

# Energy equilibration in periodically driven systems

**Kushal Shah**

IIT Delhi

**Work done jointly with :**

Vered Rom-Kedar (Weizmann Institute, Israel)

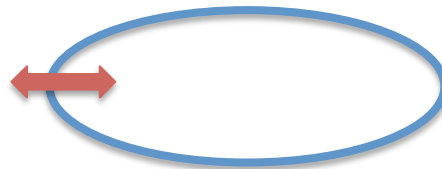
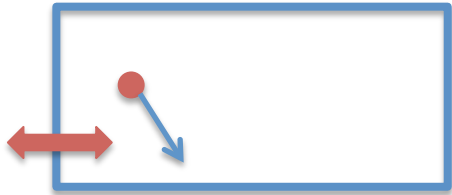
Dmitry Turaev (Imperial College, London)

Vassili Gelfreich (Warwick Univ, UK)

**Funding :** SERB-DST

**ISPC Meeting 2017 @ ICTS-TIFR, Bengaluru**

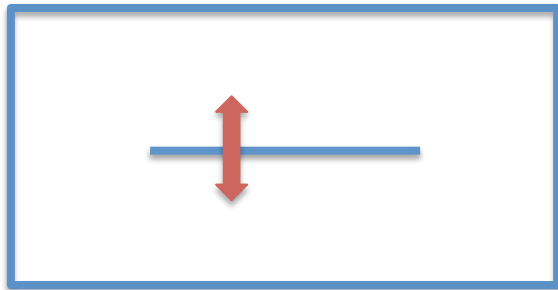
# Fermi Accelerators : Periodically Oscillating Billiards



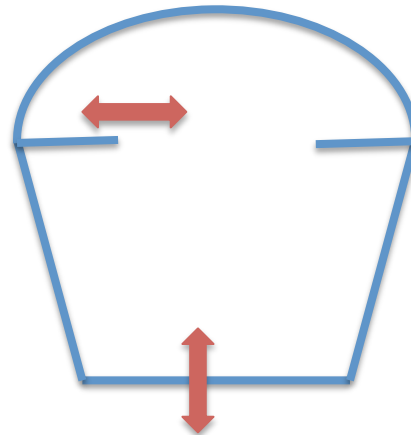
**Integrable:**

1D: Bounded energy

2D: Very slow energy growth

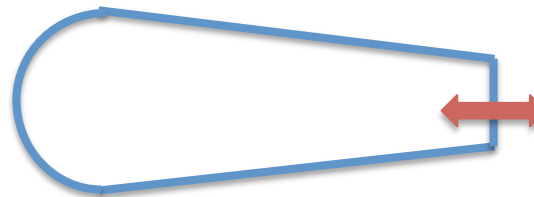
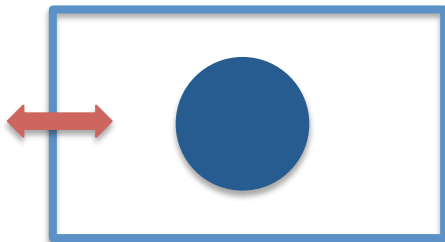


**Pseudo-Integrable**



**Mixed**

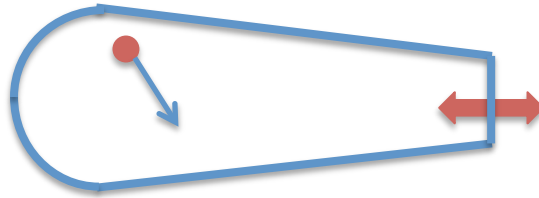
**Exponential  
energy growth**



**Chaotic:  
Quadratic**

# Energy equilibration in slow-fast systems

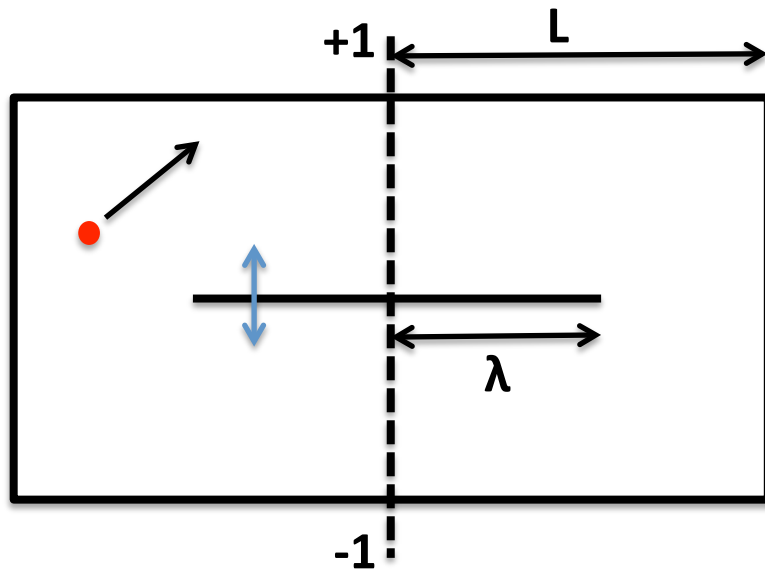
*Presence of adiabatic invariants in ergodic systems prevents equilibration on the time-scale  $(1/m)^{0.5}$*



*On longer time-scales, higher order terms **may** dominate and lead to equilibration*

[A. I. Neishtadt and Y. G. Sinai 2004; P. Wright 2007]

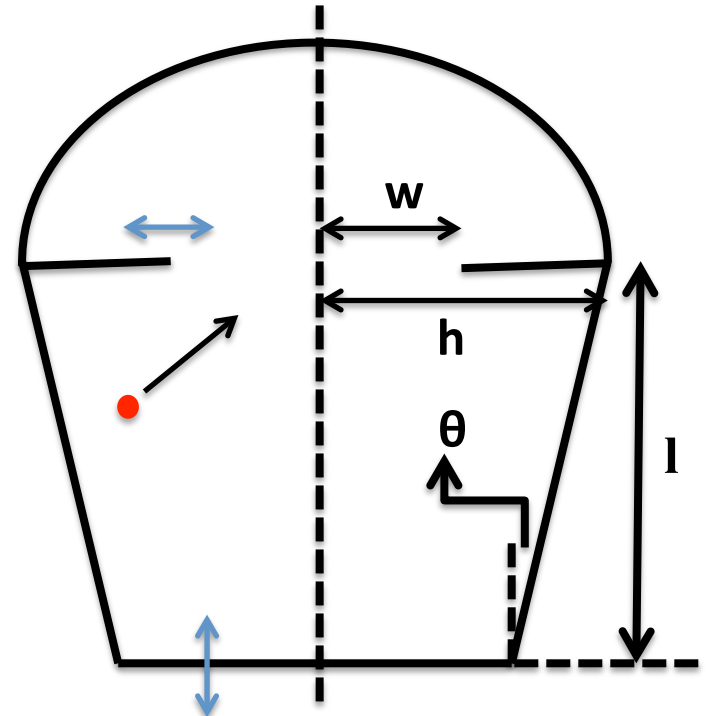
# Energy equilibration : Finite mass effects



**Springy ROB**

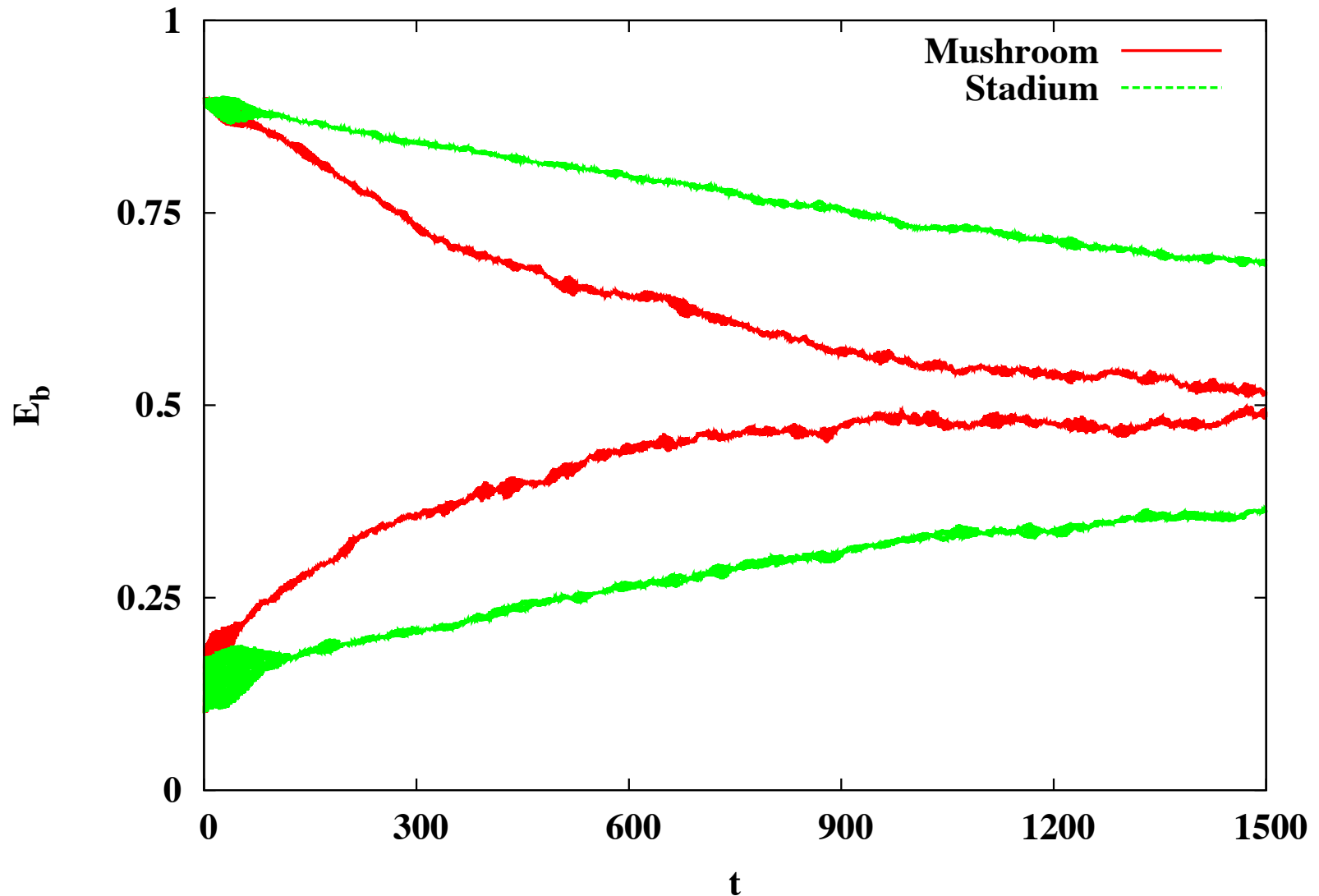
$$\ddot{y}_b + ky_b = F(y_b, E_p) = -\frac{dE_p}{dy_b}$$

$$\text{Anosov-Kasuga Invariant : } J = E_p^{d/2} V_c$$



**Springy Mushroom  
+  
Stadium  
[w=h]**

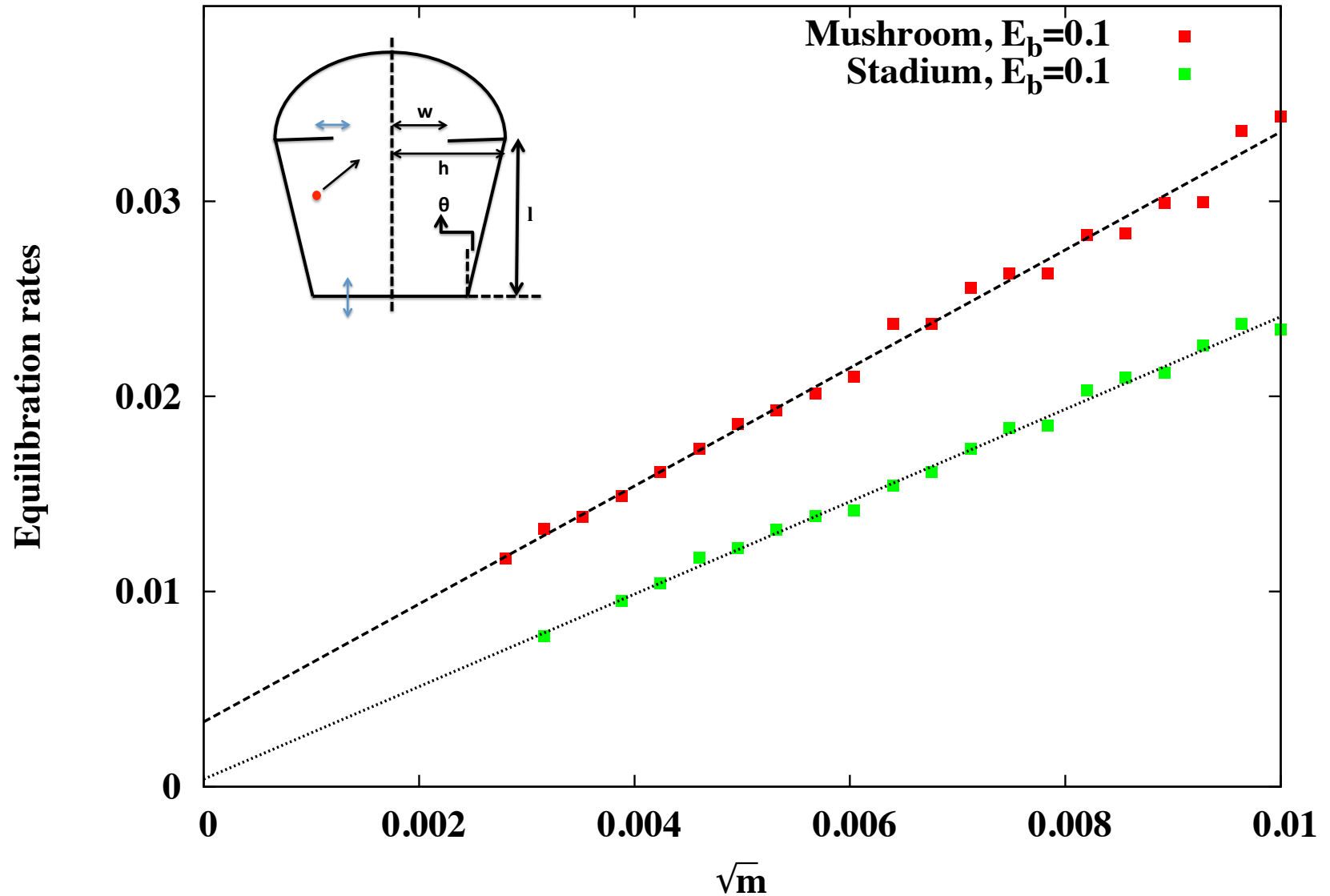
# Energy equilibration in springy Mushroom & Stadium



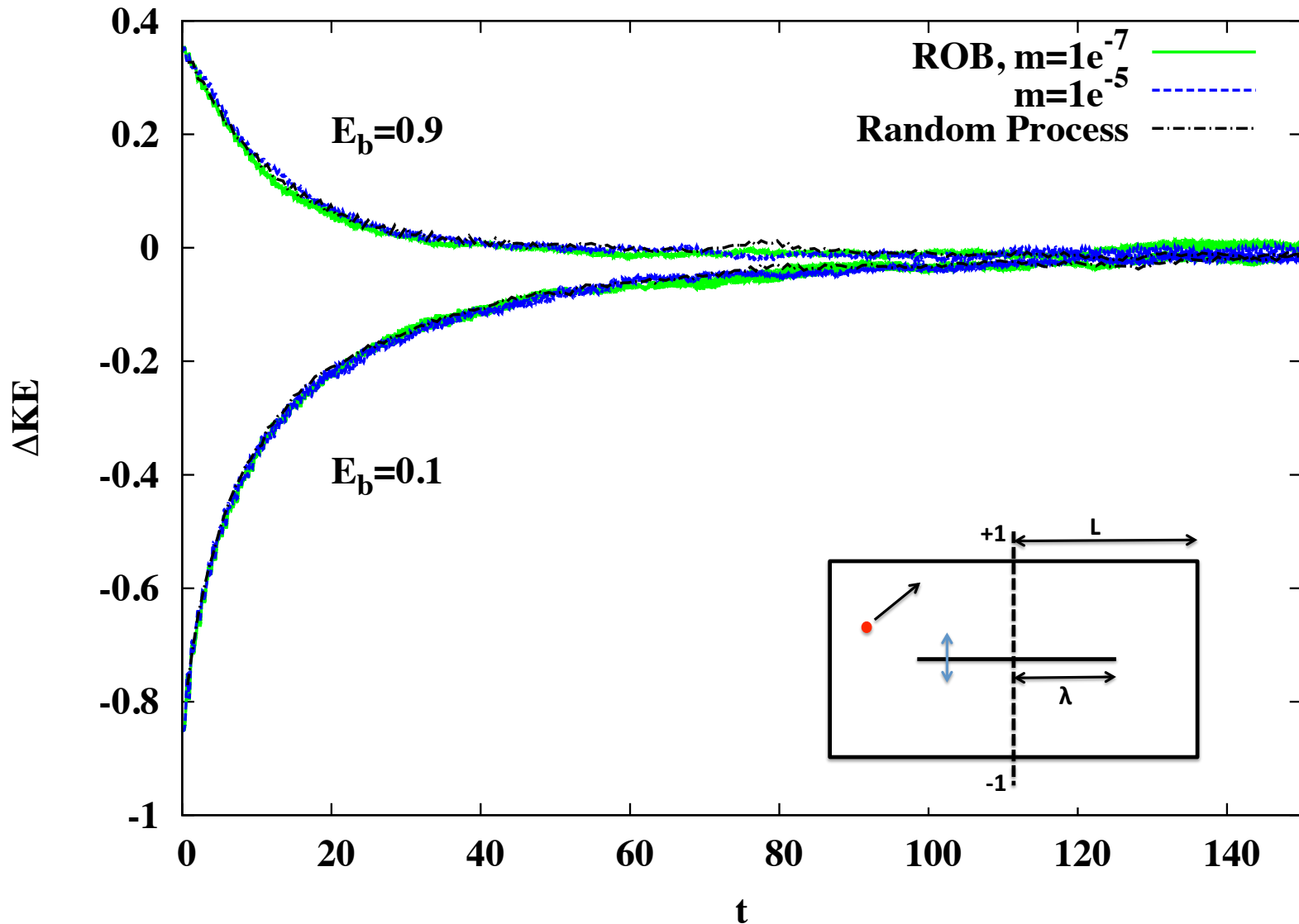
$E_b$  : Energy of oscillating bar

Total Energy,  $E = 1$

# Equilibration rates in springy Mushroom & Stadium



# Energy equilibration in springy ROB



$\Delta KE$  : Difference between KE of bar and particle (vertical)

# Energy equilibration : Finite mass effects

- ❖ Non-ergodic accelerators equilibrate faster than ergodic accelerators
- ❖ Energy equilibration rate remains non-zero for non-ergodic accelerators even when mass ratio tends to zero
  - Is the springy stadium ergodic/mixing?
- ❖ Equilibration rate in ROB accelerator is independent of mass-ratio

## Open Problem :

- ❖ How to incorporate finite mass effects into the random process model for mushroom/stadium?

[manuscript under preparation]