OTHER REDUCTION METHODS

PREVIOUSLY...

We have seen previously that MBAM is closely related to several model reduction methods.

- Renormalization Group (Ising Model)
- Singular Perturbation (Power Systems)
- Equilibrium/Steady State Approximations (Chemical Kinetics)

Other cases that we haven't shown explicitly here (although elsewhere), that are plausible:

- Thermodynamic limit ($N \to \infty$)
- Continuum limit ($a \to 0$)
- Classical limits
  ($\hbar \to 0, c \to \infty$)

CONTROL THEORY: BALANCED TRUNCATION

Background: Linear Time Invariant (LTI) systems:

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

States, x, are not observed directly, but through the observation function y, and controlled through $u(t)$.

States have different levels of controllability (how easily they can be pushed to a particular state through $u(t)$) and observability (how easily they can be inferred through y).

TRANSFORMATIONS

Observations y are invariant to the following transformation:

$$
\begin{aligned}
x &\rightarrow Tx \\
A &\rightarrow TAT^{-1} \\
B &\rightarrow TB \\
C &\rightarrow CT^{-1}
\end{aligned}
$$

There exists a transformation T that makes the new states Tx equally observable as controllable, known as the *Balanced Realization*.

Balanced Truncation: Ignoring the least controllable/observable states in the balanced realization.
Balanced Singular Perturbation: Algebraically slaving the least controllable/observable state to the others.

MBAM APPROACH

- Construct a complete parameterization of the LTI system.

- Construct geodesics on the corresponding model manifold.

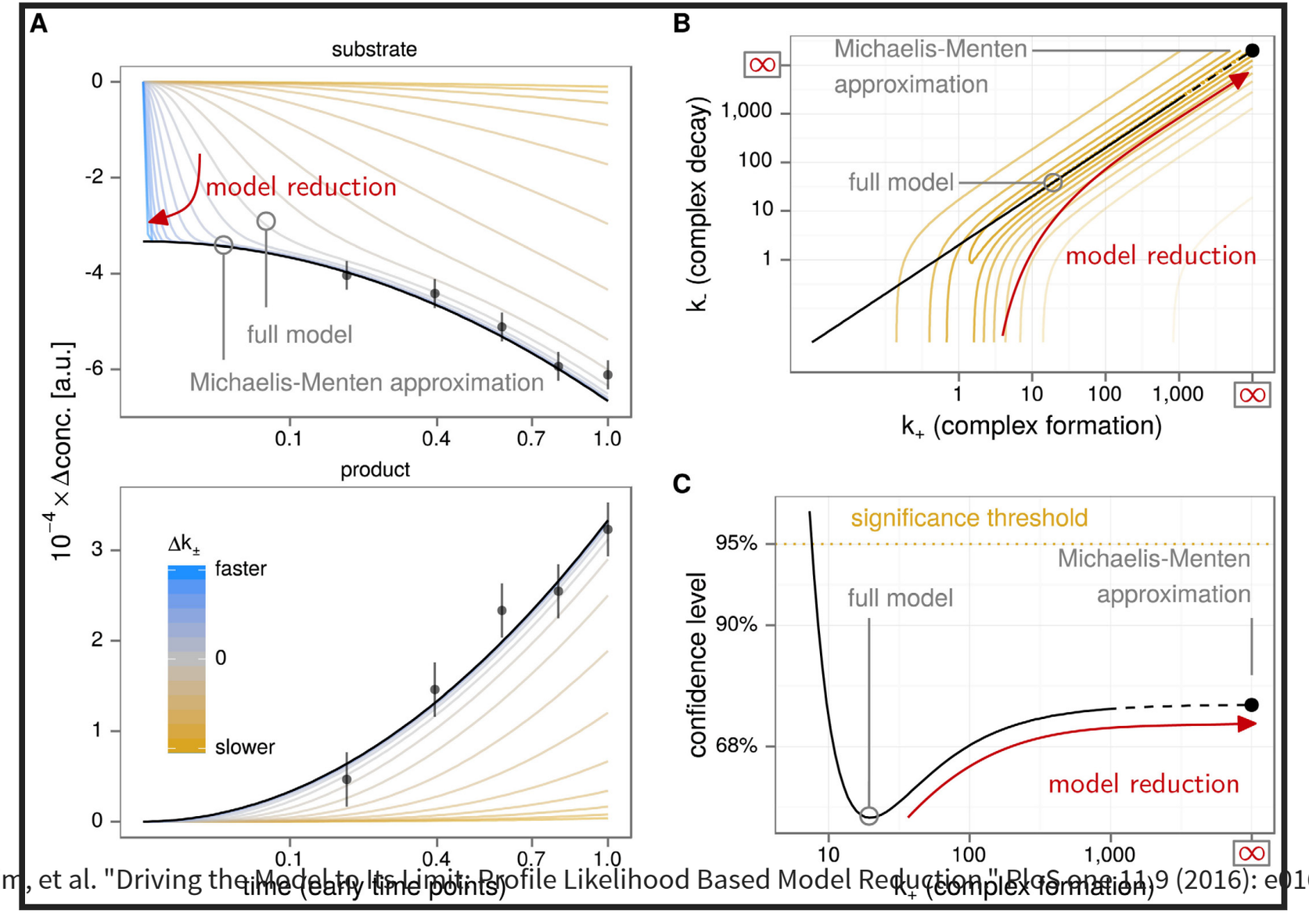
Result:

- Balanced Singular Perturbation is a boundary of the LTI system.
- Balanced Truncation is a boundary of the Balanced Singular Perturbation approximation (i.e., a corner of the LTI manifold)

Pare, Philip et al., "A unified view of balanced truncation and singular perturbation approximations". American Control Conference, Chicago, 2015.

MORE RECENT DEVELOPMENTS

LIKELIHOOD PROFILE BASED APPROACH



Maiwald, Tim, et al. "Driving the Model to Its Limit: Profile Likelihood Based Model Reduction." PloS one 11.9 (2016): e0162366.

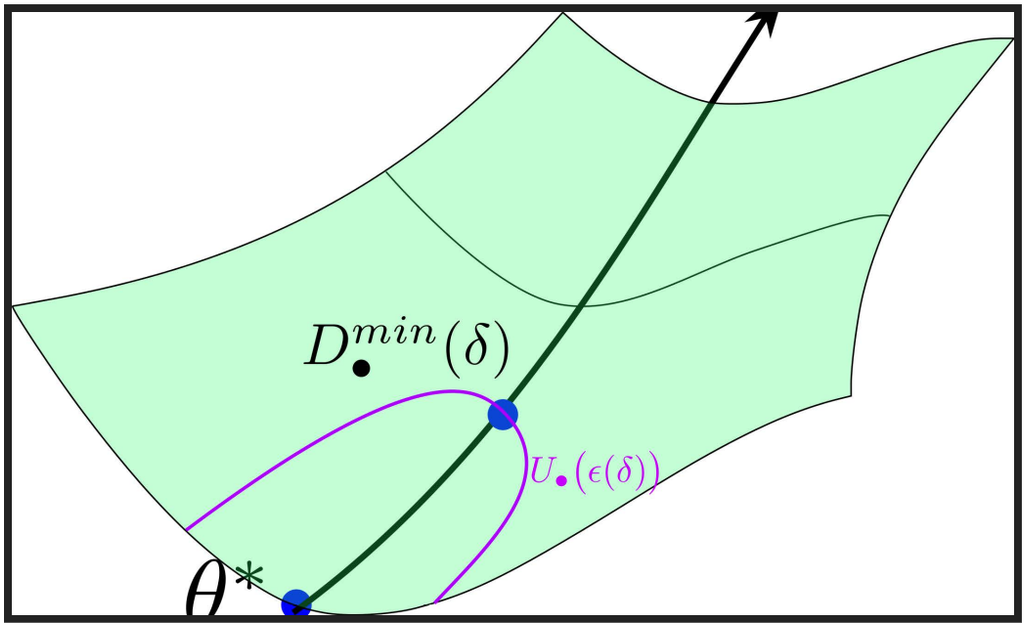
COMPARISON TO LIKELIHOOD

APPROACH

- MBAM will also identify limiting approximations
- In most cases, these methods are effectively equivalent.
- The technical details of how they are found differ.
  - Geodesics vs. Constructing Likelihood Profiles
  - Computational Efficiency?
  - MBAM = Approximation by boundary (however that boundary is found)
- Not all boundaries are extreme parameter values
  - Fold line and other symmetries
  - Restrictions of the domain

MINIMALLY DISRUPTIVE CURVES

- Multiscale Sloppiness:
- Generalize sloppiness beyond infinitesimal parameter fluctuations.
- Path through parameter space that gives least change in model behavior.
- Infer functional form of "practical unidentifiability"
- Can identify structural unidentifiabilities.



Raman, Dhruva V., James Anderson, and Antonis Papachristodoulou. "Delineating Parameter Unidentifiabilities in Complex Models." arXiv preprint arXiv:1607.07705 (2016).