Metastable minima of the Heisenberg spin glass in a random magnetic field

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Metastable minima of the Heisenberg spin glass in a random magnetic field

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We have studied zero-temperature metastable minima in classical m-vector component spin glasses in the presence of m-component random fields for two models, the Sherrington-Kirkpatrick (SK) model and the Viana-Bray (VB) model. For the SK model we have calculated analytically its complexity (the log of the number of minima) for both the annealed case where one averages the number of minima before taking the log and the quenched case where one averages the complexity itself, both for fields above and below the de Almeida-Thouless (AT) field, which is finite for $m > 2$. We have done numerical quenches starting from a random initial state (infinite temperature state) by putting spins parallel to their local fields until there is no further decrease of the energy and found that in zero field it always produces minima that have zero overlap with each other. For the $m = 2$ and $m = 3$ cases in the SK model the final energy reached in the quench is very close to the energy E_c at which the overlap of the states would acquire replica symmetry-breaking features. These minima have marginal stability and will have long-range correlations between them. In the SK limit we have analytically studied the density of states $\rho(\lambda)$ of the Hessian matrix in the annealed approximation. Despite the fact that in the presence of a random field there are no continuous symmetries, the spectrum extends down to zero with the usual $\sqrt{\lambda}$ form for the density of states for fields below the AT field. However, when the random field is larger than the AT field, there is a gap in the spectrum, which closes up as the AT field is approached. The VB model behaves differently and seems rather similar to studies of the three-dimensional Heisenberg spin glass in a random vector field.

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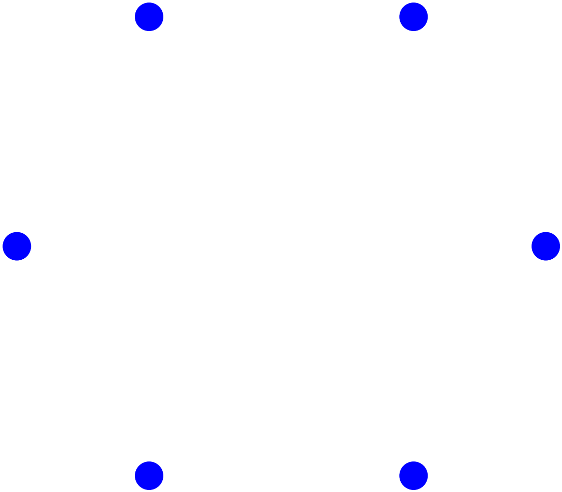
I. INTRODUCTION

In recent years there has been a resurgence of interest in the properties of metastable states, due mostly to the studies

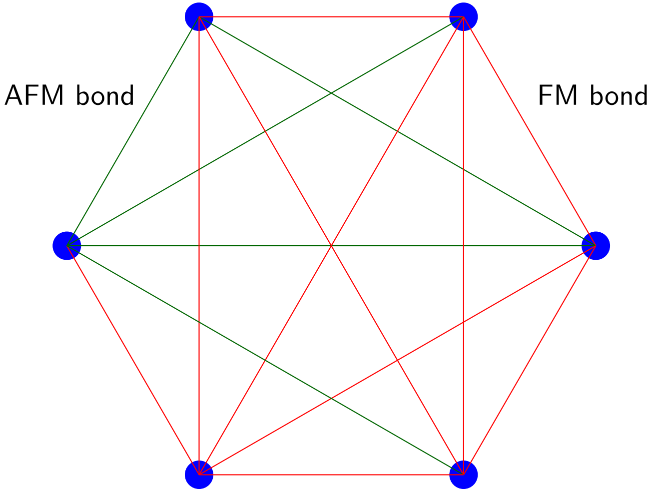
The cases that correspond to short-range models are a subject for future study.

In Sec. III we have used numerical methods to learn about

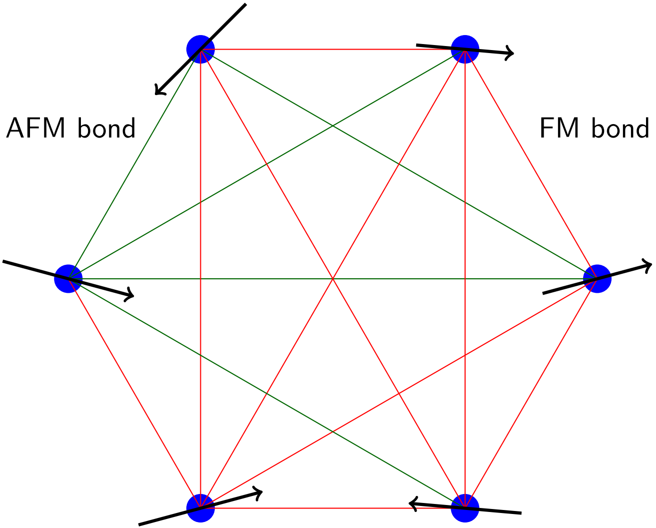
Infinite Range Spin Glass



Infinite Range Spin Glass



Infinite Range Spin Glass



Hamiltonian

The Hamiltonian we study is

$$\mathcal{H} = -m \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \sqrt{m} \sum_i \mathbf{h}_i \cdot \mathbf{S}_i \, , \qquad (1)$$

where the $\mathbf{S}_i$, $i = 1, 2, \cdots, N$, are classical m-component vector spins of unit length. The magnetic fields h_i^μ, where μ denotes a cartesian spin component, are chosen to be independent Gaussian random fields, uncorrelated between sites, with zero mean:

$$[h_i^\mu h_j^\nu]_{av} = h_r^2 \, \delta_{ij} \, \delta_{\mu\nu} \, . \qquad (2)$$

$[\cdots]_{av}$ indicates an average over the quenched disorder and the magnetic fields.

Diluted Model: The probability of there being a non-zero interaction between sites (i, j) falls off with distance as a power-law, and when an interaction does occur, its variance is independent of r_{ij}. The mean number of non-zero bonds from a site is fixed to be z.

Metastability. Quenching procedure.

At zero temperature, the metastability condition that the gradient of the Hamiltonian must be zero is exactly equivalent to aligning every spin along the local field direction:

$$\mathbf{S}_i^0 = \frac{\mathbf{H}_i}{|\mathbf{H}_i|}, \tag{3}$$

where the local fields are defined as

$$\mathbf{H}_i = \sqrt{m}\mathbf{h}_i + m \sum_j J_{ij} \mathbf{S}_j. \tag{4}$$

Overlap.

Consider the overlap between two minima A and B defined as

$$q \equiv \frac{1}{N} \sum_i \mathbf{S}_i^A \cdot \mathbf{S}_i^B. \tag{5}$$

Test for 'replica symmetry breaking': Distribution of overlaps

The form of $P(q)$ can distinguish formally between states that break replica symmetry and those that don't. Replica symmetry is characterized by a single (or two in the absence of an external field) simple delta function $P(q) = \delta(q - q_0)$. If there is replica symmetry breaking, $P(q)$ may have a continuous part, indicating the possibility of a continuum of overlaps between various phases.

Theorem of Newman and Stein (1999)

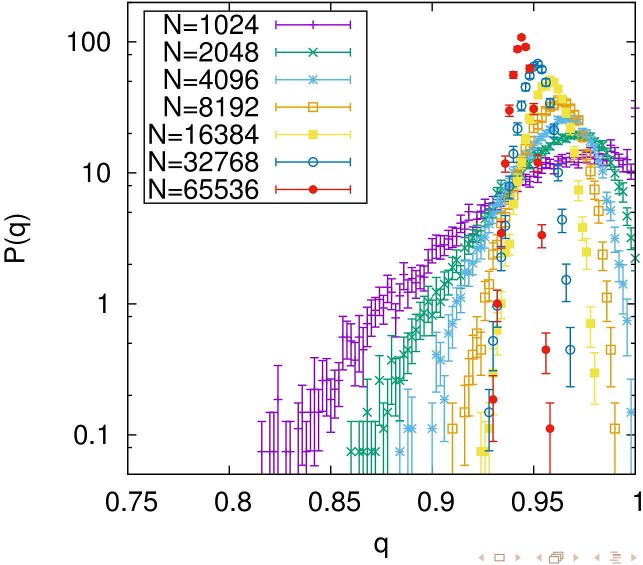
In a quench from a random initial state, the final $P(q)$ *should be trivial*, i.e., there should be no signature of replica-symmetry breaking.

Zero field problem: Long history

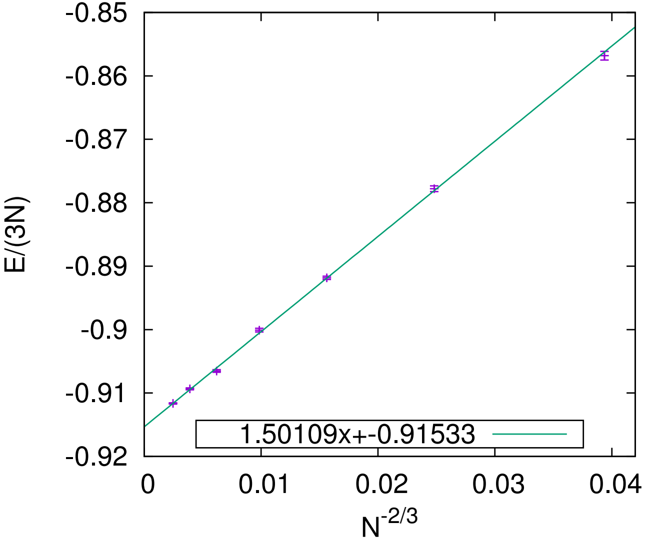
Parisi (2003) found for the Ising case that a quench from infinite temperature leads to a final state with energy per spin $\varepsilon = -0.73$. Such states show a trivial overlap distribution $P(q) = \delta(q)$.

In their studies of *one-spin flip stable* spin glasses, Bray and Moore (1981) found that such states associated with a trivial $P(q) = \delta(q)$ should not exist below a critical energy E_c and for the Ising case $E_c = -0.672$. States with an energy close to -0.73 would be expected to be have a $P(q)$ rather similar to those for full replica symmetry breaking, but those generated in the quench have a trivial $P(q)$, in adherence to the Newman-Stein theorem. There is no paradox as the states generated in the quench have more than one-spin flip stability.

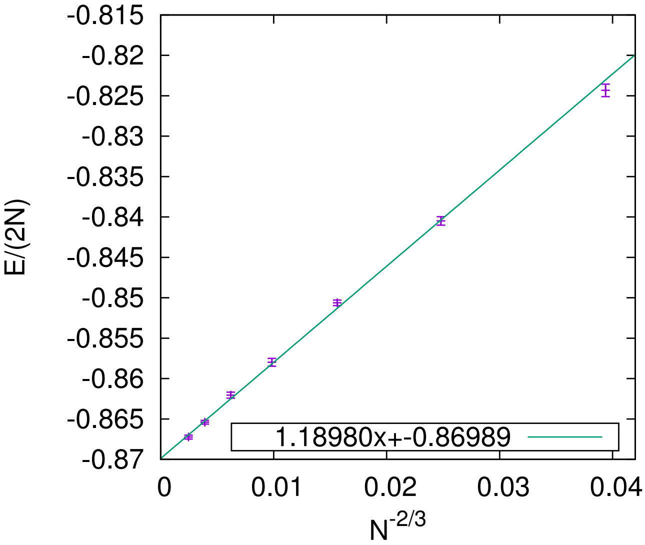
Overlap distribution (VB model)



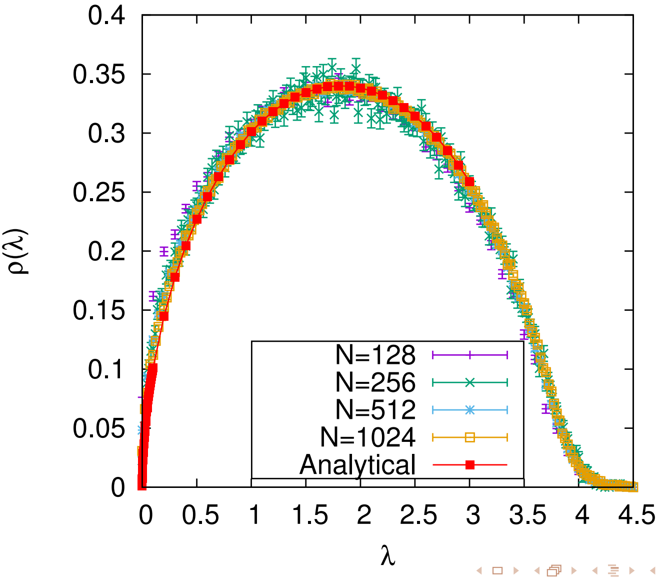
Finite-size scaling of Energy (Heisenberg) $E_c = -0.914$



Finite-size scaling of Energy (XY) $E_c = -0.866$



Density of states: Analytical and numerical results



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de Almeida–Thouless line in vector spin glasses

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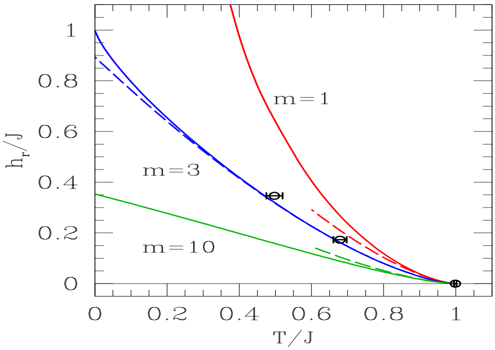
We consider the infinite-range spin glass in which the spins have $m>1$ components (a vector spin glass). Applying a magnetic field which is random in direction, there is a de Almeida-Thouless (AT) line below which the "replica symmetric" solution is unstable, just as for the Ising ($m=1$) case. We calculate the location of this AT line for Gaussian random fields for arbitrary m and verify our results by numerical simulations for $m=3$.



Density of states

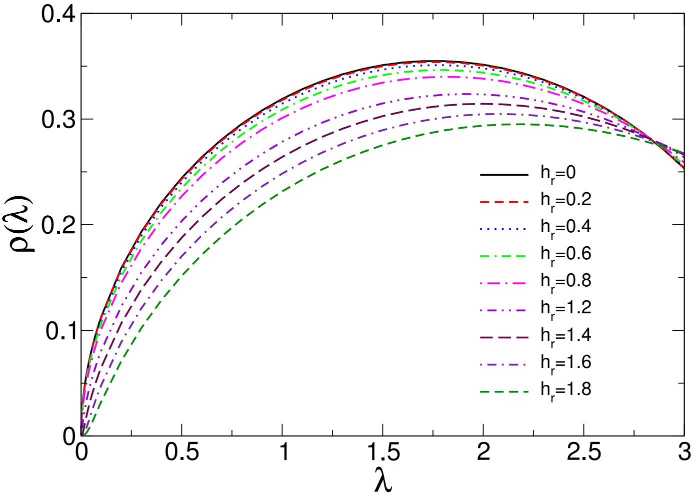


Figure:

Density of states: Gap opens up at the Almeida-Thouless critical point

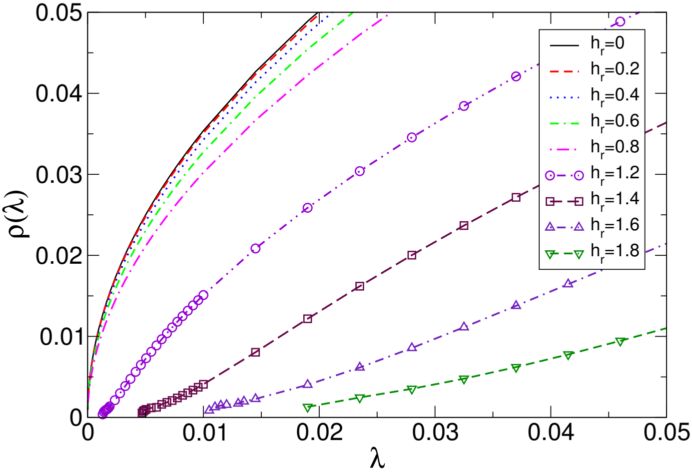
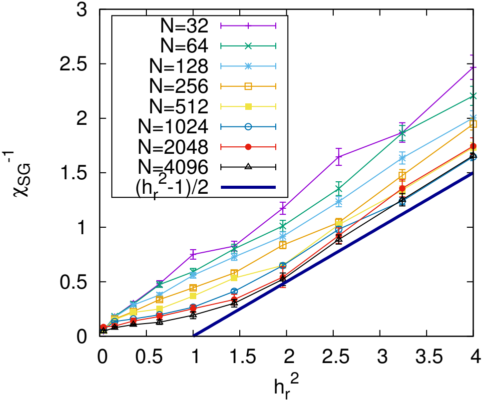


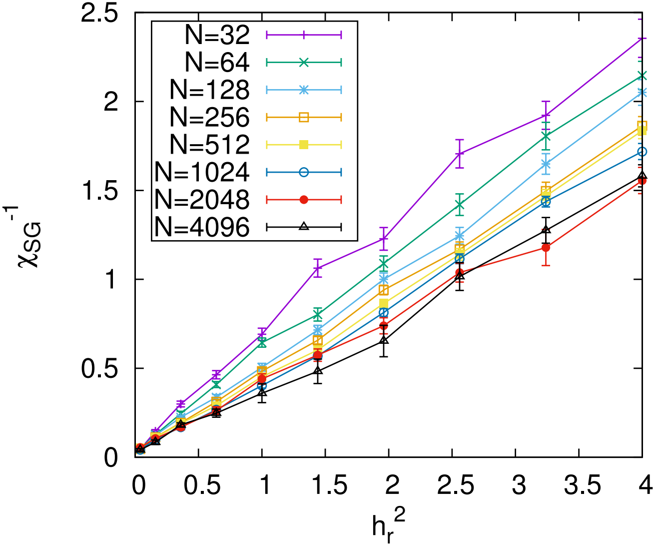
Figure:

Spin glass susceptibility: SK model

$$\chi_{SG} = \frac{1 + h_{AT}^2}{h_r^2 - h_{AT}^2}$$



Spin glass susceptibility: VB model



Conclusions and Outlook

- For the SK model in zero external fields, the quenched states reached for $m = 2$ and $m = 3$ are quite close to the critical energies E_c at which the overlap of the states would acquire features associated with a $P(q)$ with broken replica symmetry.
- Study of complexity with the replica method within an annealed approximation yields analytical expressions for both spin glass susceptibility and the density of states of the underlying Hessian, for the problem with random external magnetic fields. The annealed approximation is exact above the Almeida-Thouless critical point, where the complexity is zero.
- The density of states of the Hessian has a gap above the AT critical field h_{AT}. The point at which the gap vanishes could in fact be a method to identify the critical field.
- Below the critical field, the annealed approximation is only an approximation, but the density of states calculated from this approach seems to agree surprisingly well with numerical simulations.

Thank You