Revisiting the Szilard engine

Abhishek Dhar
International centre for theoretical sciences (TIFR), Bangalore
www.icts.res.in

Deepak Bhat (ICTS)
Anupam Kundu (ICTS)
Sanjib Sabhapandit (RRI)

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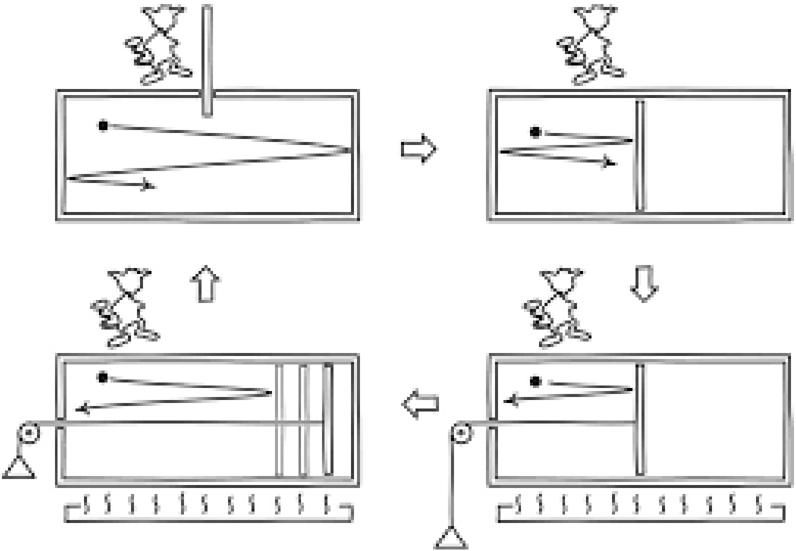
The Szilard engine



The Szilard engine- Apparent violation of second law

- Work done by one-particle gas:

$$W = \int_{V/2}^{V} dV \frac{k_B T}{V} = k_B \ln 2$$

.

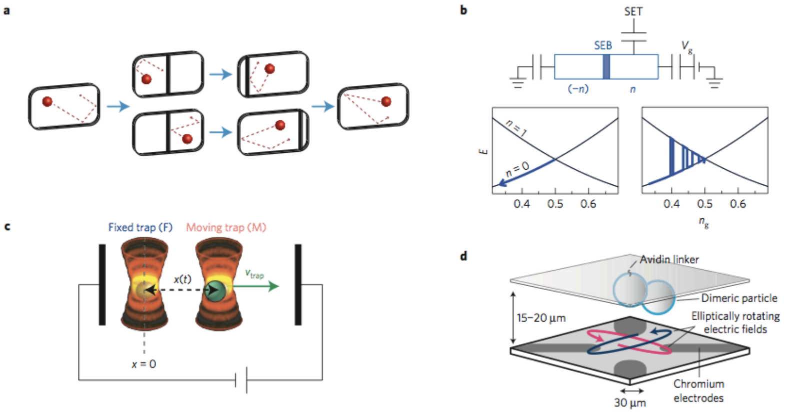
- Wall is thermal so that particle remains at constant temperature and heat flows into the system during the expansion.
- Energy conservation implies $\rightarrow$ Heat absorbed BY system $Q = W = k_B T \ln 2$.
- Cyclic process. System unchanged at end of process. Change in entropy of the universe

$$\Delta S = -k_B \ln 2 < 0$$

Violation of second law.

- Popular resolution: measurement leads to information of 1 bit which corresponds to generation of entropy $k_B \ln 2$.

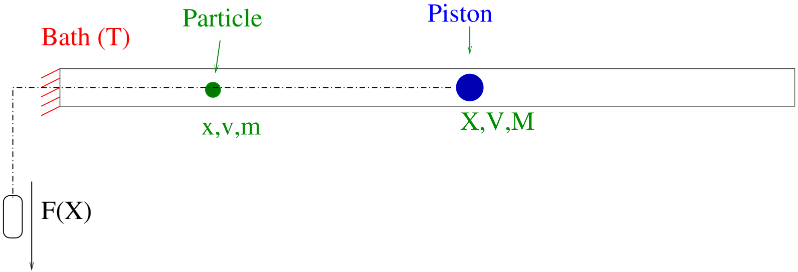
The Szilard engine



Review article: Thermodynamics of information: Nature Physics (2015).

Present study

- Forget philosophocal aspects....look at practical aspects!
- A microscopic understanding of the motion of the piston — In particular we expect that fluctuations are important in this “small” system and we want to understand their role in the motion of the piston.
- Focus on the following one-dimensional version



- $M >> m$, X, V slow degrees, x, v fast degrees.

Main result

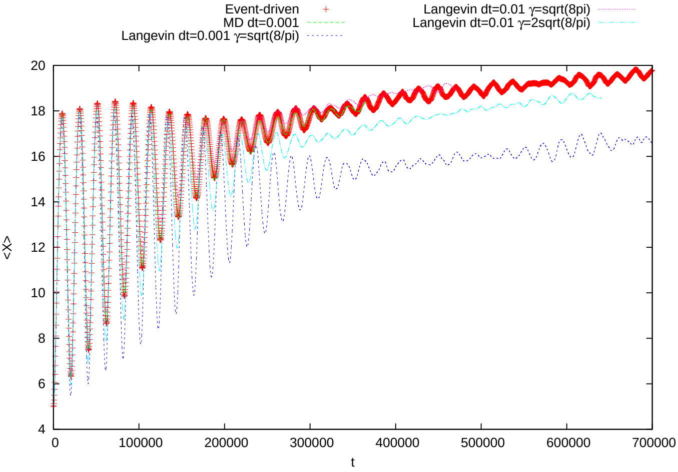
- CLAIM — Langevin equation of motion for the piston (heavy mass)

$$M\frac{dV}{dt} = -\frac{\partial U}{\partial x} + \frac{k_B T}{X} - \frac{\alpha}{X}V + \left(\frac{2\alpha k_B T}{X}\right)^{1/2}\eta(t),$$
$$\text{where } \alpha = c\left(\frac{8k_B T m}{\pi}\right)^{1/2},$$

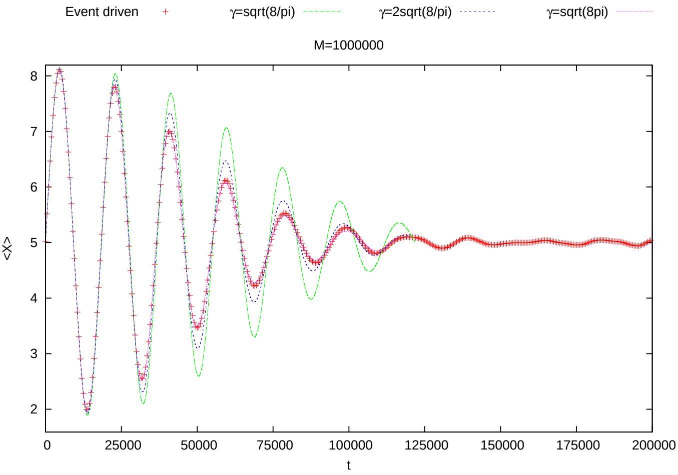
  and $\eta(t)$ is Gaussian white noise.

- Naive theory gives $c = 1$.
- Simulations give $c = \pi$.
- Derivation from the master equation for $P(x, X, v, V, t)$.
  - Eliminate fast variables (x, v).
  - Write effective Fokker-Planck equation for $P(X, V, t)$.
  - Read off the corresponding Langevin equation.
- OTHER results: Energetics of the Szilard engine, optimum protocols.

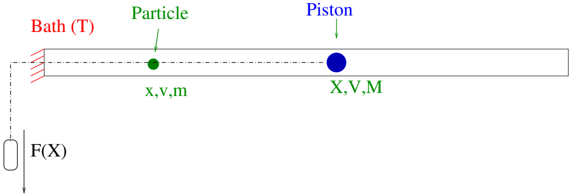
Comparision of direct simulations with solution of Langevin equations.
Example (I)— Szilard-engine case



Example (II) — Three particle case



Master equation approach



$$\frac{\partial P(x,X,v,V,t)}{\partial t} = -v\frac{\partial P}{\partial x} - V\frac{\partial P}{\partial X} + \frac{U'(X)}{M}\frac{\partial P}{\partial V}$$

$$+ \quad \delta(x)\int\Big\{\big[-v'\,\theta(-v')\phi(v)\big]\,P(x,X,v',V,t) - \big[-v\,\theta(-v)\phi(v')\big]\,P(x,X,v,V,t)\Big\}dv'$$

$$+ \quad \delta(X-x)\Big\{\theta(V-v)(V-v)\,P\left[x,X,\frac{(\mu-1)v+2V}{1+\mu},\frac{(1-\mu)V+2\mu v}{1+\mu},t\right]$$

$$-\theta(v-V)(v-V)\,P(x,X,v,V,t)\Big\},$$

where $\phi(v) = \sqrt{\beta m}ve^{-\beta mv^2/2}$ — distribution of particles emerging from a thermal bath and $\mu = m/M$.

GREEN — collision of small particle with thermal wall.

BLUE — collision of small particle with piston.

Master equation approach

- Introduce new $O(1)$ variables $u = \sqrt{\beta m}v$, $y = \sqrt{\beta M}V$, $\tau = t/\sqrt{\beta m}$ and let us denote the small parameter in the problem as $\epsilon = \sqrt{m/M}$.

- Small ϵ-expansion of the master equation results in:

$$\frac{\partial P(x, X, u, y, \tau)}{\partial \tau} = \mathcal{L}^{(0)}P + \epsilon\mathcal{L}^{(1)}P + \epsilon^2\mathcal{L}^{(2)}P \; .$$

- Then write

$$\begin{aligned} P(x, X, u, U, \tau) &= \Phi(x, u, \tau|X, U, \tau)\Psi(X, U, \tau) \\ \Phi &= \Phi_0 + \epsilon\Phi_1 + \epsilon^2\Phi_2 \\ \Psi &= \Psi_0 + \epsilon\Psi_1 + \epsilon^2\Psi_2 \end{aligned}$$

- Write equations at each order, use solutions to establish equation at next order.

- Finally write Fokker-Planck equation for $\Psi(X, U, \tau)$.

.....Solution in time-Laplace transform possible....inversion problem remains!

MORE DETAILS — Deepak Bhat's Poster.