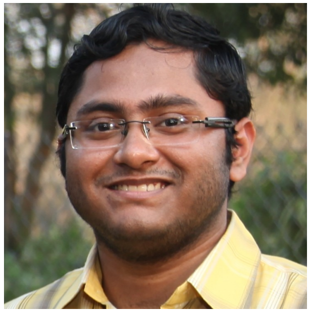
Pairing of renormalized eigenstates in disordered *cuprate* superconductors



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Goal:

- Observed: strongly correlated d-wave superconductors robust to impurities.
  [NJP, *16*, 103018 ('14), PRB, *95*, 14516 ('17] + others

- Can results be grasped in a simple ***pairing theory*** of the single-particle eigenstates of the underlying ***effective*** Hamiltonian?

Recall: for s-wave superconductors, such a pairing idea yields:

Anderson theorem: "As long as the system remains fairly large, no amount of scattering which leaves the substance a metal, would be capable of actually destroying superconductivity".

Strongly correlated d-wave superconductor: $U \gg t$

$$\mathcal{H}_{\rm Hubb} = -t \sum_{\langle ij\rangle\sigma} (c^\dagger_{i\sigma}c_{j\sigma} + h.c.) + U\sum_j n_{j\uparrow}n_{j\downarrow} \Rightarrow \mathcal{H}_{\rm t-J} = -t\sum_{\langle ij\rangle\sigma} \left(\tilde{c}^\dagger_{i\sigma}\tilde{c}_{j\sigma} + \mathrm{h.c.} \right) + \sum_{\langle ij\rangle} J\left(\tilde{\mathbf{S}}_i.\tilde{\mathbf{S}}_j - \frac{\tilde{n}_i\tilde{n}_j}{4}\right)$$

$$+ \text{ “Other terms”}$$

Ph.D. Thesis,
Rajdeep Sensarma (2007)

$$t \rightarrow g_t t \quad \text{with} \quad g_t = \frac{(1-\rho)}{(1-\rho/2)}$$

$$J \rightarrow g_J J \quad \text{with} \quad g_J = (1-\rho/2)^{-2}$$

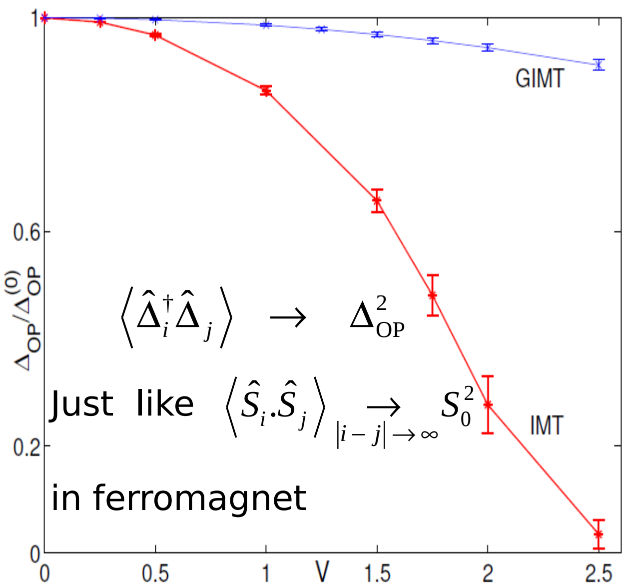
- Renormalized theory
“Gutzwiller Approx”

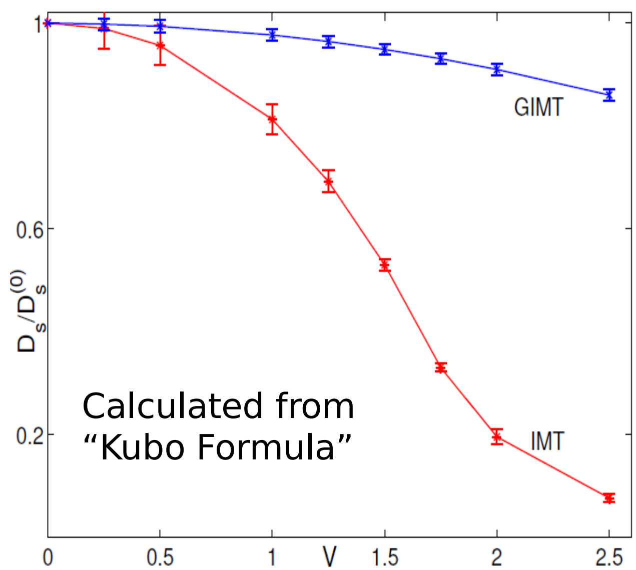
- **Now add disorder:**
  Garg, Randeria, Trivedi, Nat. Phys. ('07)

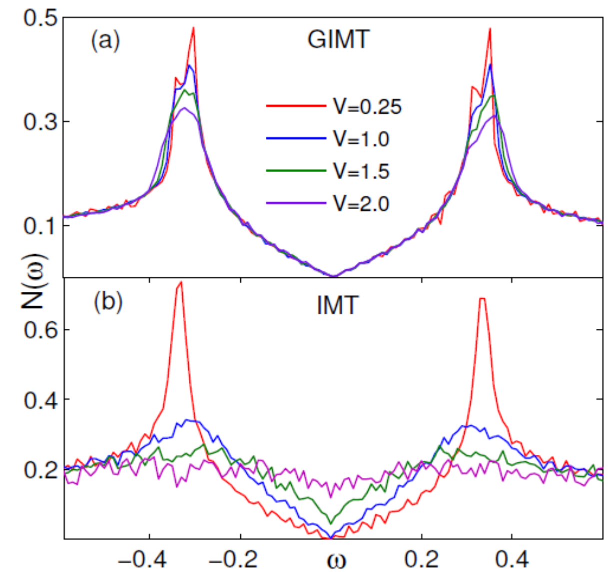
$$+\sum_{i,\sigma} (V_i - \mu)\, n_{i\sigma}$$

Local Gutzwiller factors depend on local density!

- Robustness of strongly correlated dSC to disorder ($V \leq 3t$) in (Gutzwiller + IMT=) GIMT







D. Chakraborty, AG, NJP, **16**, 103018 ('14); D. Chakraborty, R. Sensarma, AG, PRB, **95**, 14516 ('17)

- **Found:** d-SC amazingly rubust to non-magnetic impurities for V ≤ 3t

MFT: Δ ~ T_c is not affected by disorder!

Anderson's Theorem for sSC!

- **Anderson's theorem for cuprates?**

Anderson theorem: "As long as the system remains fairly large, no amount of scattering which leaves the substance a metal, would be capable of actually destroying superconductivity".

Metal: "Normal state" of (sSC)

$$\mathcal{H}_{\mathrm{BCS}} = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} \left(\Delta^{*} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + \mathrm{h.c.}\right) \Rightarrow \mathcal{H}^{\mathrm{NS}}_{\mathrm{BCS}} = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$$

- **<u>What is the "Normal" state of our strongly correlated dSC??</u>**

- **How does our effective one-particle problem look in terms of "Gutzwiller" renormalized parameters?**

"Bare" *H*: $\mathcal{H}_{\mathrm{Hubb}} = -t \sum_{\langle ij \rangle \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_j n_{j\uparrow} n_{j\downarrow}$ $\boxed{+ \sum_{i,\sigma} (V_i - \mu)\, n_{i\sigma}}$

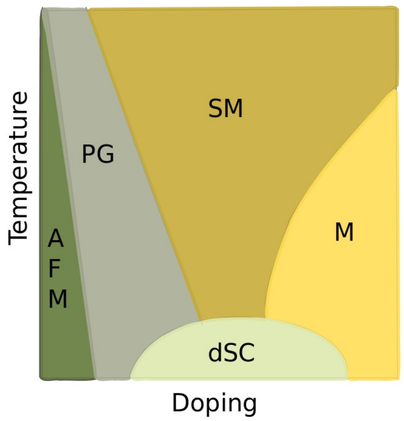
Effective *H*:

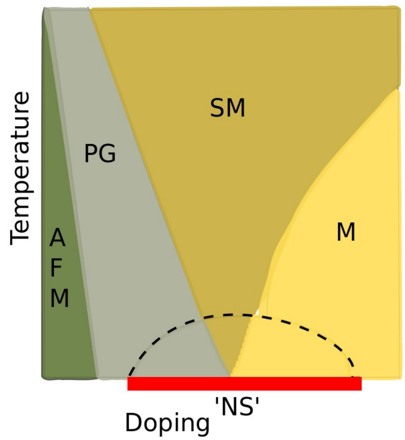
$$H_{MF} = -\sum_{i,\delta,\sigma} t_{eff}(i,\delta) c^\dagger_{i\sigma} c_{i+\delta\sigma} + \sum_{i,\sigma} V_{eff}(i) n_{i\sigma}$$

$$+ \{\text{pairing terms involving } \Delta_i^\delta\}$$

Normal State $H_{MF} = -\sum_{i,\delta,\sigma} t_{eff}(i,\delta) c^\dagger_{i\sigma} c_{i+\delta\sigma} + \sum_{i,\sigma} V_{eff}(i) n_{i\sigma}$

- **Do not confuse our 'normal-state' with other 'normal states'!**





Pairing of "Normal State" - I

Check:

1. Solve: $\hat{\mathcal{H}}_{\rm NS}(t_{\rm eff}, V_{\rm eff})$ self-consistently.

2. Construct BdG-type Hamiltonian:

$$\mathcal{H}_{\rm BdG} = \begin{pmatrix} \hat{\mathcal{H}}_{\rm NS}(t_{\rm eff}, V_{\rm eff}) & \hat{\Delta}_{\rm BdG} \\ \hat{\Delta}^{\dagger}_{\rm BdG} & -\hat{\mathcal{H}}^{\dagger}_{\rm NS}(t_{\rm eff}, V_{\rm eff}) \end{pmatrix}$$

With $\hat{\Delta}_{\rm BdG}$ taken from BdG-GIMT output.

3. Diagonalize: $\mathcal{H}_{\rm BdG}$ $\underline{\textbf{once}}$ (no self-consistency!) & re-calculate $\hat{\Delta}$.

Resulting $\hat{\Delta}$ same as input $\hat{\Delta}_{\rm BdG}$ to good accuracy!.

⇨ **GIMT results can be thought to arise from pairing of NS!!**

Implication?

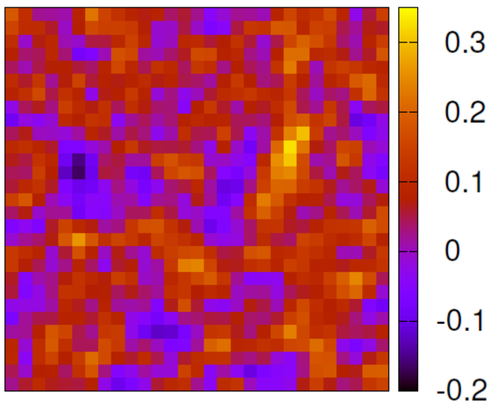
- **Effective single-particle “NS”-parameters unaffected by *Pairing*?**

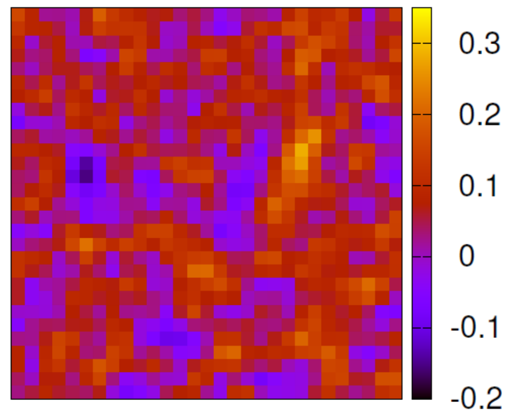
$\boxed{V=1.75}$

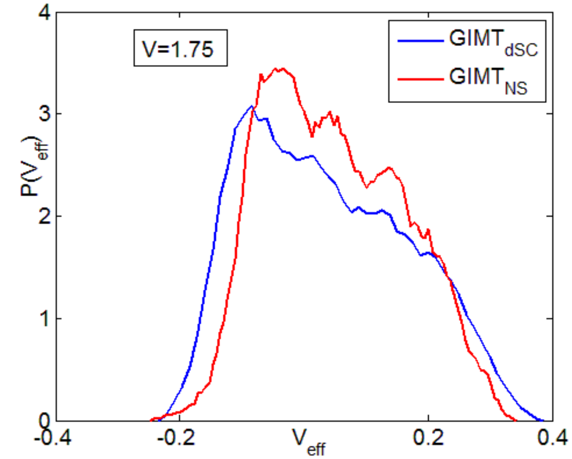
$GIMT_{dSC}$ $GIMT_{NS}$

... Indeed!!

V_{eff}

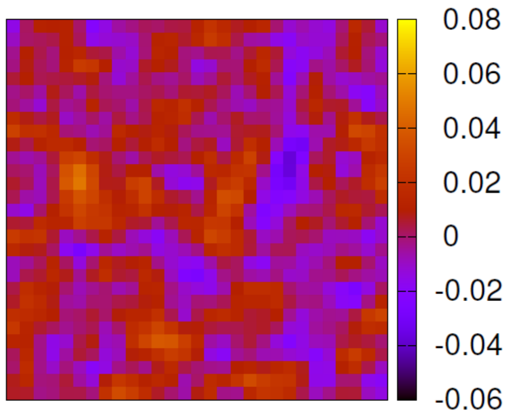


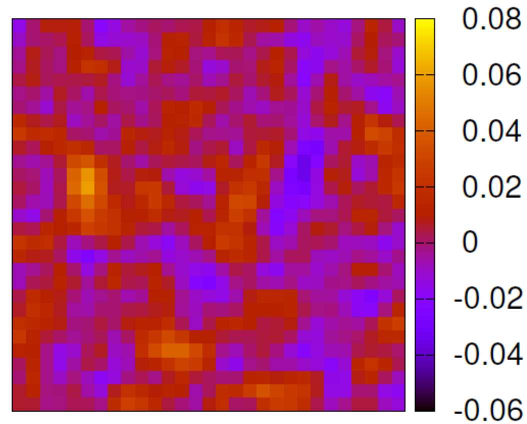


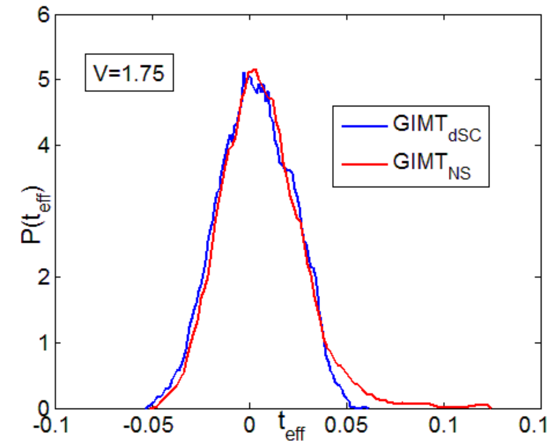


- Note: Spatial anti-correlation of V_{eff} and t_{eff} !

t_{eff}







Pairing of “Normal State” - II

Develop Anderson-type pairing with “effective” single-particle NS!

Anderson's pairing theory (for s-SC) based on two key ideas:

1. 1-particle problem of electrons in impurity potential first solved to obtain “exact eigenstates” which are then (at the next stage) paired by BCS-type effective electronic attraction.

The decoupling of the two steps demands that the pairing interaction has no role in determining the 1-particle “exact eigenstates” (these are set before switching on “pairing”).

2. Only pairing between the time-reversed “exact eigenstates” (in the 2^{nd} stage) are kept in the formalism.

Pairing of “Normal State” - II

In the context of cuprtes ...

1. MF Decomposition $\Rightarrow \mathcal{H}_{\mathrm{t-J}} = \mathcal{H}_{\mathrm{NS}} + \mathcal{H}_{\mathrm{Pair}}$

2a. $\mathcal{H}_{\mathrm{NS}} = \sum_{ij\sigma} t_{ij}^{\mathrm{eff}} (c_{i\sigma}^{\dagger} c_{j\sigma} + \mathrm{h.c.}) + \sum_{i\sigma} (V_i^{\mathrm{eff}} - \mu) n_{i\sigma} \equiv \sum_{\alpha\sigma} \xi_\alpha c_{\alpha\sigma}^{\dagger} c_{\alpha\sigma}$

2b. And, $\mathcal{H}_{\mathrm{Pair}} = \frac{1}{2} \sum_{\langle ij \rangle} \Delta_{ij} (c_{i\uparrow}^{\dagger} c_{j\downarrow}^{\dagger} - c_{i\downarrow}^{\dagger} c_{j\uparrow}^{\dagger}) + \mathrm{h.c}$

$$= \frac{1}{2} \sum_{\alpha\beta} \Delta_{\alpha\beta} \{ c_{\alpha\uparrow}^{\dagger} c_{\beta\downarrow}^{\dagger} - c_{\alpha\downarrow}^{\dagger} c_{\beta\uparrow}^{\dagger} \} + \mathrm{h.c}$$

$$\boxed{c_{i\sigma} = \sum_{\alpha=1}^{N} \psi_i^\alpha c_{\alpha\sigma}}$$

Where, $\Delta_{\alpha\beta} = \sum_{\langle ij \rangle} \Delta_{ij} (\psi_i^\alpha)^* (\psi_j^\beta)^*$

“Pairing Hamiltonian” for dSC:

4. $\mathcal{H}_{\mathrm{t-J}} = \mathcal{H}_{\mathrm{NS}} + \mathcal{H}_{\mathrm{Pair}}$

$$= \sum_{\alpha\sigma} \xi_\alpha c_{\alpha\sigma}^{\dagger} c_{\alpha\sigma} + \frac{1}{2} \sum_{\alpha\beta} \left(\Delta_{\alpha\beta} \{ c_{\alpha\uparrow}^{\dagger} c_{\beta\downarrow}^{\dagger} - c_{\alpha\downarrow}^{\dagger} c_{\beta\uparrow}^{\dagger} \} + \mathrm{h.c} \right)$$

$$\boxed{\text{Where, } \Delta_{\alpha\beta} = \sum_{\langle ij \rangle} \Delta_{ij} (\psi_i^\alpha)^* (\psi_j^\beta)^*}$$

Note: All info on inhomogeneity Due to disorder contained in $\Delta_{\alpha\beta}$

Results: Pairing Amplitude

$$g_{ij}^{xy} = \frac{4}{(2-\rho_i)(2-\rho_j)}$$

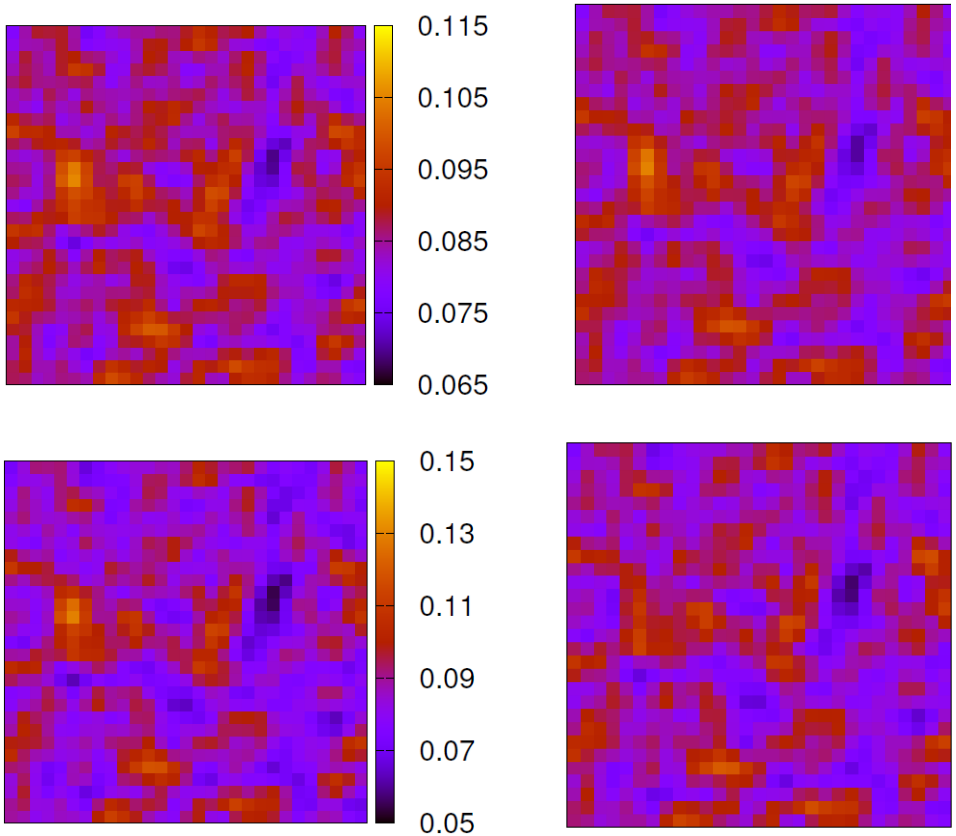
choice of ρ not crucial for pairing!

- Uniform average ρ
- Inhomogeneous "NS" ρ
- Inhomogegeous ρ, self-organized at pairing

GIMT$_{dSC}$ | **NS-pairing**

V=1.0

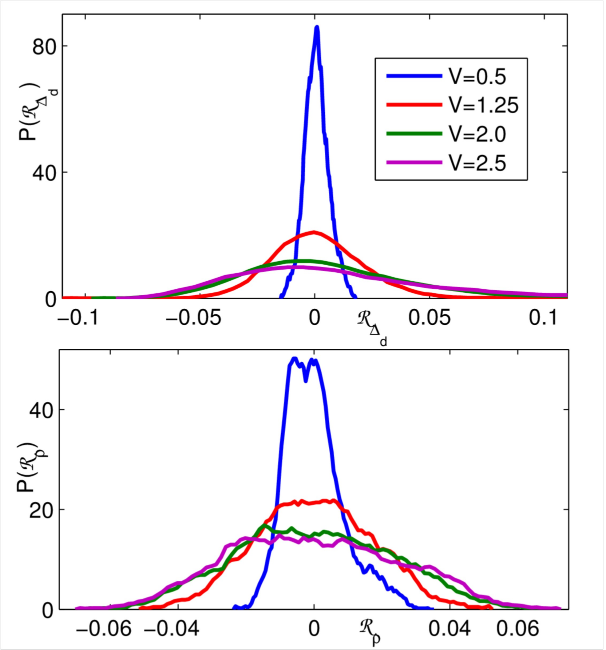
V=2.0



- **Relative difference of order parameters (R):**

$$R_{OP}(i) = \frac{OP^{GIMT_{dSC}}(i) - OP^{PNS}(i)}{\langle OP^{GIMT_{dSC}} \rangle}$$

Here, $OP \equiv \Delta_d, \rho$

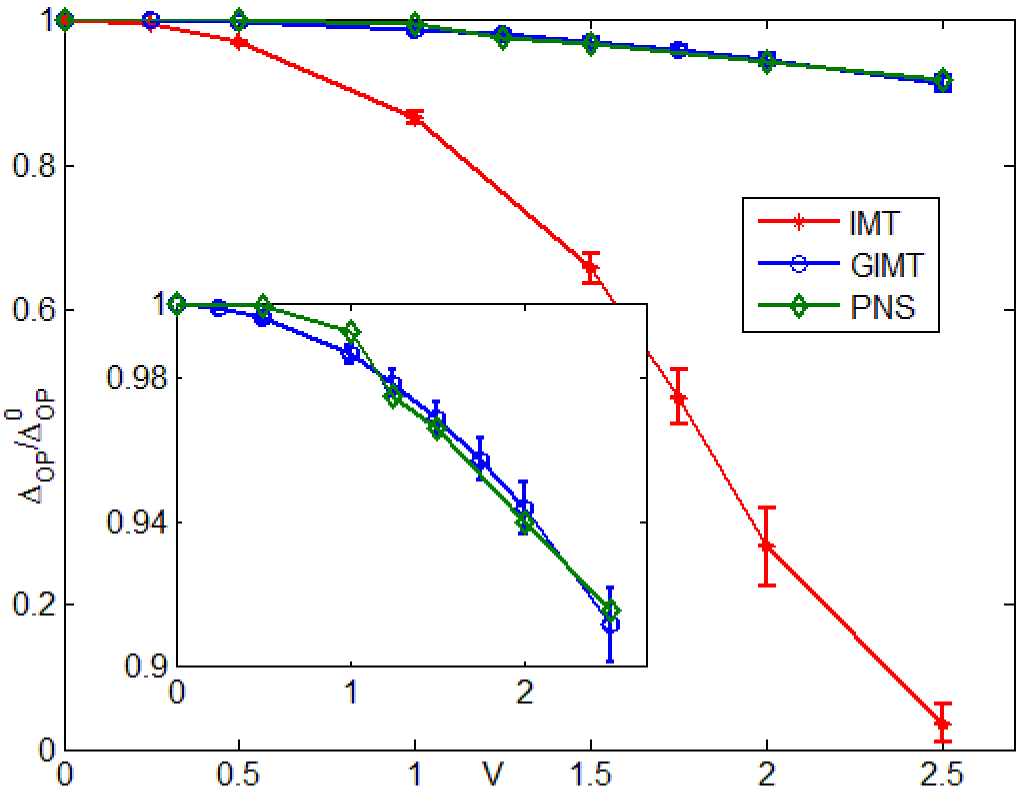


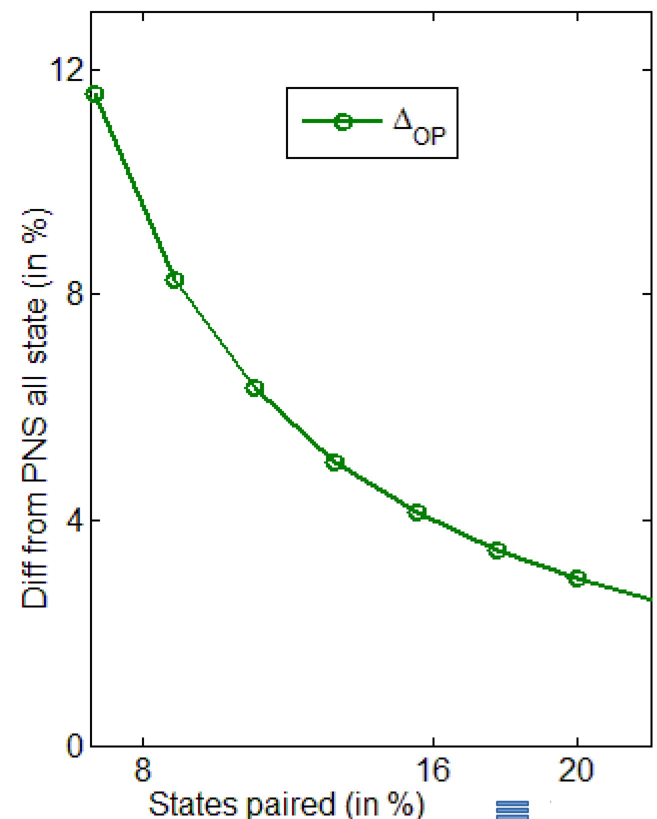
Δ_d

ρ

- **First idea of "pairing theoory" (2-step mechanism) works well to deccribe disordered cuprates!!**

Physical observables: ODLRO





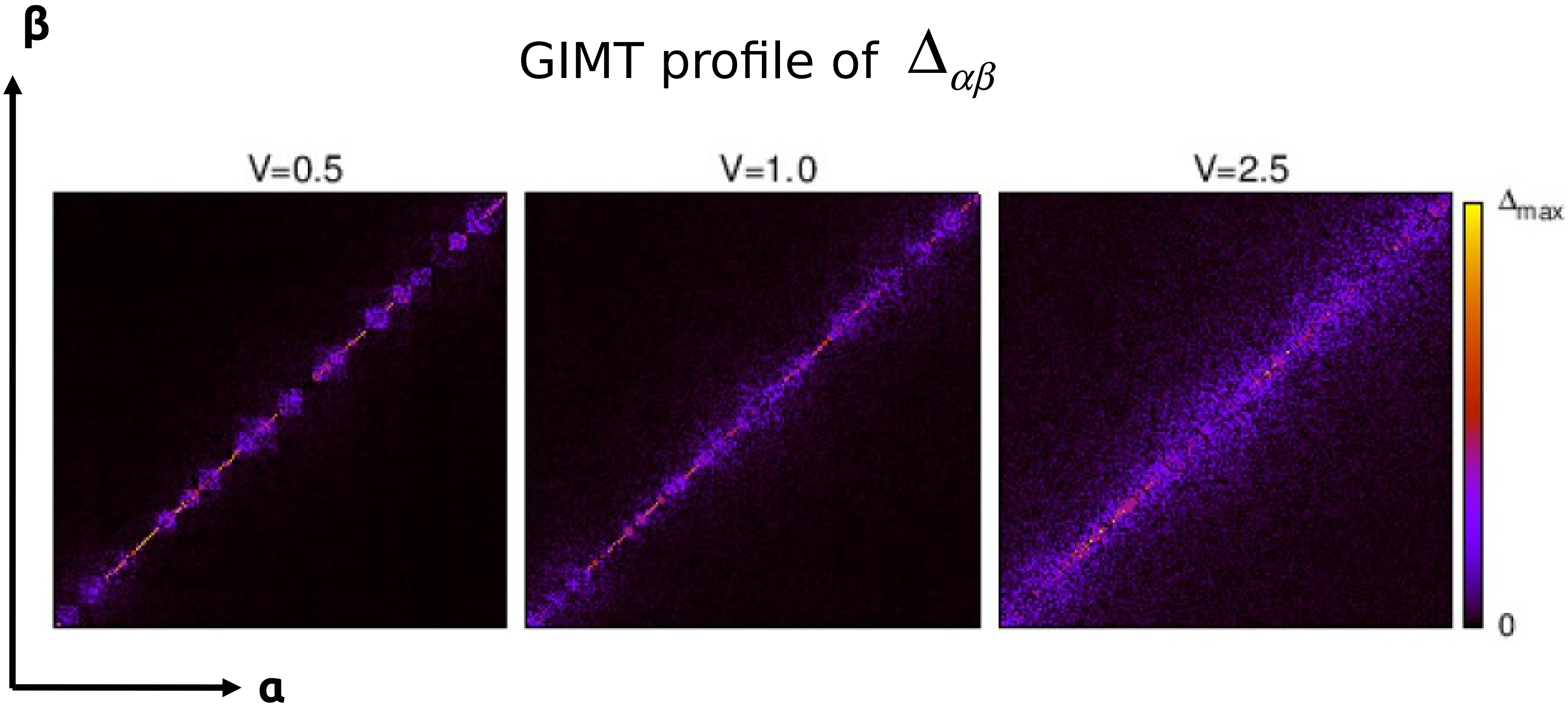
- Similar conclusions drawn from $D_s, N(\omega), etc$

Validates the 2nd idea: Get by
With pairing only limited states

Pairing theory for dSC requires pairing all states, though
dominant contribution comes from "diagonal" channel.

Pairing in *exact* "eigen-space"

GIMT profile of $\Delta_{\alpha\beta}$



Only $\alpha \in [350, 550]$ is plotted from clarity

Conclusion

(1) Anderson's “pairing theory” provides a simple description of complex physics of strongly correlated disordered cuprate superconductors.

(2) The “correlated normal states” are crucial for making the pairing concept work with only a limited subset of such states. Normal states that ignore strong correlations turn out inadequate (not shown here).

(3) The intriguing physics of the “correlated normal states” is currently under investigation. They appear quite different from the eigenstates of the disordered Anderson model, due to *spatially correlated disorder*.