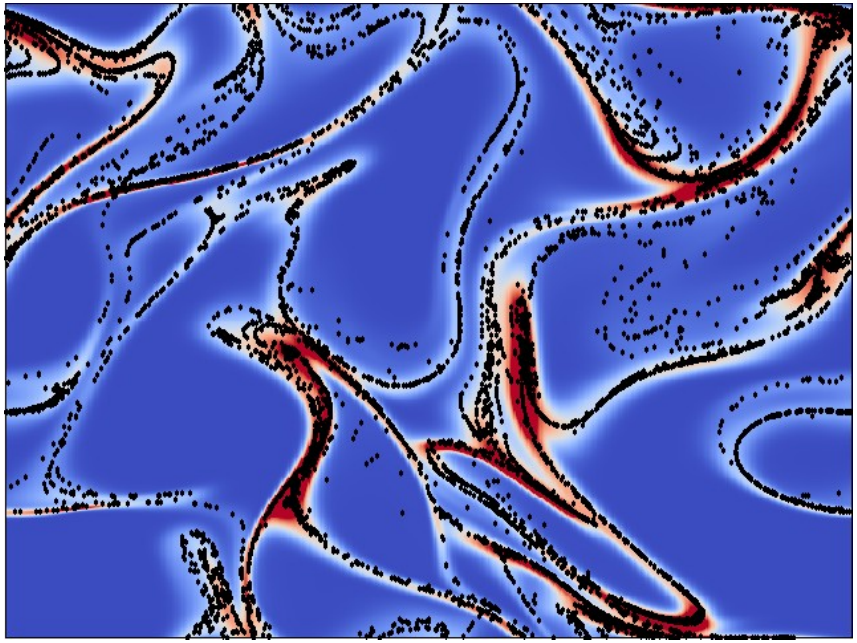
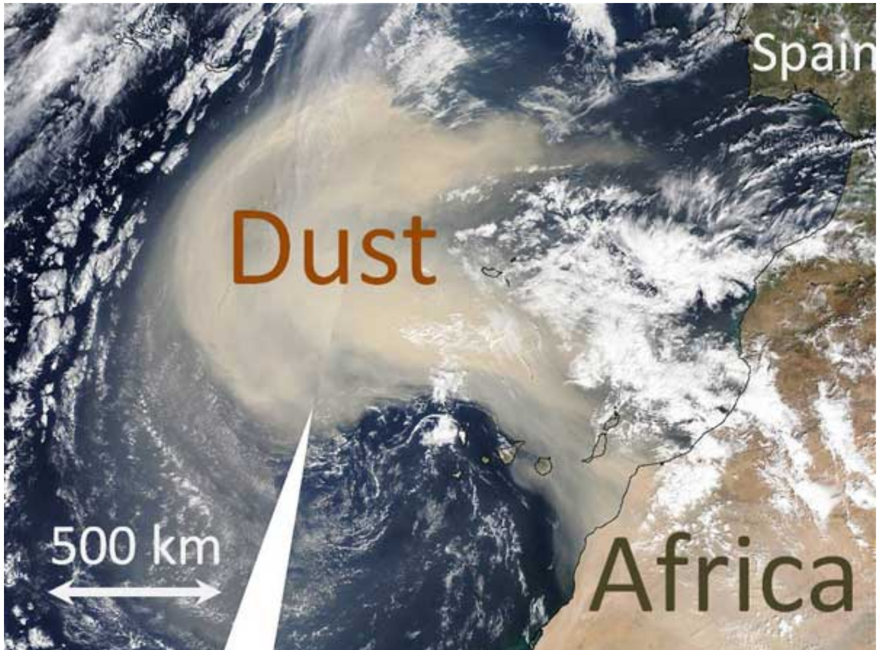
Topological properties of inertial particles in 2d turbulence

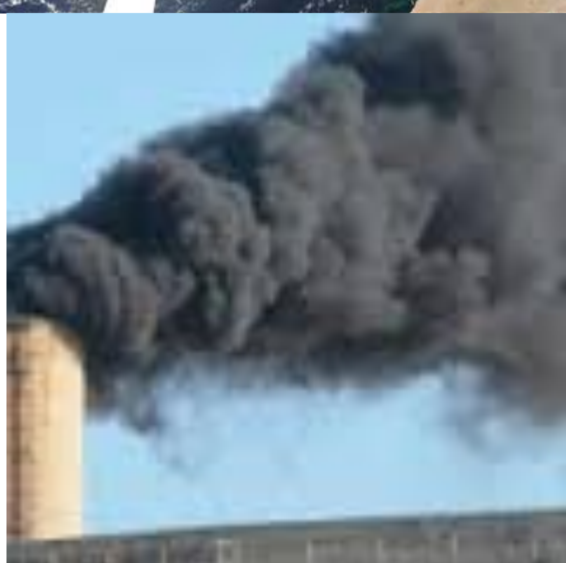


Prasad Perlekar, TCIS, TIFR Hyderabad

Dhrubaditya Mitra, NORDITA, Stockholm

Inertial particles

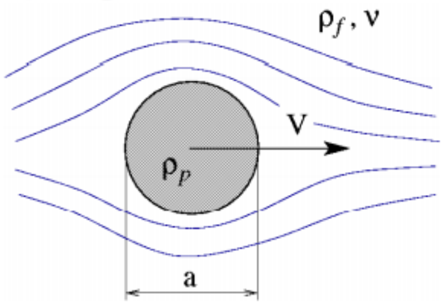




Equation of motion for a small rigid sphere in a nonuniform flow

Martin R. Maxey and James J. Riley

Citation: Physics of Fluids (1958-1988) **26**, 883 (1983); doi: 10.1063/1.864230



$$Re = \frac{a(u-V)}{\nu} << 1$$

$$m_p \frac{dV_i}{dt} = (m_p - m_f) g_i + m_f \left. \frac{Du_i}{Dt} \right|_{\boldsymbol{X}(t)}$$ bouyancy

$$-6\pi a \mu \left[V_i(t) - u_i(\boldsymbol{X}(t), t) - \frac{1}{6} a^2 \left. \nabla^2 u_i \right|_{\boldsymbol{X}(t)} \right]$$ Stokes drag Faxen correction

$$-\frac{m_f}{2} \frac{d}{dt} \left[V_i(t) - u_i(\boldsymbol{X}(t), t) - \frac{1}{10} a^2 \left. \nabla^2 u_i \right|_{\boldsymbol{X}(t)} \right]$$ Added mass

$$-6\pi a \mu \int_0^t ds \left(\frac{d/ds \left[V_i(s) - u_i(\boldsymbol{X}(s), s) - \frac{1}{6} a^2 \left. \nabla^2 u_i \right|_{\boldsymbol{X}(s)} \right]}{\sqrt{\pi \nu (t-s)}} \right)$$ Basset memory term

Maxey & Riley (1983)

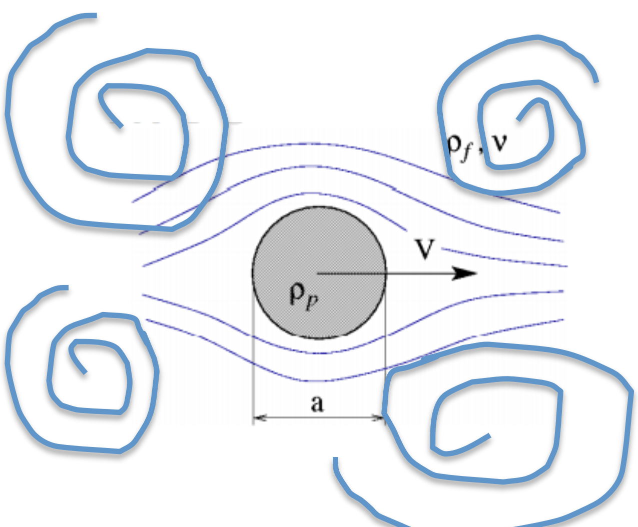
Auton et al (1988)

Today's talk: Heavy particles

Use $\rho_p >> \rho_f$, $a \ll 1$ —Maxey-Riley→

$$\frac{dx}{dt} = V$$
$$\frac{dV}{dt} = -\frac{(V-U)}{\tau}$$

U: Fluid velocity at the particle position



Stokes number : $St = \frac{\tau}{\tau_f}$

τ_f : Flow timescale

Turbulent flows:

$$Re_p \ll 1, Re_f >> 1$$

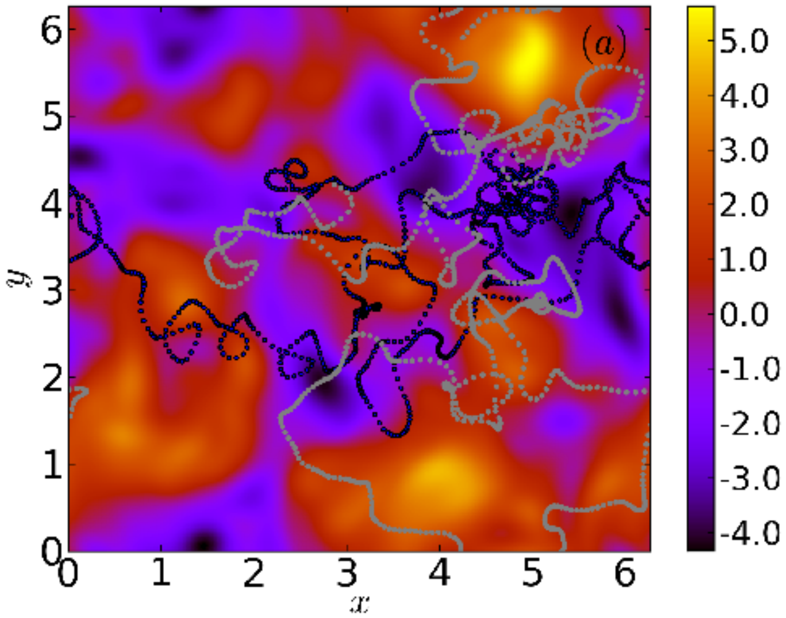
Inertial particles in turbulence

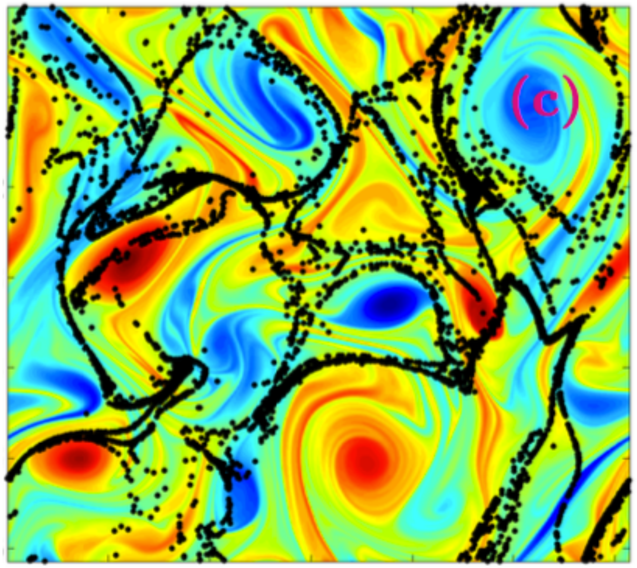
Solve NS equation:

$$D_t \mathbf{u} = \nu \nabla^2 \mathbf{u} - \nabla p + f$$

f : Large scale forcing

$$\frac{dx}{dt} = V$$
$$\frac{dV}{dt} = -\frac{(V-U)}{\tau}$$





Perlekar et al., PRL, **106**, 054501 (2011).

Preferential sampling

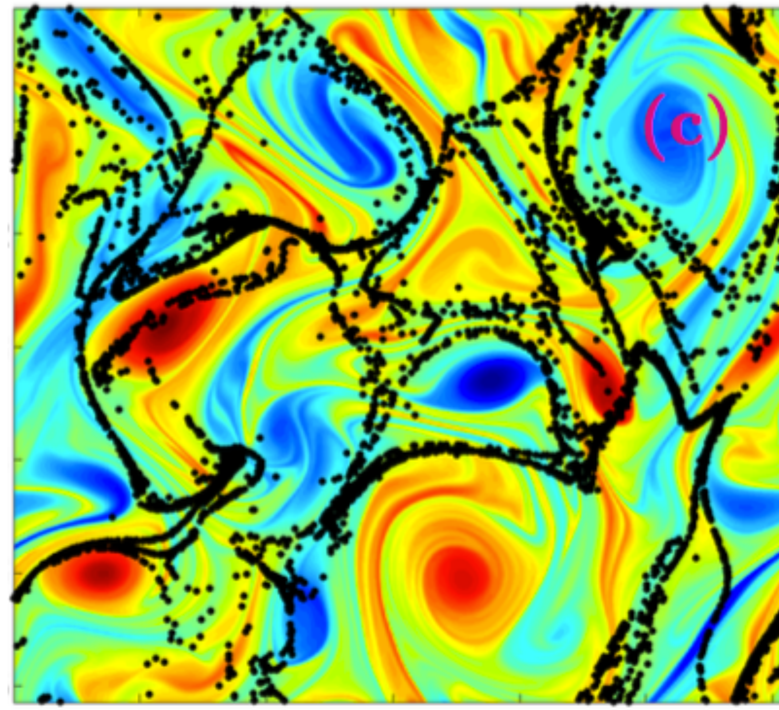
Inertial Particles are thrown out of vortices

$$\frac{dx}{dt} = V$$

$$\frac{dV}{dt} = -\frac{(V-U)}{\tau}$$

$$\nabla \cdot V = \nabla \cdot \left[-\frac{dV}{dt} \right]$$

$$\nabla \cdot V > 0$$



Lagrangian vs Eulerian

Lagrange

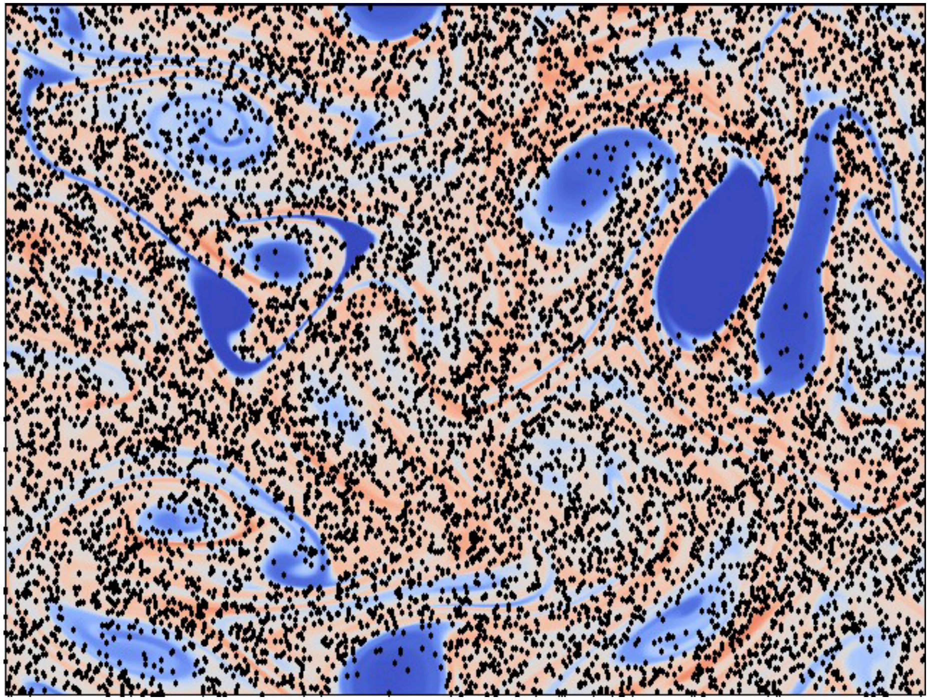
$$\frac{dX_i(t)}{dt} = V_i$$
$$\frac{dV_i(t)}{dt} = -\frac{1}{\tau}[V_i(t) - U(X_i, t)]$$

$$\rho(x,t) = \frac{1}{N_p}\sum_i \delta(x - X_i(t))$$

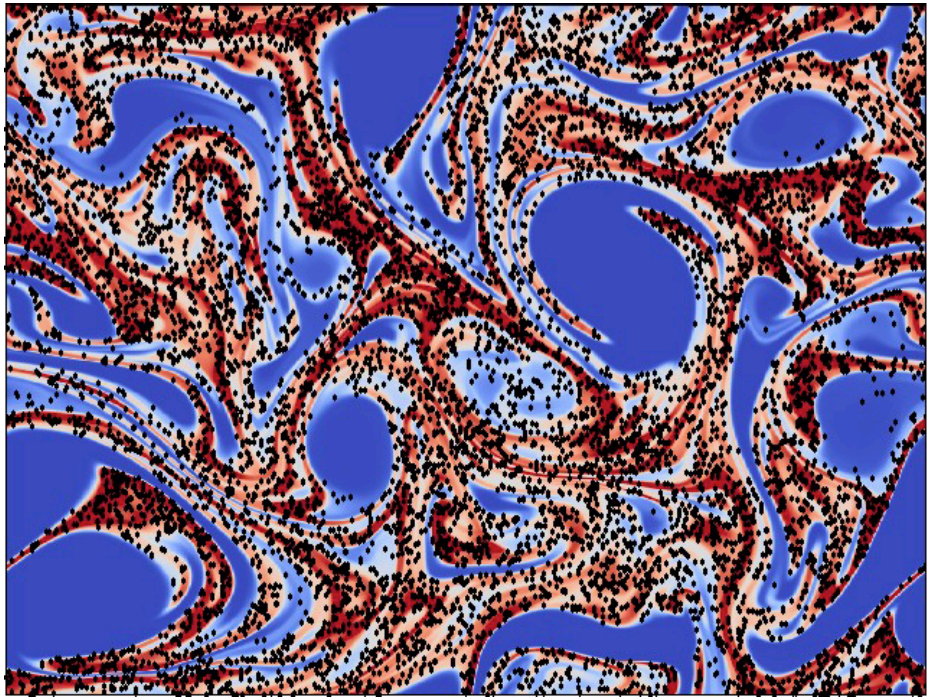
Euler

$$\frac{\partial \rho(x,t)}{\partial t} + \nabla \cdot [\rho(x,t) v(x,t)] = 0$$
$$\frac{\partial v(x,t)}{\partial t} + v(x,t) \cdot \nabla v(x,t) = -\frac{v(x,t) - U(x,t)}{\tau}$$

Comparison: Lagrangian vs Eulerian



$St = 1.7 \cdot 10^{-2}$



$St = 8.6 \cdot 10^{-2}$

Dots

Density field $(St < 1)$

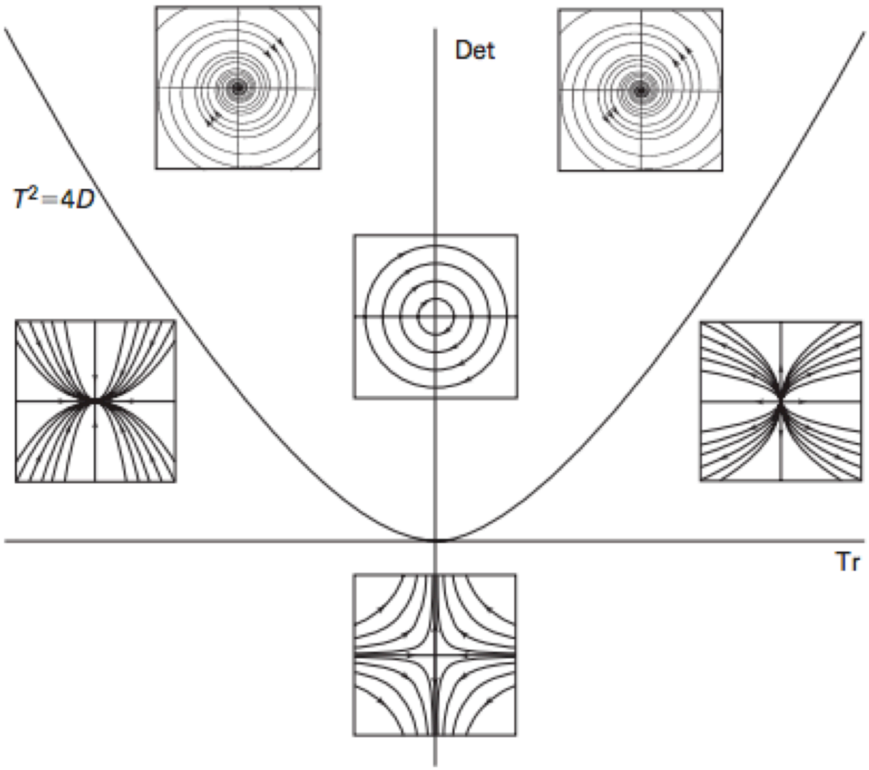
$$\frac{dX_i(t)}{dt} = V_i$$

$$\frac{dV_i(t)}{dt} = -\frac{1}{\tau}[V_i(t) - U(X_i, t)]$$

$$\frac{\partial \rho(x,t)}{\partial t} + \nabla \cdot [\rho(x,t) v(x,t)] = 0$$

$$\frac{\partial v(x,t)}{\partial t} + v(x,t) \cdot \nabla v(x,t) = -\frac{v(x,t) - U(x,t)}{\tau}$$

Topological properties of the flow



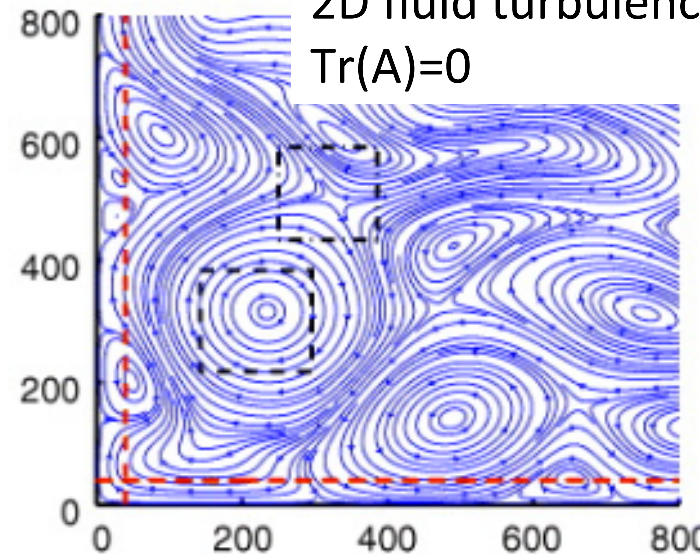
$$A = \begin{bmatrix} \partial_x u_x & \partial_x u_y \\ \partial_y u_x & \partial_y u_y \end{bmatrix}$$

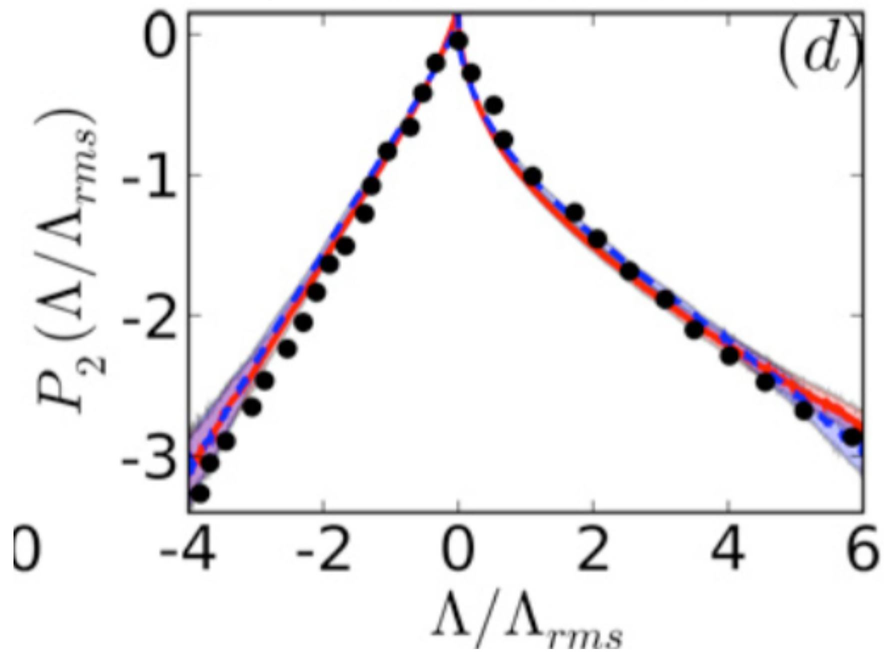
$$det(A) = \frac{1}{4}(\omega^2 - \sigma^2)$$

$$Tr(A) = \nabla \cdot u$$

2D fluid turbulence

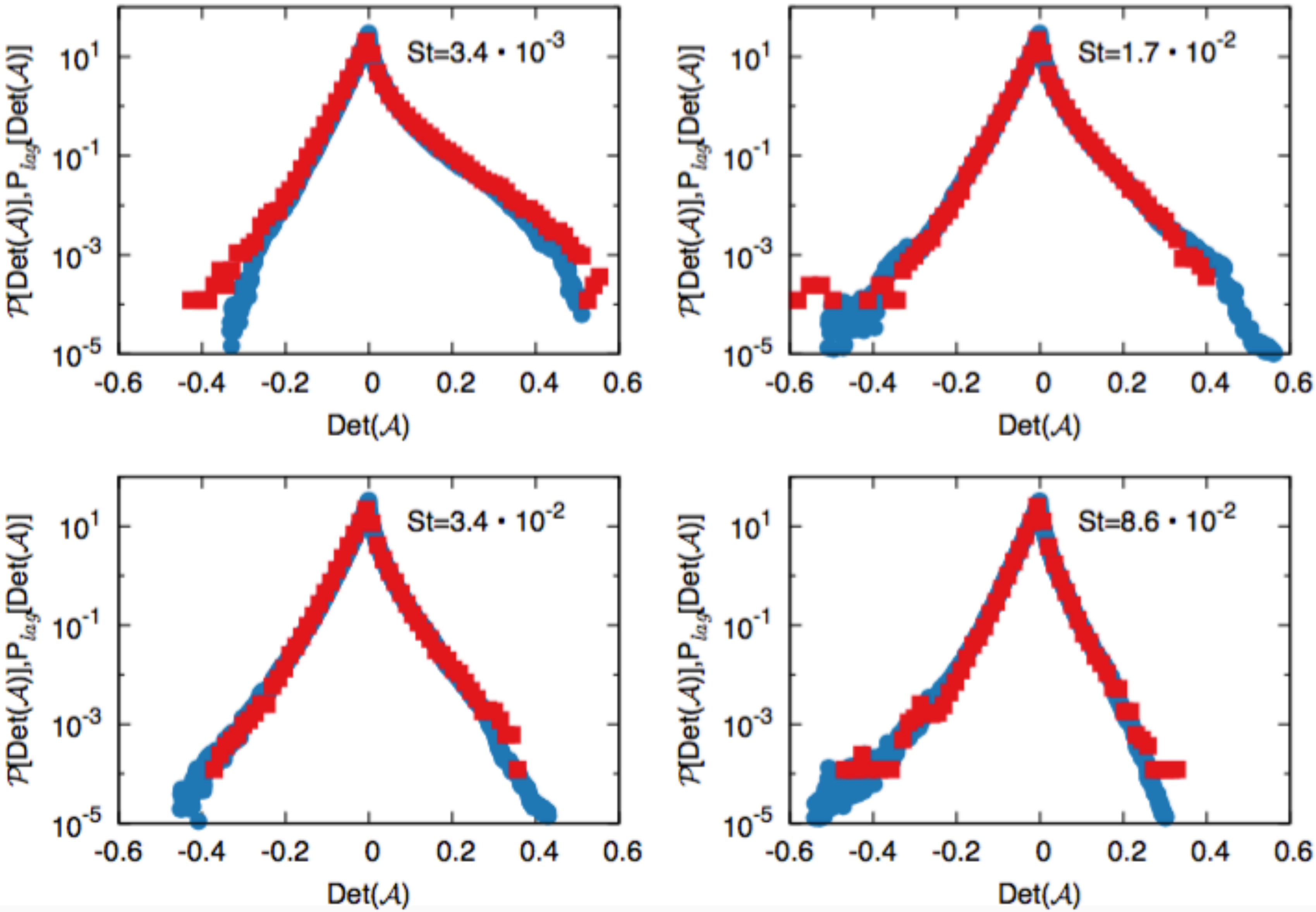
Tr(A)=0





PP and RP, **NJP**, 11, 073003 (2011).

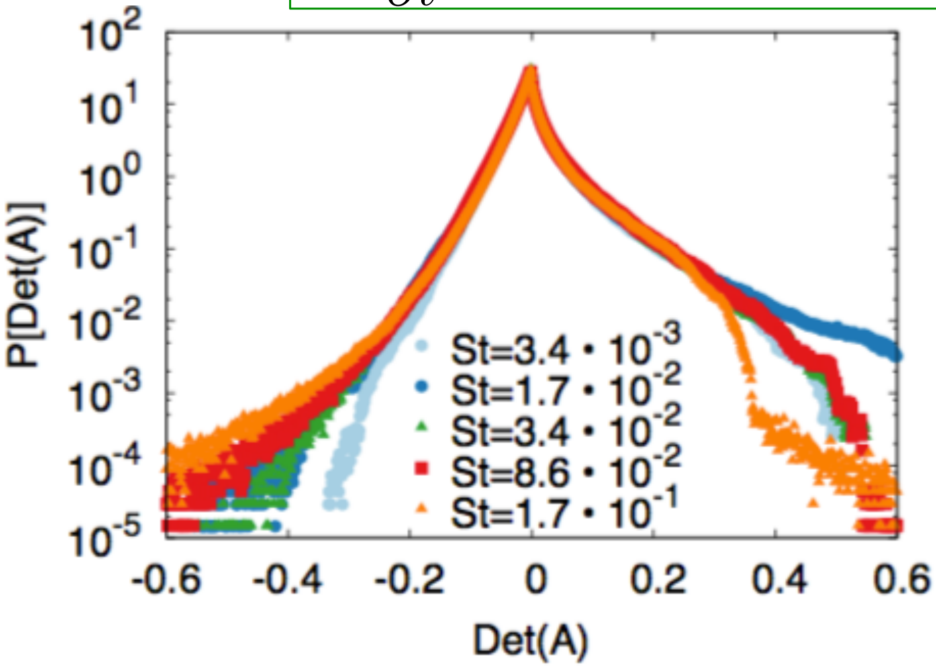
Fluid Det($\mathcal{A}$) along inertial particle tracks



Dust/Particle Det(A)

$$\frac{\partial \rho(x,t)}{\partial t} + \nabla \cdot [\rho(x,t)v(x,t)] = 0$$

$$\frac{\partial v(x,t)}{\partial t} + v(x,t) \cdot \nabla v(x,t) = -\frac{v(x,t) - U(x,t)}{\tau}$$

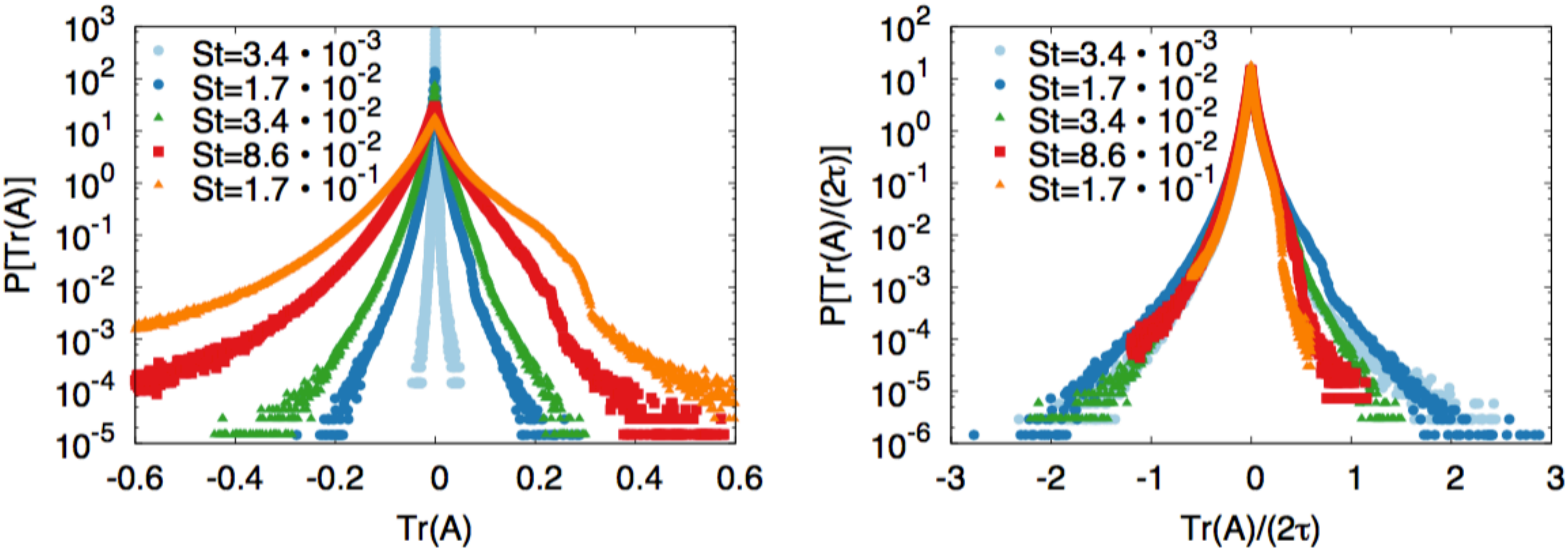


$$v = u - \tau \frac{dv}{dt} \xrightarrow{\tau \to 0} v = u - \tau \frac{Du}{Dt}$$

$$Det(A) \approx Det(\mathcal{A})$$

- P[Det(A)] does not depend on St.
- P[Det(A)] is approximately equal to P[Det($\mathcal{A}$)]

Dust/Particle Tr(A)

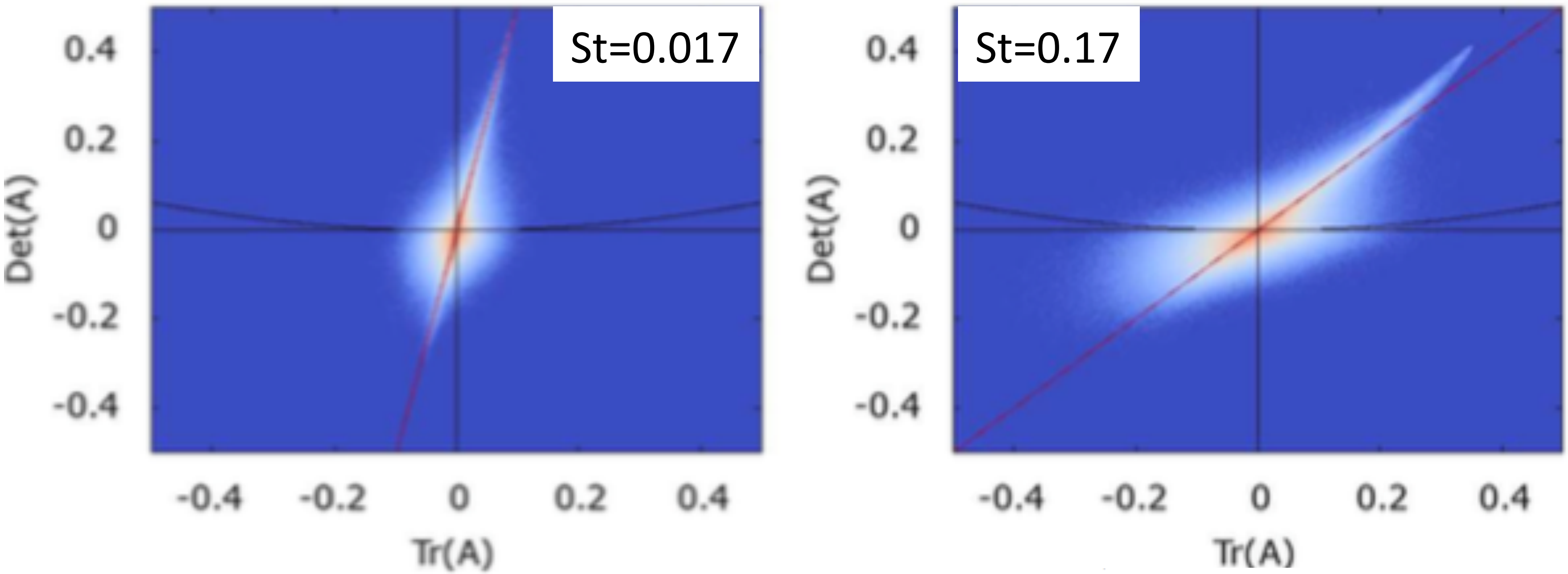


$$v = u - \tau \frac{dv}{dt} \xrightarrow{\tau \to 0} v = u - \tau \frac{Du}{Dt}$$

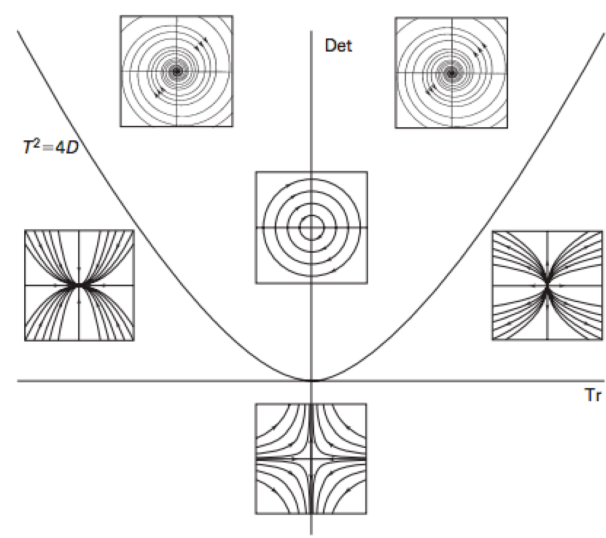
$$Tr(A) = 2\tau Det(\mathcal{A})$$

- Regions with non-zero Tr(A) increase with St
- Small St approximation correctly rescales the pdf.

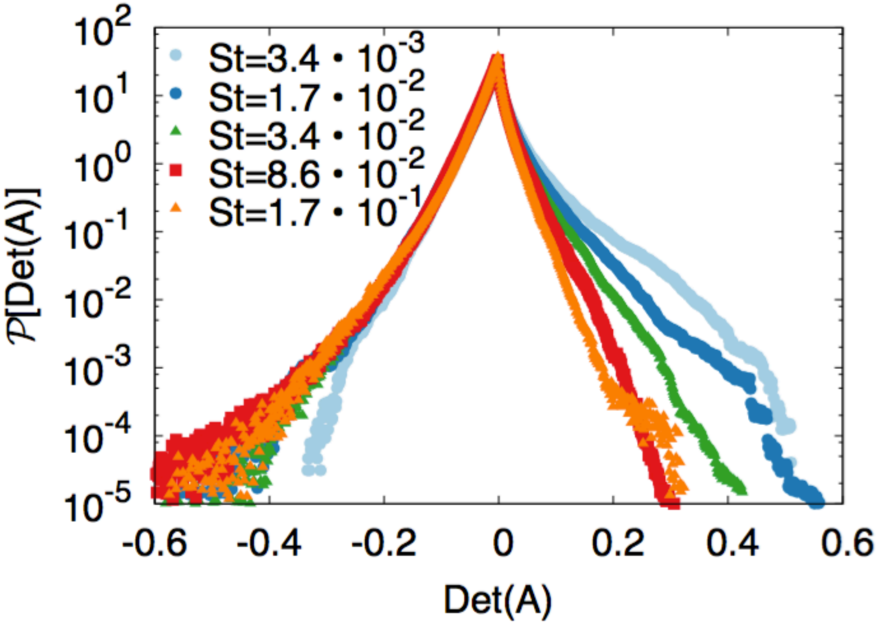
Joint PDF [Tr(A), Det(A)]

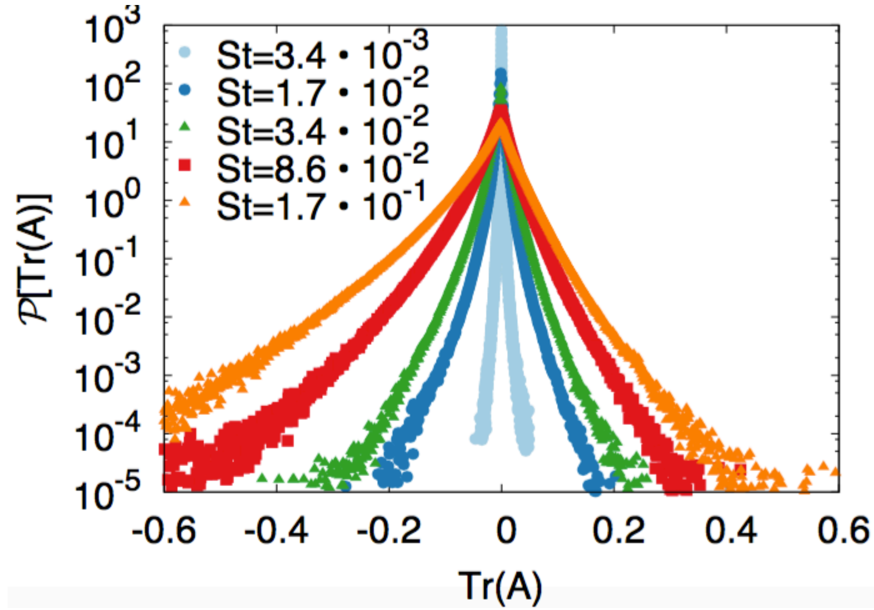


- Dust flow consists of vortices spiralling outwards and extensional (saddle) flow with negative divergence.



Density weighted Dust/Particle Det(A), Tr(A)





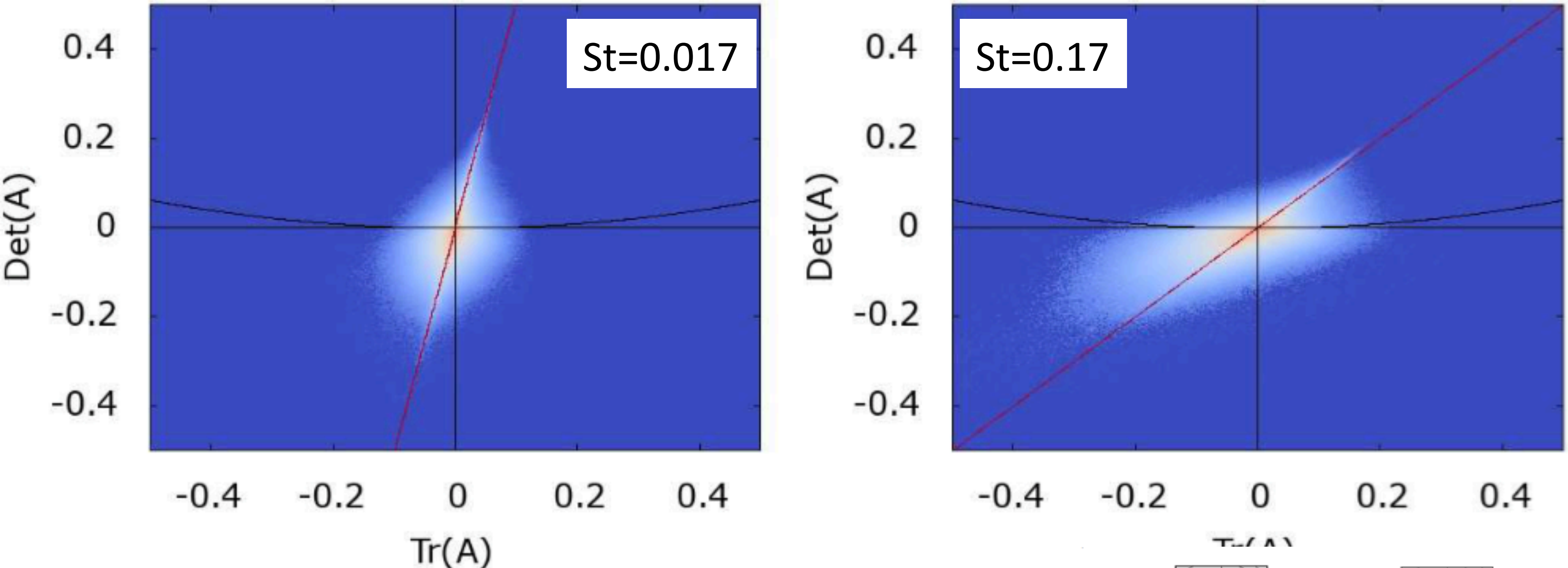
$$v = u - \tau \frac{dv}{dt} \xrightarrow{\tau \to 0} v = u - \tau \frac{Du}{Dt}$$

$$Tr(A) = 2\tau Det(\mathcal{A})$$

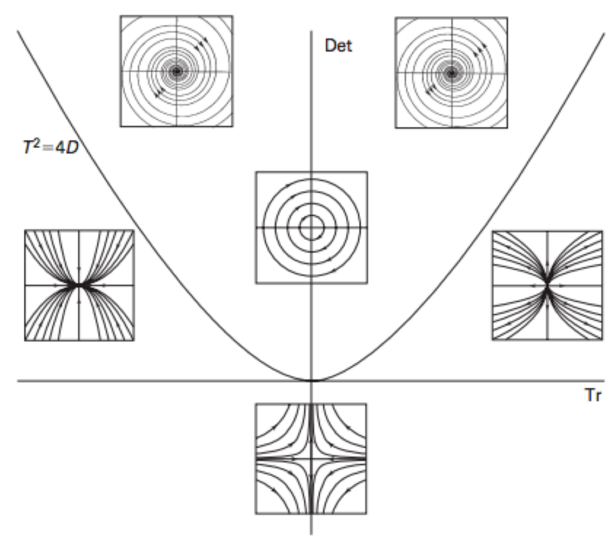
Inertial particle preferentially sample:

- Regions of large strain.
- Regions with negative divergence.

Joint density weighted PDF [Tr(A), Det(A)]



- Particles sample extensional (saddle) flow regions with negative divergence.



Conclusions

- Density weighted Eulerian and Lagrangian pictures identical
- Inertial particles sample regions of large strain and negative divergence.