

Plastic events in sheared amorphous solids

Srikanth Sastry

Jawaharlal Nehru Centre for Advanced Scientific
Research, Bengaluru, India



ISPCM 2017
ICTS, Bengaluru

Ongoing work (**3D model glass**) with:

P. Leishangthem (JNCASR)
Anshul D. S. Parmar
(JNCASR/TCIS)



Earlier work (**2D model glass**) In Collaboration with:

Itamar Procaccia, Prabhat Jaiswal, George Hentschel (Weizmann)

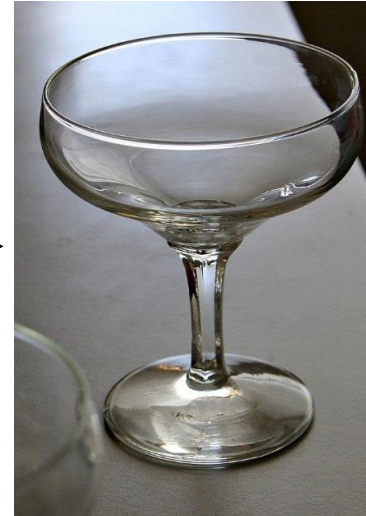
Hentschel, Jaiswal, Procaccia, and Sastry
Phys Rev E 92, 062302 (2015)

Plasticity in Amorphous Solids

Liquid

Cooling

Glass Formation



Applied Stress

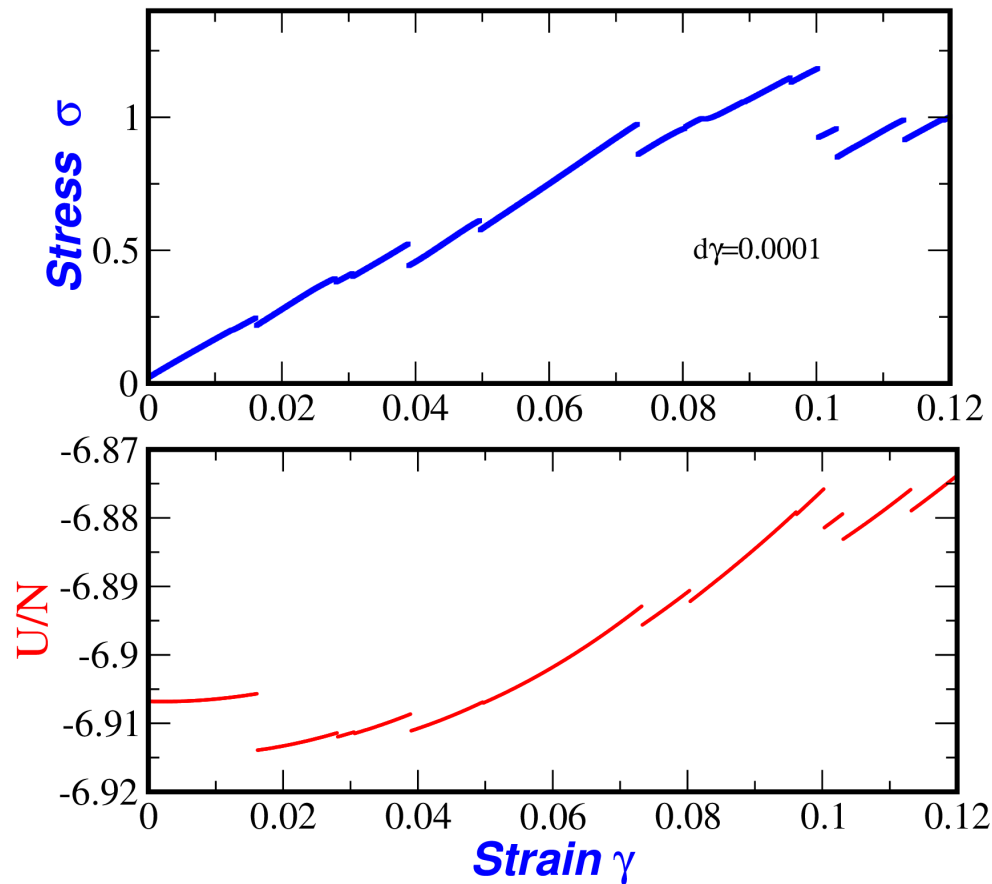
Plasticity and Mechanical Failure



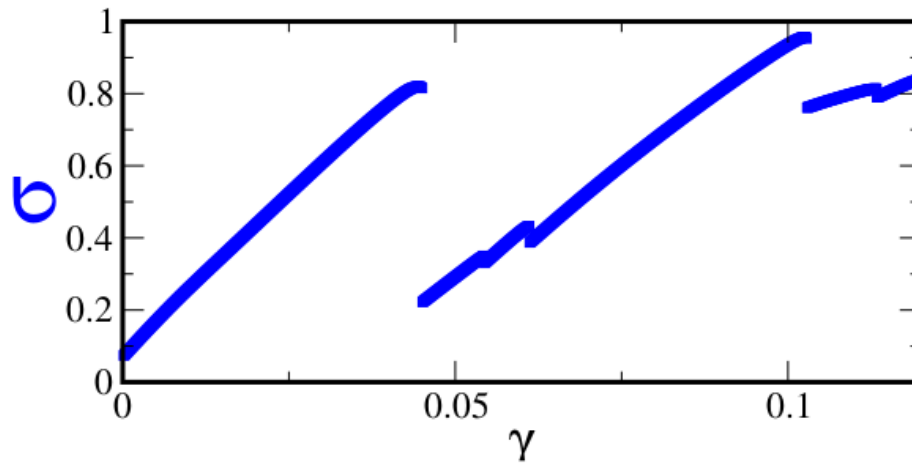
Focus of this talk..

- **"Plastic Events"** that lead eventually to mechanical failure.
- Distributions of strain intervals between plastic rearrangement events.
- Size dependence of mean interval size.
- What do they tell us about the nature of plastic rearrangements en route to failure?

Plastic Events



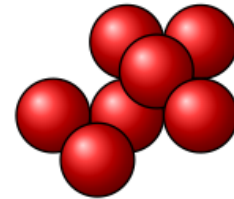
• **Amorphous solids** when strained, the linear elastic branches (before yielding) have plastic jumps as shown above. Local stability is studied by measuring the distance between two **consecutive plastic drops**, through Athermal Quasistatic Shear (AQS) simulations of an atomic glass former (KA binary mixture).



Plastic Instabilities:

Stress Released

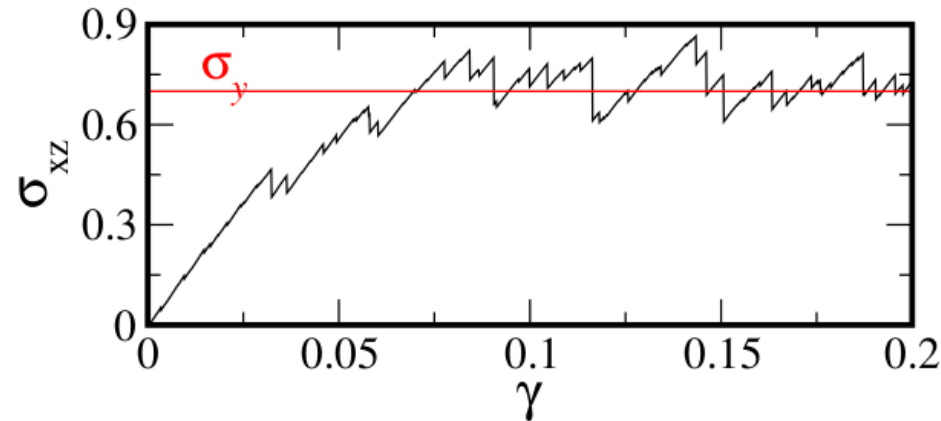
Energy Dissipated



Clusters of
Active Particles

- **The size and structure of plastic drops** have been studied in our earlier work from the statistics of the avalanches and energy drops.
- Here, we focus on the **gap of the plastic drops $\Delta\gamma$** . (First, is there a relation between the two? Yes. For the steady states.)

Relation between plastic strain interval and drop size



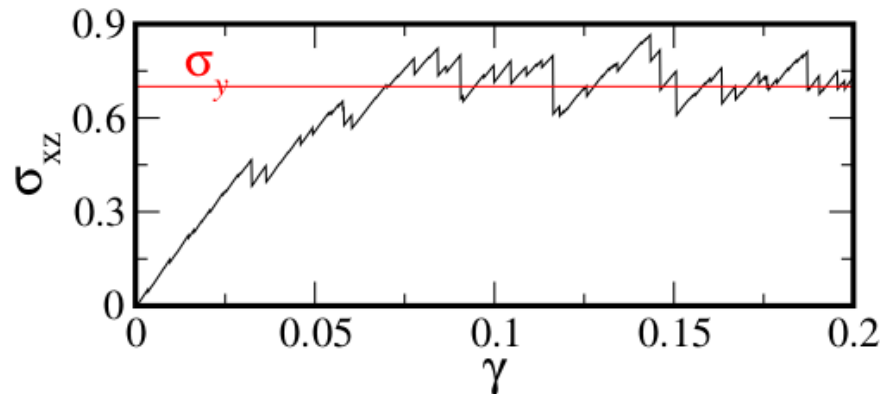
- In the steady flow states:

$$\begin{aligned}\sigma_y \langle \Delta \gamma \rangle V &= \sigma_y \frac{\langle \Delta \sigma \rangle}{\mu} V \\ &= \langle \Delta U \rangle.\end{aligned}$$

- The mean of the input energy density multiplied by the volume during the elastic rise is dissipated by releasing the stress of $\Delta \sigma$.

(In the steady state, mean of the input and the output (dissipated energy) are the same.)

Relation between plastic strain interval and drop size



- In the steady flow states:

$$\begin{aligned}\sigma_y \langle \Delta \gamma \rangle V &= \sigma_y \frac{\langle \Delta \sigma \rangle}{\mu} V \\ &= \langle \Delta U \rangle.\end{aligned}$$

- Let's say:

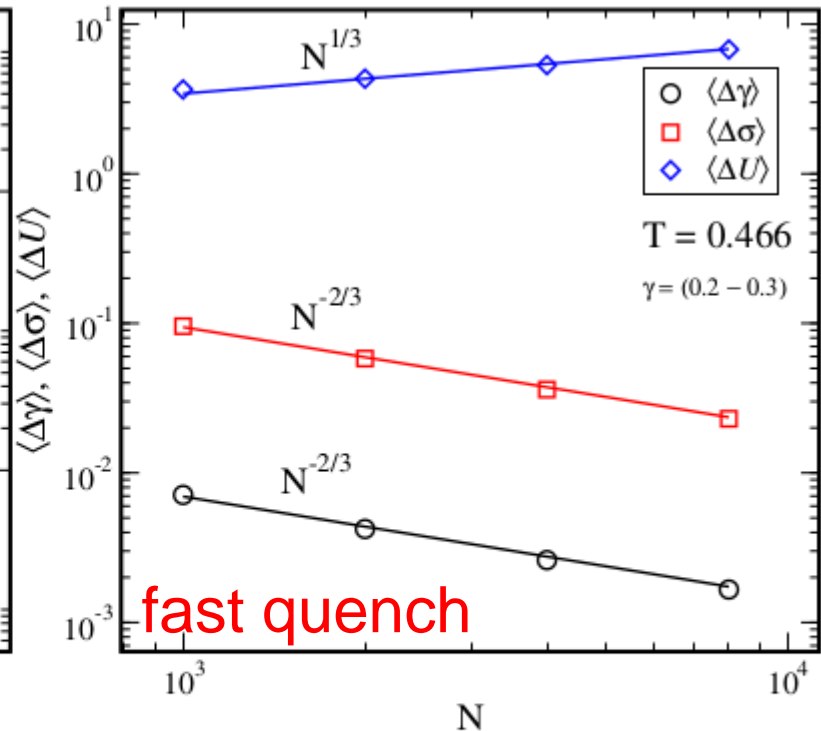
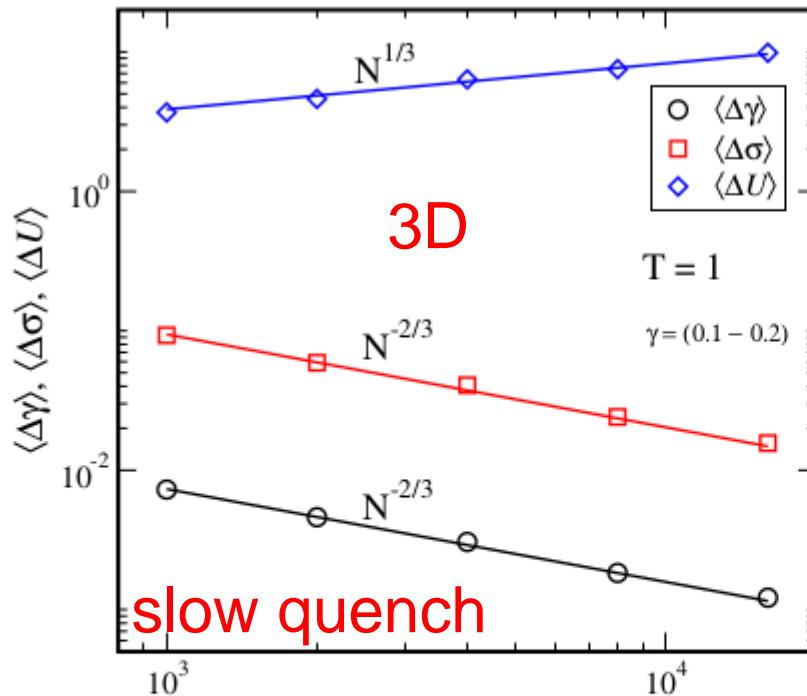
$$\langle \Delta \gamma \rangle \sim N^\beta; \quad \langle \Delta U \rangle \sim N^\alpha.$$

- Therefore,

$$\boxed{\alpha - \beta = 1}$$

We will focus on these exponents.

Steady State (Flow Regime)



$$\alpha = 1/3, \beta = -2/3$$

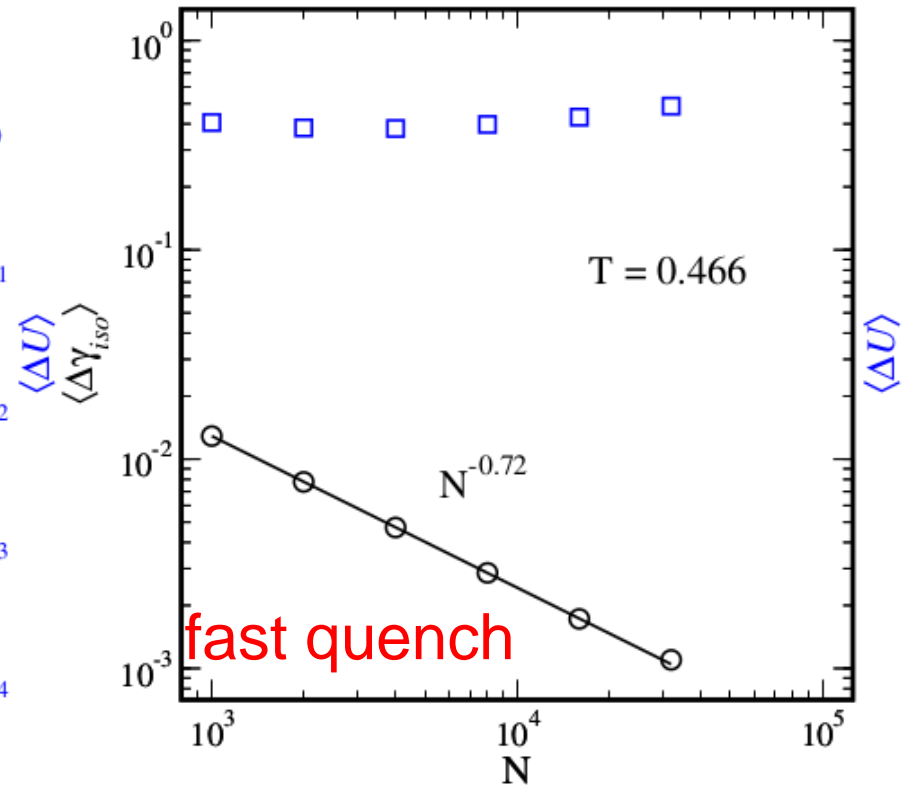
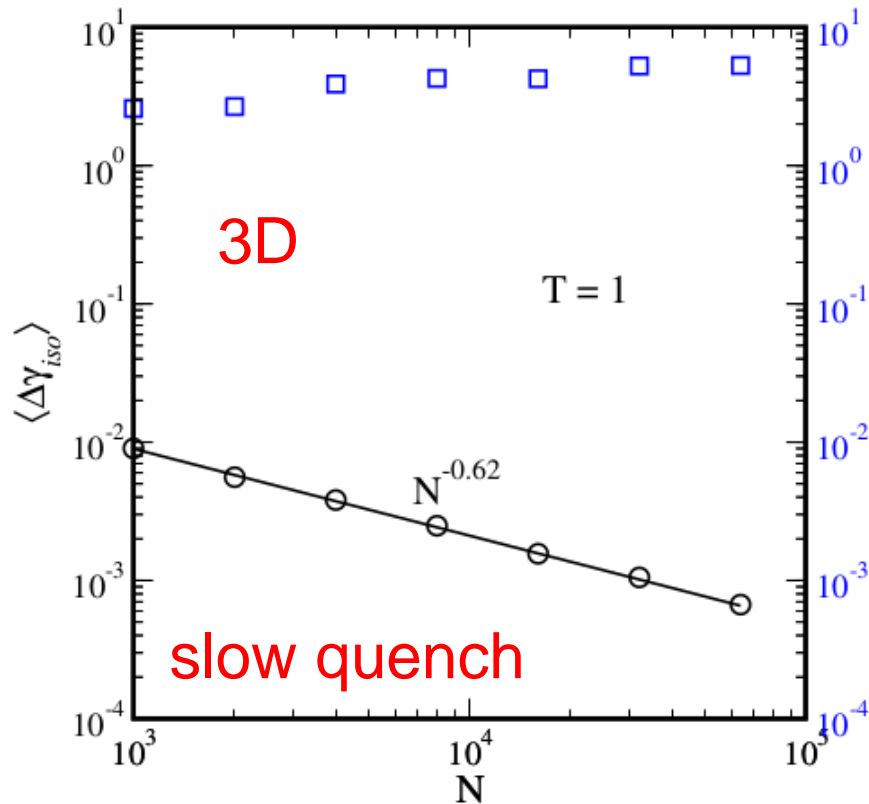
Observed in 2D and 3D. Universal?

$$\alpha - \beta = 1 \quad \checkmark$$

(We show here the validity of the relation for the two differently annealed glasses.)

The behaviour preceding flow is different: (i) Initial plastic event of isotropic configurations, (ii) events preceding failure.

The First Plastic Events (Isotropic Configurations)

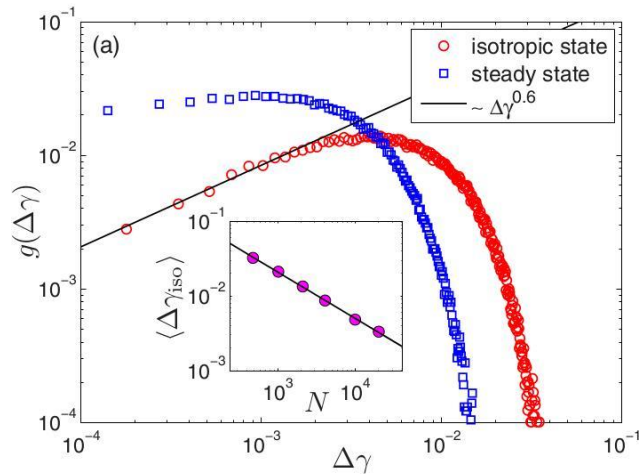


- $\beta = 0.62$ for $T=1$ and 0.72 for $T=0.466$
- $\Delta U_{iso} \sim N^0$. No system size dependence.

Consistent with
previous results.

Not universal.

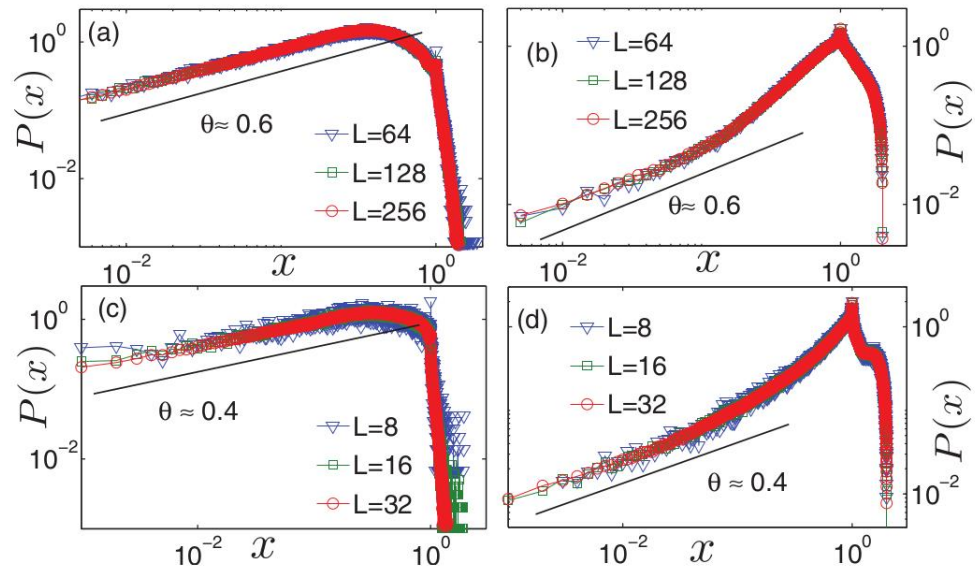
The First Plastic Events



S. Karmakar, et al.
PRE 82, 055103 (2010)

2D Atomic Glass Former

- $g(\Delta\gamma) \sim \Delta\gamma^{0.6}$ for 2d
- $g(\Delta\gamma) = \frac{1+\eta}{a_0} \left(\frac{\Delta\gamma}{a_0}\right)^\eta \exp\left[-\left(\frac{\Delta\gamma}{a_0}\right)^{1+\eta}\right]$



J. Lin, et al.
EPL 105, 26003 (2014)

2D and 3D Elastoplastic Model

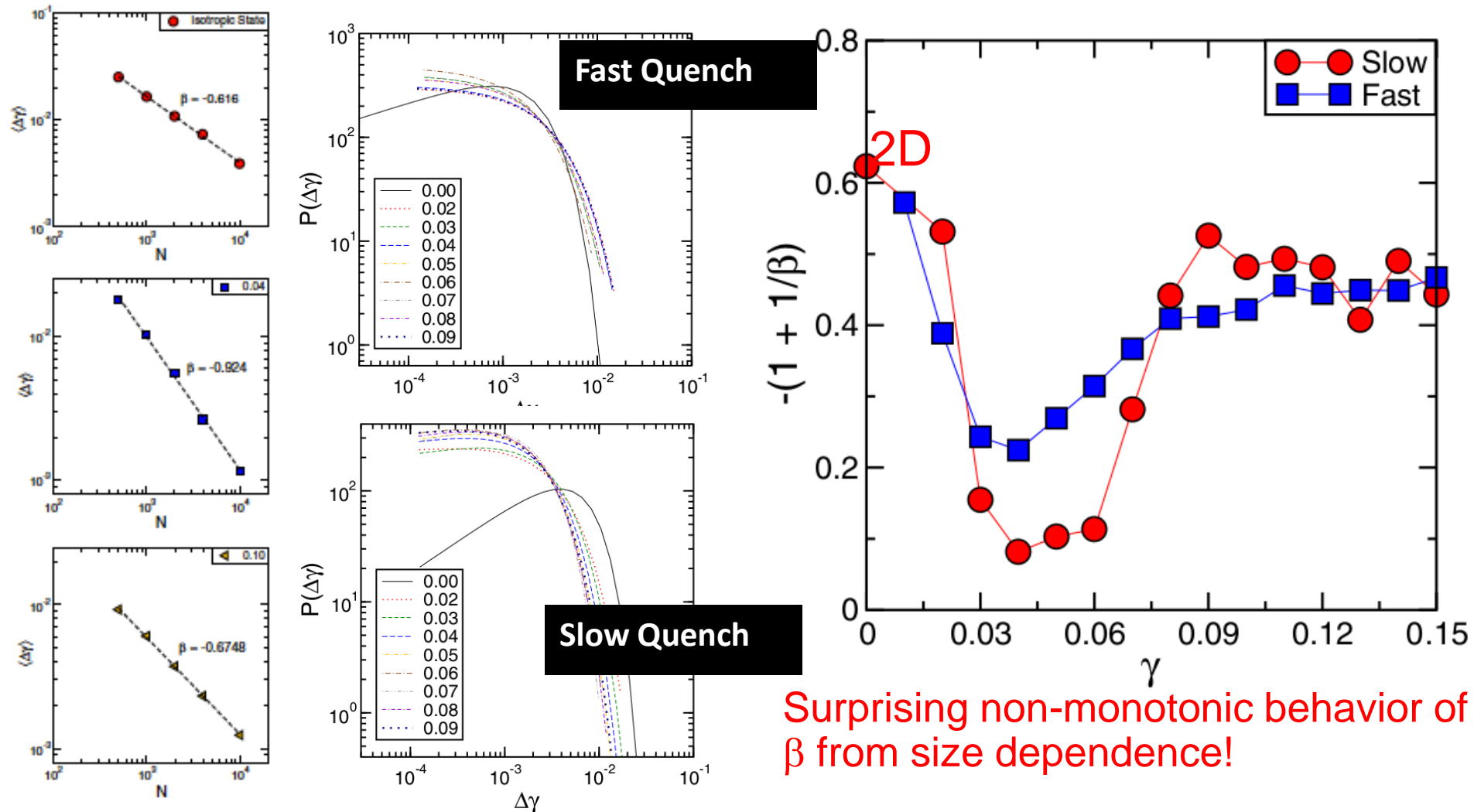
$\eta = 0.6$ in 2D and 0.4 in 3D

Well described by Weibull distribution. • Here, $\theta = \eta = -(1 + 1/\beta)$

$\eta = 0.6/0.4$ (2D/3D) for first event, $= 1/2$ in the steady state.

What about in between?

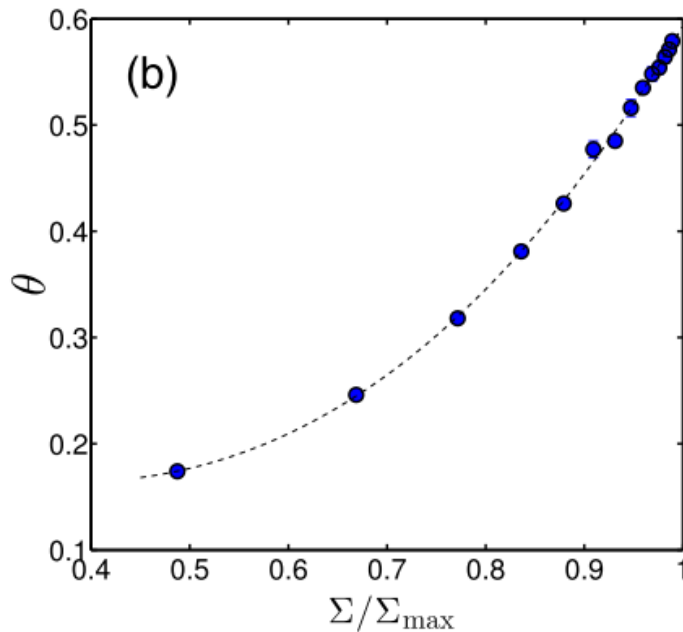
Exponent β as a function of strain



Surprising non-monotonic behavior of β from size dependence!

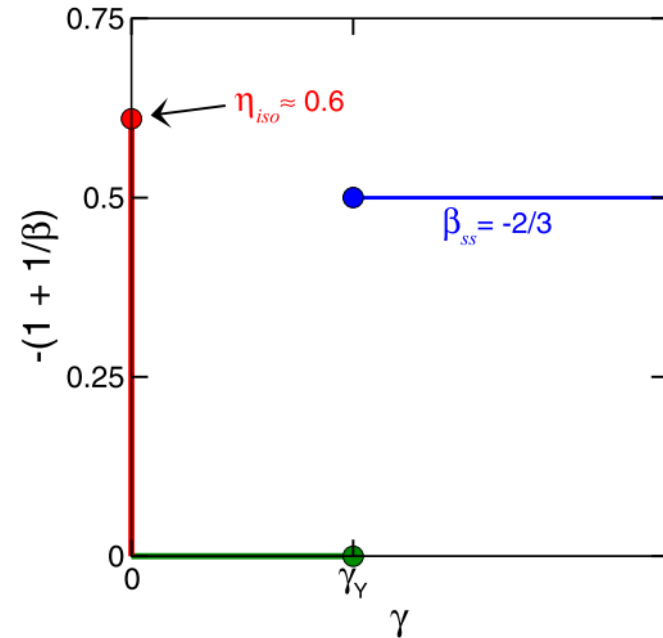
$\eta = 0$ from distributions.

Exponent β as a function of strain: Expectations



- **The exponents changes gradually.**
(Elastoplastic model)

Lin, Gueudre, Rosso, and Wyart
Phys Rev. Lett. 115, 168001 (2015)



- **Discontinuities at $\gamma=0+$ and $\gamma=\gamma_Y$.**
(Theoretical result using Fokker-Plank)

Hentschel, Jaiswal, Procaccia, and Sastry
Phys Rev E 92, 062302 (2015)

A prediction of $\beta = -1$ in the pre-yield behaviour, with $\eta = 0$.

Fokker-Planck Equation Analysis

Fokker Planck equation for the eigen value of Hessian.

$$\partial P / \partial \gamma = -\partial [v(\lambda, \gamma) P] / \partial \lambda + D(\gamma) \partial^2 P / \partial \lambda^2 + j_{\text{in}}(\lambda, \gamma).$$

Saddle-node bifurcation $\Delta \gamma \sim \lambda^2$ $\eta = (\theta - 1)/2$

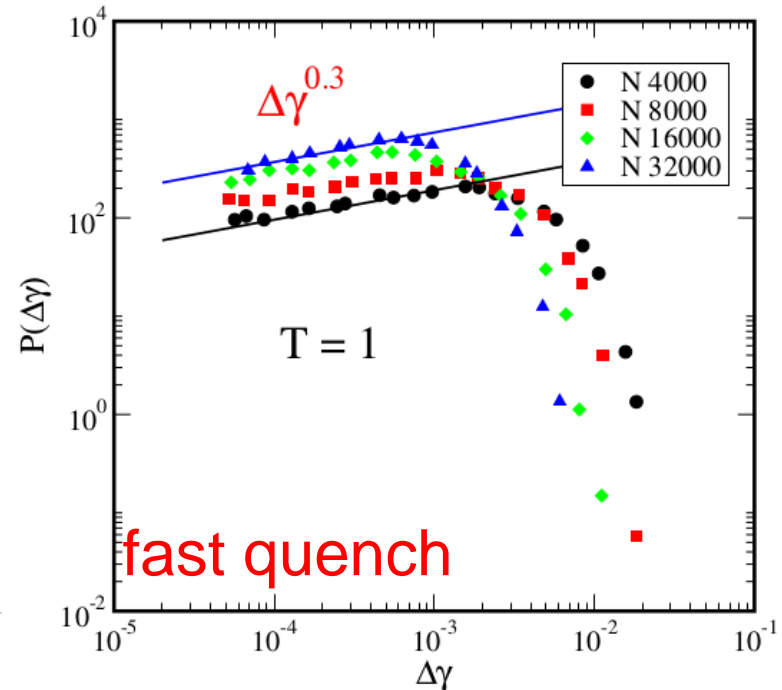
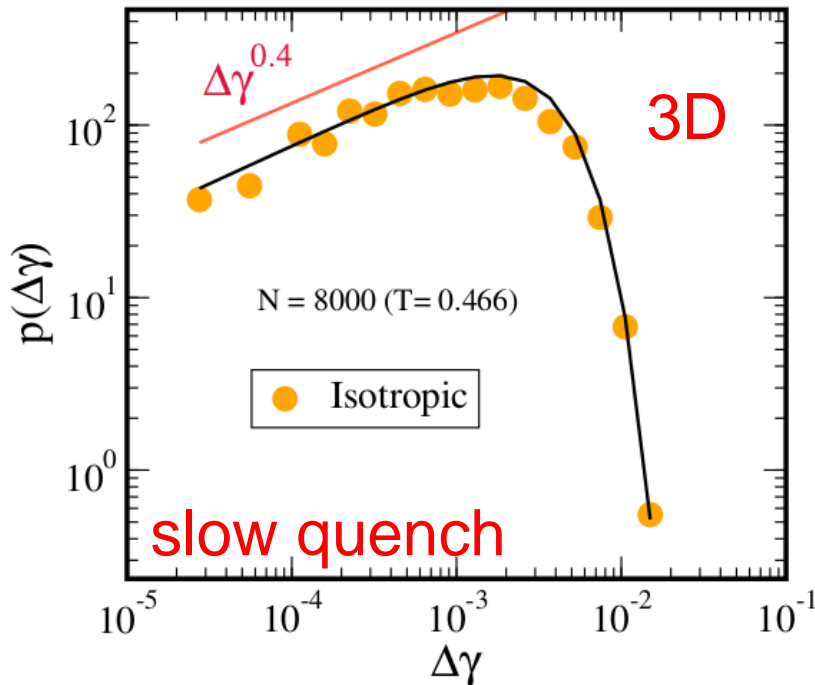
$\frac{d\lambda}{d\gamma} \simeq -\frac{a(\gamma)}{\lambda}$ Equation for “velocity” from earlier work.

$$\lim_{\lambda \rightarrow 0} P(\lambda; \gamma) \sim \lambda^\theta$$

In order to have finite fluxes $\lim_{\lambda \rightarrow 0} P(\lambda; \gamma) \sim \lambda$

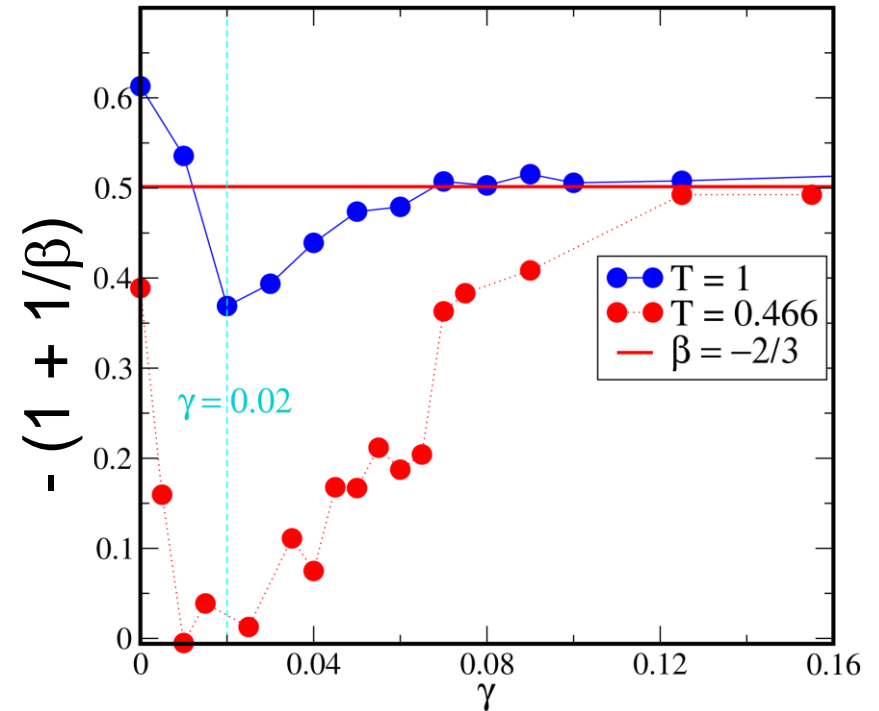
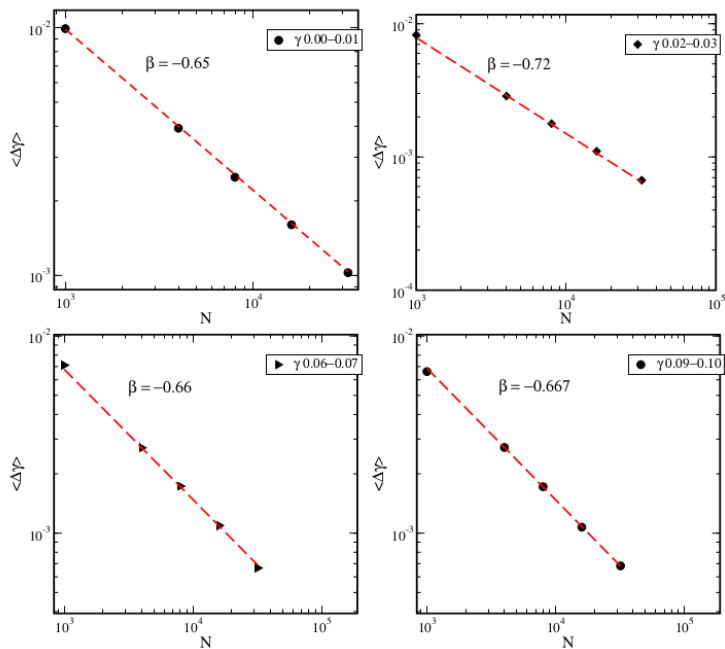
This leads to $\eta = 0$, $\beta = -1$

The First Plastic Events (3D)



The distribution and exponents in 3D consistent with previous results. Exponent depends on the quench rate of the glass.

Exponent β as a function of strain

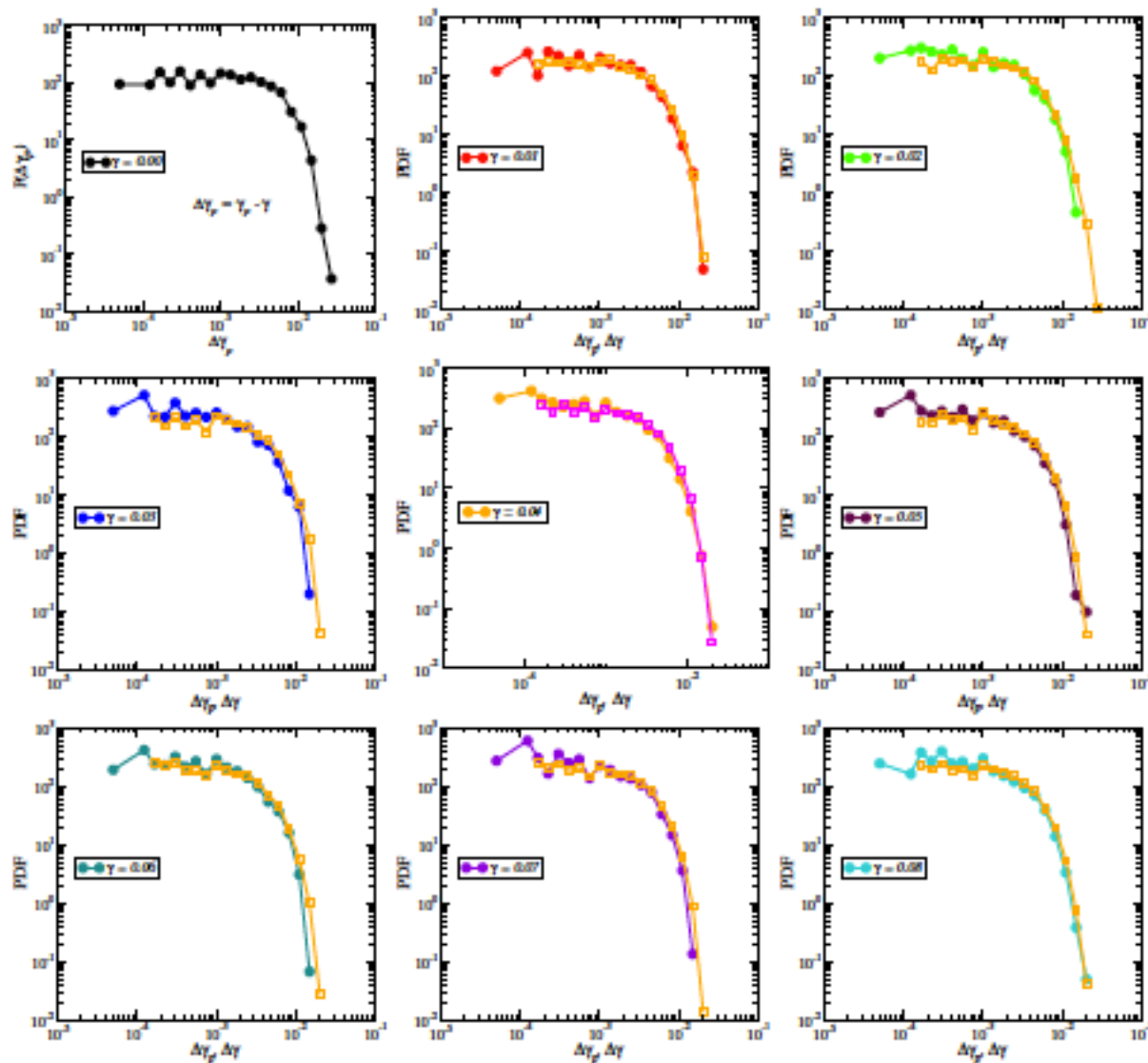


Behaviour similar to 2D simulations.

To what extent do these data support the idealized picture of Hentschel et al?

What does this mean? Criticality in the pre-yield regime?

Distribution of $\Delta\gamma$



Summary

Size dependence of strain intervals between plastic rearrangements exhibits interesting, non-monotonic behaviour.

Numerical evidence and arguments suggesting finite probability of zero barrier instability directions in the sheared state before yield.

Meaning of these observations to approach to yielding needs further analysis and understanding.