

# Kinetics of Swelling and Collapse in Polyelectrolyte Systems

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# Charged polymers – energy and entropy:

Do all Na<sup>+</sup> ions dissolve? **NO!**

Counterion condensation. **WHY?**

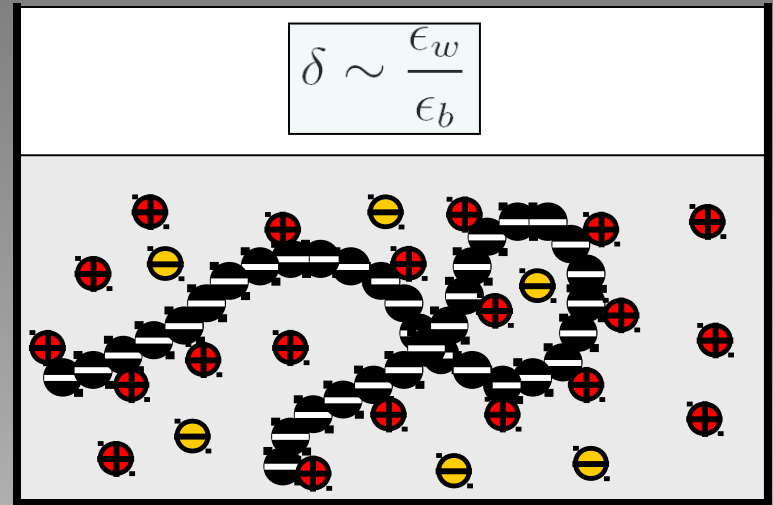
Effective  $\epsilon$  low close to backbone

Condense  $\longrightarrow$  gain in **Coulomb energy**

Let's predict **qualitatively**:

**Remember:**  $E - TS$

$$-\frac{e^2}{4\pi\epsilon_0\epsilon d}$$



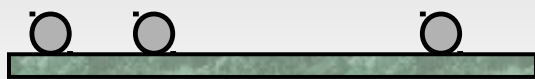
$E$  = Coulomb energy,  $S$  = free ion entropy  $\longrightarrow k_B T$

1.  $T \downarrow \longrightarrow S$  fixed,  $E/\text{ion-pair}$  fixed  $\longrightarrow$  more ion-pairs  $\longrightarrow$  less charge
2.  $\epsilon \downarrow \longrightarrow S$  fixed,  $T$  fixed,  $E/\text{ion-pair}$  increases  $\longrightarrow$  more ion-pairs
3.  $V \uparrow \longrightarrow S$  increases,  $T$  fixed,  $E/\text{ion-pair}$  fixed  $\longrightarrow$  less ion-pairs

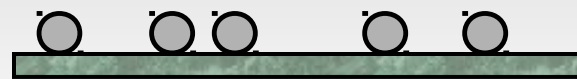
Similar to **DEW** formation: at low  $T$ , air can't hold vapor  $\longrightarrow$  condensation



**SALT**  $\longleftrightarrow$  **WET WEATHER**



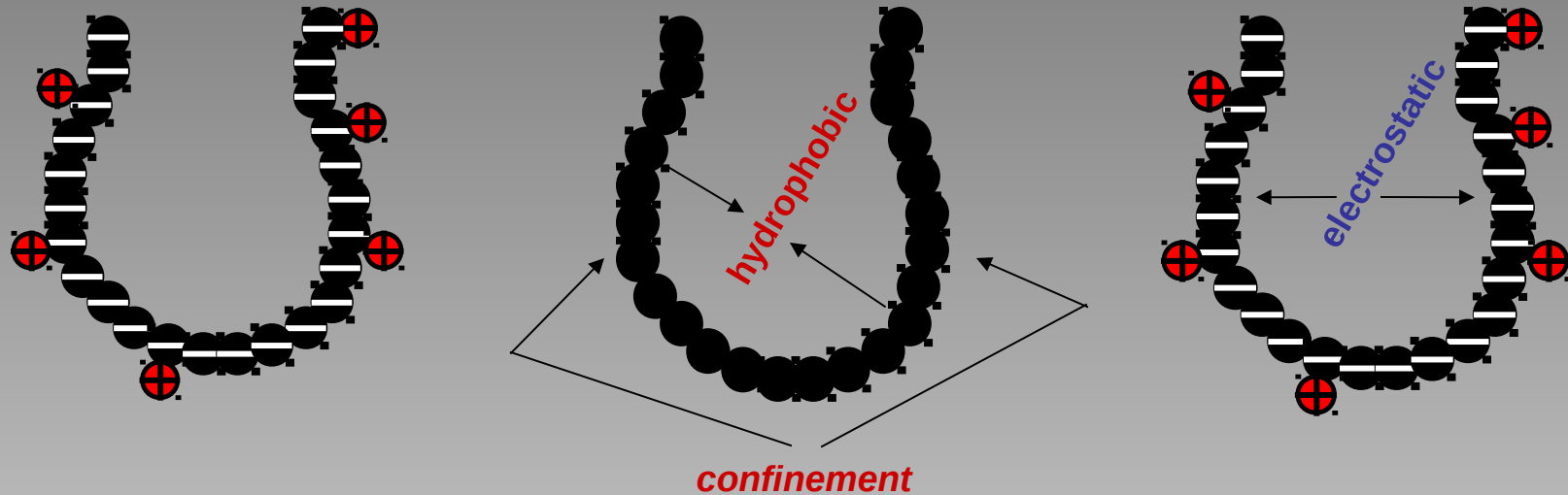
Cold surface



Cold surface

# Interactions (energy) in a charged polymer chain:

$E - TS$ , but is it so simple? What are the contributions?



**Chain entropy:** maximized if Gaussian coil

**Excluded volume:** chemical affinity (**hydrophilicity**), or mismatch (**hydrophobicity**)

**Coulomb repulsion:** between bare charges  $\longrightarrow$  chain expansion

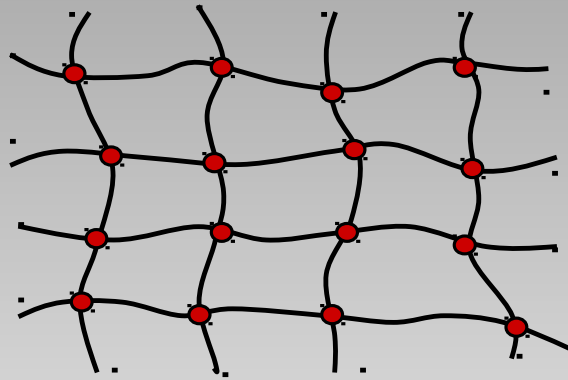
**EXAMPLE – Mutual Dependency:**

poor solvent  $\longrightarrow$  Collapsed chain  $\longrightarrow$  Ion condensation

**SCHEME:**  $E - TS$  must be **MINIMIZED**. But, **SIZE** and **CHARGE** coupled.

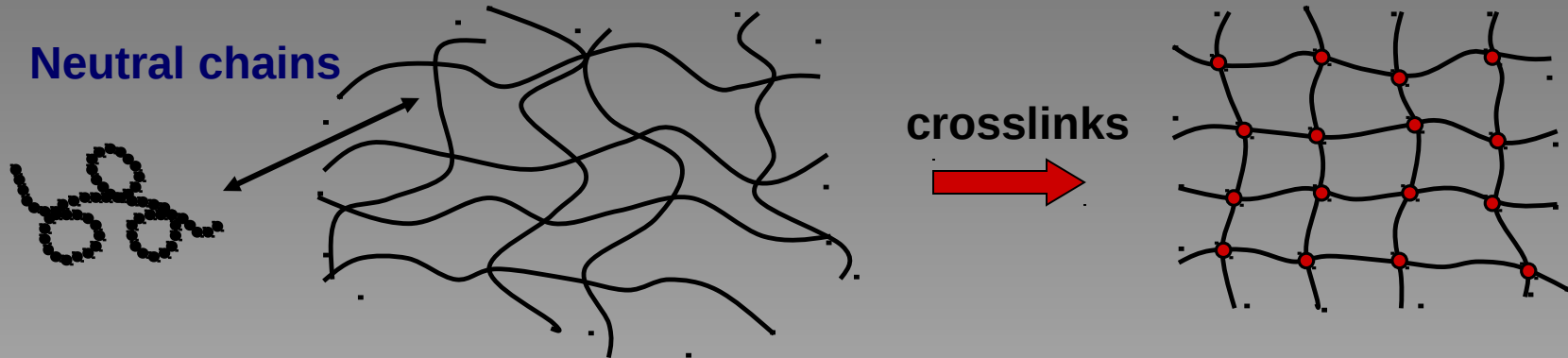
**DOUBLE MINIMIZATION : SELF-CONSISTENCY**

# Kinetics of swelling of polymer gels



More details: Poster by Swati Sen

# Polymer gels - uncharged:

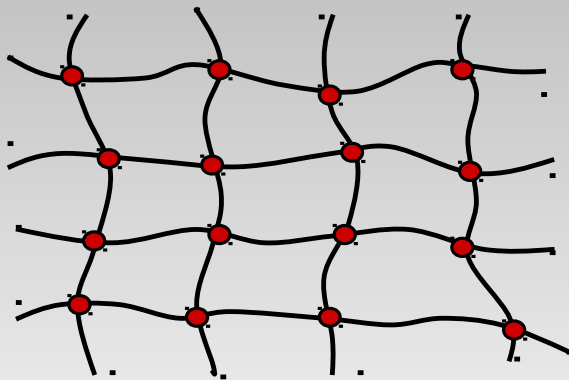


**Gel:** large single molecule – different kind

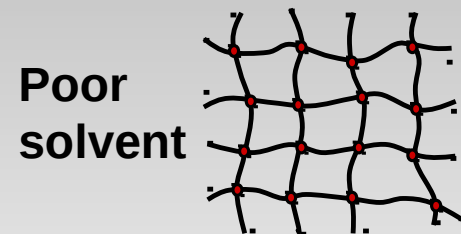
Many chains connected at different points

Strand between two crosslinks  $\longrightarrow$  similar to single chain

Follows all properties of a single chain  $\longrightarrow$  one-to-one correspondence



Good  
solvent



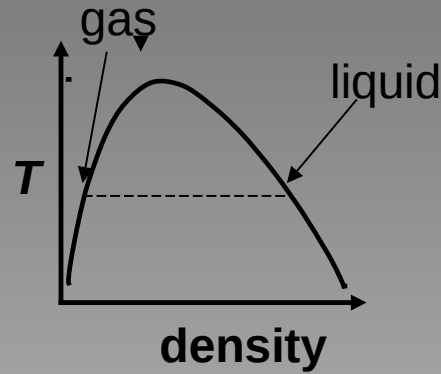
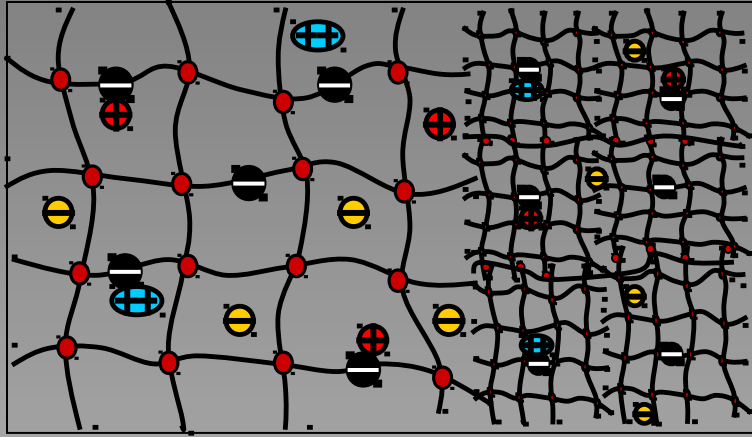
Poor  
solvent

Contribution to free energy ( $E - TS$ )?

**Energy:** mixing (hydrophobicity)

**Entropy:** chain entropy  $\longrightarrow$  elasticity

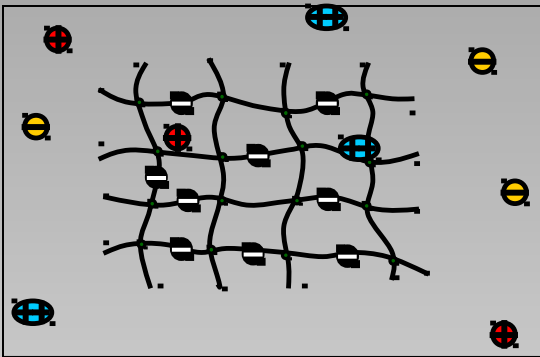
# Phase transition – charged gels - schematic:



## Issues:

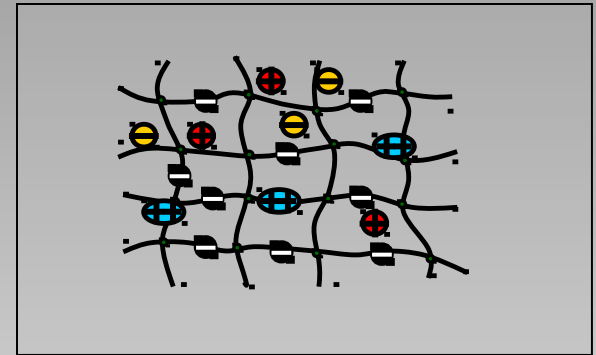
A. **Coexistence** in charged polymer gels,

B. Interplay of **hydrophobic** and **electrostatic** interactions



Repulsion - monomers

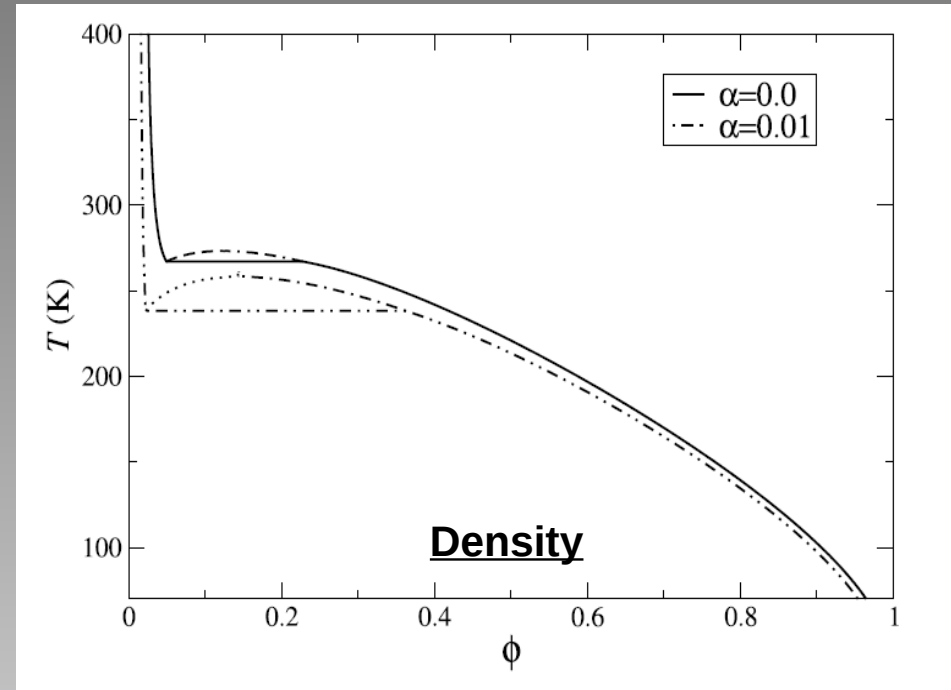
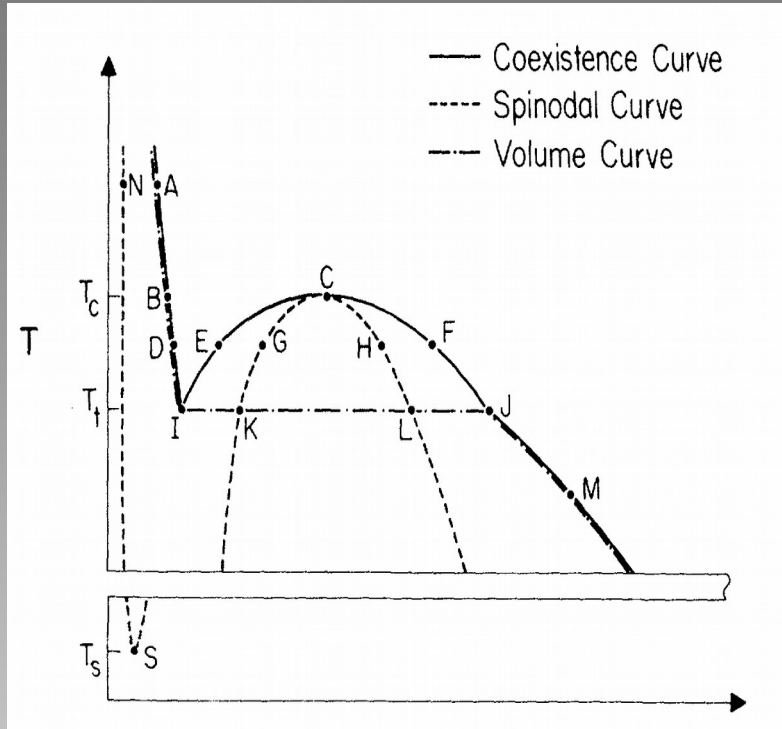
What does swell the gel? Electrostatics or free ion entropy?



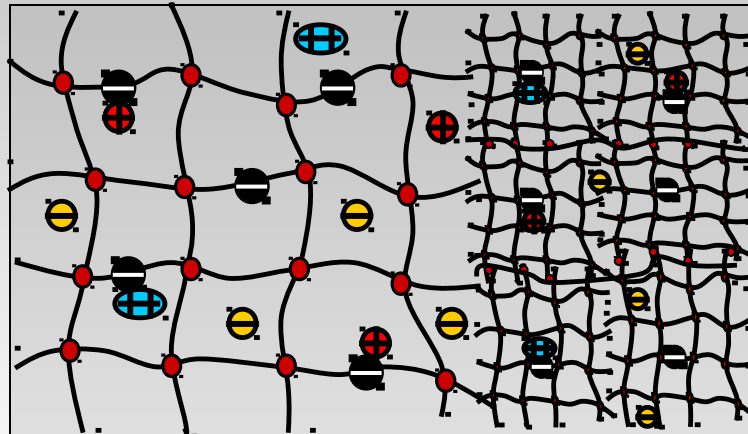
Entropy - counterions

1. P. J. Flory, *Principles of Polymer Chemistry*, Cornell University Press
2. Jing Hua, Mithun K. Mitra, and M. Muthukumar JCP, 136, 134901 (2012).

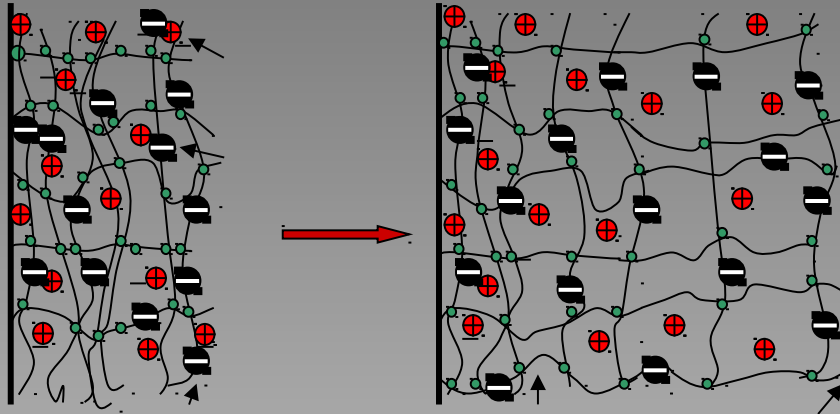
# Phase behaviour – charged gels - theory:



Swati Sen and A. Kundagrami,  
JCP, 143, 224904 (2015)



# Swelling kinetics of a charged gel – Aim of study:



Swelling starts with a homogenous gel, ends with a homogeneous gel

In between,  
density, charge, osmotic pressure/stress,  
– inhomogeneous and evolves with time

We want to find:

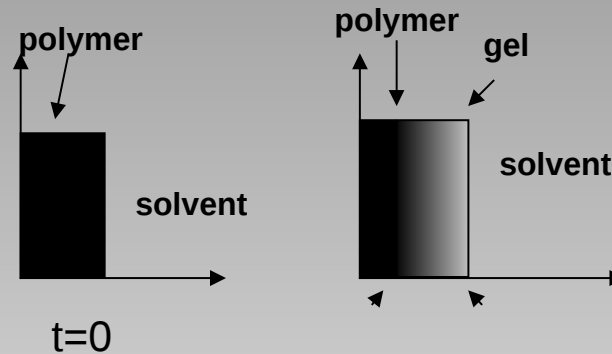
Spatial and temporal profiles of  
density, charge, osmotic pressure/stress

as functions of

a) charge content / dielectric constant

b) hydrophobicity

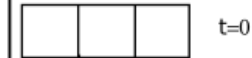
c) cross-link density



Displacement (strain)

$$\langle \mathbf{u}(\mathbf{r}, t \rightarrow \infty) \rangle = 0$$

(a) Initial Homogeneous State



(b) Final Swollen State : Neutral Gel



(c) Final Swollen State : Polyelectrolyte Gel

$t=t' (\alpha_2 > \alpha_1) (t' \rightarrow \infty)$





# Effective Bulk Modulus of a Polyelectrolyte (PE) Gel:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \tilde{\sigma} - f \frac{\partial \mathbf{u}}{\partial t}$$

## Bulk Modulus Method

$$u_{ik} = \frac{1}{2} \left( \frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k} \right)$$

$$\sigma_{ik} = K \nabla \cdot \mathbf{u} \delta_{ik} + 2\mu \left( u_{ik} - \frac{1}{3} \nabla \cdot \mathbf{u} \delta_{ik} \right)$$

$$\frac{\partial \mathbf{u}}{\partial t} = \left( K + \frac{4\mu}{3} \right) \nabla (\nabla \cdot \mathbf{u}) + \frac{\mu}{f} \nabla^2 \mathbf{u}$$

$$\frac{\partial \mathbf{u}}{\partial t} = \frac{K}{f} \frac{\partial^2 \mathbf{u}}{\partial x^2}$$

$$\sigma_x = K \frac{\partial u}{\partial x}$$

## Stress Relaxation Method

$$\frac{\partial \mathbf{u}}{\partial t} = \frac{1}{f} \frac{\partial}{\partial x} \sigma_{xx} (\phi, \alpha, \delta, \chi, S)$$

$$\Pi(\phi, \alpha, \chi, S, T) = \frac{K_B T}{\nu_c} \left[ \phi \left( \frac{\partial f_{\text{en}}}{\partial \phi} \right)_T - f_{\text{en}} \right]$$

$$\phi(x, t) = \frac{\phi_f}{1 - \frac{\partial u}{\partial x}}$$

$$F \sim f_{\text{en}} \Omega \quad F(\phi, \alpha, \chi, S, T)$$

$$K \frac{\partial u}{\partial x} \longrightarrow \sigma_{xx} (\phi, \alpha, \delta, \chi, S)$$

**Aim:** To find an **effective bulk modulus** for the Polyelectrolyte gel from the **kinetics of relaxation of osmotic stress**

Swati Sen and A. Kundagrami, JCP, 143, 224904 (2015).

Acknowledgment: T. Tanaka and D. J. Fillmore, JCP, 70, 1214 (1979), E. S. Matsuo and T. Tanaka, JCP, 89, 1695 (1988)

# Polyelectrolyte gel - free energy:

$$f_s = \frac{\phi}{N} \log \phi + \phi_c \log \phi_c + \phi_s \log \phi_s$$

$$\phi = nN\ell^3 / \Omega$$

$$\phi_c = \alpha nN\ell^3 / \Omega$$

$$f_{sa} = [\alpha \log \alpha + (1 - \alpha) \log(1 - \alpha)] \phi$$

$$\phi + \phi_c + \phi_s = 1$$

$$f_\chi = \chi \phi \phi_s$$

**FLORY**

**SALT FREE**

$$\tilde{\kappa}^2 = 4\pi \tilde{\ell}_B \alpha \phi$$

**Lever rule**

$$\phi = x\phi^a + (1 - x)\phi^b$$

**Minimize the TOTAL free energy (the sum of both coexisting phases), w.r.t. 4 variables – 2 densities, 2 charges of two phases.**

$$f_{el} = 2\pi\alpha^2\ell_B\phi^2 \frac{N^{2/3}}{\left[ \frac{3^{4/3}\pi^{7/6}}{2^{5/3}}\phi^{2/3} + \tilde{\kappa}^2 N^{2/3} \right]}$$

$$f_{fl,i} = -\frac{1}{4\pi} \left[ \log(1 + \tilde{\kappa}) - \tilde{\kappa} + \frac{1}{2}\tilde{\kappa}^2 \right]$$

$$f_{ad} = -(1 - \alpha)\phi\tilde{\ell}_B\delta$$

$$f = f_s + f_{sa} + f_\chi + f_{el} + f_{ad} + f_{fl,i}$$

$$f_{elast} = \frac{3}{2}S\phi_0^3 \left[ \left( \frac{\phi}{\phi_0} \right)^{1/3} - \frac{\phi}{\phi_0} + \frac{1}{3} \frac{\phi}{\phi_0} \ln \frac{\phi}{\phi_0} \right]$$

**M. Muthukumar, J. Hua, and  
A. Kundagrami  
JCP, 132, 084901 (2010).**

# Osmotic pressure from free energy of a PE gel:

$$\Pi(\phi, \alpha, \chi, S, T)$$

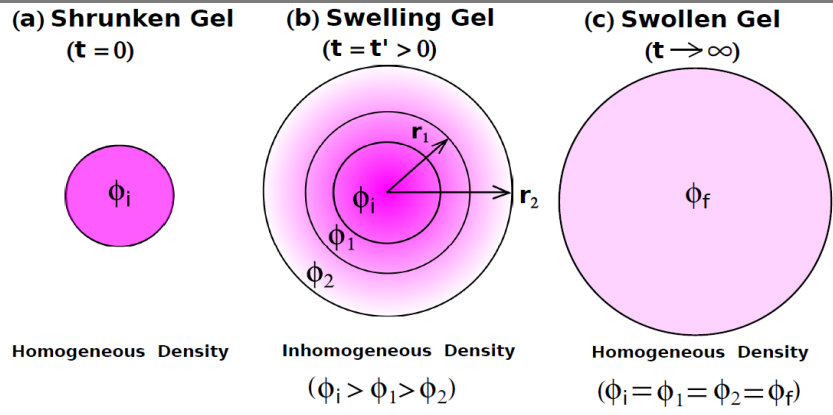
## Polyelectrolyte gel

$$\sigma_{xx} = \frac{K_B T}{\nu_c} \left[ -\phi - \ln(1 - (1 + \alpha)\phi) - \chi\phi^2(1 + \alpha) + S\phi_0^3 \left( \frac{\phi}{2\phi_0} - \left( \frac{\phi}{\phi_0} \right)^{\frac{1}{3}} \right) + \frac{1}{4\pi} \left\{ \ln(1 + \tilde{\kappa}) - \frac{\tilde{\kappa}}{2(1 + \tilde{\kappa})} - \frac{\tilde{\kappa}}{2} \right\} + \frac{2\pi b\alpha^2 N^{\frac{2}{3}} \tilde{l}_B}{3} \frac{\phi^{8/3}}{(b\phi^{2/3} + N^{\frac{2}{3}} \tilde{\kappa}^2)^2} \right],$$

## Polymer (uncharged) gel

$$\sigma_{xx} = \pi_{os} = \frac{K_B T}{\nu_c} \left[ -\phi - \ln(1 - \phi) - \chi\phi^2 + S\phi_0^3 \left\{ \frac{\phi}{2\phi_0} - \left( \frac{\phi}{\phi_0} \right)^{\frac{1}{3}} \right\} \right].$$

# Swelling of PE gels – variable degree of ionization:



**Schematic of swelling**

$$\frac{\partial u}{\partial t} = \frac{K}{f} \frac{\partial}{\partial r} \left( \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) \right) \quad \text{BMM}$$

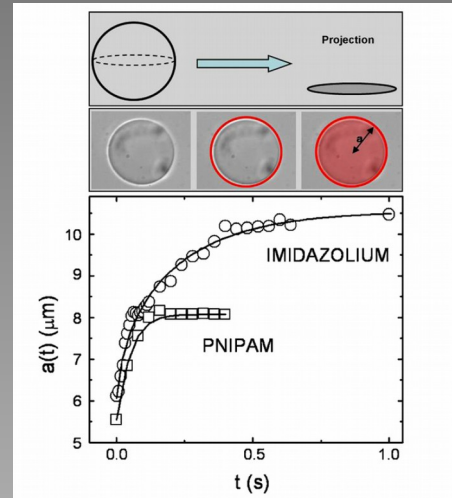
$$\frac{\partial u}{\partial t} = \frac{1}{f} \frac{\partial}{\partial r} \Pi_{rr} \quad \text{SRM w/ charge-regularization}$$

$$\partial f_{\text{en}} / \partial \alpha|_{\phi} = 0$$

$$\Pi = \Pi_0 + K \left( \frac{\partial u}{\partial r} + 2 \frac{u}{r} \right)$$

$$\longrightarrow K(\phi, \alpha, \chi, S)$$

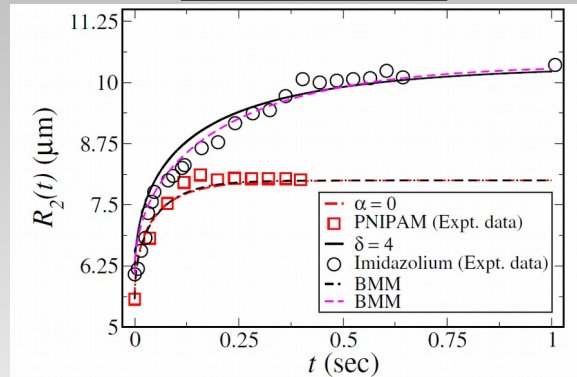
Swati Sen and A. Kundagrami, to be submitted



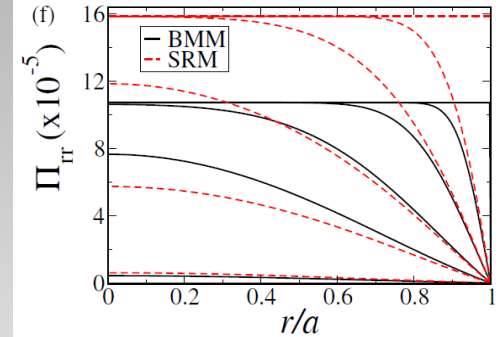
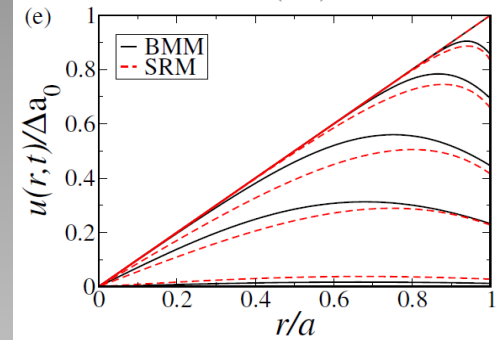
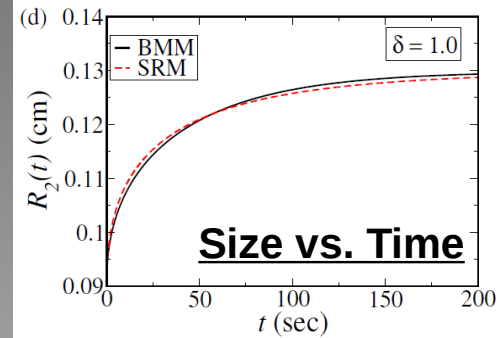
**Matching of gel-front w/ Experiments: BMM**

I. J. Suarez et al.  
Colloids and Surfaces A:  
Phys. Eng. Asp. 343, 33 (2009)

**Size vs. Time**

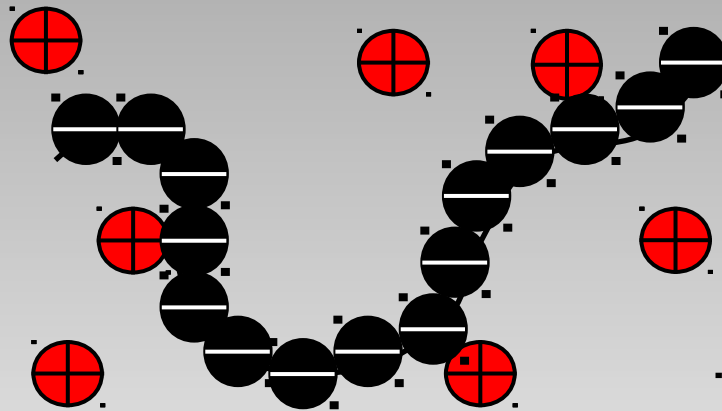


**Matching of gel-front w/ Experiments: SRM**

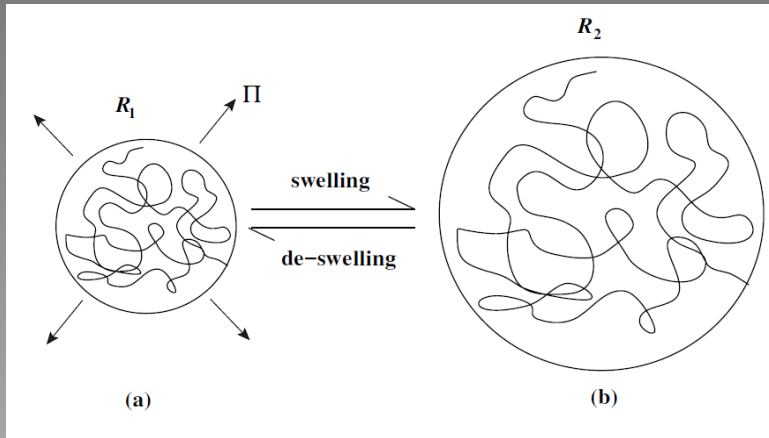


**Matching between BMM & SRM: Size & Osmotic Pressure**

# Kinetics of swelling and collapse of a single polymer chain



# Equation of motion – osmotic and viscous forces:



**Swelling and collapse of:**

**Single, isolated, flexible  
polyelectrolyte (PE) chain**

## Uniform spherical expansion model

**EOM for surface element – osmotic stress and viscous force**

$$\sigma_s \Delta S \frac{d^2 R}{dt^2} = -\zeta \Delta S \frac{dR}{dt} + \Pi \Delta S$$

**Osmotic stress obtained through the free energy**

$$\Pi = - \left( \frac{\partial F}{\partial V} \right)_{N,T} = - \frac{1}{4\pi R^2} \frac{\partial F}{\partial R} \Big|_{N,T}$$

**Free energy**

$$F(\tilde{l}_1, f, N, T)$$

**Equation of Motion:**

$$\zeta \frac{d\tilde{l}_1}{dt} + \frac{1}{\pi} \left( \frac{6}{Nl^2} \right)^2 \frac{\partial F}{\partial \tilde{l}_1} = 0$$

$$\tilde{l}_1 = \left( \frac{6}{Nl^2} \right) R_g^2$$

# Charge regularization:

Assumption: Motion of counterions much faster than that of monomers

1. Counterions re-adjust themselves with virtually frozen configuration of polymer
2. Gel is stable with respect to counterion density variation – chemical equilibrium

Kinetics:

Downhill in free energy with size

But, minimum in charge

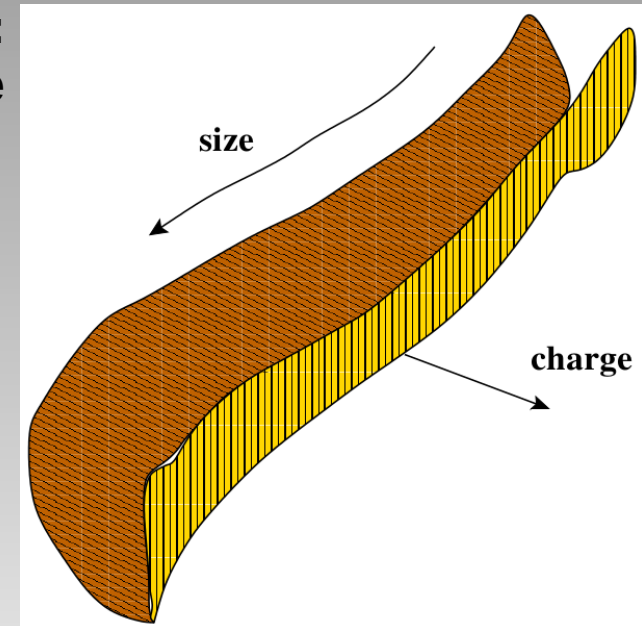
**General expression:**

$$\left. \frac{\partial F}{\partial \tilde{l}_1} \right|_{N,T} = \left. \frac{\partial F}{\partial \tilde{l}_1} \right|_{f,N,T} + \left. \frac{\partial F}{\partial f} \right|_{\tilde{l}_1,N,T} \left( \frac{\partial f}{\partial \tilde{l}_1} \right)$$

**Special condition applicable to this system:**

$$\left. \frac{\partial F}{\partial f} \right|_{\tilde{l}_1,N,T} = 0$$

**Expression of derivative of free energy for fixed charge applies!**



## A free-energy to derive the osmotic pressure:

$$F(\tilde{l}_1, f, N, T)$$

$$F_1 = f \log f + (1 - f) \log(1 - f)$$

$$F_2 = (f\tilde{\rho} + \tilde{c}_s) \log(f\tilde{\rho} + \tilde{c}_s) + \tilde{c}_s \log \tilde{c}_s - (f\tilde{\rho} + 2\tilde{c}_s)$$

$$F_3 = -\frac{1}{3} \sqrt{4\pi} \tilde{l}_B^{3/2} (f\tilde{\rho} + 2\tilde{c}_s)^{3/2}$$

$$F_4 = -(1 - f) \delta(l_B/l)$$

$$F_5 = \frac{3}{2N} [\tilde{l}_1 - 1 - \log \tilde{l}_1] + \frac{4}{3} \left( \frac{3}{2\pi} \right)^{3/2} \frac{w}{\sqrt{N}} \frac{1}{\tilde{l}_1^{3/2}} + \frac{w_3}{N \tilde{l}_1^3} + 2 \sqrt{\frac{6}{\pi}} f^2 \tilde{l}_B \frac{N^{1/2}}{\tilde{l}_1^{1/2}} \Theta_0(a)$$

$$\Theta_0(a) = \frac{\sqrt{\pi}}{2} \left( \frac{2}{a^{5/2}} - \frac{1}{a^{3/2}} \right) \exp(a) \operatorname{erfc}(\sqrt{a}) + \frac{1}{3a} + \frac{2}{a^2} - \frac{\sqrt{\pi}}{a^{5/2}} - \frac{\sqrt{\pi}}{2a^{3/2}}$$

$$a = \tilde{\kappa}^2 N \tilde{l}_1 / 6 \quad \tilde{\kappa}^2 = 4\pi \tilde{l}_B (f\tilde{\rho} + 2\tilde{c}_s) \quad \tilde{l}_B = e^2 / 4\pi \epsilon \epsilon_0 l k_B T$$

**M. Muthukumar, JCP, 120, 9343 (2004)**

**A. Kundagrami and M. Muthukumar, Macromolecules, 43, 2574 (2010)**



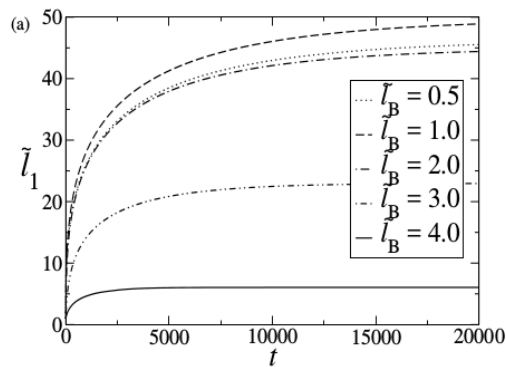
# Swelling profiles:

$$f = \frac{-(\tilde{c}_s + e^{-\delta \tilde{l}_B}) + \sqrt{(\tilde{c}_s + e^{-\delta \tilde{l}_B})^2 + 4\tilde{\rho}e^{-\delta \tilde{l}_B}}}{2\tilde{\rho}}$$

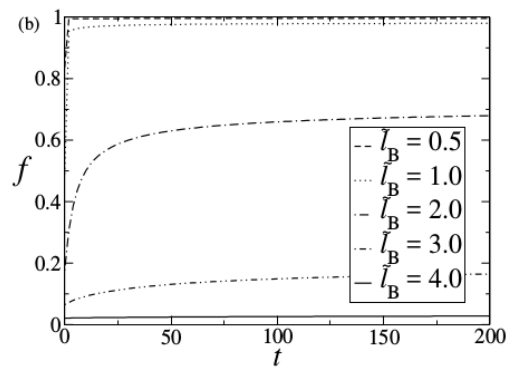
Expression of charge – expanded state

A. Kundagrami and M. Muthukumar, *Macromolecules*, 43, 2574 (2010)

## Size vs. Time



## Charge vs. Time



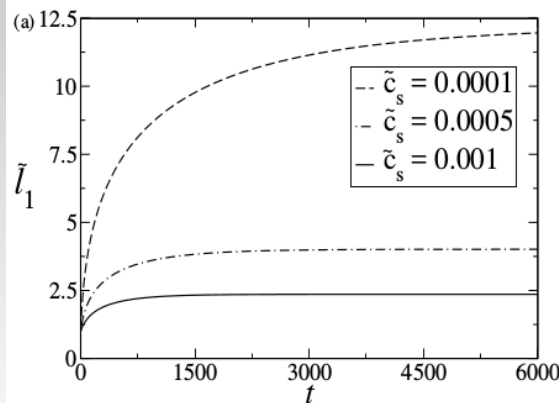
## Swelling at different temperatures:

1. Swells faster and farther for higher  $T$ .
2. Lower temperature – condensation reduces final size
3. Extended chain – charge is not dependent on size

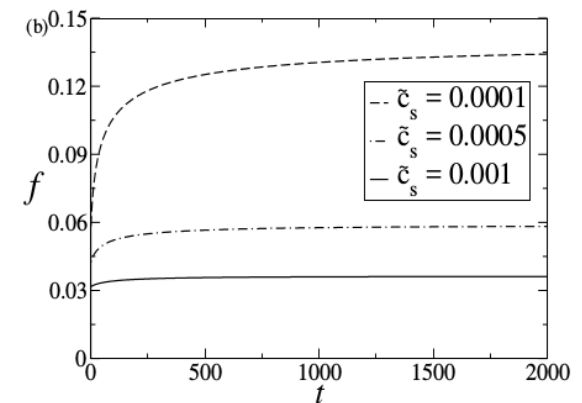
## Swelling at different salt:

1. Swells faster and farther for lower monovalent salt.
2. Extended chain – charge is not dependent on size

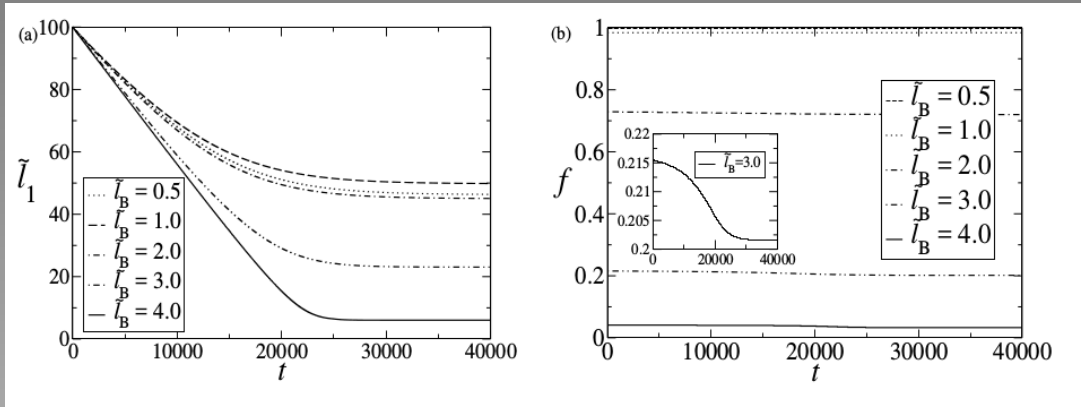
## Size vs. Time



## Charge vs. Time

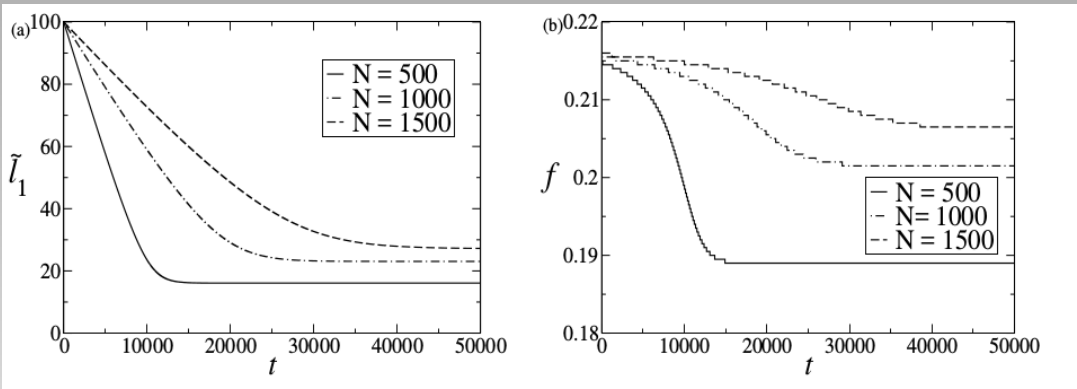


# De-swelling profiles:



Size vs. Time

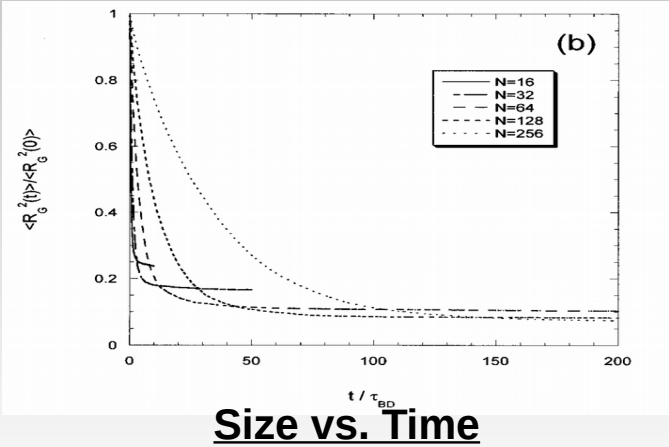
Charge vs. Time



- De-swelling for different molecular weights:**
1. De-swells slower for higher molecular weight
  2. Matches qualitatively with experimental results with PMMA gels

- De-swelling at different temperatures:**
1. De-swells faster and deeper for lower  $T$
  2. Lower temperature – condensation reduces final size
  3. Extended chain – charge is not dependent on size

**Simulations:**  
 Chang and Yethiraj,  
 JCP, 114, 7688 (2001)

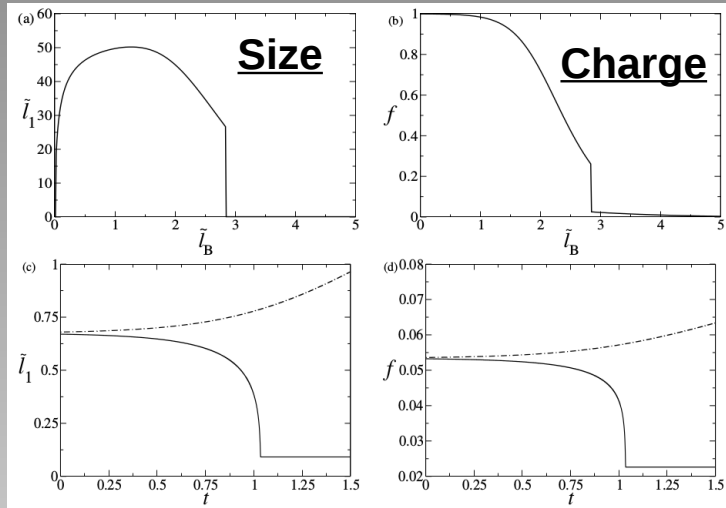


Size vs. Time

# Collapse profiles:

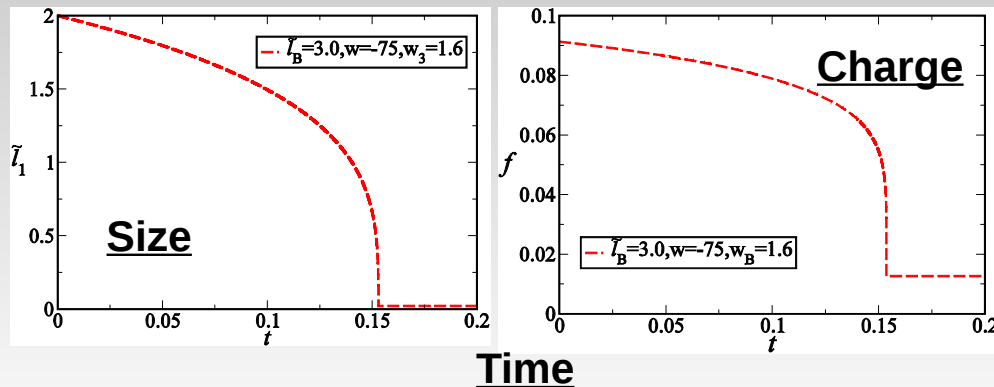
1. Poor solvent – hydrophobic – negative two-body parameter
2. Size goes below Gaussian -  $\tilde{l}_1 < 1$

## Equilibrium



## Kinetics

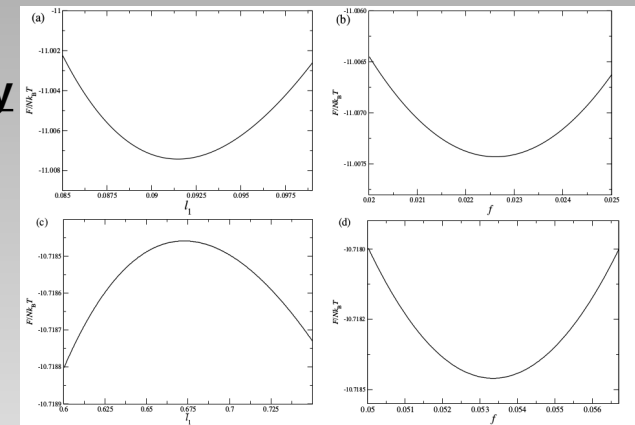
Direct collapse: extended to globule



Equilibrium result – first order transition  
– both in size and charge

Local maximum prohibits direct collapse to the global minimum

## Free energy



Size

Charge

Local  
maximum

Global  
minimum

# Low- and high-salt limits – equations of motion:

**Low-salt limit:**

$$\zeta' \frac{d\tilde{l}_1}{dt} + \frac{T}{N} \left\{ \frac{3}{2N} \left[ 1 - \frac{1}{\tilde{l}_1} \right] - 2 \left( \frac{3}{2\pi} \right)^{3/2} \frac{w}{\sqrt{N}} \frac{1}{\tilde{l}_1^{5/2}} - \frac{3}{N} \frac{w_3}{\tilde{l}_1^4} - \frac{2}{15} \sqrt{\frac{6}{\pi}} f^2 \tilde{l}_B \frac{N^{1/2}}{\tilde{l}_1^{3/2}} \right\} = 0$$

**High-salt limit:**

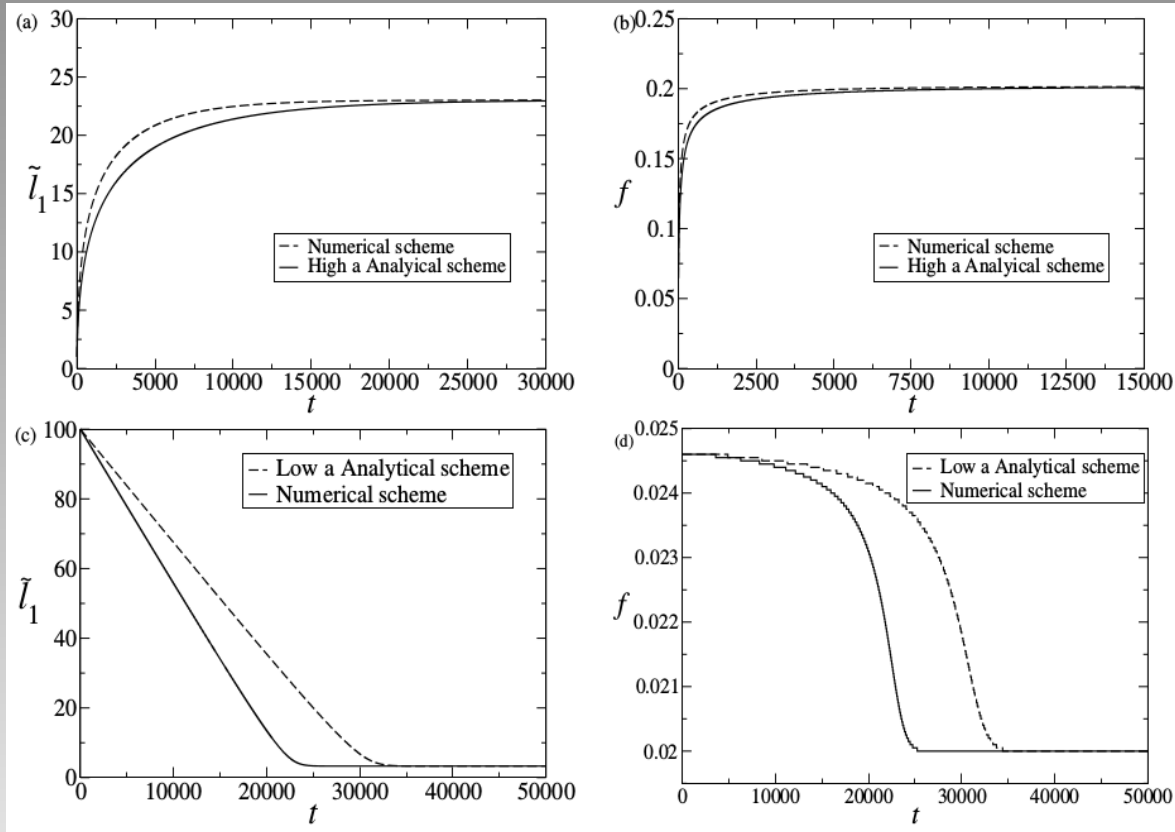
$$\zeta' \frac{d\tilde{l}_1}{dt} + \frac{T}{N} \left\{ \frac{3}{2N} \left[ 1 - \frac{1}{\tilde{l}_1} \right] - 2 \left( \frac{3}{2\pi} \right)^{3/2} \frac{w}{\sqrt{N}} \frac{1}{\tilde{l}_1^{5/2}} - \frac{3}{N} \frac{w_3}{\tilde{l}_1^4} - \frac{3}{2} \left( \frac{6}{N} \right)^{1/2} \frac{1}{\pi^{3/2}} \frac{f^2}{(f\tilde{\rho} + 2\tilde{c}_s)} \frac{1}{\tilde{l}_1^{5/2}} \right\} = 0$$

$$w' = w + \frac{f^2}{(f\tilde{\rho} + 2\tilde{c}_s)}$$

1. Simpler differential equations – analytical expressions for derivatives of free energy
2. In high-salt limit, electrostatic interaction is screened and becomes Short-ranged. Hence, just the two-body interaction parameter is re-scaled

# Limiting results – comparison to full numerical results:

High-salt



Size vs. Time

Charge vs. Time

Low-salt

# Analytical Expressions – Size vs. Time:

## Size vs. Time

### Swelling:

$$\tilde{l}_1^{5/2} - \tilde{l}_{10}^{5/2} = \frac{5}{2} \frac{T}{N\zeta'} \frac{2}{15} \sqrt{\frac{6}{\pi}} f^2 \tilde{l}_B N^{1/2} t \quad \text{Low-salt}$$

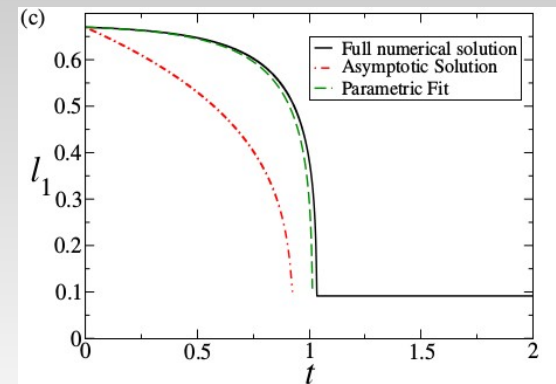
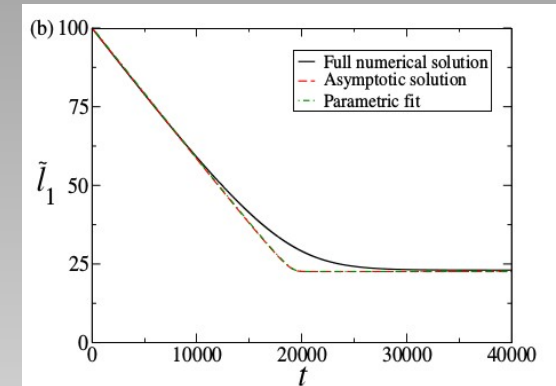
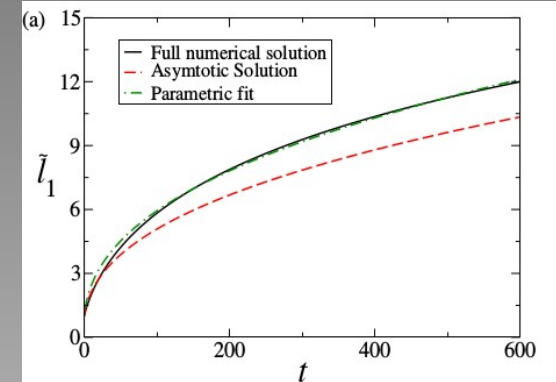
$$\tilde{l}_1^{7/2} - \tilde{l}_{10}^{7/2} = \frac{7}{8\zeta'} \left( \frac{6}{N\pi} \right)^{3/2} \frac{T f^2}{f\tilde{\rho} + 2\tilde{c}_s} t \quad \text{High-salt}$$

### De-swelling:

$$(\tilde{l}_1 - \tilde{l}_{1f}) \exp(\tilde{l}_1) = \exp(\tilde{l}_{10}) (\tilde{l}_{10} - \tilde{l}_{1f}) \exp \left( -\frac{3T}{2N^2\zeta'} t \right)$$

### Collapse:

$$\tilde{l}_1^{7/2} - \tilde{l}_{10}^{7/2} = \frac{7}{2} \frac{2T}{N\zeta'} \left( \frac{3}{2\pi} \right)^{3/2} \frac{wt}{\sqrt{N}}$$



# Conclusions:

1. Swelling of polyelectrolyte systems – both gels and isolated chains  
– can be treated in the same footing – motion of polymer through the solvent – osmotic stress vs. viscous damping
2. Motion of small-ion charge species much faster than polymer:  
- charge is regularized (self-adjusted) all along the kinetics
2. Swelling of a polymer gel: for small deformation – is diffusive  
- single chain: sub-diffusive
3. Effective bulk-modulus of polyelectrolyte gels decreases with charge  
- small deformation
4. Single polyelectrolyte chain:
  - a) swells faster and farther for higher temperature
  - b) de-swells faster and deeper for higher salt
  - c) kinetics is slower for higher molecular weight
  - d) self-consistent dependency between size and charge strong in the vicinity of the Gaussian size

# Charged polymers – energy and entropy:

Do all Na<sup>+</sup> ions dissolve? **NO!**

Counterion condensation. **WHY?**

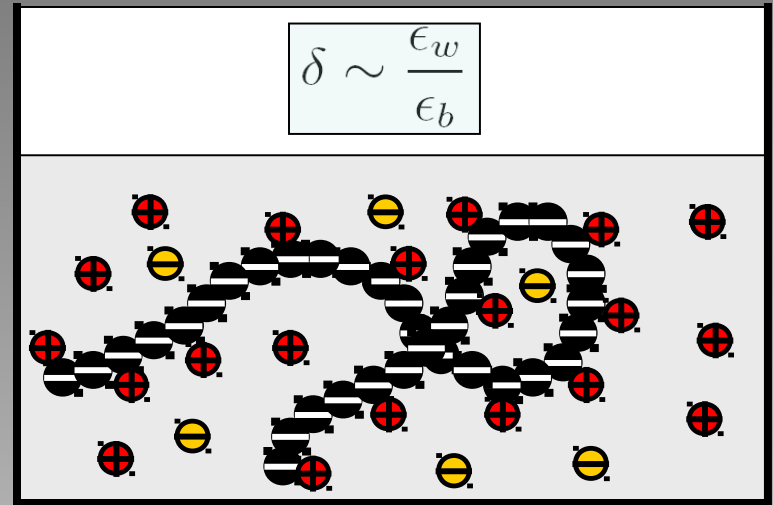
Effective  $\epsilon$  low close to backbone

Condense  $\longrightarrow$  gain in **Coulomb energy**

Let's predict **qualitatively**:

**Remember:**  $E - TS$

$$-\frac{e^2}{4\pi\epsilon_0\epsilon d}$$



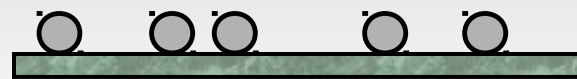
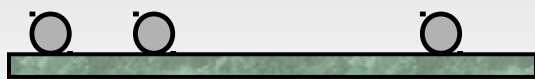
$E$  = Coulomb energy,  $S$  = free ion entropy  $\longrightarrow k_B T$

1.  $T \downarrow \longrightarrow S$  fixed,  $E/\text{ion-pair}$  fixed  $\longrightarrow$  more ion-pairs  $\longrightarrow$  less charge
2.  $\epsilon \downarrow \longrightarrow S$  fixed,  $T$  fixed,  $E/\text{ion-pair}$  increases  $\longrightarrow$  more ion-pairs
3.  $V \uparrow \longrightarrow S$  increases,  $T$  fixed,  $E/\text{ion-pair}$  fixed  $\longrightarrow$  less ion-pairs

Similar to **DEW** formation: at low  $T$ , air can't hold vapor  $\longrightarrow$  condensation

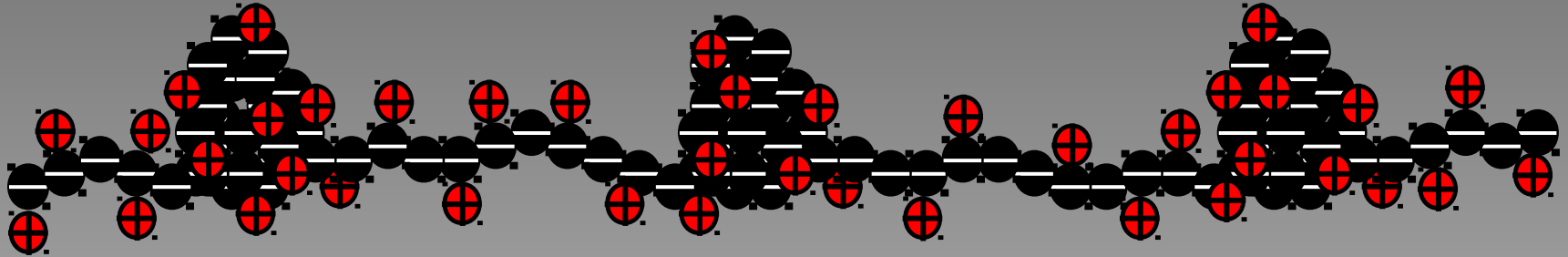


**SALT**  $\longleftrightarrow$  **WET WEATHER**

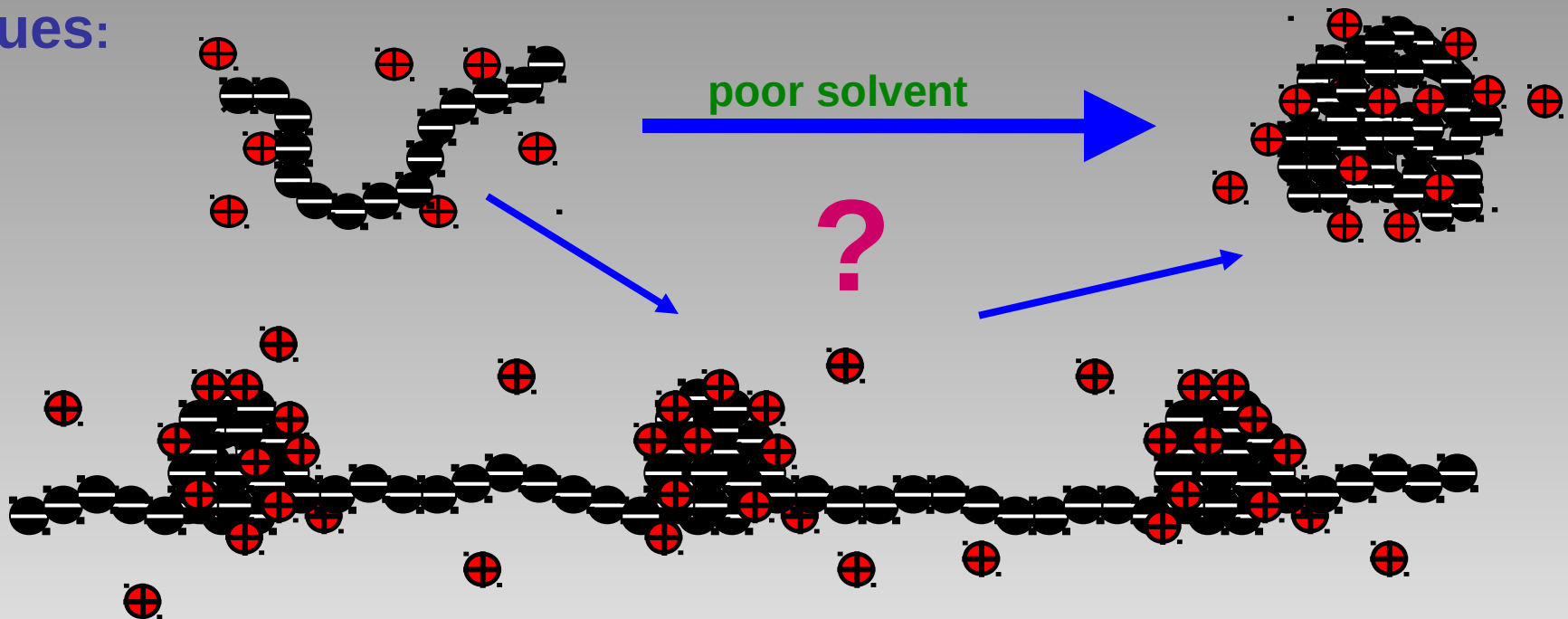




# Charged chain in poor solvent – a pearl-necklace phase?:



Issues:



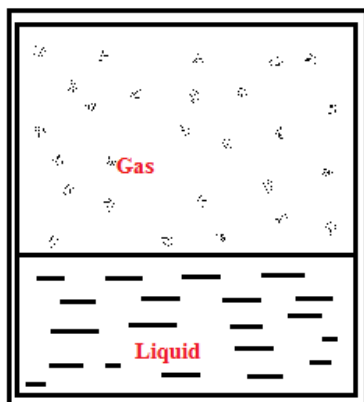
Simulations **do not** consider oily backbone  $\longrightarrow$  Low dielectric constant

Can free ion entropy win over electrostatic energy gain of bound pairs?

IPhD student: Sourav Sadhukhan (joined August 2014)

# Coexistence of coil and globule in a single chain – stability? :

Liquid-Gas Phase

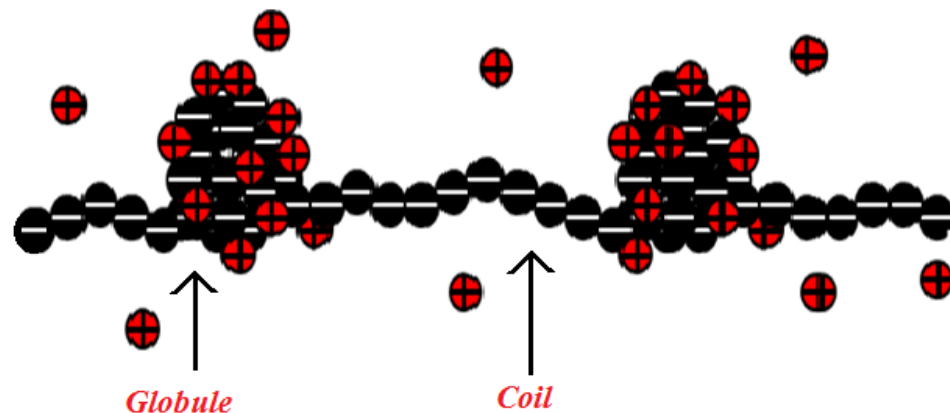


V

Similarly  
Possible ??



Pearl-Necklace Phase



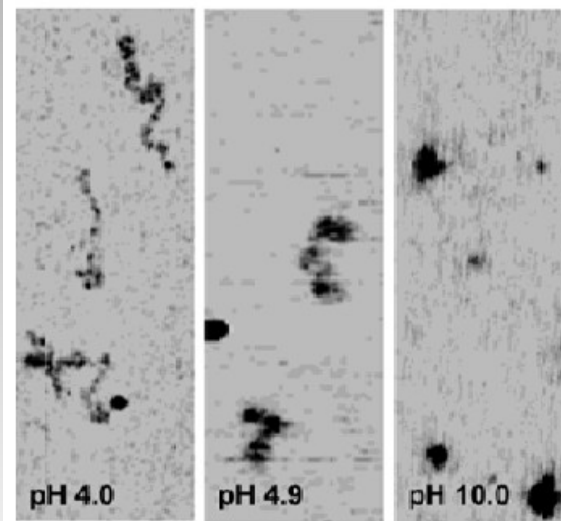
GLOBULE PART

COIL PART

$$N = N' + N''$$

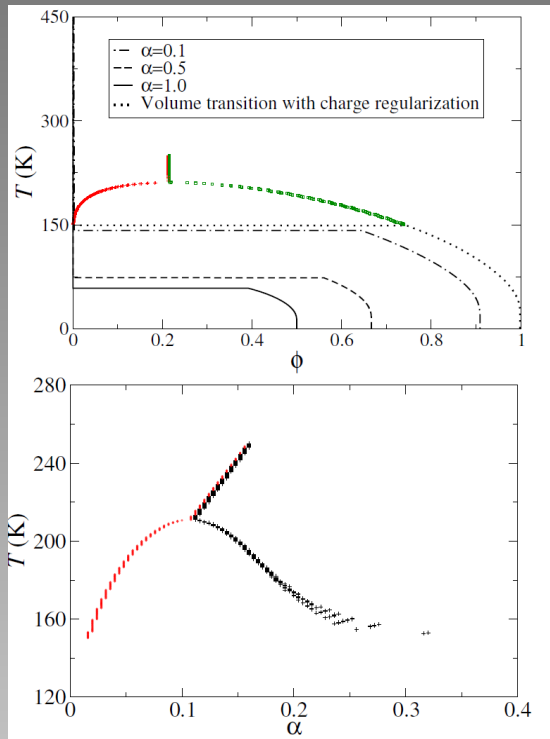
$$N', f', \tilde{l}_1'$$

$$N'', f'', \tilde{l}_1''$$

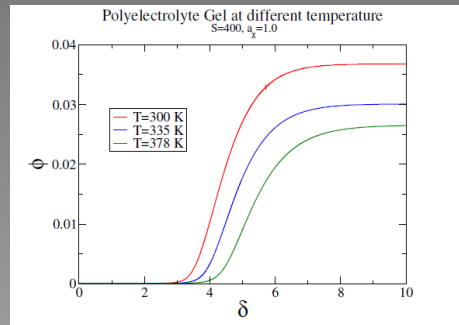


L. J. Kirwan, G. Papastavrou,  
M. Borkovec, Nano Lett., 4, 149 (2004)

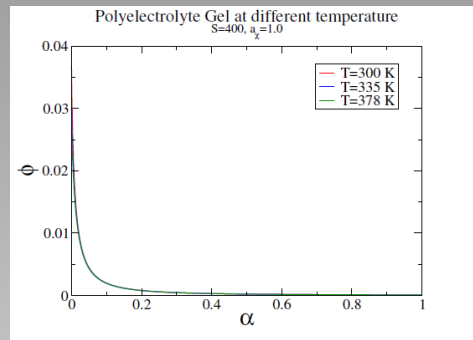
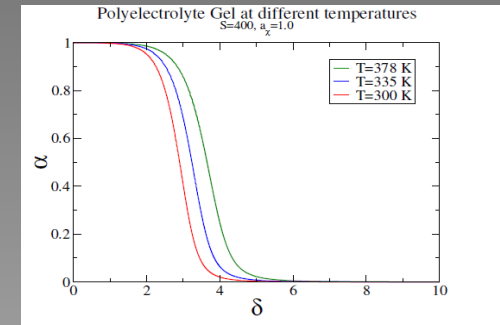
# Polyelectrolyte gels: equilibrium results:



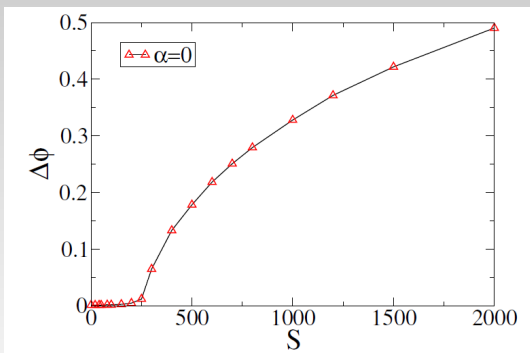
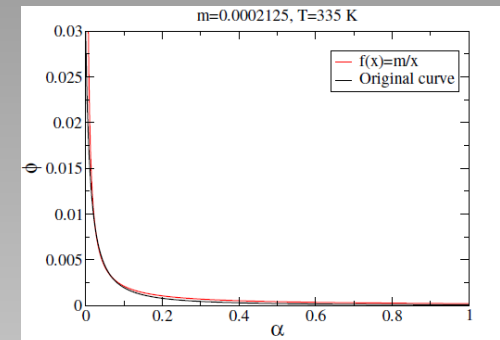
**Coexistence curves:  
Density and charge**



**Density and charge vs. Coulomb strength**

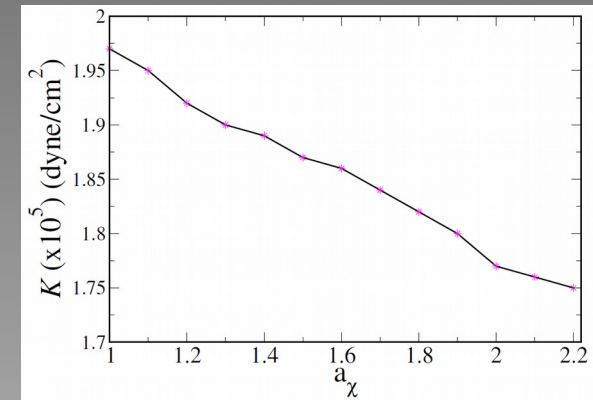
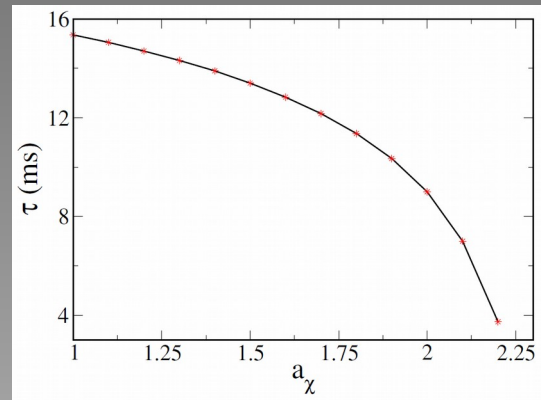
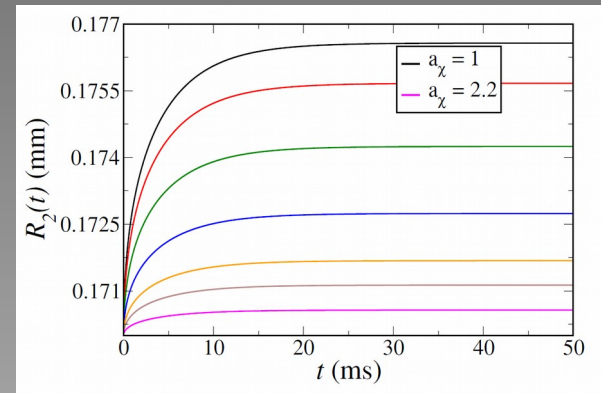


**Density vs. charge:  
invariant with Coulomb strength; product is constant**

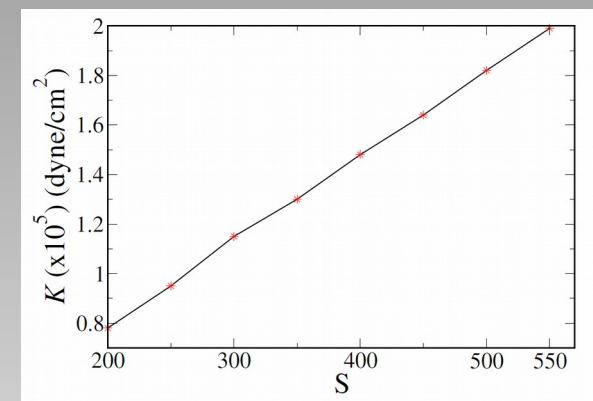
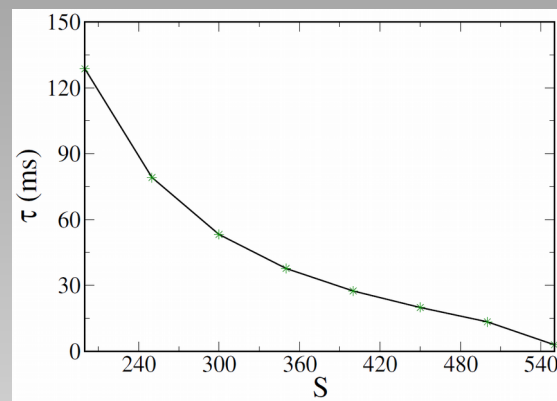
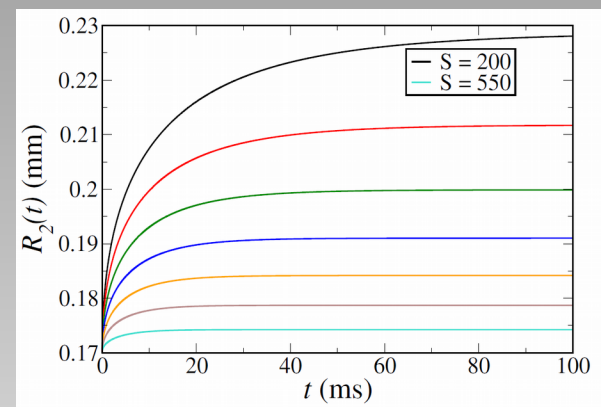


**Collapse volume change with crosslink density:  
Critical exponent?**

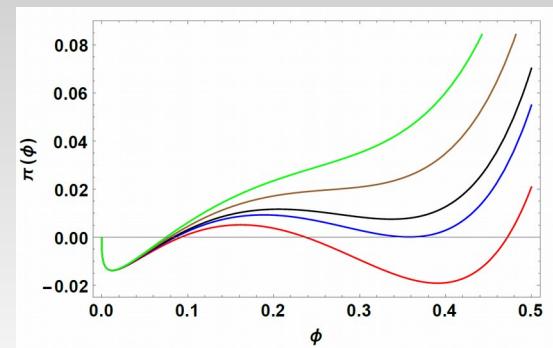
# Swelling: solvent quality, crosslink density, critical dynamics:



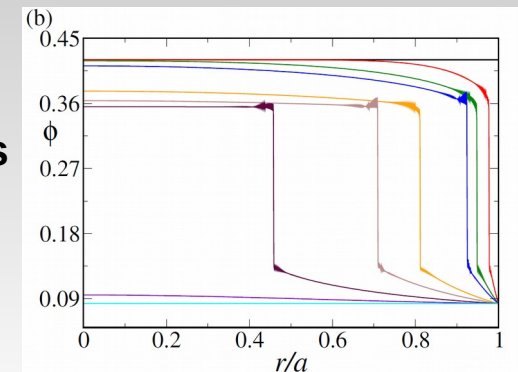
## Swelling characteristics: solvent quality



## Swelling characteristics: crosslink density



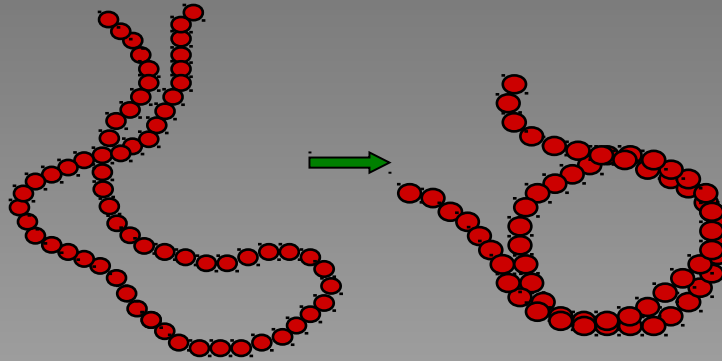
## Critical dynamics



PhD student: Swati Sen (SRF)

# **Future Directions**

# Collapse of a semi-flexible chain - toroids:

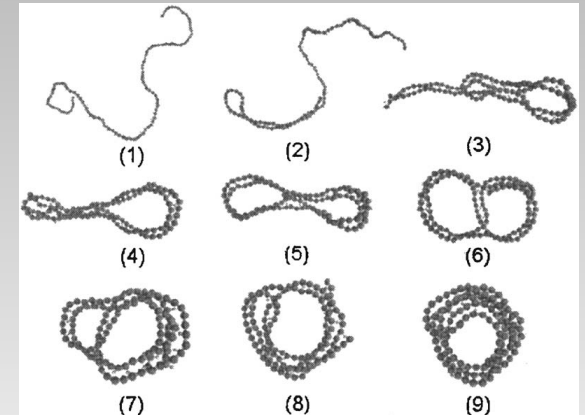
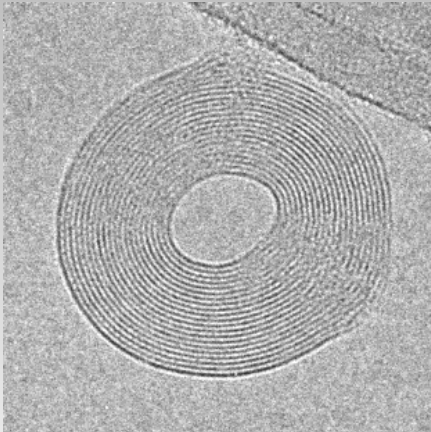


## Issues:

stiff chain  $\xrightarrow{\text{condensing agent}}$  **no** globules  
toroids

Kinetics of **nucleation and growth** of toroids

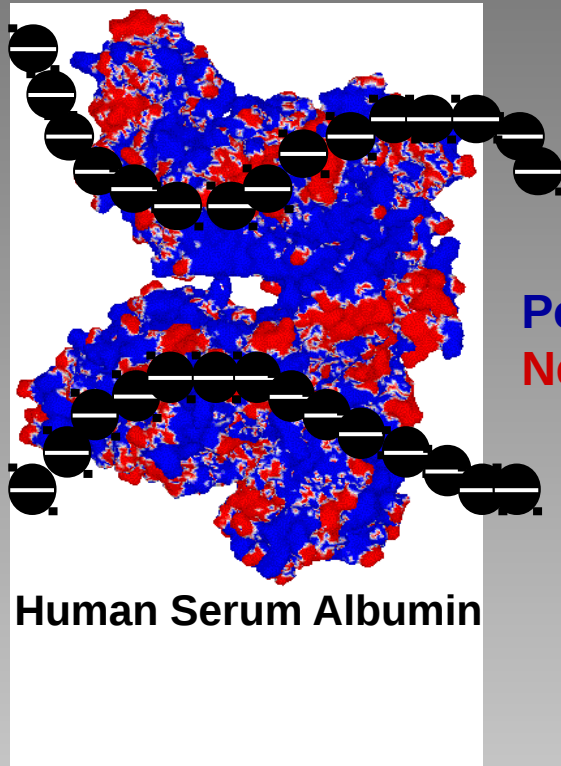
All PE issues: free energy, phase diagrams – **important for dsDNA**



**Experiments: Nick Hud, Georgia Tech.**

**Simulations: Ou, Muthukumar, UMass**

# Complexation of protein and flexible charged chain:



Human Serum Albumin

Issues:

Protein - both **positive** and **negative** charged zones

Role of **chain entropy**: benefactor or prohibitor?

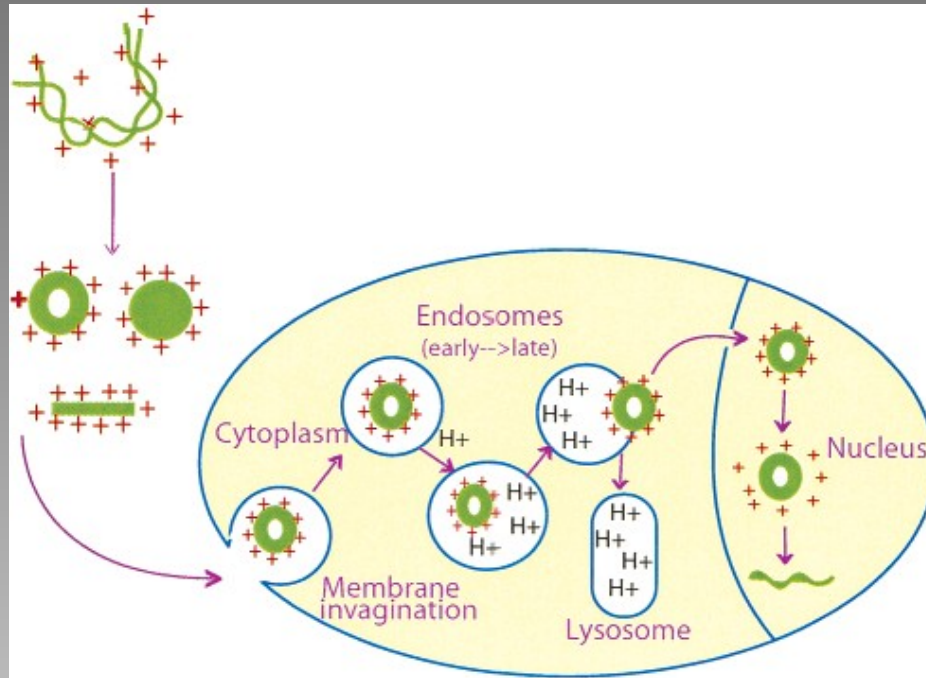
Simple theories of overcharging inapplicable

Possible **aggregation** and **detachment** depending on salt

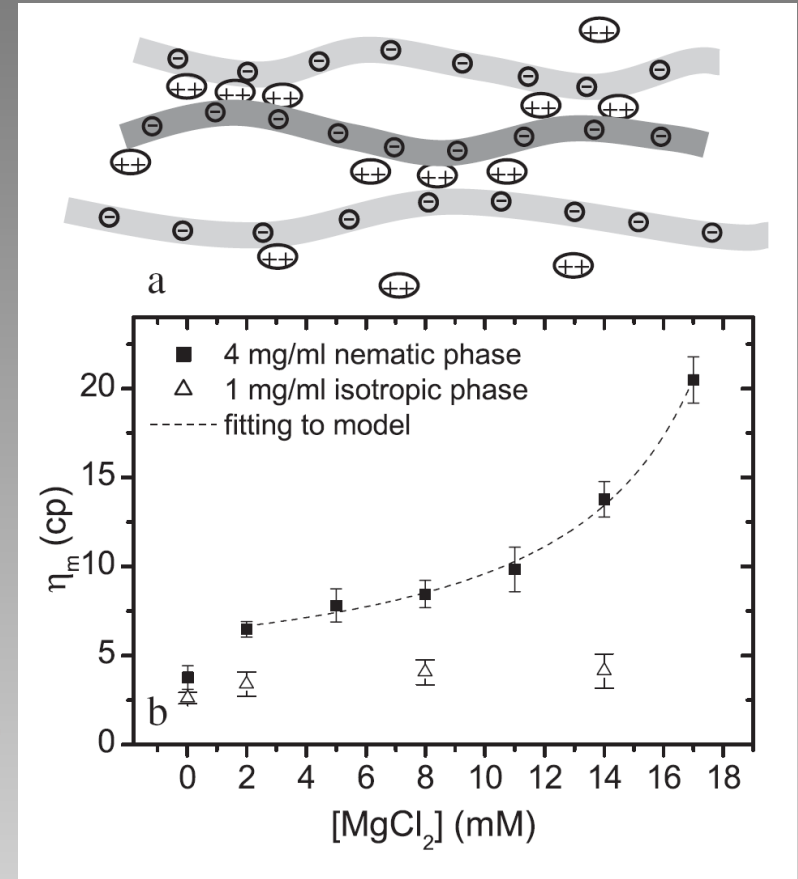
**Examples:** aggregation of flexible anionic polyion (sodium polyacrylate) and DOTAP lipids

**Extensions:** **anionic dendrimers-cationic polymers**, **colloid-anionic polymer**

# Charge inversion and ion-bridging – applications, effects:



V. Vijayanathan *et al.*, *Biochemistry*, **41**, 14085 (2002)



J. He *et al.* *PRL* **99**, 068103 (2007)

1. DNA uptake for **gene therapy**: condensing DNAs to nanoparticles
2. **Reduced diffusivity** of F-actin filaments near isotropic-nematic transition