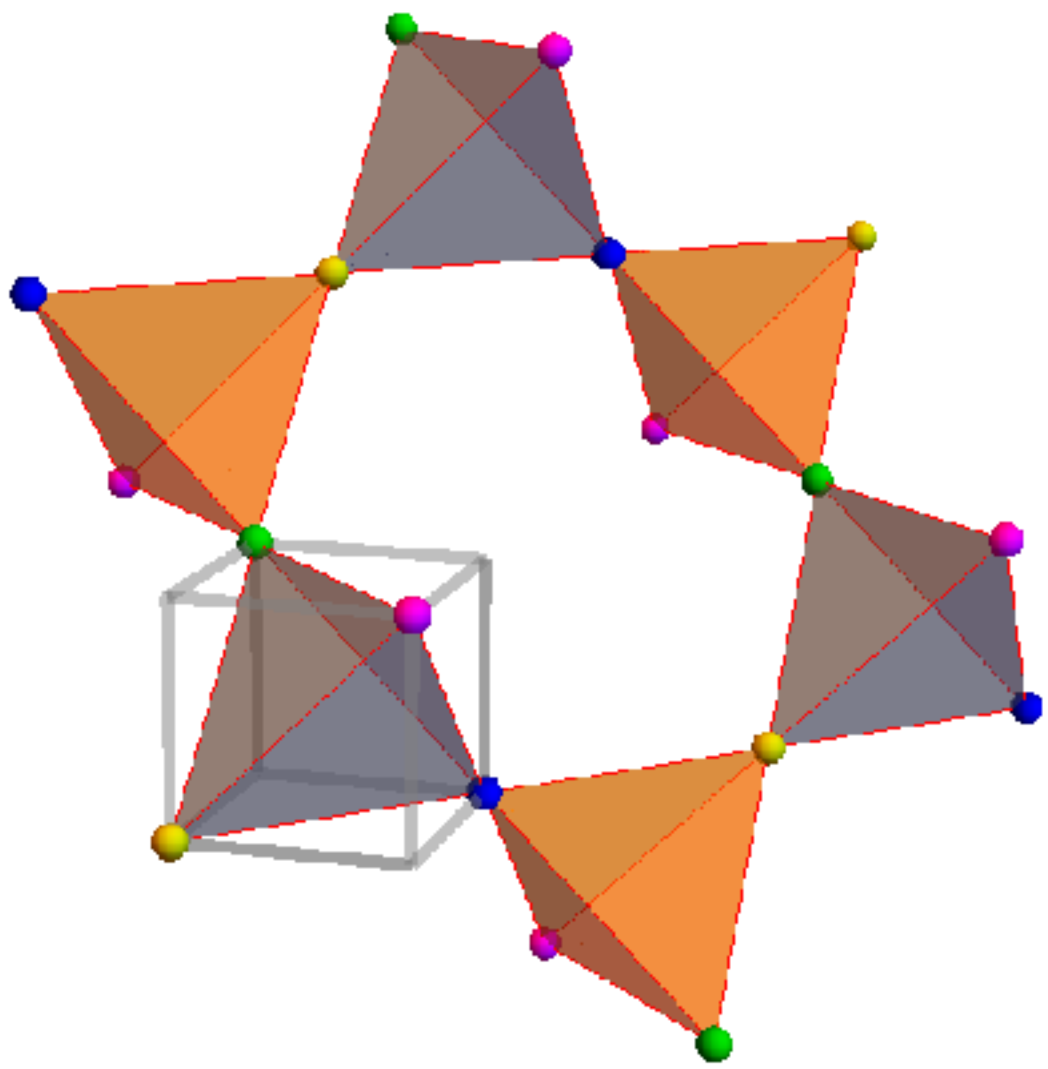
are secondary in a sense that there are other prominent

effect that characterises the low energy properties

What if the system does not order ?

Quantum spin liquids

Three dimensional U(1) quantum spin liquid on Pyrochlore lattice : Quantum spin ice



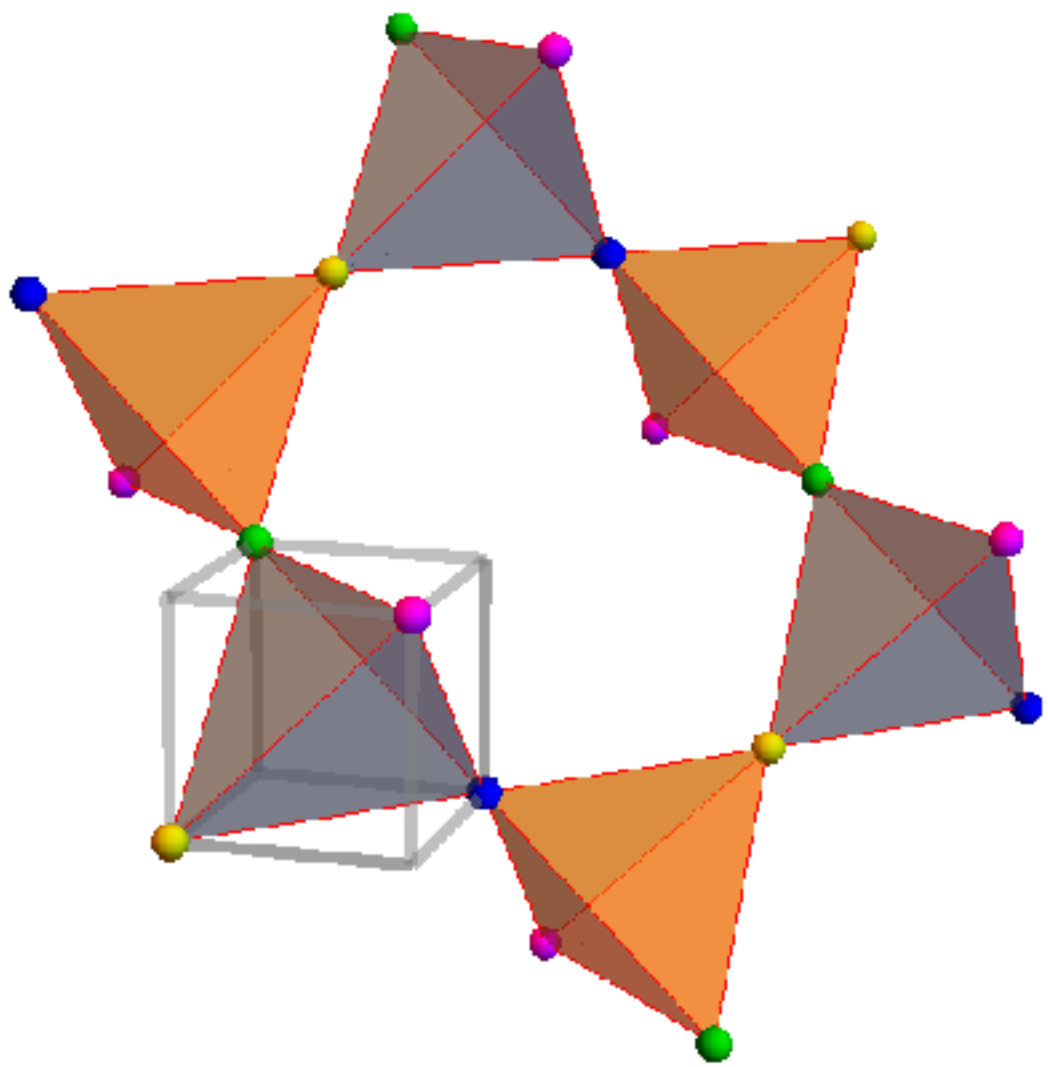
$$H = J_{zz} \sum_{\langle ij \rangle} s_i^z s_j^z$$

$$- J_{\perp} \sum_{\langle ij \rangle} \left(s_i^+ s_j^- + h.c. \right)$$

$$J_{zz} \gg J_{\pm}$$

[Hermele et. al., 2002]

Three dimensional U(1) quantum spin liquid on Pyrochlore lattice : Quantum spin ice

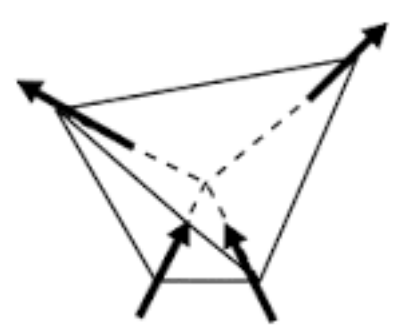


$$H = J_{zz} \sum_{\langle ij \rangle} s_i^z s_j^z$$

Corner sharing geometry

$$H = \frac{J}{2} \sum_{\boxtimes} \left(\sum_{i \in \boxtimes} s_i^z \right)^2 + const.$$

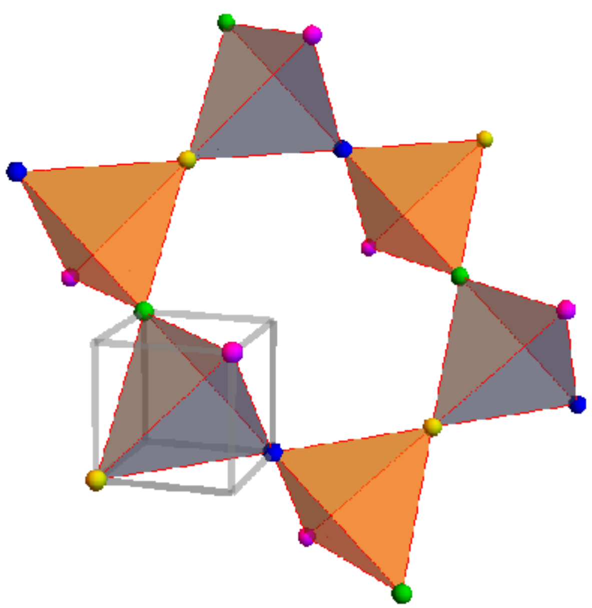
$$\sum_{i \in \boxtimes} s_i^z = 0 \quad \forall \boxtimes$$

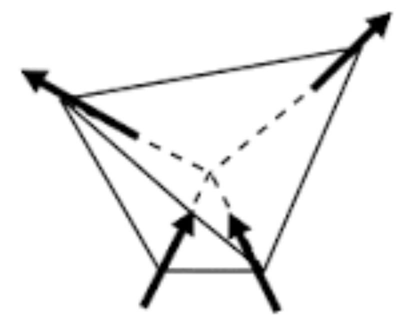


two-in-two-out-rule

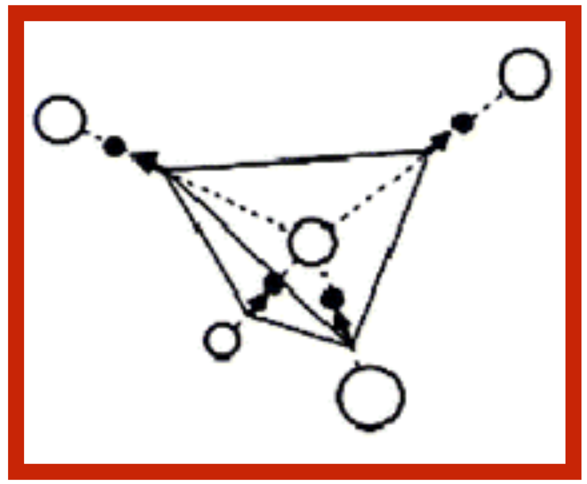
Spin-ice : Ice rules

two-in-two-out-rule





ice

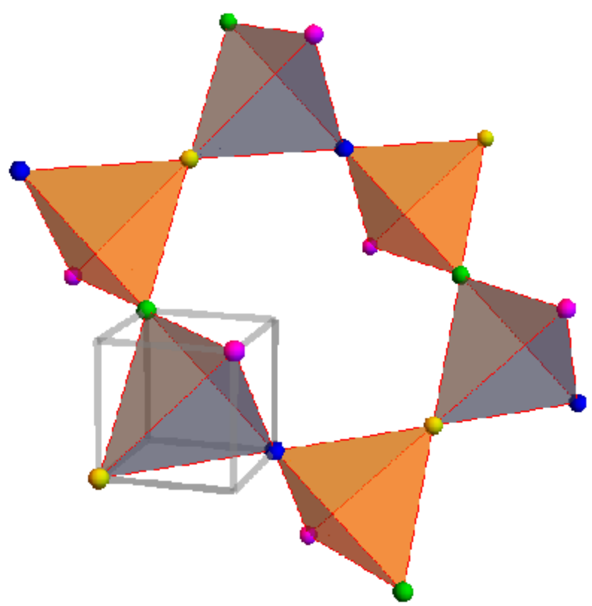


[Pauling, 1956]

Macroscopically degenerate ground state manifold

T= 0 Entropy $S_0 \approx \frac{1}{2} \ln 1.5 \text{ per site}$

Spin-ice : Quantum effects

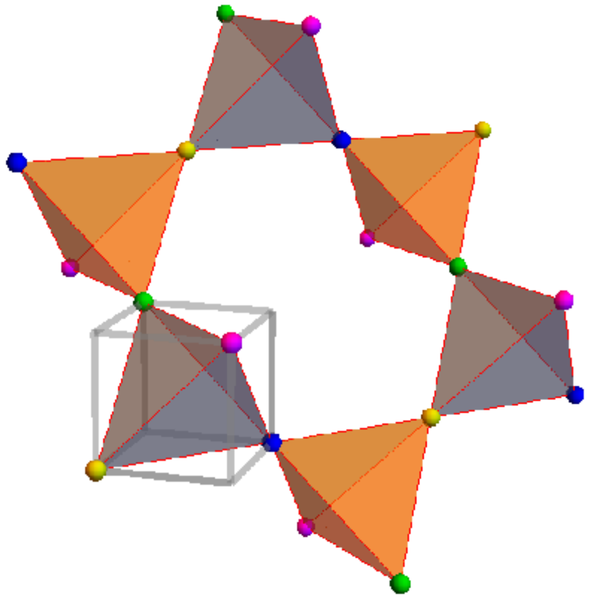


$$-J_{\perp} \sum_{\langle ij \rangle} (s_i^+ s_j^- + h.c.)$$

Two ways to quench the zero temperature entropy

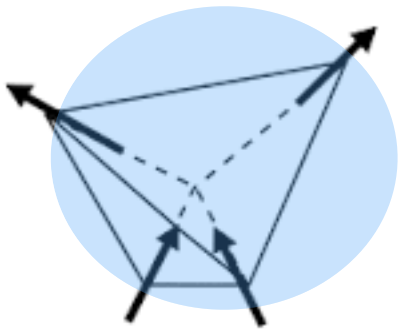
- Quantum order by disorder (quench by ordering)
- Quantum spin liquid (quench by long range quantum entanglement $\Longrightarrow$ Topological order)

Spin-ice : Ice rules



two-in-two-out-rule

= zero flux

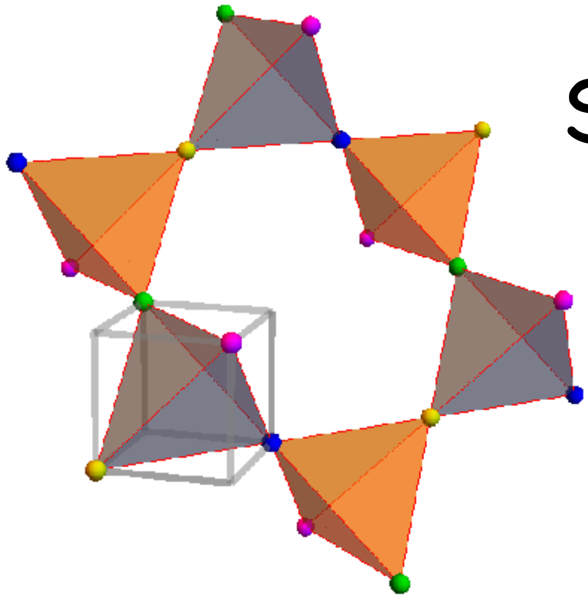


$$\sum_{i\in\boxtimes} s_i^z = 0 \quad \forall \boxtimes$$

Emergent Magnetic Field

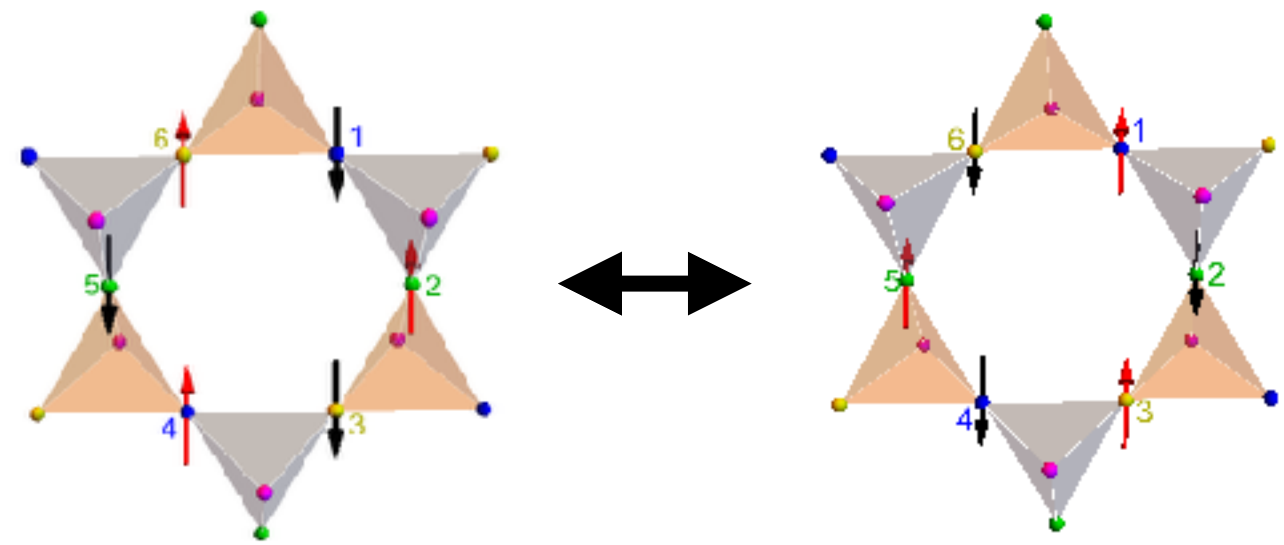
$$\nabla \cdot \mathbf{b} = 0$$

Gauss's Law



Spin-ice : Quantum Dynamics $J_\perp \sum_{\langle ij \rangle} (s_i^+ s_j^- + h.c.)$

spin-flips along loops



Gauss's Law

$$\nabla \cdot \mathbf{b} = 0$$

$$\sim \frac{J_\perp^3}{j_{zz}^2} s_1^+ s_2^- s_3^+ s_4^- s_5^+ s_6^-$$

Emergent Electric field

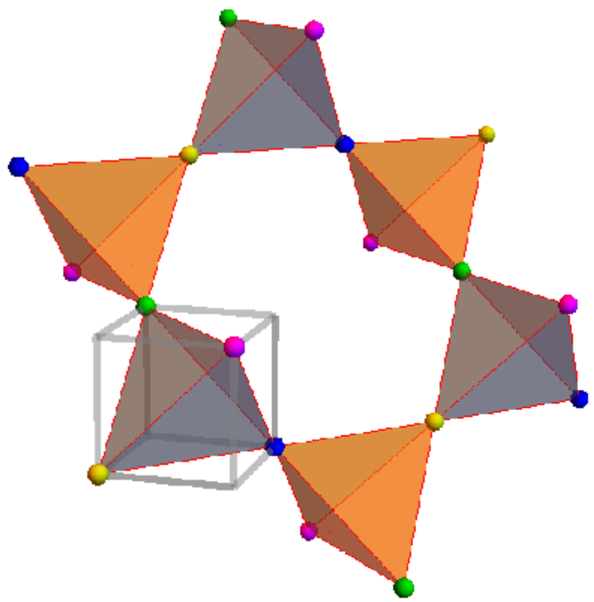
Single spin flips
violates Gauss's Law
(magnetic monopole)

[Castelnovo et. al. 2008]

$$\rightarrow \quad \frac{J_\perp^3}{J_{zz}^2} \cos\left(\mathbf{e} \cdot \hat{\mathbf{n}}\right)$$

Quantum spin ice : Emergent quantum electrodynamics

(without charges)

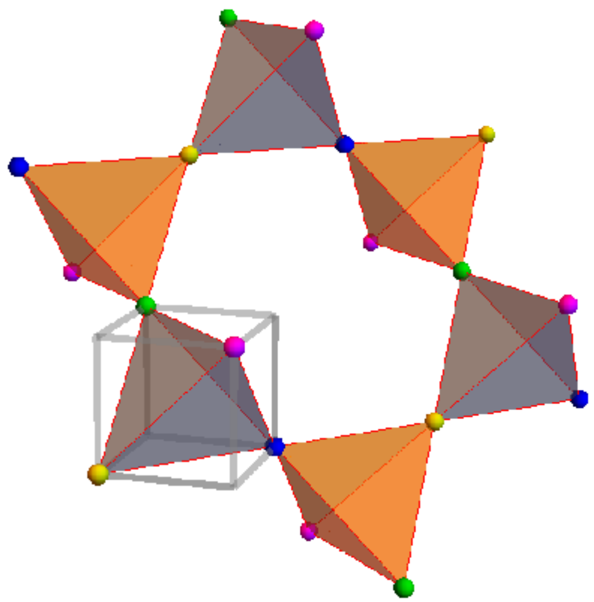


$$\mathcal{H}_{\rm eff} = \frac{U}{2}\sum_{\langle \mathbf{rr}' \rangle} b_{\mathbf{rr}'}^2 - \frac{J_{\pm}^3}{J_{zz}^2}\sum_{\langle \mathbf{ss}' \rangle} \cos[e_{\mathbf{ss}'}]$$

Compact U(1) Gauge theory in 3D

- Confined (area) phase : Some ordered phase
- Deconfined (perimeter) phase : U(1) QSL

Quantum spin ice : Emergent quantum electrodynamics



(without charges)

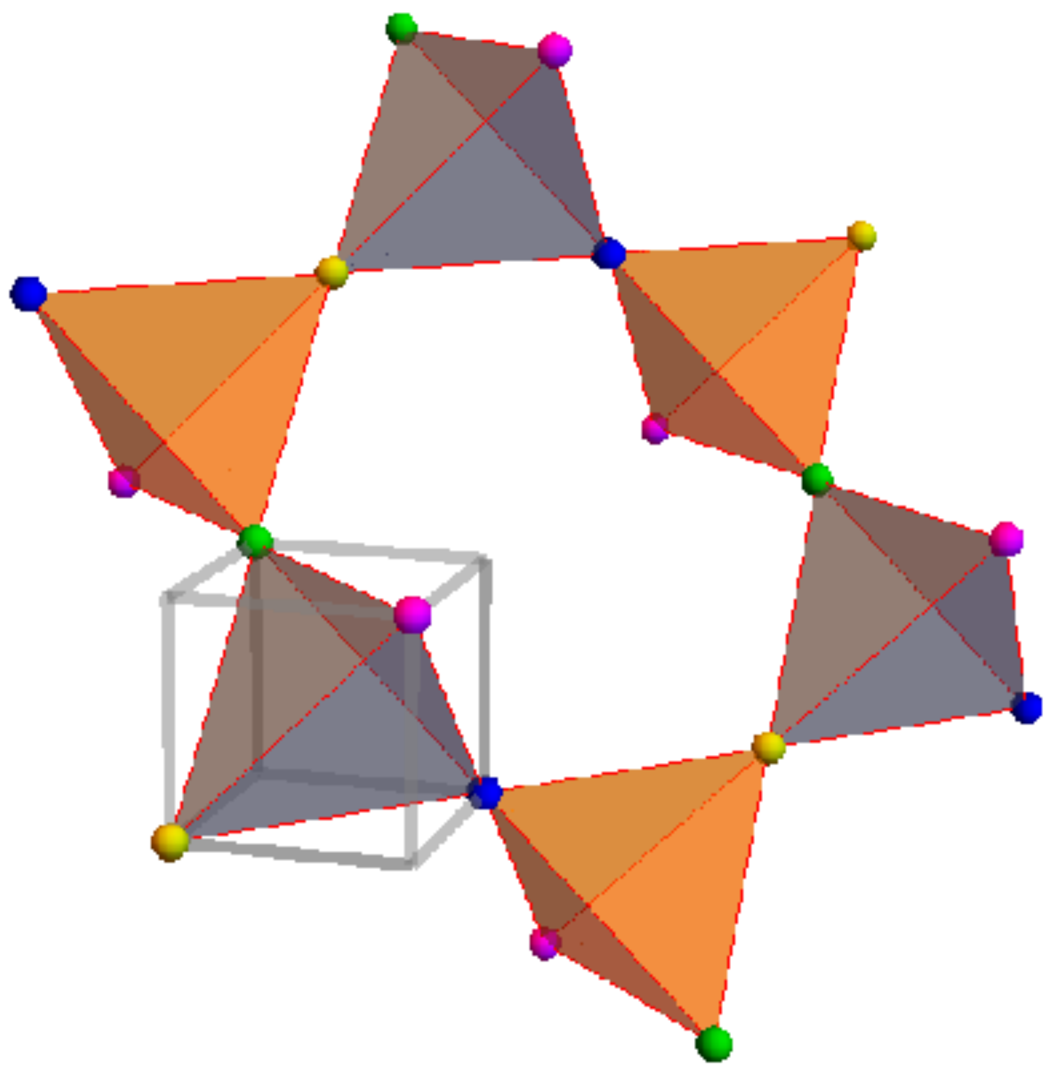
$$\mathcal{H}_{\text{eff}} = \frac{U}{2} \sum_{\langle \mathbf{rr}' \rangle} b_{\mathbf{rr}'}^2 - \frac{J_{\pm}^3}{J_{zz}^2} \sum_{\langle \mathbf{ss}' \rangle} \cos[e_{\mathbf{ss}'}]$$

Compact U(1) Gauge theory in 3D

Deconfined (perimeter) phase : U(1) QSL

- Emergent gauge invariance (at low energy)
- Gapless emergent gauge boson (photon)

Quantum spin ice in external Electric field



$$H = J_{zz} \sum_{\langle ij \rangle} s_i^z s_j^z$$

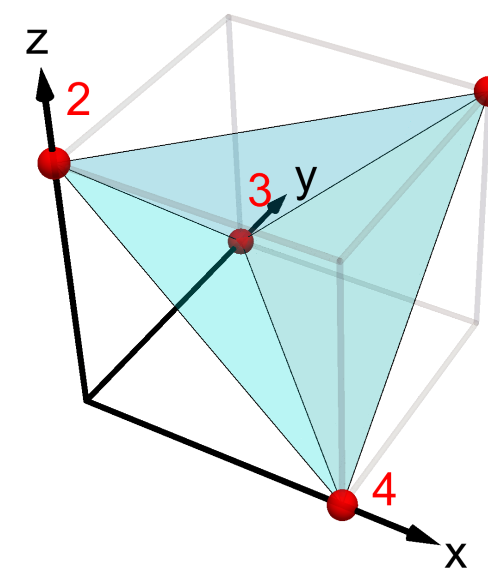
$$- J_{\perp} \sum_{\langle ij \rangle} \left(s_i^+ s_j^- + h.c. \right)$$

$$\mathcal{H}_{pert} = -\mathbf{p} \cdot \mathbf{E}$$

Electric polarisation operator

$$\mathbf{P} = \mathbf{P}^{(L)} + \mathbf{P}^{(T)}$$

$$P_x^{(L)} = A(s_1^z s_4^z - s_2^z s_3^z)$$
$$P_y^{(L)} = A(s_1^z s_3^z - s_2^z s_4^z)$$
$$P_z^{(L)} = A(s_1^z s_2^z - s_3^z s_4^z)$$

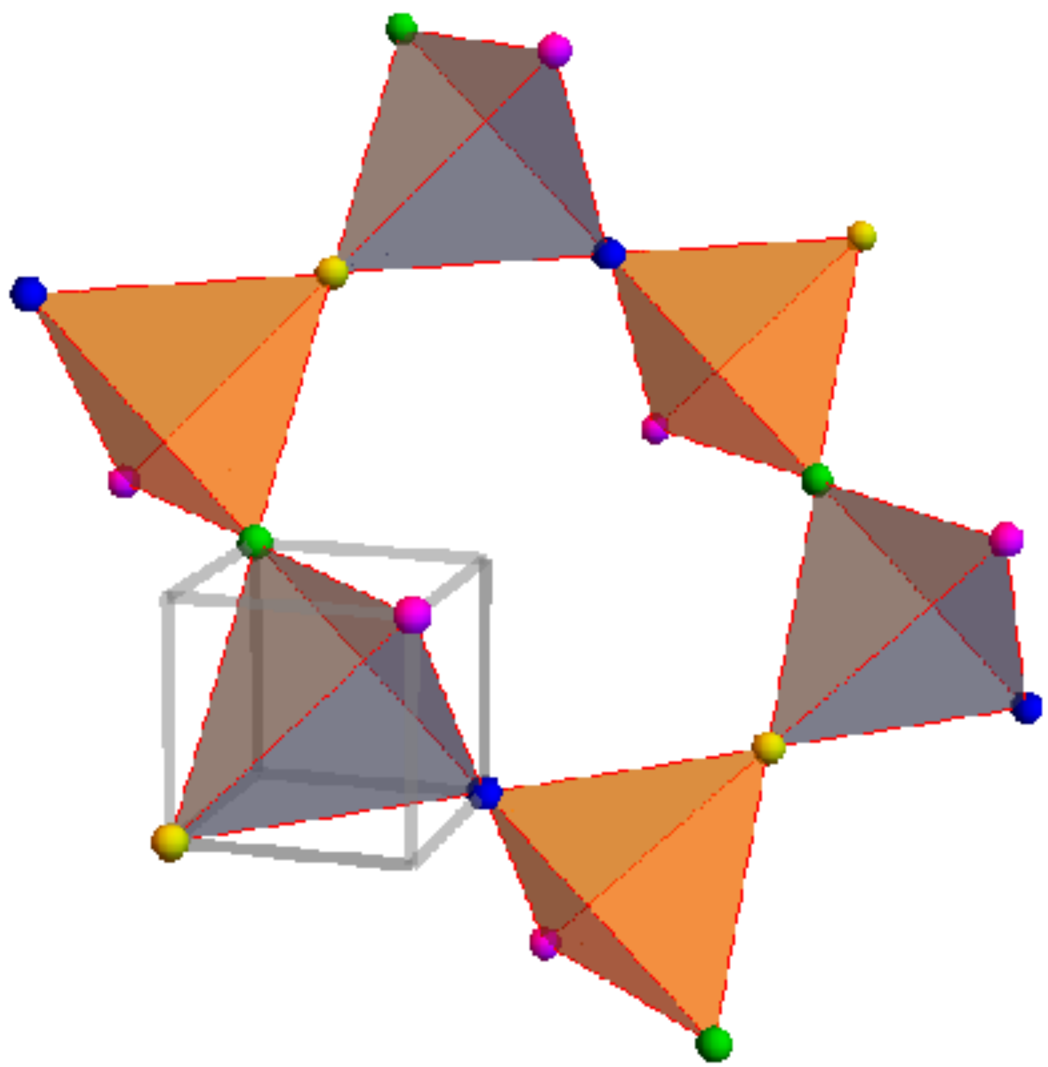


$$\begin{aligned}
P_x^{(T)} =& L(-s_1^x - s_4^x + s_2^x + s_3^x) + B\left[(s_1^+ s_4^- + h.c.) - (s_2^+ s_3^- + h.c.)\right] \\
&+ C\left[(\omega s_1^+ s_4^+ + h.c.) - (\omega s_2^+ s_3^+ + h.c.)\right] + D\left[\{s_1^z(\omega^2 s_4^+ + \omega s_4^-) + (1 \leftrightarrow 4)\} - \{s_2^z(\omega^2 s_3^+ + \omega s_3^-) + (2 \leftrightarrow
\end{aligned}$$

$$\begin{aligned}
P_y^{(T)} =& L\left[\frac{1}{2}(s_1^x + s_3^x - s_2^x - s_4^x) - \frac{\sqrt{3}}{2}(s_1^y + s_3^y - s_2^y - s_4^y)\right] + B\left[(s_1^+ s_3^- + h.c.) - (s_2^+ s_4^- + h.c.)\right] \\
&+ C\left[(\omega^2 s_1^+ s_3^+ + h.c.) - (\omega^2 s_2^+ s_4^+ + h.c.)\right] + D\left[\{s_1^z(\omega s_3^+ + \omega^2 s_3^-) + (1 \leftrightarrow 3)\} - \{s_2^z(\omega s_4^+ + \omega^2 s_4^-) + (2
\end{aligned}$$

$$\begin{aligned}
P_z^{(T)} =& L\left[\frac{1}{2}(s_1^x + s_2^x - s_3^x - s_4^x) + \frac{\sqrt{3}}{2}(s_1^y + s_2^y - s_3^y - s_4^y)\right] + B\left[(s_1^+ s_2^- + h.c.) - (s_3^+ s_4^- + h.c.)\right] \\
&+ C\left[(s_1^+ s_2^+ + h.c.) - (s_3^+ s_4^+ + h.c.)\right] + D\left[\{s_1^z(s_2^+ + s_2^-) + (1 \leftrightarrow 2)\} - \{s_3^z(s_4^+ + s_4^-) + (3 \leftrightarrow 4)\right]
\end{aligned}$$

Quantum spin ice in external Electric field



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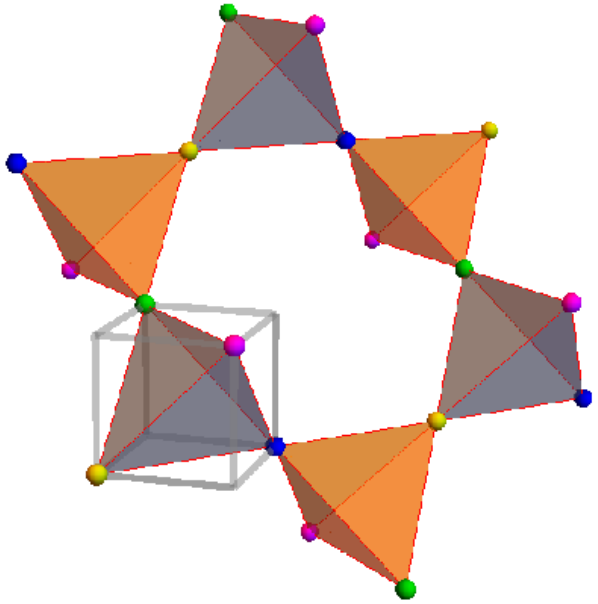
$$- J_{\perp} \sum_{\langle ij \rangle} \left(s_i^+ s_j^- + h.c. \right)$$

$$\mathcal{H}_{pert} = -\mathbf{p} \cdot \mathbf{E}$$

calculate the Effective Hamiltonian in the limit

$$J_{zz} \gg J_{\perp}, |\mathbf{E}|$$



Quantum spin ice in external Electric field

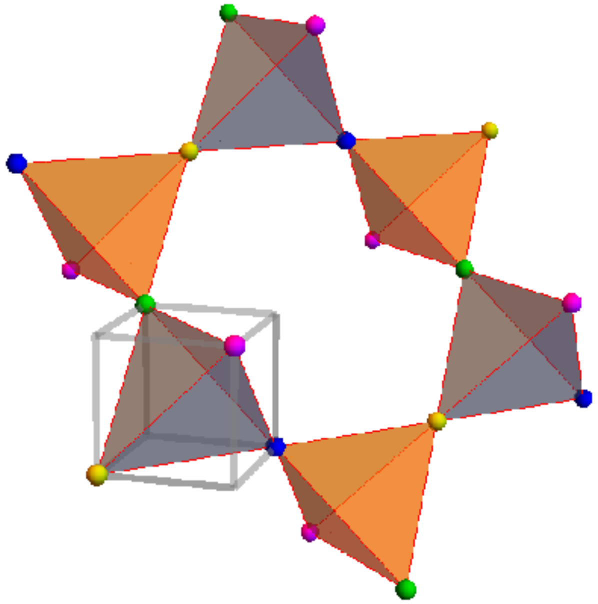
Effective Hamiltonian

$$\mathcal{H}_{\rm eff} = \frac{U}{2} \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} b_{\mathbf{r}\mathbf{r}'}^2 - \frac{J_\pm^3}{J_{zz}^2} [1-\chi] \sum_{\langle \mathbf{s}\mathbf{s}' \rangle} \cos[e_{\mathbf{s}\mathbf{s}'}]$$

$$- \frac{3J_\pm^3 \chi}{J_{zz}^2} \sum_{m=1}^{4} (\hat{\mathbf{E}} \cdot \hat{\mathbf{t}}_m)^2 \sum_{\langle \mathbf{s}\mathbf{s}' \rangle \| \hat{\mathbf{t}}_m} \cos[e_{\mathbf{s}\mathbf{s}'}]$$

$$\chi \sim \frac{|\mathbf{E}|^2}{J_\perp^2}$$

Ratio of strengths of external electric field and quantum term



Quantum spin ice in external Electric field

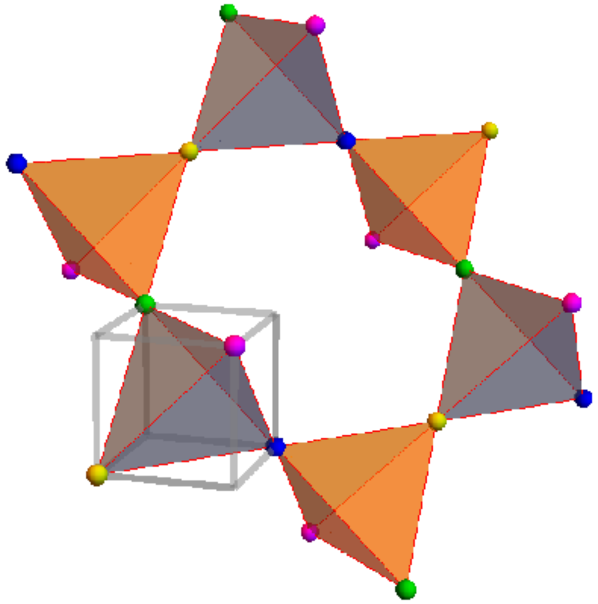
Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{U}{2}\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} b^2_{\mathbf{r}\mathbf{r}'} - \frac{J_\pm^3}{J_{zz}^2}\left[1-\chi\right]\sum_{\langle \mathbf{s}\mathbf{s}'\rangle}\cos[e_{\mathbf{s}\mathbf{s}'}]$$

$$- \frac{3J_\pm^3\chi}{J_{zz}^2}\sum_{m=1}^{4}(\hat{\mathbf{E}}\cdot\hat{\mathbf{t}}_m)^2 \sum_{\langle \mathbf{s}\mathbf{s}'\rangle\|\hat{\mathbf{t}}_m}\cos[e_{\mathbf{s}\mathbf{s}'}]$$

$$\chi \sim \frac{|\mathbf{E}|^2}{J_\perp^2}$$

$\chi < \chi_0$: Expand the cosine about zero



Quantum spin ice in external Electric field

Effective Hamiltonian

$$\chi \sim \frac{|\mathbf{E}|^2}{J_\perp^2}$$

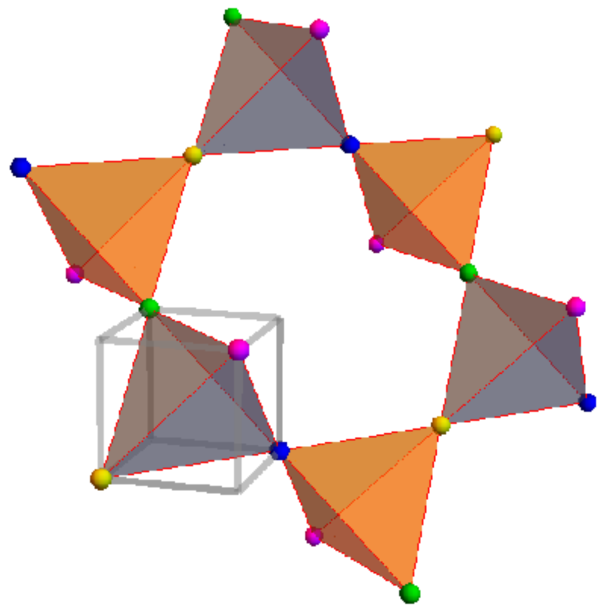
$$\mathcal{H}_{\rm eff} = \frac{U}{2}\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} b^2_{\mathbf{r}\mathbf{r}'} - \frac{J_\pm^3}{J_{zz}^2}\left[1-\chi\right]\sum_{\langle \mathbf{s}\mathbf{s}'\rangle}\cos[e_{\mathbf{s}\mathbf{s}'}]$$
$$- \frac{3J_\pm^3\chi}{J_{zz}^2}\sum_{m=1}^{4}(\hat{\mathbf{E}}\cdot\hat{\mathbf{t}}_m)^2 \sum_{\langle \mathbf{s}\mathbf{s}'\rangle \| \hat{\mathbf{t}}_m}\cos[e_{\mathbf{s}\mathbf{s}'}]$$

$\chi < \chi_0$

$$\mathcal{H}_{\rm eff} = \frac{U}{2}\sum_{\mathbf{r},n} b^2_{\mathbf{r},n} + \frac{1}{2}\sum_{\mathbf{s},m}\mathcal{M}_{mm} e^2_{\mathbf{s},m}$$

$$\mathcal{M}_{mm} = \frac{J_\pm^3}{J_{zz}^2}\left(1-\chi+3\chi(\hat{\mathbf{E}}\cdot\hat{\mathbf{t}}_m)^2\right)$$

Quantum spin ice in external Electric field

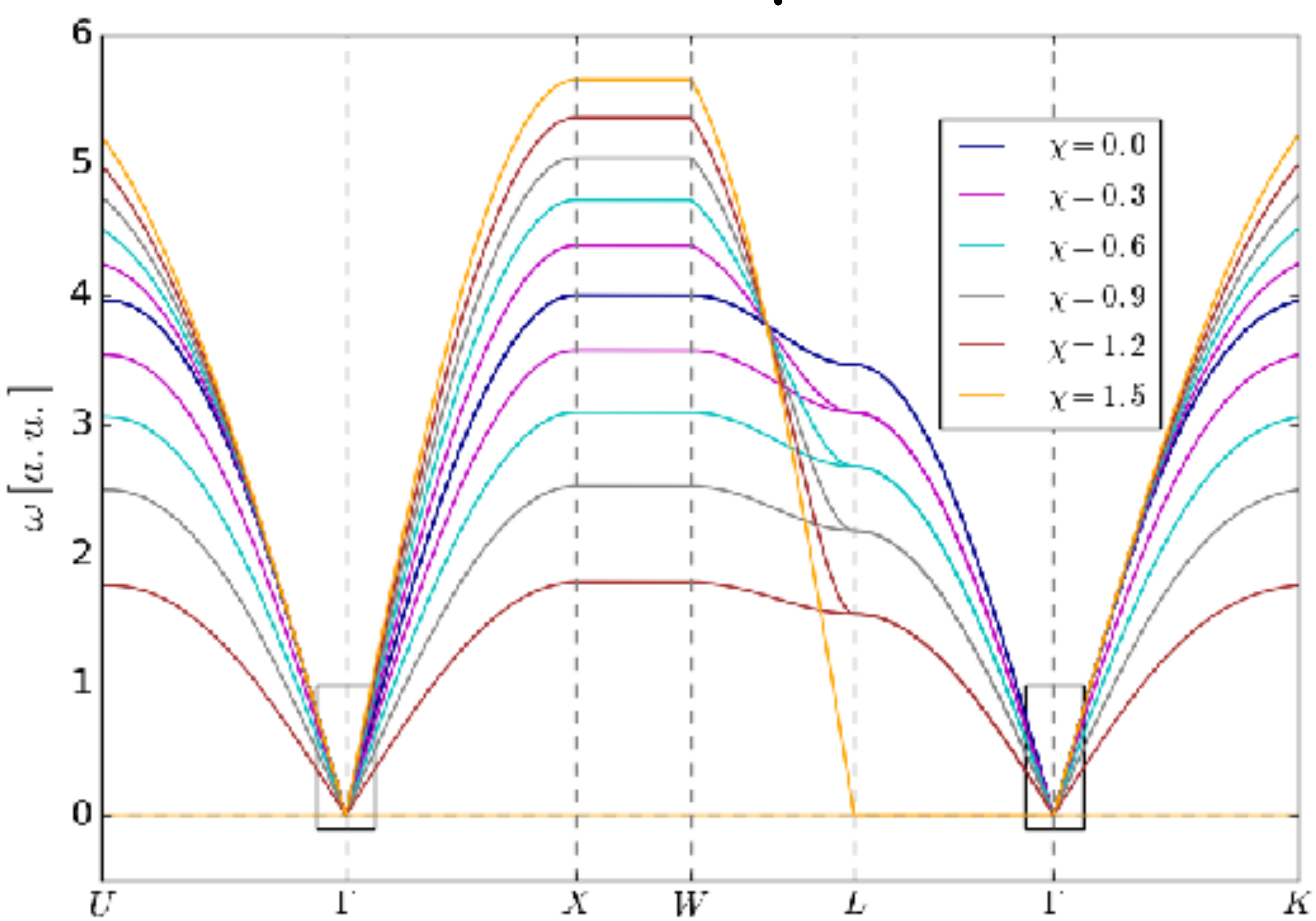


$$\chi \sim \frac{|\mathbf{E}|^2}{J_\perp^2} \qquad \chi < \chi_0$$

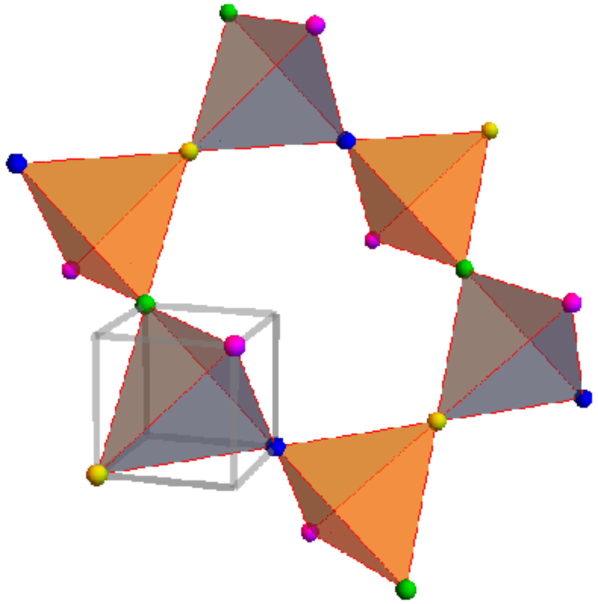
$$\mathcal{H}_{\text{eff}} = \frac{U}{2}\sum_{\mathbf{r},n} b_{\mathbf{r},n}^2 + \frac{1}{2}\sum_{\mathbf{s},m} \mathcal{M}_{mm} e_{\mathbf{s},m}^2$$

$$\mathcal{M}_{mm} = \frac{J_\pm^3}{J_{zz}^2}\left(1 - \chi + 3\chi(\hat{\mathbf{E}}\cdot\hat{\mathbf{t}}_m)^2\right)$$

"Photon" dispersion



- emergent birefringence
- Polarisation and direction dependent velocity of "light"

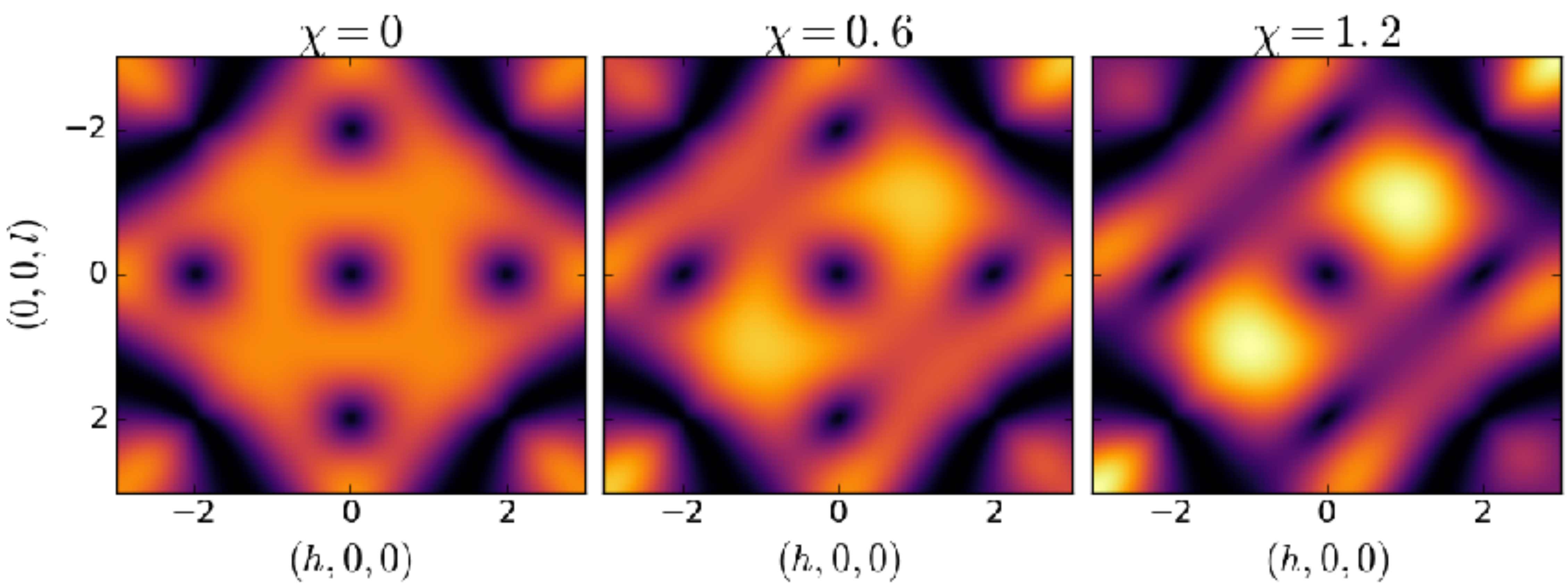


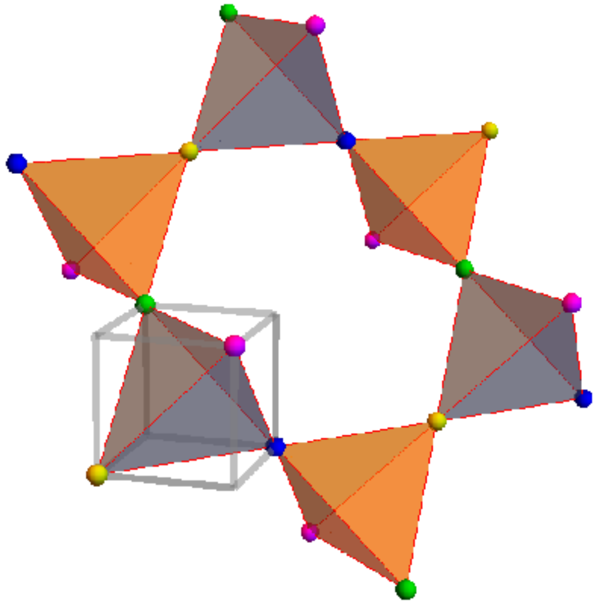
Quantum spin ice in external Electric field

$\chi \sim \frac{|\mathbf{E}|^2}{J_\perp^2}$ $\chi < \chi_0$ $\mathbf{E} \sim [111]$

- Polarisation and direction dependent velocity of "light"
- What happens to underlying spins ?

Equal-time spin structure factor



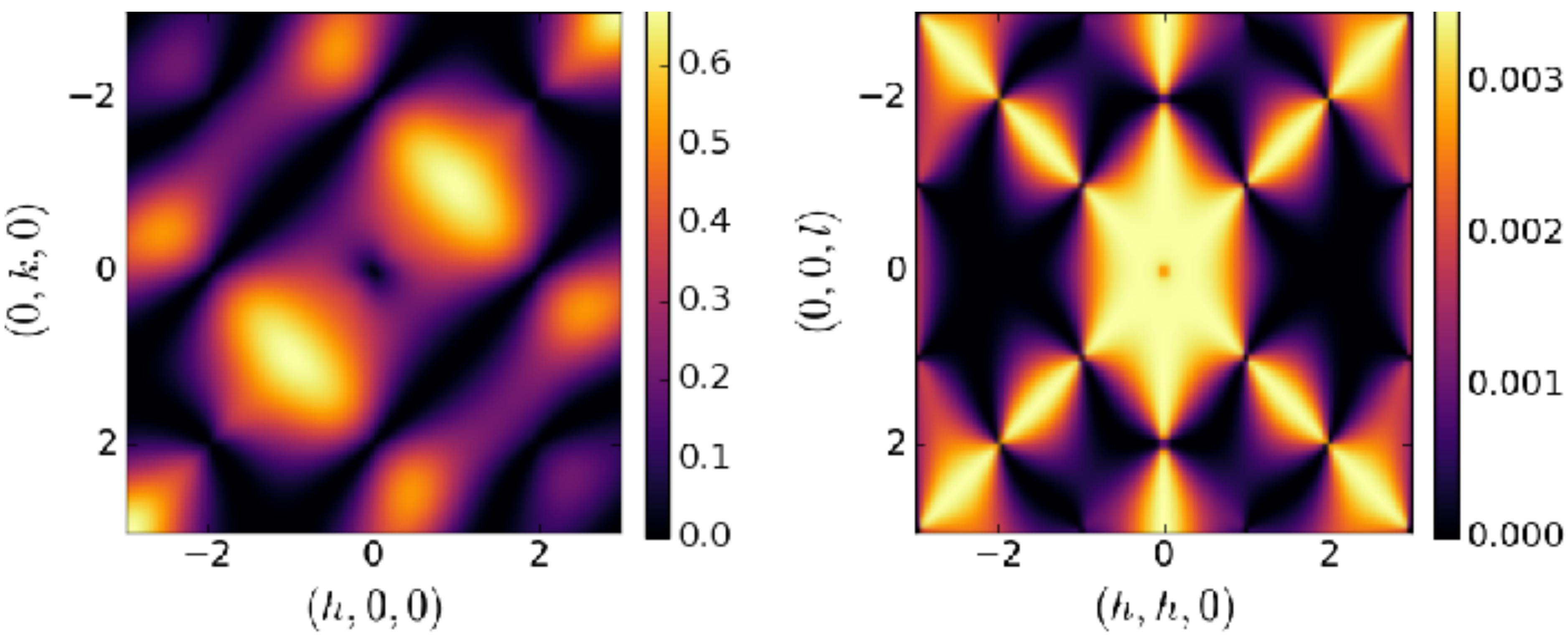


Quantum spin ice in external Electric field

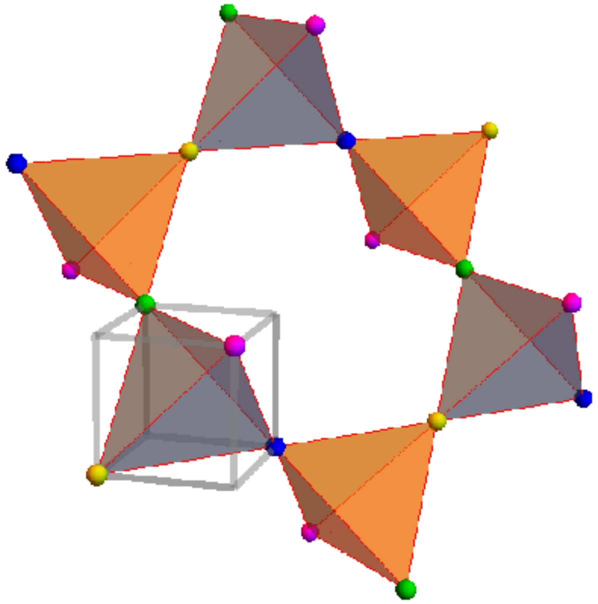
$$\chi \sim \frac{|\mathbf{E}|^2}{J_\perp^2} \qquad \chi \to \chi_0 \qquad \mathbf{E} \sim [111]$$

- Polarisation and direction dependent velocity of "light"
- What happens to underlying spins ?

Equal-time spin structure factor



pinch-points



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Effective Hamiltonian

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$$- \frac{3J_\pm^3\chi}{J_{zz}^2}\sum_{m=1}^{4}(\hat{\mathbf{E}}\cdot\hat{\mathbf{t}}_m)^2 \sum_{\langle \mathbf{ss}'\rangle \| \hat{\mathbf{t}}_m}\cos[e_{\mathbf{ss}'}]$$

$$\chi \sim \frac{|\mathbf{E}|^2}{J_\perp^2}$$

$\chi > \chi_0$: Expand the cosine about π

$\Downarrow$

Drives transition to a U(1) QSL with electric flux
(Phase transition between two long ranged entangled states)

Is this observable ?

In candidate materials (rare earth pyrochlores)

$$J_{zz} \sim 1K$$

$$J_{\perp} \sim 100mK$$

$$|\mathbf{E}| \sim 10^4 \ V/mm$$

Thank you