Heat transport in a disordered nonlinear chain: exploring signatures of many-body localization

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Outline

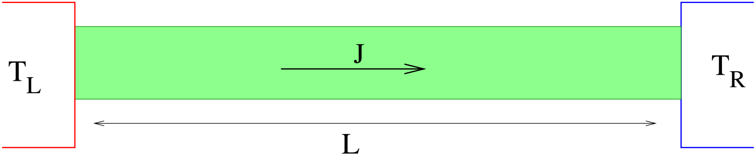
Main question

- A classical pinned disordered harmonic chain is a heat insulator. Does the introduction of a arbitrarily small amount of anharmonicity make the system conducting.

  Quantum systems: It is believed that for one-dimensional disordered interacting systems a many-body localized (MBL) phase exists — for sufficiently small interactions, the system continues to be an insulator.

  We address this question via non-equilibrium simulations of a one-dimensional disordered anharmonic chain.

Heat conductors and insulators



For a truely thermodynamically large system,
a heat conductor has a finite size-independent thermal conductivity while
an insulator has zero thermal conductivity.

For large but finite systems:

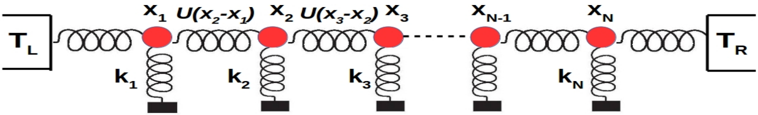
- Heat conductor

$$J = \kappa \frac{\Delta T}{L} \implies \kappa = JL/\Delta T \to \text{ finite}$$

- Heat insulator

$$J = Ae^{-cL} \implies \kappa \to 0$$

A simple one-dimensional microscopic model of heat conduction



$$H = \sum_{\ell=1,N} \left[\frac{p_\ell^2}{2} + k_\ell \frac{x_\ell^2}{2} \right] + \sum_{\ell=1}^{N+1} k \frac{(x_\ell - x_{\ell-1})^2}{2} + \sum_{\ell=1}^{N+1} \nu \frac{(x_\ell - x_{\ell-1})^4}{4} .$$

Fixed boundary conditions: $x_0 = x_{N+1} = 0$.

DISORDER: k_i (i.i.d variables) chosen uniformly in the interval $[1 - \Delta, 1 + \Delta]$

NONLINEARITY: ν

Metal - to - insulator transition

- ORDERED ANHARMONIC CHAIN: $\Delta = 0, \nu \neq 0$ - Conductor (Numerics)

- DISORDERED HARMONIC CHAIN: $\Delta \neq 0, \nu = 0$ - Anderson Insulator

Anderson localization

Oscillator chain with random masses.

$$H = \sum_{l=1,N} [\frac{p_l^2}{2m} + k_l \frac{x_l^2}{2}] + \sum_{l=1,N+1} k \frac{(x_l - x_{l-1})^2}{2}$$

$k > 0$, $\{k_l\} = [1 - \Delta, 1 + \Delta]$.

Finding normal modes in this system is closely related to the problem of finding electronic eigenstates in a disordered potential as described by the following Hamiltonian:

Electrons on a $1D$ lattice with onsite disorder.

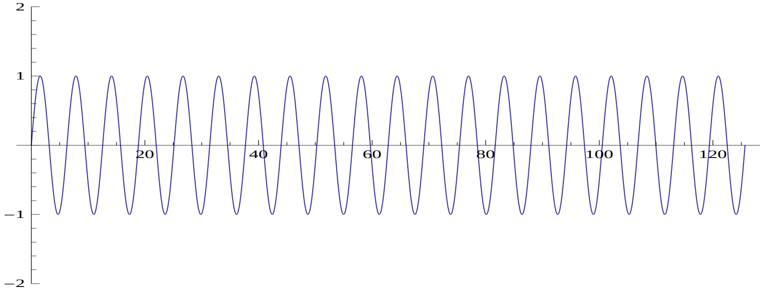
$$H = \sum_{l=1}^{N-1} -t[c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l] + \sum_{l=1}^{N} \epsilon_l c_l^\dagger c_l$$

$\{\epsilon_l\} = [-\Delta, \Delta]$.

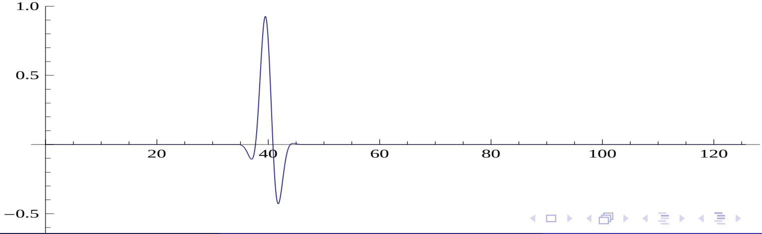
In both cases: all states are exponentially localized.

Character of normal modes of a disordered crystal

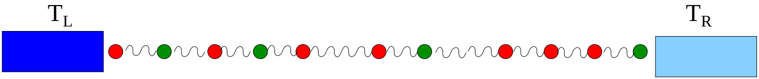
An extended periodic normal mode



A localized normal mode



Disordered Harmonic systems: Results in $1D$



The exact formula for current is given by

$$J = \frac{k_B \Delta T}{2\pi} \int_0^\infty d\omega \mathcal{T}(\omega) ,$$

where $\mathcal{T}(\omega)$ is the phonon transmission function, known in terms of the Green's function $G = [-\omega^2 M + \Phi - \Sigma]^{-1}$.

To understand the N-dependence of J we thus need to understand the N-dependence of the transmission coefficient $\mathcal{T}(\omega)$.
Anderson localization implies: $T(\omega) \sim e^{-L/\ell(\omega)}$ with $\ell(\omega) \sim 1/\omega^2$ for $\omega \to 0$.
Hence $J \sim e^{-L/\ell}$.

AD (2001)

Disorder + Anharmonicity

With Anharmonicity —- NEED NUMERICAL SIMULATIONS

Equations of motion with Langevin type heat baths

Equations of motion:

$$\ddot{x}_1 = -k_1 x_1 - \nu[x_1^3 + (x_1 - x_2)^3] - \gamma \dot{x}_1 + \sqrt{2\gamma k_B T_L}\eta_L \ ,$$
$$\ddot{x}_\ell = -k_\ell x_\ell - \nu[(x_\ell - x_{\ell-1})^3 + (x_\ell - x_{\ell+1})^3] \ , \quad 2 \le \ell \le N-1 \ ,$$
$$\ddot{x}_N = -k_N x_N - \nu[x_N^3 + (x_N - x_{N-1})^3] - \gamma \dot{x}_N + \sqrt{2\gamma k_B T_R}\eta_L \ ,$$

where η_L and η_R are white noise terms with zero mean and unit variance.
Observables of interest:

$$\langle J_N \rangle = \frac{1}{(N-1)} \sum_{l=2}^{N} \langle f_{l,l-1} \dot{x}_l \rangle \ , \qquad \text{with} \ \ f_{l,l-1} = \nu (x_{l-1} - x_l)^3$$

Let $T = (T_L + T_R)/2, \ \delta T = T_L - T_R$. The thermal conductivity is given by

$$\kappa(T, \nu, \Delta) = \lim_{\delta T \to 0} \lim_{N \to \infty} \frac{J \, N}{\delta T} \ , \ \text{provided the limit exists.}$$

$$\text{Temperature profile} \quad T_l = \langle p_l^2 \rangle$$

$\langle ... \rangle$ denotes averaging over time in the steady state. We also do a disorder average.

Scaling relations

The Langevin equations are invariant under the transformation

$$T_{L,R} \to sT_{L,R}, \ \{x_l\} \to \{s^{1/2}x_l\}, \ \nu \to \nu/s \, .$$

Ths implies that the steady state heat current $J = \nu\langle(x_{l-1} - x_l)^3 \dot{x}_l\rangle$ satisfies the scaling relation

$$J(sT_L, sT_R, \nu/s) = sJ(T_L, T_R, \nu)$$

$$\text{Hence} \ \ \kappa(T, \nu/s) = \kappa(T/s, \nu)$$

Thus changing nonlinearity ν equivalent to changing temperatures.

We set $\nu = 1$ and measure the heat current at different temperatures and for different system sizes N.
Define finite size conductivity as

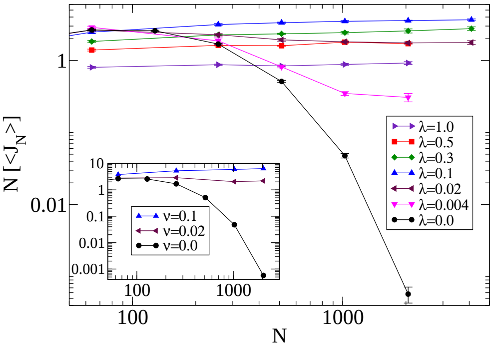
$$\kappa(T) = \frac{JN}{\delta T}$$

Effect of interaction on localization (Numerical results)

Disordered ϕ^4 model [Dhar and Lebowitz (2008)]

$$H = \sum_{l=1,N} \left[\frac{p_l^2}{2m_l} + k_o \frac{q_l^2}{2} \right] + \sum_{l=1,N+1} k \frac{(q_l - q_{l-1})^2}{2} + \sum_{l=1,N} \lambda \frac{q_l^4}{4} .$$

$\{m_l\} = [m - \Delta, m + \Delta]$. Disorder $\rightarrow \Delta$, Anharmonicity $\rightarrow \lambda$.



Dramatic transition: $e^{-cN/\ell} \rightarrow \frac{1}{N}$
for small amount of interaction.

$\kappa \sim T^{1/2}$

Effect of interaction on localization (Numerical results)

Flach, Ivanchenko, Li (2011)

$$H = \sum_{l=1,N} \left[\frac{p_l^2}{2} + k_l \frac{q_l^2}{2} \right] + \sum_{l=1,N+1} \nu \frac{(q_l - q_{l-1})^4}{4} .$$

$\nu = 0$ corresponds to independent oscillators with random frequencies.

$\kappa \sim T^4$

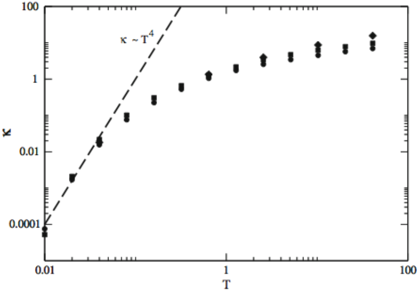


Figure 3. FSW chain: $\kappa(T)$ for system sizes $N = 16$ (filled circles), $N = 64$ (filled squares) and $N = 256$ (filled diamonds). The dashed line corresponds to the T^4 law.

Effect of interaction on localization (Numerical results)

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$$H = J\sum_{\ell=1}^{N} \mathbf{S}_\ell . \mathbf{S}_{\ell+1} + \sum_{\ell=1}^{N} \mathbf{h}_\ell . \mathbf{S}_\ell \ .$$

Oganesyan, Huse, Pal (2009) — Computation of diffusivity using Green-Kubo.

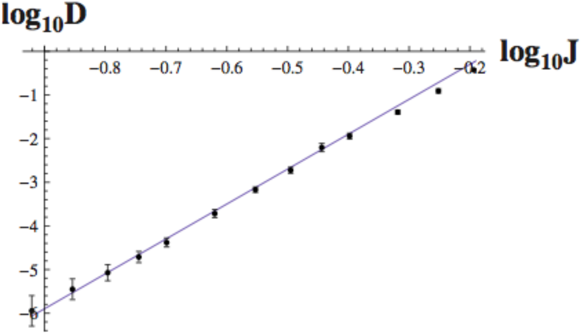

FIG. 1. (Color online) Disorder-averaged energy diffusion constant D as a function of the spin-spin interaction J. The line has slope 8 on this log-log plot.

Effect of interaction on localization (Theoretical ideas)

Huse, Basko, Huveneers, Flach...

- In the absence of interactions, transfer of energy between reservoirs is via isolated localized states talking to both reservoirs—energy transfer rate $\sim \epsilon^{-L/\xi}$.
- As one swithces on the interaction, chaotic islands of few particles are formed which act like local heat baths.
  - These arise from 3-particle resonances.
  - The chaotic islands are separated by lengths $\ell_c \sim 1/T$ (??).
  - One then has a diffusive energy transfer channel via these chaotic islands at a rate $\sim e^{-\ell_c/\xi}$.

  Hence Expect

$$\kappa_N(\Delta, T) = A\, e^{-N/\xi} + B\, e^{-\ell_c/\xi}.$$

- Huveneers (2013): With decreasing anharmonic coupling ϵ, the conductivity decays faster than any power of ϵ.

System-size dependence of conductivity

Δ = Disorder strength;
T =Interaction strength.

Simulation results for κ versus N for different disorder strengths at three values of temperature.

Observe that at low enough temperatures, there is a transition in the form of the temperature dependence.

Solid lines are fits to specific forms (see later).

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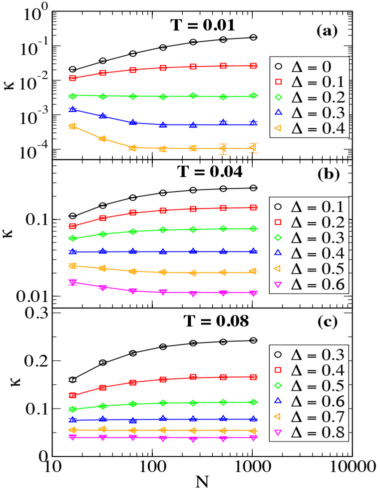


System-size scaling of conductivity

(a) Weak disorder (strong chaos) regime: Finite conductivity, but with finite size effects arising from scattering at the boundaries.

$$\frac{1}{\kappa_N(\Delta, T)} = \frac{1}{\kappa_\infty(\Delta, T)} + \frac{c}{N} \quad \text{for } \Delta < \Delta_c(T),$$

(b) Strong disorder (weak chaos) regime: Two parallel modes of transport —

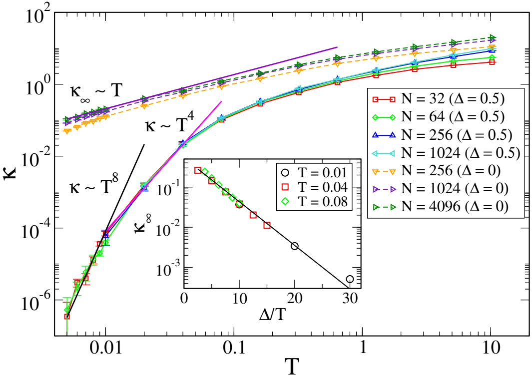
(i) through localized modes ($\kappa \sim e^{-N/\xi}$)

(ii) through energy diffusion between chaotic islands separated by lengths $\ell_c \sim 1/T^s$ leading to $\kappa \sim e^{-\ell_c/\xi}$ — (Huse, Basko).

$$\begin{aligned} \kappa_N(\Delta, T) &= A\, e^{-N/\xi} + \kappa_\infty(\Delta, T), \quad \text{for } \Delta > \Delta_c(T). \\ \kappa_\infty(\Delta, T) &= B e^{-\ell_c(T)/\xi}\ . \end{aligned}$$

These fitting forms are used to extract the asymptotic ($N \to \infty$) conductivity.

Temperature dependence of κ



Main figure: We see $\kappa \sim T^a$ with a larger "a" as $T \to 0$.

Ordered system ($\Delta = 0$): At low temperatures, $\kappa \sim T$.

Main Questions Now

Numerics becomes extremely slow in the low temperature “localized” regime.

How do we establish the proposed mechanism of transport throught chaotic islands and the true temperature dependence (as $T \to 0$).

- Propagation of chaos - OTOC
- Look for signatures in the temperature profile — expect steps at the location of the chaotic islands.
- Look at the power spectrum and see if one can see broadening of lines around resonant frequencies.

Electronic conductivity: Variable range hopping scenario — similar picture and similar issues arise.

Conclusions

- Looked for signatures of many-body localized phase in a disrodered ANharmonic chain, through study of non-equilibrium heat conducting steady state.
- Observation of different system-size scaling properties at “weak” and “strong” disorder. Possibly a transition at $\Delta_c(T)$ with different mechanisms of transport in the two regimes.
- Temperature dependence of conductivity at very low temperatures—difficult numerical problem since the heat current becomes very small. Evidence for the form $\kappa \sim e^{-\alpha/T}$.
- Studied forms of the steady state temperature profile—consistency with temperature dependence of κ.
- Looked at spread and growth of chaos in the system. We did not see any signatures of a logarithmic spread at small times.