Effective Time-independent Hamiltonian of Periodically Driven Quantum Systems

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- In NLD, periodically driven systems are studied extensively: classically + quantum mechanically
- Particularly, *delta*-kicked systems: Eg. kicked rotor
- These systems display very rich dynamics:
  - ✓ Integrable-to-chaos transition
  - ✓ Dynamical localization
- Recent interest: periodic driving is proposed as a flexible experimental tool to realize new phases of matter, which may not easily be accessible in time-independent systems
  - ✓ Irradiated graphene
  - ✓ Floquet topological insulator

- These systems are usually analyzed for the two extremes: slower driving and faster driving

  - Slower driving: the driving frequency is slower than the natural frequencies of the non-driven model--the system almost adiabatically follows the instantaneous Hamiltonian.

  - Faster driving: the driving frequency is faster than the natural frequencies of the non-driven model-- the system typically feels an effective static Hamiltonian dependent on the driving.

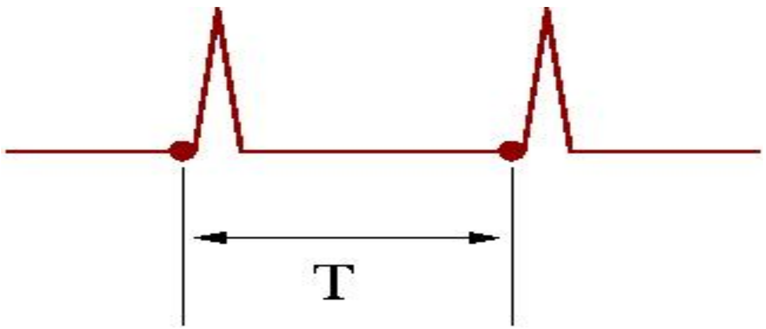
- What will be the effective time-independent Hamiltonian for the faster driving (kicking)?

✓This question was asked about 3 decades back by Scharf: in the context of 'kicked' quantum chaotic system

✓A CBH based perturbation method was proposed (Scharf, 1988)

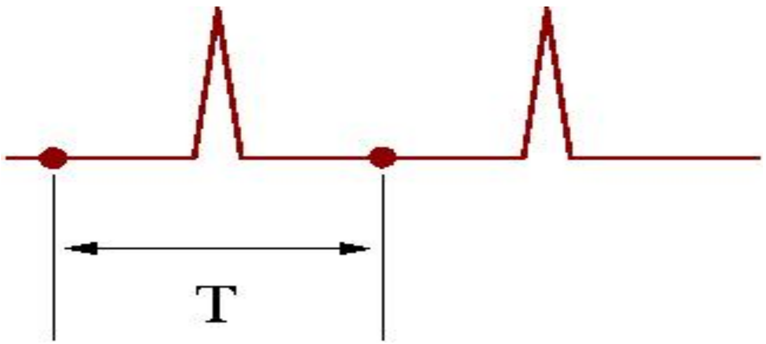
✓However, not many quantum chaotic systems were studied using Scharf's method: its merit/ demerit was not tested that much

• Recently, Scharf's method is applied to calculate the effective Hamiltonian for the kicked top Hamiltonian: $H(t) = pJ_x + \frac{\kappa}{2j} J_z^2 \sum_n \delta(t - nT)$



• $U(T) = \exp(-ipJ_x T)\exp\left(-i\frac{\kappa}{2j}J_z^2\right)$

$= \exp\left(-iH_{eff}[t_0]T\right)$



• $U(T) = \exp\left(-\frac{i}{2}pJ_x T\right)\exp\left(-i\frac{\kappa}{2j}J_z^2\right)$

$\exp\left(-\frac{i}{2}pJ_x T\right) = \exp\left(-iH_{eff}[t_0]T\right)$

• The above two $H_{eff}[t_0]$ are connected by some gauge like symmetry ("Floquet gauge"): equivalent

- Using Scharf's method, the recent work has found (for the kicked top), up to ω^{-2} :

$$\hat{H}_{\mathrm{E}} = \frac{\kappa}{2j} J_z^2 + \frac{p}{2} \left[\frac{-i \frac{\kappa}{2j} J_+ (2J_z + 1)}{\exp[-i \frac{\kappa}{2j}(2J_z + 1)] - 1} + \mathrm{H.c.} \right]$$

(Bastidas et al, 2014)

- Some matrix elements of the above Hamiltonian can be singular due the term at the denominator

- The energy spectrum also shows some singular behavior

• Instead, for the analysis of the faster driving case, one has to use Floquet theorem

• The Floquet theorem states that the time evolution operator can be expressed as:

$$U(t_0+T, t_0) = e^{-iF(t_0+T)} e^{-iH_F T} e^{iF(t_0)}$$

where $F(t_0) = F(t_0+T)$ is a periodic time-dependent Hermitian operator and $H_F \equiv H_{eff}$ is the time-independent Floquet (effective) Hamiltonian.

- Recently, Goldman & Dallibard (suggested originally by Fishman et al) has proposed a perturbative scheme based on Floquet theorem such that the approximate H_{eff} will be t_0 independent.

CBH Vs. Fishman/Goldman-Dallibard

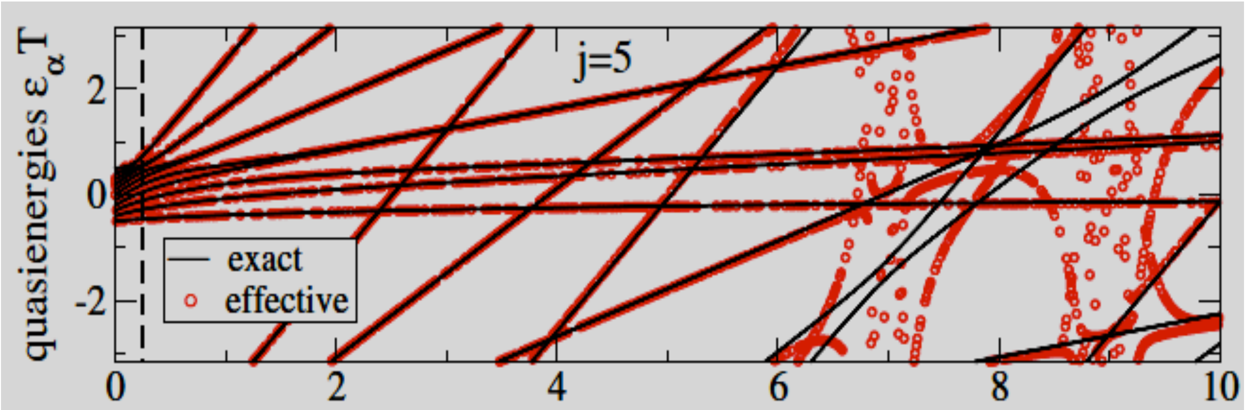
- CBH based expansion gave:

$$\hat{H}_{\mathrm{E}} = \frac{\kappa}{2j} J_z^2 + \frac{p}{2}\left[\frac{-i\frac{\kappa}{2j} J_+ (2J_z + 1)}{\exp[-i\frac{\kappa}{2j}(2J_z + 1)] - 1} + \mathrm{H.c.}\right]$$

- Fishman et al and Goldman-Dallibard method gives:

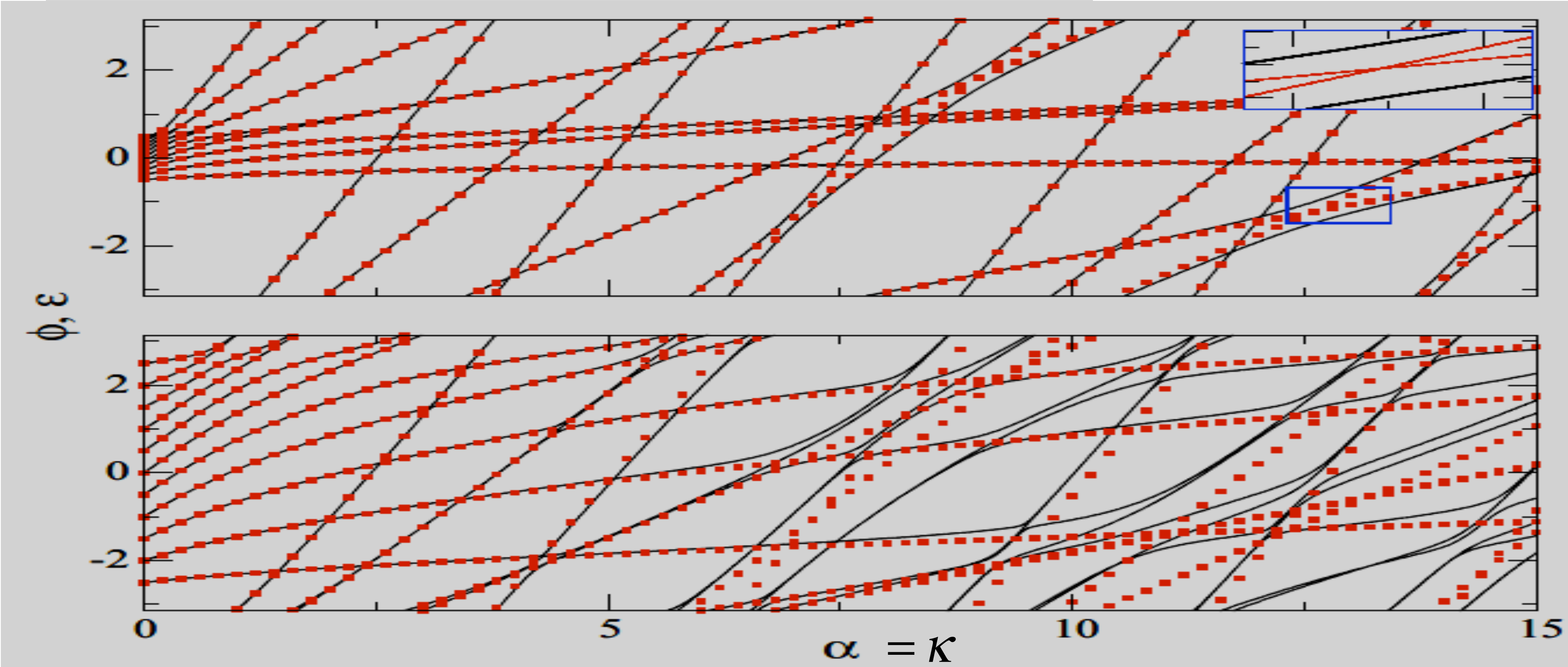
$$H_{eff} = \frac{\kappa}{2j} J_z^2 + pJ_x - \frac{\kappa p^2}{24j}\left(J_z^2 - J_y^2\right)$$

- No singularity appears.



Bastidas et al

Our work



Exact quasienergy spectrum of U(T) Vs. Energy spectrum of H_{eff}

Conclusion

- Nowadays, we are extensively calculating effective time-independent Hamiltonian of many different periodically (fast) driven Hamiltonians.

✓ Caution: We should be careful about the perturbative scheme which we are using

Thanks!