Localization to Deloclization Transition: is Rosenzweig-Porter ensemble the hidden skelton?

Pragya Shukla[1]

Department of Physics and Meteorology,
Indian Institute of Technology
Kharagpur

Feb 17, 2017

Localization vs Delocalization

- **Localized dynamics: Absence of diffusion**

  confinement of a dynamical variable to a small part of available space due to certain constraints

  Quantum: a particle in a disordered potential,
  a scar (increased wavefunction intensity around short periodic orbits in phase space)

  Classical: a stable island in classical phase space (due to constants of motion),
  a standing wave,
  localization of energy in non-ideal springs

- **extended dynamics: dynamical variable is able to access large part of the available space (very few constraints)**

  Quantum: a plain wave, free electrons in metals

  Classical: a chaotic trajectory

- Dynamics: localization $\longrightarrow$ non-ergodic (extended) $\longrightarrow$ ergodic (extended) $\longrightarrow$ chaotic

- **Ergodic dynamics:** almost all parts of available space accessed:
- **non-ergodic extended state:** $\lim_{t\to\infty} \frac{V_{state}}{V} \to 0$
- **ergodic extended state:** $\lim_{t\to\infty} \frac{V_{state}}{V} \to 1$

V_{state}: volume occupied by a state and V available volume to system

Criteria for LD transition: scaling theory

Based on the initial values of characteristic system parameters (scaling variables) for a system size L_0, the system flows with increasing size L to an insulating state or a delocalized state or it stays scale-indpendent at a critical state.

Single parameter scaling: only a single system parameter required to distinguish localized and delocalized phases and to characterize the LD transition

Example: dimensionless conductance $g = \frac{E_{th}}{\Delta}$

- a ratio of two characteristic energy scales of the problem:

  a transport related energy sacle $E_{th} = \frac{h}{t_D} = \frac{hD}{L^2}$ as the energy scale associated with time t_D to diffuse across the system

  a thermodynamically defined energy scale Δ as the mean level spacing.

- Localized phase: $E_{th} \gg \Delta \rightarrow g \ll 1$
- Delocalized phase: $E_{th} \ll \Delta \rightarrow g \gg 1$
- Transition: $E_{th} \sim \Delta \rightarrow g \sim O(1)$,

Criteria for LD transition: scaling theory of Distribution

- mesocopic regime: scaling variable fluctuates from system to system

  a knowledge of mean value of the scaling variable is not enough, one needs to know the distribution needed.

- appropriate scaling approach: in terms of distribution of the scaling variable

  A distribution of a physical quantity X that depends on system size L and a set of n initial parameters $t_1, t_2, \ldots, t_n$ obeys one parameter scaling if for large L it is approximately a function of only X and one scale dependent parameter Λ_L

$$P(X; L, t_1, \ldots, t_n) \approx F(X; \Lambda_L)$$

- Λ_L: scaling variable
- Flow of Λ_L: $\beta(\ln \Lambda_L) = \frac{\mathrm{d} \ln \Lambda_L}{\mathrm{d} L}$.
- **critical point: a universal limiting distribution at the critical value of the $\Lambda^* = \lim_{L \to \infty} \Lambda_L$**

what it implies?

a multi-parametric system: represented by a multi-parametric ensemble

distribution of a typical property of a multi-parametric ensemble: a function of many system parameters

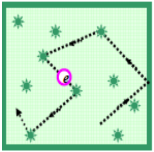
single parametric scaling of the distribution requires a mapping of the multi-parametric ensemble to a single parameter ensemble.

question: does it exist and how to find it?

answer: Rosenzweig-Porter ensemble in case of localization to delocalization transition

Anderson Hamiltonian

- **quantum particle moving in a random potnetial**



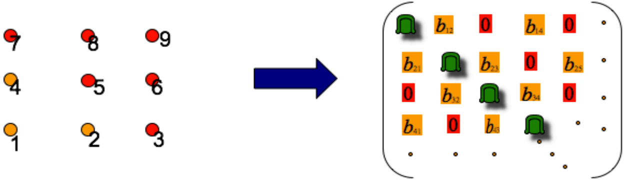
- equation of motion $\left[-\frac{\nabla^2}{2m} + V(\mathbf{r})\right] \Psi(\mathbf{r}) = E\Psi(\mathbf{r})$
- $V(\mathbf{r})$: random potential e.g. white noise
- $H = \sum_{i,j} \varepsilon_i \; c_i^+ . c_i + \sum_{i,j} V_{ij} \; c_i^+ . c_j$
  ε_i: randomized site energies, V_{ij}: hopping due to mean potential of all atomic sites

  Anderson (1957): absence of diffusion of wavefunction even for a weak disorder for dimensionality $d = 1, 2$, diffusion above a critical disorder for dimensionality $d > 2$

- **many system parameters dimensionality, disorder, hopping (anisotropic or isotropic), boundary conditions etc but physical properties are governed by a single parameter, a ratio of system size L and correlation length ξ.**

H-matrix in site basis: $H = \sum_{i,j} \varepsilon_i \; c_i^+ . c_i + \sum_{i,j} V_{ij} \; c_i^+ . c_j$

Localized Wave Dynamics:



In the basis associated with the space

1. $H_{kl} \neq 0$ if $k = l$ or $k = l \pm L^{d-1}$

a. H_{kk} : Randomly Distributed $\rho_{kk} = e^{-H_{kk}^2/2v_{kk}^2}$

b. Nearest Neighbor interactions may be random or non random

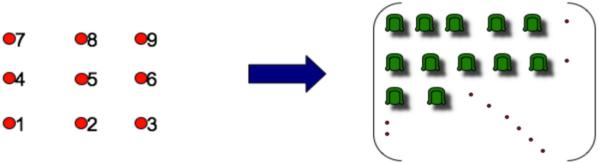
If non-random $\rho_{kl} = \delta(H_{kl} - b_{kl})$

If random $\rho_{kl} = e^{-(H_{kl} - b_{kl})^2/2v_{kl}^2}$

Hopping Strength $b_{kl} = 0$ if $k \neq l$ or $k \neq l \pm L^{d-1}$

H-matrix in site basis: ergodic limit

Delocalized Wave Dynamics: waves extended in entire system



In the basis associated with the space

$\langle k|H|l\rangle = H_{kl}$: Similar Distribution about most probable value

$\langle H_{kl}^2\rangle$: Indicator of determinstic uncertainty

If available information : $\langle H_{kl}^2\rangle = \sigma^2, \quad \langle H_{kl}\rangle = 0$

$$\rho(H) = \exp\left[-\frac{1}{2\sigma^2}\sum_{k\le l} H_{kl}^2\right] = \exp\left[-\frac{1}{2\sigma^2}\mathrm{Tr}\,H^2\right]$$

Wigner-Dyson Ensembles

H-matrix in LD transition is a sparse matrix

- **Anderson Hamiltonian is a sparse matrix in site basis.**
- **degree of sparsity is fixed and depends on the dimensionality and range of hopping.**
- **effective sparsity increases as disorder increases or hoping strength decreases.**

- **weak disorder**: diagonals almost zero, almost all matrix elements are relatively of same order i.e nearly zero, or **long-range hopping**: number of non-zero off-diagonals is much larger than the diagonals.
  H effectively behaves as a full matrix: an average off-diagonal is of same strength as the diagonal
  Eigenstates are ergodic and level-statistics corresponds to the universality class based on symmetry e.g. GOE for time-reversal symmetry.
- **strong disorder or weak hopping**: diagonal $\gg$ off-diagonals. H effectively behaves as a diagonal matrix with localized eigenstates.
- level-statistics of H changes from a basis-independent, stationary ensemble (of maximum level-repulsion) to diagonal matrix (independent levels).

probability distribution of the matrix elements H_{kl}

$$\rho(H) \propto \prod_{k=1}^{N} \mathrm{e}^{-\frac{H_{kk}^2}{2w^2}} \prod_{k\neq l} \delta(H_{kl} - b_{kl})$$

$$\delta(H_{kl} - b_{kl}) \rightarrow \mathrm{e}^{-(H_{kl}-b_{kl})^2/2v_{kl}^2}$$

Is a common mathematical formulation of various complex systems possible?

Consider uncorrelated multiparametric Gaussian ensemble

$$\rho(H, v, b) = C \exp\left[-\sum_{k\le l} \frac{1}{2v_{kl}} \left(H_{kl} - b_{kl}\right)^2\right]$$

v_{kl}: **variance of** H_{kl} **, zero for non-random cases, finite for random cases**

b_{kl}: **mean-value of** H_{kl}

ensemble-parameters v, b influenced by system conditions:

- Dimensionality due to b-dependence on localization length,
- Boundary conditions due to their effect on interaction of basis-states close to boundaries,
- Symmetry conditions, system-size, disorder and scattering conditions

System information is contained in the sets of parameters v, b

- Different systems will correspond to different v, b sets.

Localization to Deloclization Transition: is Rosenzweig-Porter ensemble the hidden skelton?

Pragya Shukla[1,2]

Single parametric diffusion of matrix elements

Changing complexity i.e. v, b leads to evolution of $\rho(H) \propto \mathrm{e}^{-\sum \frac{(H_{kl}-b_{kl})^2}{2v_{kl}}}$

possible to define a function Y **such that** (with $g_{kl} = 1 + \delta_{kl}$)

$$\frac{\partial \rho}{\partial Y} = \sum_{k,l} \frac{\partial}{\partial H_{kl}} \left[\frac{g_{kl}}{2} \frac{\partial}{\partial H_{kl}} + H_{kl} \right] \rho$$

where

$$Y = -\frac{1}{N(N+1)} \ln \left[\prod_{k \le l} |g_{kl} - 2v_{kl}| \quad |b_{kl}|^2 \right] + const.$$

multi-parametric diffusion of $\rho(H)$ **can be reduced to a single parameter**

Y= average distribution parameter, a measure of average uncertainty of system

Y : a measure of complexity of the system $\rightarrow$ **complexity parameter**

Y_0: complexity parameter for the initial state (e.g. localized state with Poisson statistics).

Diffusion of eigenvalues

Eigenvalues undergo a diffusion process governed by a single parameter Y

Spectral statistics depends on system information only through

$$\Lambda_e = \frac{Y - Y_0}{\Delta_{local}^2} = -\frac{1}{N^2\, \Delta_{local}^2}\, \ln \left[\prod_{k \le l} |1 - (2 - \delta_{kl}) v_{kl}| |b_{kl}|^2 \right]$$

.

Δ_{local}: local mean level spacing (depends on localization length, dimensionality)

$\Lambda_e \to \infty$ leads to steady state i.e Wigner-Dyson ensemble,

$\Lambda_e \to 0$ corresponds to initial state (e.g. localized state with Poisson statistics)

$\Lambda(N)$ as the scaling variable for spectral distribution

As system size N increases, $\Lambda(N)$ flows to 0 (Poisson) or ∞ (GOE).

For infinite system sizes, an intermediate state can occur only if Λ remains finite.

critical state of the statistics $\Lambda^* = \lim_{N \to \infty} \Lambda(N)$

Rosenweig-Porter ensemble: Poisson $\to$ GOE Brownian ensemble

$$\rho(H) \propto \exp\left[-\frac{1}{2} \sum_{i=1}^{N} H_{ii}^2 - (1+\mu) \sum_{i,j=1;i<j}^{N} H_{ij}^2 \right]$$

ensemble of diagonal matrices: $\mu \to \infty$, Wigner-Dyson ensemble: $\mu \to 0$

Spectral fluctuations around average level density $R_1(e)$ are governed by Λ_e

$$\Lambda_e(e) = \frac{Y - Y_0}{\Delta(e)^2} = \frac{R_1^2(e)}{\mu}.$$

finite N: changing μ at a fixed energy e results in a cross-over from Poisson ($\Lambda \to 0$) to GOE ($\Lambda \to \infty$) statistics.

arbitrary μ: $\lim_{N\to\infty} \Lambda(e)$ varies abruptly between $0 \to \infty$, ruling out possibility of any intermediate statistics.

Critical point: **limit** $\Lambda^*(e) \equiv \lim_{N\to\infty} \Lambda(e)$,

$R_1(e;\mu)$ changes from $\sqrt{N}$ to $N \to$ existence of two critical points

$\mu = c_2 N$: here $R_1(e) = (b\pi)^{-1} \sqrt{2bN - e^2}$ $\quad \Lambda(e) = \frac{2bN - e^2}{\pi^2 b^2 N c_2}$ with $b \sim 2$.

$\mu = c_1 N^2$: here $R_1 = \frac{N}{\sqrt{2\pi}} \mathrm{e}^{-e^2}$ $\quad \Lambda(e) = \frac{1}{4\pi c_1} \mathrm{e}^{-e^2}$.

Characteristics: multifractal wavefunctions

criticality in the wavefunctions:

- manifest through large fluctuations of their amplitudes at all length scales,
- characterized by an infinite set of critical exponents related to the scaling of the moments of the wave-function intensity $|\Psi(r)|^2$ with system size.

- **Inverse participation ratio**: $I_q(\psi) = \sum_n |\psi_n|^{2q}$

  $I_q(\psi) = 1$ for completely localized ψ on a single basis state

  $I_q(\psi) = 1/N^{q-1}$ for ψ extended over all basis space.

  I_2 in the bulk is inversely proportional to the average localization length.

  at transition, I_q reveals an anamalous scaling with size N:
  $\langle I_q \rangle = N \, \langle | \Psi |^{2q} \rangle \sim N^{-\tau_q/d}$.

- **multifractal spectrum**

  generalized fractal dimension D_q of the wave-function structure: $D_q = \frac{\tau_q}{q-1}$.

  $D_q = d$ for the eigenfunctions extended in a d-dimensional space,
  $D_q = 0$ for completely localized ones.

  at critical point, D_q is a non-trivial function of q,

Characteristics: critical level statistics

The basis-variant nature of an ensemble, often the case at the critical point, implies correlations between the eigenvalues and the eigenfunctions. The special features of the eigenfunctions at the criticality are therefore expected to manifest in spectrum too.

- **nearest neighbor spacing distribution $P(s)$:**

  $P(s) = e^{-s}$ for Poisson (localized wavefunctions),

  $P(s) = A s^{\beta} \, e^{-Bs^2}$ for Wigner-Dyson (delocalized wavefunctions),

  in critical regime $P(s)$ a universal hybrid of the GOE at small-s and Poisson at large-s, with an exponentially decaying tail: $P(s) \sim e^{-\kappa s}$ for $s \gg 1$ with κ a constant

- **number variance $\Sigma^2(r)$:** variance of the number of levels in a interval of r unit mean spacings, $\Sigma^2(r) = r$ for localized phase, $\Sigma^2(r) \approx \ln r$ for delocalized phase)

  in critical regime, number variance is linear but with fractional coefficient $\Sigma^2 r \sim \chi \; r$

- **level compressibility $\chi = \lim_{r \to \infty} \frac{d\Sigma^2(r)}{dr}$**

  $\chi = \frac{d - D_2}{2d}$ with D_2 as the fractal dimension and d as the system-dimension (valid only in the weak-multifractality limit). Also $\kappa \approx \frac{1}{2\chi}$

  D_2, a measure of the spatial correlation of the intensity $|\Psi|^2$ (with $N^2 \langle |\Psi^2(r)\Psi^2(r')|\rangle \sim (|r - r'|/N)^{(D_2/d)-1}$) is related to compressibility χ.

Numerical Comparison of fluctuations for 3 different systems

Multiparametric Gaussian ensemble

$$\rho(H, v, b) = C \exp\left[-\sum_{k\leq l} \frac{1}{2v_{kl}} (H_{kl} - b_{kl})^2\right]$$

- knowledge of their complexity parameter Λ

$$\Lambda = -\frac{1}{N^2\ \Delta_{local}^2} \ln\left[\prod_{k\leq l} |1 - (2 - \delta_{kl}) v_{kl}| |b_{kl}|^2\right]$$

  requires mean level density and localization length

- a similar initial statistics
  possible for systems under same global constraints

Three examples

Three examples

- **Anderson ensembe (AE) with on site disorder and non-random nearest neighbor hopping**

$$v_{kk} = \frac{w^2}{12}, \qquad v_{kl} = 0 \quad (k \neq l)$$

$$b_{kl} = t \quad (k, l = n.n), \qquad b_{kl} = 0 \quad (k, l \neq n.n)$$

$$\Lambda(E) = -\frac{\xi^{2d}}{L^d \Delta^2} \ln|1 - w^2/12||t|$$

Three examples

- **Anderson ensembe (AE) with on site disorder and non-random nearest neighbor hopping**

$$v_{kk} = \frac{w^2}{12}, \qquad v_{kl} = 0 \quad (k \neq l)$$
$$b_{kl} = t \quad (k, l = n.n), \qquad b_{kl} = 0 \quad (k, l \neq n.n)$$
$$\Lambda(E) = -\frac{\xi^{2d}}{L^d \Delta^2} \ln|1 - w^2/12||t|$$

- **Brownian Ensemble (BE)**

$$v_{kl} = 2 \left(1 + c\, N^2\, \delta_{kl}\right)^{-1}$$
$$b_{kl} = 0$$
$$\Lambda(E) = -(4\pi c \Delta^2)^{-1}$$

Three examples

- **Anderson ensembe (AE) with on site disorder and non-random nearest neighbor hopping**

$$v_{kk} = \frac{w^2}{12}, \qquad v_{kl} = 0 \quad (k \neq l)$$
$$b_{kl} = t \quad (k, l = n.n), \qquad b_{kl} = 0 \quad (k, l \neq n.n)$$
$$\Lambda(E) = -\frac{\xi^{2d}}{L^d \Delta^2} \ln|1 - w^2/12||t|$$

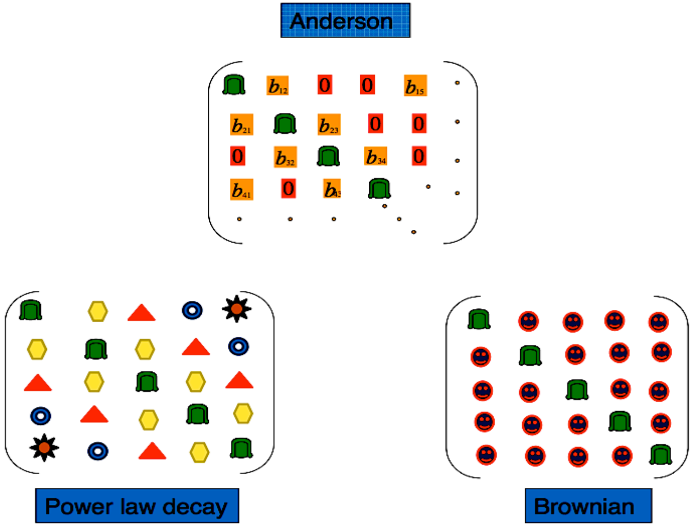
- **Brownian Ensemble (BE)**

$$v_{kl} = 2\left(1 + c\, N^2\, \delta_{kl}\right)^{-1}$$
$$b_{kl} = 0$$
$$\Lambda(E) = -(4\pi c \Delta^2)^{-1}$$

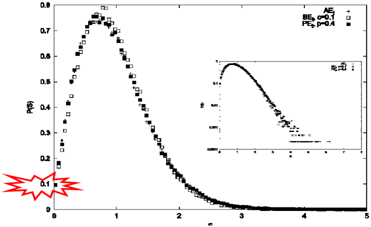
- **Power law random banded matrix ensemble (PRBM or PE)**

$$v_{kl} = \left[(2 - \delta_{kl})(1 + |k - l|^2/p^2)\right]^{-1}$$
$$b_{kl} = 0$$
$$\Lambda(E) = -(N\Delta^2)^{-1} \sum_{r=1}^{N} (1 - r/N)\ln|1 + p/r|^2$$

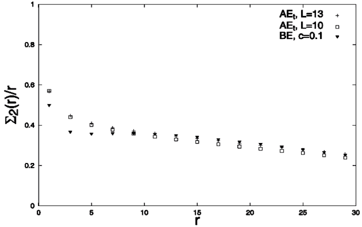
numerics: three different ensembles



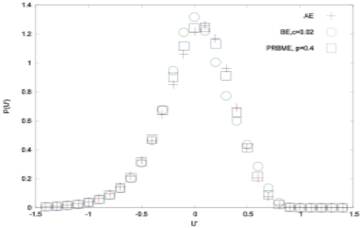
spacing distrbution



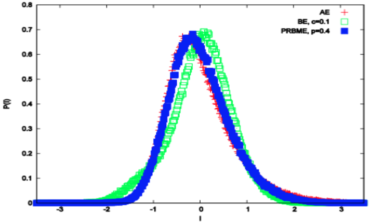
number variance



local intensity distribution



IPR distribution



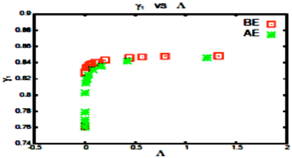
Time-reversal symmetry: disordered system and Brownian ensemble

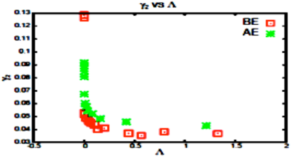
AE : $\Lambda(E) = -\frac{1}{\Delta^2}\left(\frac{\xi^2}{L}\right)^d \ln\left[\, |1 - w^2/12|\; |1 - w_1^2/6|^z \; t^z \right]$

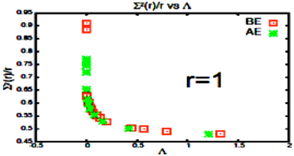
BE : $\Lambda(E) = -\left(4\pi c\, \Delta^2\right)^{-1}$

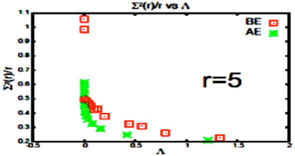
Cumulative spacing distribution $\gamma_n = \int_0^{x_n} P(s)\, ds$

Number variance $\Sigma_2(r) = \langle (r - \langle r \rangle)^2 \rangle$



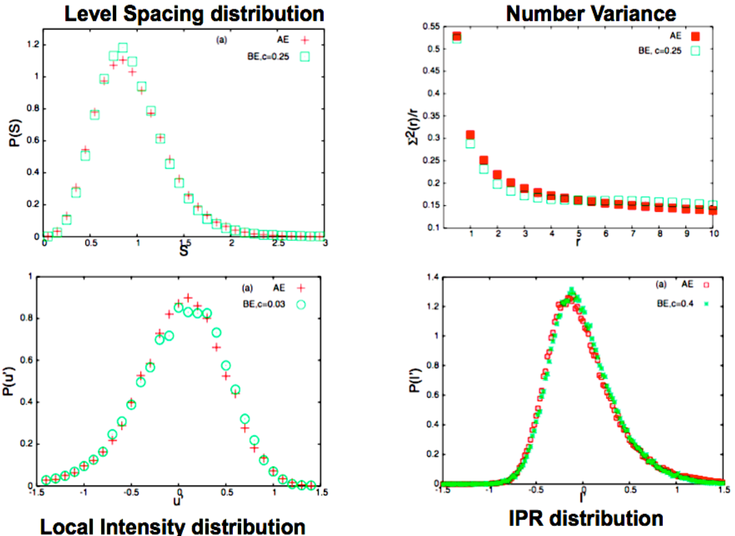






spin-orbit coupling: symplectic case

Anderson H (d=2) (with on site disorder & spin orbit coupling) and BE



Dynamical localization: Quantum Kicked Rotor

Quantum kicked rotor

$$H = \frac{1}{2}(p+\gamma)^2 + K \sum_n \cos(\theta+\theta_0)\delta(t-n)$$

- No disorder or particle-particle interaction !
- Matrix elements in position or momentum representation are random due to mixed classical dynamics which originates due to periodic kicks of strength K

Semiclassical derivation (periodic orbit theory) of complexity parameter:

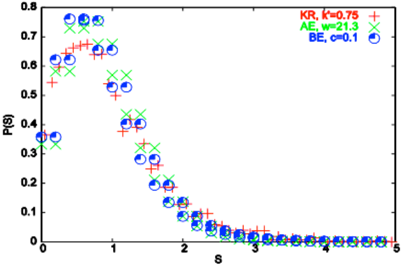
Regular to chaotic Transition

$$\Lambda(E) \approx \frac{K^2\theta_0^2 N^3}{64\pi^4 M^2} = \frac{\theta_0^2 N^2 k}{4\pi^2}$$

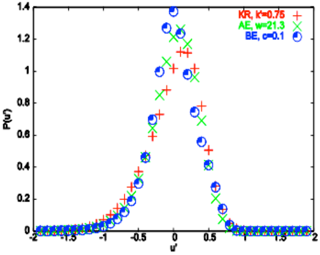
classical diffusion in momentum space suppressed due to quantum dynamical localization

similar to Anderson localization (at least based on enegry level statistics).

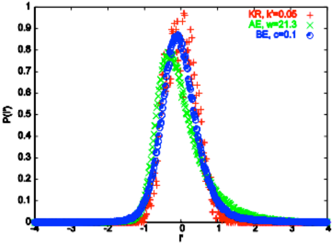
Comparison of dynamical systems, disordered systems and Brownian ensemble



Spacing Distribution



Local Intensity Distribution



IPR Distribution

Localization to Deloclization Transition: is Rosenzweig-Porter ensemble the hidden skelton?

Pragya Shukla[26]

Goldstone modes: localization due to column constraints

An ensemble of replicas of the disordered Hamiltonian H with column constraints

- $\rho(H)$: ensemble density

$$\rho(H) = \rho_s(H) \prod_{l=1}^{N} \delta(\sum_k H_{kl}). \tag{1}$$

what is ρ_s ?:

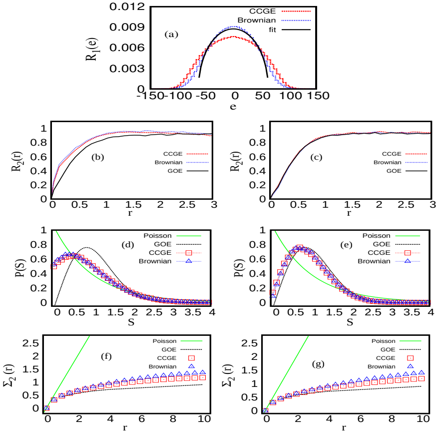
Based on information about moments, many distributions possible (maximum entropy hypothesis):

- other constraints only on 1st, 2nd moments: Gaussian form of ρ_s .

$$\rho_s(H) = C \exp\left[- \sum_{kjl;k,j\neq l} c_{kljl} H_{kl} H_{jl} - 2 \sum_{kl;k\neq l} d_{kl} H_{kl} \right]. \tag{2}$$

column constrained ensembles and RP ensembles

comparison of level-statistics in two different energy-regimes (S.sadhukhan and PS, J.Phys. A, 2015)



many body interaction: Foch basis

Many body Hamiltonian of N interacting electrons

$$H = \sum_{i,j} \varepsilon_i \; c_i^\dagger . c_i + \sum_{i>j} V_{ij} \; c_i^\dagger . c_j + \sum_{i,j,k,l} U_{ijkl} \; c_j^\dagger . c_l^\dagger . c_k . c_i$$

- **Fock space basis:** $\begin{pmatrix} M \\ N \end{pmatrix}$ dimensional basis, consisting of all NI states Φ_α (Slater determinant of N electrons).
- An NI state Φ_α: a M-length string with entries 1 and 0 labeling the M single-particle states which do or do not participate in it, respectively. If $n_1, n_2, \ldots, n_S$ electrons are in $\phi_1, \phi_2, \ldots, \phi_S$ single e-states, respectively, with $n_k = 0, 1$

$$|\Phi_\alpha\rangle = |\phi_1\rangle^{n_1} |\phi_2\rangle^{n_2} \ldots |\phi_S\rangle^{n_S} \equiv |n_1 n_2 \ldots n_s\rangle$$

Example: 2 electrons in 3 single e-states ϕ_1, ϕ_2, ϕ_3: $\Phi_1 = |110\rangle, \quad \Phi_2 = |101\rangle, \quad \Phi_3 = |011\rangle$

- **Distance in Fock space:** The distance between two NI states is the number of positions in which the corresponding strings differ. Example: distance between $\Phi_i, \Phi_j = 2$ for any pair of $i, j = 1, 2, 3$
- **Foch basis as "site" basis:** each "site" an entity with wavefunction Φ_α.
- **nearest neighbor hopping in Foch basis:** $V_{\alpha\beta} \equiv \langle \Phi_\alpha | V | \Phi_\beta \rangle \neq 0$ only if the distance between Φ_α and Ψ_β is $0, 2, 4$ (they differ in occupancy of $0, 2, 4$ single e-states).
- **Interacting eigensates**: eigenstates of H: can be expanded in the eigenfunction basis of non-interacting eigenstates Φ_α $\Psi = \sum_\alpha C_\alpha \; \Phi_\alpha$
- **Localization in Fock Space:** each interacting eigenstate is composed out of a small number of NI eigenstates Φ_α. (In Anderson case, each wavefunction of H was spread to a few sites with expoential decay).
- **extended interacting eigenstate:** composed of many NI eigenstates Φ_α

Anderson again

Initial State: choose insulator limit of the system, with zero hopping and Gaussian site disorder $\Rightarrow \; v_{kl} = (1/2)\delta_{kl}, \; b_{kl} = 0$

$$Y_0 = -\alpha_0 N^{-1} + \text{constant}, \text{ where } \alpha_0 = \ln 2$$

$$\Lambda(E,N,d,w,t,z) = F^2(\mathrm{E}) \; \ln\left[\frac{1}{2}\left|1-\frac{w^2}{12}\right| \; t^z\right] \left(\frac{\xi^2}{L}\right)^d$$

Delocalized Limit: $\xi^2 > L \;\; \Rightarrow \Lambda \to \infty \;\; : \text{Wigner-Dyson Statistics}$

Insulator limit $\xi^2 << L \;\; \Rightarrow \Lambda \to 0 \;\; : \text{Poisson Statistics}$

Finite Systems: Infinite class of statistics intermediate between Poisson and Wigner-Dyson, characterized by finite Λ.

Infinite systems: Only one intermediate state possible (critical state) if a finite Λ can be achieved for a specific set of parameters.

Anderson again

Critical Point: existence of a critical state of the system requires

$$\lim N \to \infty \quad \Lambda \to \text{Finite i.e } \Lambda \text{ size-independent}$$

Case d=1: $\xi \approx \pi l$ (mean free path) $\approx o(L^0) \quad \Rightarrow \lim N \to \infty \; \Lambda \to 0$

Case d=2: $\xi \approx l \exp|k_F l \pi / 2| \approx o(L^0) \quad \Rightarrow \lim N \to \infty \; \Lambda \to 0$

- For finite L, a crossover of statistics from Poisson to WD statistics
- No critical point for infinite size disordered system of dimension d=1,2 occurs as ξ^2/L can be kept nonzero (for a larger L-range for d=2).

Case d> 2: $\xi \approx \xi_0 L^{D2}$; $D2$ = multifractality exponent

$\Rightarrow$ $\Lambda^{critical} = \beta^{-1} F^2 |\alpha - \alpha_0| \xi_0^{2d} L^{2D2-d}$

$\Rightarrow$ Λ is size independent if $D2 = d/2$

Level statistics at critical point depends on various system parameters e.g. topological and boundary conditions, symmetry and dimensionality.

Conclusion

Conclusion

- energy level statistics during Anderson transition can be mapped to single parameteric Brownian ensemble.

Conclusion

- energy level statistics during Anderson transition can be mapped to single parameteric Brownian ensemble.
- energy level statistics during dynamical transition undergoes same path as that of Anderson transition.

Conclusion

- energy level statistics during Anderson transition can be mapped to single parameteric Brownian ensemble.
- energy level statistics during dynamical transition undergoes same path as that of Anderson transition.
- energy level statistics during many-body metal $\rightarrow$ insulator transition undergoes same path as that of Anderson transition.

Conclusion

Pragya Shukla[32]

- energy level statistics during Anderson transition can be mapped to single parameteric Brownian ensemble.
- energy level statistics during dynamical transition undergoes same path as that of Anderson transition.
- energy level statistics during many-body metal $\rightarrow$ insulator transition undergoes same path as that of Anderson transition.
- it seems all known localization $\rightarrow$ delocalization transition can be modeled by a single parameter Brownian ensemble (Rosenzweig-Porter ensemble)?

Conclusion

- energy level statistics during Anderson transition can be mapped to single parameteric Brownian ensemble.
- energy level statistics during dynamical transition undergoes same path as that of Anderson transition.
- energy level statistics during many-body metal $\rightarrow$ insulator transition undergoes same path as that of Anderson transition.
- it seems all known localization $\rightarrow$ delocalization transition can be modeled by a single parameter Brownian ensemble (Rosenzweig-Porter ensemble)?
- metal $\rightarrow$ insulator transition in presence of strong e-e interaction?

References

- **P. Shukla, Phys. Rev. E, 62, 2098, (2000).**
  (Common mathematical formulation for Gaussian ensembles of Hermitian operators)
- **P. Shukla, Phys. Rev. Lett. (2001).**
  (for Gaussian ensembles of non-Hermitian operators)
- **P. Shukla, Phys. Rev. E. 71, 026226, (2005).**
  (for non-Gaussian ensembles of Hermitian operators)
- **P.Shukla, J.Phys. C: Condens. Matter, 17, 1653, (2005).**
  (application to disordered systems)
- **P. Shukla and I.P.Batra, Phys. Rev. B, (2005).**
  (for transfer matrices)
- **P. Shukla, Phys. Rev. E, 75, 051113, (2007).**
  (for eigenfunction correlations)
- **Rina Dutta and P. Shukla, Phys. Rev. E,76, 051124, (2007).**
  (application to spin systems)
- **Rina Dutta and P.Shukla, Phys. Rev. E 78, 031115 (2008).**
  (application to kicked rotor)
- **P.Shukla, J.Phys. A, (2008).**
  (accuracy approach)
- **M V Berry and P. Shukla, J. Phys. A, 42, 485102, (2009).**
  (2×2 case)

Implictaions of single parameteric diffusion

Pragya Shukla[35]

Implictaions of single parameteric diffusion

- Distributions of eigen correlations of a complex system described by a multiparametric Gaussian ensemble is a non-stationary state of a diffusion governed by the complexity parameter.

Implictaions of single parameteric diffusion

- Distributions of eigen correlations of a complex system described by a multiparametric Gaussian ensemble is a non-stationary state of a diffusion governed by the complexity parameter.
- Different Systems with same Λ, although different system-conditions (local constraints), will have same eigenvalue statistics (PS, Phys.Rev. E, 2000). (valid for same global constraint class).

Implictaions of single parameteric diffusion

- Distributions of eigen correlations of a complex system described by a multiparametric Gaussian ensemble is a non-stationary state of a diffusion governed by the complexity parameter.

- Different Systems with same Λ, although different system-conditions (local constraints), will have same eigenvalue statistics (PS, Phys.Rev. E, 2000). (valid for same global constraint class).

- Valid also for eigenfunction statistics (PS, Phys.Rev. E, 2007). (different Λ for different measures)
  $\rightarrow$ Possible to classify complex systems into continuum of universality classes characterized by Λ

Implictaions of single parameteric diffusion

- Distributions of eigen correlations of a complex system described by a multiparametric Gaussian ensemble is a non-stationary state of a diffusion governed by the complexity parameter.
- Different Systems with same Λ, although different system-conditions (local constraints), will have same eigenvalue statistics (PS, Phys.Rev. E, 2000). (valid for same global constraint class).
- Valid also for eigenfunction statistics (PS, Phys.Rev. E, 2007). (different Λ for different measures)
  $\rightarrow$ Possible to classify complex systems into continuum of universality classes characterized by Λ
- For infinite system sizes, an intermediate state can occur only if Λ remains finite (due to $\Lambda = \Lambda(N)$)

Implictaions of single parameteric diffusion

- Distributions of eigen correlations of a complex system described by a multiparametric Gaussian ensemble is a non-stationary state of a diffusion governed by the complexity parameter.

- Different Systems with same Λ, although different system-conditions (local constraints), will have same eigenvalue statistics (PS, Phys.Rev. E, 2000). (valid for same global constraint class).

- Valid also for eigenfunction statistics (PS, Phys.Rev. E, 2007). (different Λ for different measures)
  $\rightarrow$ Possible to classify complex systems into continuum of universality classes characterized by Λ

- For infinite system sizes, an intermediate state can occur only if Λ remains finite (due to $\Lambda = \Lambda(N)$)

  critical state of the statistics if Λ remains finite i.e size-independent in lim $N \rightarrow \infty$