RFIM once more!

(general criteria for infinite avalanches)

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most work done by Diana Thongjaomayum for her PhD
please visit her poster for details

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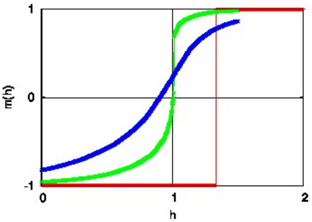
$$H = -J \sum_{i,j} s_i s_j - h \sum_i s_i - \sum_i h_i s_i$$

$s_i = \pm 1$; $m(h) = \frac{1}{N} \sum_i s_i$ at $T = 0, -\infty < h < \infty$; $h_i \Rightarrow N(0, \sigma^2)$



if no disorder, infinite avalanche at $J = 2d$; infinte avalanche $\Rightarrow$ discontinuity in $m(h)$! does it survive in presence of quenched disorder?

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Is there a general criteria?

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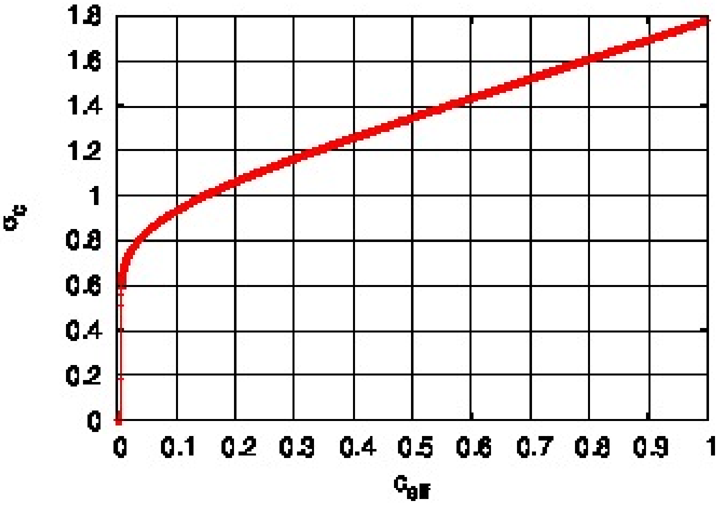
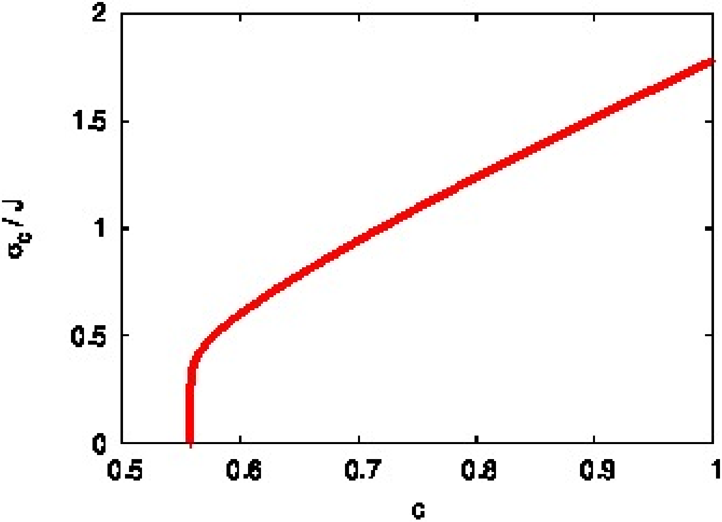
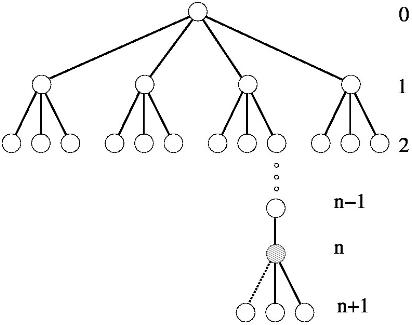


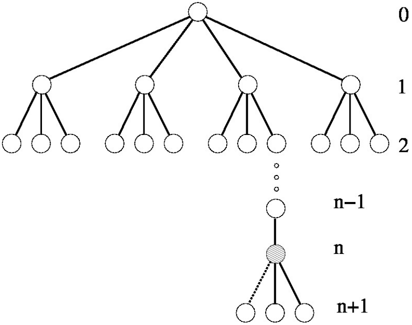
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1



2
1.5
1
0.5
0
0.5
0.6
0.7
0.8
0.9
1
σ_c / J
c



we can solve the model exactly on a Cayley tree. Start with all spins down. Relax surface, then next layer, then next....
if $h < J$, $Q^0(h,\sigma) < Q^1(h,\sigma) < Q^2(h,\sigma) \ldots < Q^*(h,\sigma)$



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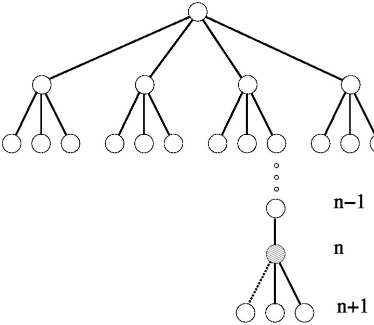

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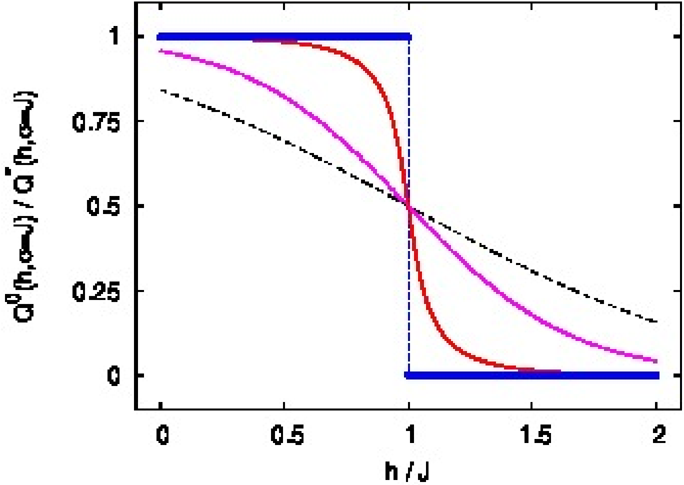
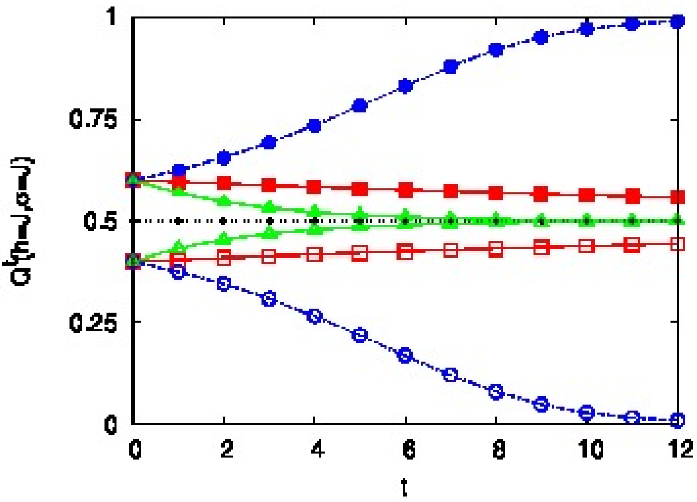
$Q^*(J, \sigma) = 0.5$ is stable for $z = 2$ and $z = 3$ but unstable for $z = 4$! why?

turn down an up spin on surface. what is the prob that its neighbor if up would also turn down?
$\delta Q^1(J,\sigma) = B_z \delta Q^0(J,\sigma)$.

$$B_z = (z-1)\frac{1}{2^{z-2}} \sum_{m=0}^{z-2} \binom{z-2}{m} (q_{z,m+2} - q_{z,m+1})$$

$q_{z,k}$ is the prob that a z-coordinated spin is down if k of its neighbors are down.
In the limit $\sigma \to 0$, $B_2 \to 1$, $B_3 \to 1$, and $B_4 \to 3/2$. $B2$, $B3$, $B4$ all decrease with increasing σ. $B2 < 1$ and $B3 < 1$ for $\sigma > 0$ making $Q^*(J,\sigma) = 1/2$ a stabe fixed point. Hence no infinite avalanche on $z = 2$ or $z = 3$ lattice. $B_4 \to 1$ as $\sigma \to \sigma_c \approx 1.781$. Thus $Q^*(J,\sigma) = 1/2$ is unstable on a $z = 4$ lattice if $\sigma < \sigma_c$ and there is an infinite avalanche.


1
0.75
0.5
0.25
0
0
0.5
1
1.5
2
h / J



1
0.75
0.5
0.25
0
0
2
4
6
8
10
12
t

dilute $z4$ lattice

the devil is in details!

As $\sigma \to 0$, perturbation $\delta Q^0(J,0)$ to $Q^*(J,\sigma) = 1/2$ passes throgh $z3$ and $z2$ sites unchanged. It gets enhanced by a factor $3/2$ by a $z4$ site. Here $z1$ sites also play a role. They break a spanning path. The perturbation is boosted with the probability $\frac{3}{2}z_4$, and terminated with probability $\frac{3}{4}z_1$. At the critical point $z_1 = 2z_4$. Using $z_4 = c^5$, $z_1 = 4c^2(1-c)^3$, the critical value of c is given by $c^3 = 2(1-c)^3$, or $c_{min} = 2^{1/3}/(1+2^{1/3}) \approx 0.5575$.
On the $z3 + z4$ lattice, the path from the surface to the center is never broken, therefore an arbitrarily small fraction z_4 creates a gap in $Q^*(J,\sigma)$ in the deep interior of the tree.

finally, the criteria for infinite avalanches

Following conditions must be fulfilled:

- σ must be sufficiently small
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THANK YOU FOR YOUR PATIENCE