What is Quantum Mechanics? A Minimal Formulation

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- Introduction:
  - Why, after ninety years, is the interpretation of quantum mechanics still a matter of controversy?
- Formulating classical mechanics: microscopic theory (MICM)
  - Phase space: states and properties
- Microscopic formulation of quantum mechanics (MIQM):
  - Hilbert space: states and properties
  - quantum incompatibility: noncommutativity of operators
- The simplest examples: a single spin-½; two spins-½
- The measurement problem
- Macroscopic quantum mechanics (MAQM):
  - Testing the theory. Not new principles, but consistency checks
- Other treatments: not wrong, but laden with excess baggage
- The ten commandments of quantum mechanics

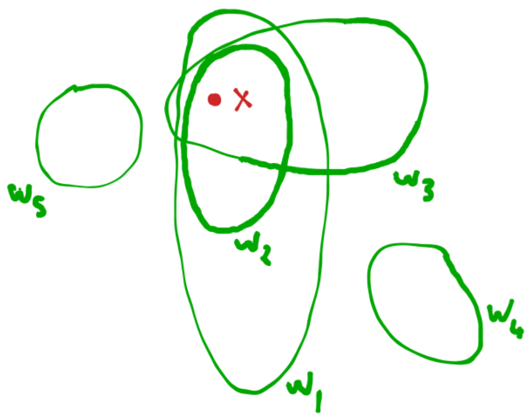
Foundations of Quantum Mechanics

- I. What is quantum mechanics (QM)? How should one formulate the theory?
- II. Testing the theory. Is QM the whole truth with respect to experimental consequences?
- III. How should one interpret QM?
  - Justify the assumptions of the formulation in I
  - Consider possible assumptions and formulations of QM other than I
  - Implications of QM for philosophy, cognitive science, information theory,…
- IV. What are the physical implications of the formulation in I?

- In this talk I shall only be interested in I, which deals with foundations
- II and IV are what physicists do. They are not foundations
- III is interesting but not central to physics
- Why is there no field of Foundations of Classical Mechanics?
  - We wish to model the formulation of QM on that of CM

Formulating nonrelativistic classical mechanics (CM)

- Consider a system S of N particles, each of which has three coordinates (x_i, y_i, z_i) and three momenta (p_{xi}, p_{yi}, p_{zi}) .
- Classical mechanics represents this closed system by objects in a Euclidean phase space of $6N$ dimensions.
- A state of the system is a point x in phase space
- A property is a subset w of points in phase space, e.g. the set of all points for which the energy E of the system has the value E_1.
- If the state is x_0 at time $t = t_0$ then the Hamiltonian determines the trajectory $x(t)$ of the system for all $t < t_0$ and $t > t_0$.
- Predictions: the property w is true at time t if $x(t) \epsilon w$ and false otherwise.



If the state is x then the properties w_1 , w_2, w_3 are simultaneously true, and the properties w_4, and w_5 are simultaneously false.

- Determining which physically interesting properties are true given the state at any time is the full content of classical mechanics.

Comments regarding classical mechanics:

- The preceding formulation is what we call microscopic classical mechanics (MICM), since it applies to any closed system of arbitrary size N, and uses only concepts pertaining to the system itself.
- States are assumed to exist. They are not observed. Only properties are observed, by noting whether they are true or false.
- MICM makes no direct reference to how states are prepared, nor how the predictions might be tested.
- Those questions can be answered by macroscopic classical mechanics (MACM), which is not a separate theory, but a special case ($N \rightarrow \infty$), in which one assumes the existence of macroscopic measurement or preparation devices, which interact with, but are external to, the system under study.
- Note that MICM is logically complete by itself.
- We wish to formulate quantum mechanics (QM) in as close analogy as possible to classical mechanics.

Microscopic (nonrelativistic) quantum mechanics (MIQM)

- MIQM also defines states and properties, but these are now objects in Hilbert space, rather than phase space.

- Hilbert space contains vectors $|\psi>$ and operators O_1 , O_2. If you multiply two operators you get another operator…

- BUT in general: $O_1 O_2 \neq O_2 O_1$ Quantum incompatibility! 

- Any vector $|\psi>$ (more precisely any ray $\alpha|\psi>$) or the corresponding projector $[\psi]$ can be selected as the unique (pure) state. More generally, a state ρ (pure or mixed) is a positive operator of trace one, called the density matrix.

- Any vector $|\psi>$ or its projector $[\psi]$ can also represent a property. More generally a property is a subspace A of Hilbert space, or the projector $[A]$ satisfying $[A]^2 = [A]$. The subspace may have any dimension $d_H \geq d_A \geq 1$

- Incompatible properties A and B are ones whose projectors don't commute: $[A]\,[B] \neq [B]\,[A]$

States and properties (cont'd)

- In classical mechanics states confer truth on properties
- Bell/Kochen-Specker Theorem: in Hilbert space no truth function can be consistently defined to apply to incompatible properties. It follows that the state cannot confer truth on all properties. It can at most confer the probability of being true.
- Gleason's Theorem: the only consistent way to define the probability that the state ρ confers truth on property A is via the Born rule:

$$P_\rho(A) = \mathrm{Tr}\,(\rho\,[A]), \quad \text{(Tr is the 'trace')}$$

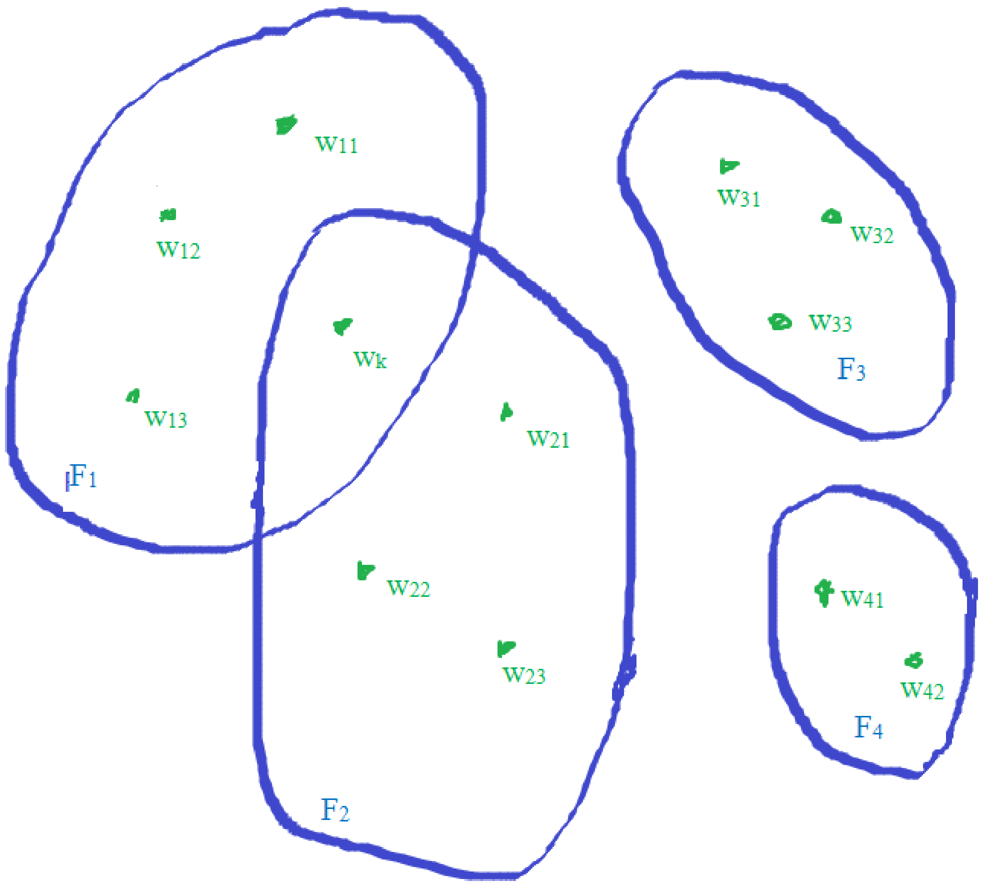
- BUT: the Born rule does not define a probability function over the whole space of (possibly incompatible) properties since it violates the Kolmogorov condition:

$$P_\rho(A \vee B) + P_\rho(A \wedge B) \neq P_\rho(A) + P_\rho(B) \quad \text{if} \quad [A][B] \neq [B][A].$$

- Every probability function requires a sample space of compatible properties $\{w_1, w_2, \ldots\}$ called a framework:

Framework: an Exhaustive Set of Exclusive Alternatives (ESEA)

$$w_1 \wedge w_2 \wedge \ldots = I \qquad\qquad < w_1 | w_2 > = 0$$

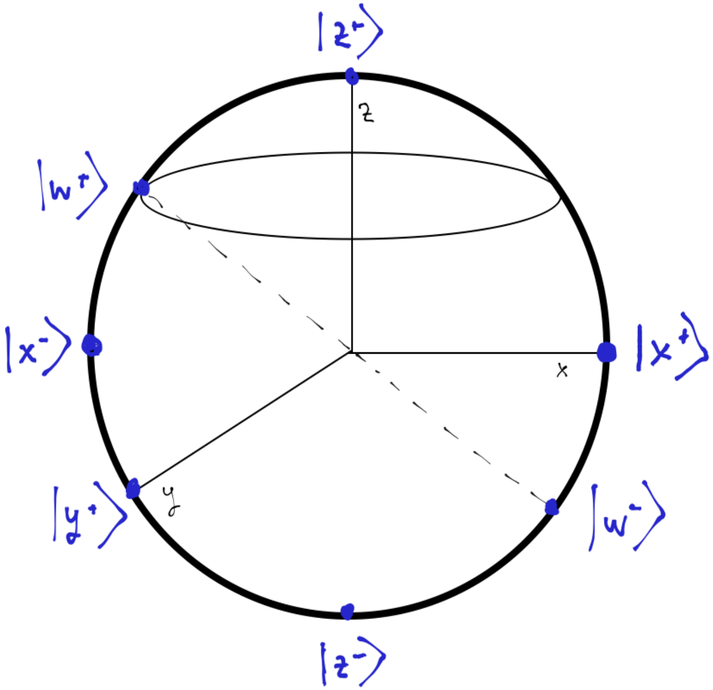


- Frameworks $F_1 = \{w_{11}, w_{12}, \ldots\}$, $F_2 = \{w_{21}, w_{22}, \ldots\}$, … are mutually incompatible
- The properties $w_{11}, w_{12}, \ldots$ within the framework F_1 are mutually compatible
- Given a state ψ and a framework F_1 the probability function $P_{\psi,F1}(w_{ij})$ is defined in the sample space of mutually compatible properties $\{w_{11}, w_{12}, \ldots\}$ belonging to F_1

Spin-½

- Let $S_x = ½\, \sigma_x$, $S_y = ½\, \sigma_y$, $S_z = ½\, \sigma_z$
- S_x has eigenstates $|x^+>$, $|x^->$ with eigenvalues $\pm ½$, etc.

Bloch sphere:



Each $\{|w^+>, |w^->\}$ is a framework F_w (ESEA):

$|w^+>$ and $|w^->$ are mutually compatible and mutually orthogonal: $<w^+|\,|w^-> = 0$.

$|z^+>$ is incompatible with all $|w^+>$, $|w^->$, for $w = x, y, v, \ldots$

- Let the state be $\psi = |z^+>$, or equivalently $[z^+]$.
- In order to define the truth or the probability of truth of a property, I must choose a framework of mutually compatible properties.
- Let me first choose a framework $\{[z^+], [z^-]\}$ that is compatible with $|z^+>$:

$P_{z,z}([z^+]) = \mathrm{Tr}([z^+][z^+]) = 1$, $[z^+]$ is true

$P_{z,z}([z^-]) = \mathrm{Tr}([\psi][z^-]) = \mathrm{Tr}([z^+][z^-]) = 0$, $[z^-]$ is false

- No other property is true or false. Now choose $\{[x^+], [x^-]\}$ as the framework:

$P_{z,x}([x^+]) = \mathrm{Tr}([z^+][x^+]) = P_{z,x}([x^-]) = \mathrm{Tr}([z^+][x^-]) = ½$

- The properties $[x^\pm]$ belonging to the framework F_x each have a probability ½ of being true (or false) in the state $\psi = |z^+>$.
- A quantum state defines a multiplicity of mutually incompatible probability functions, each one associated with a particular framework (diameter on the Bloch sphere) of mutually compatible properties.
- In order for the state to assign truth or the probability of truth to a property, some choice of framework (ESEA) must be made. The probabilities associated with different frameworks are incompatible. The choice of a framework is the breaking of framework symmetry.

Conditioning and selection

- From the standard Bayesian definition of conditional probabilities one can prove that the state $\psi = |z^+>$, conditioned on the truth of the property $[x^+]$ is a new state $\psi_x = |x^+>$.
- The transition from ψ to ψ_x illustrates a fundamental principle:

Information is physical

- Conditioning on the property $[x^+]$ adds information to the state ψ and thereby changes the state physically. This is referred to as the 'collapse of the wavefunction' in the orthodox interpretation. It occurs in 'logical time', not in dynamical (physical) time and does not require the intervention of any external apparatus or agent.
- Note that the state ψ does not determine the truth, falsehood or even probability of any property, until a framework has been chosen. This breaking of framework symmetry is necessary before classical information can be extracted from a quantum state.

The von Neumann-Lüders Rule

- More generally, conditioning a state ρ on a framework F_A defines a probability function $P_\rho(\{A_i\})$, with a unique outcome A_k, say. By unique we mean A_k or A_j , not A_k and A_j.

- Further conditioning on the outcome A_k (or equivalently selecting that outcome), produces the state

$$\rho_A = [A_k]\, \rho\, [A_k] / \mathrm{Tr}(\rho[A]) \qquad \text{(von-Neumann Lüders rule)}$$

- These conditioning operations are what we call 'microscopic measurement'. Similarly, conditioning and selection can be used to define 'microscopic state preparation'.

- All of these operations occur in logical time.

The Measurement Problem

- Since we have not talked about macroscopic measurements in MIQM, one might think we have avoided the measurement problem, which is usually phrased in terms of macroscopic measurements.
- We can, however, identify a 'microscopic measurement problem', by noting that the collapse mechanism ($\rho \rightarrow \rho_A$) of the von Neumann-Lüders rule violates the unitary dynamics of the Schrödinger equation.
- Our 'resolution' is to note that this rule is a theorem about conditioning and selection in Hilbert space. It is the only way for the state to confer truth on a property. The collapse is a direct consequence of the quantum incompatibility of ρ and A and the physical nature of information (i.e. of conditioning and selection).
- The transformation from ρ to ρ_A occurs in logical time, which can be simultaneous in dynamical time. The two are distinct.
- Thus the 'measurement problem' is neither about macroscopic measurements, nor is it a problem.
- This is QM made ESEA

Macroscopic quantum mechanics (MAQM)

- The preceding is a quick sketch of MIQM, with all its supposed weirdness and paradox, manifested in any closed system. It depends on a single fundamental assumption: Hilbert space.

- Just as in classical mechanics, in order to test the theory or to prepare the quantum state, the system must be put into contact with a macroscopic measurement or preparation apparatus.

- This is the domain of MAQM, a special case of MIQM, applicable to large systems, which can display classical behavior (the classical behavior of large systems can be considered a phenomenological assumption, but it can also be justified from the theory).

Macroscopic Measurements

- Once MIQM is accepted then macroscopic measurements can be analyzed using the standard (textbook) theory.

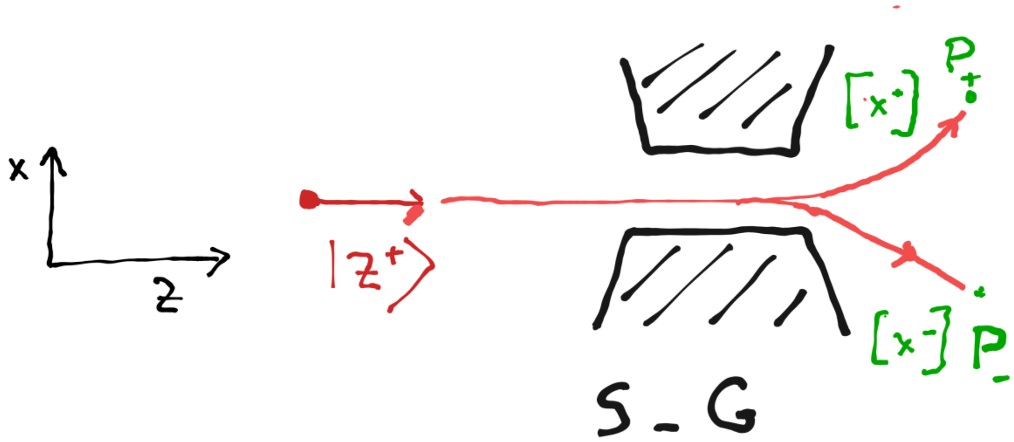
- The measurement apparatus $\mathcal{M}$, when placed in contact with the quantum system $\mathcal{S}$ in an environment $\mathcal{E}$, selects a unique framework $F_M = \{[w_1],[w_2], \ldots\}$ of $\mathcal{S}$ appropriate to that measurement.

- The properties $[w_i]$ of $\mathcal{S}$ are correlated with individual properties $\{P_i\}$ of $\mathcal{M}$ called 'pointer readings'. These readings are 'decohered' by interaction of $\mathcal{M}$ with its environment $\mathcal{E}$.

- The 'true' property $[w_k]$ selected in $\mathcal{S}$ according to MIQM then selects the true pointer reading P_k of $\mathcal{M}$.

- The above is an application of QM, a consistency check, not a new theory.

Example: Stern-Gerlach apparatus for spin ½



- The particle in the state $|z^+>$ enters the Stern-Gerlach apparatus which is pointed in the *x*-direction, thereby choosing the framework F_x of the spin system and defining the probability function $P_{z,x}$ with sample space (ESEA) $\{[x^+], [x^+]\}$.
- The microscopic outcome $[x^+]$ is then selected with probability ½, and this outcome is correlated to the macroscopic position P^+ on the apparatus. Similarly, the outcome $[x^-]$ leads to the 'pointer reading P^-.
- A similar procedure describes the preparation of a quantum state, using a macroscopic apparatus to select a single outcome, which becomes the new state.

Other formulations or interpretations of QM

- Copenhagen and/or orthodox QM (textbooks): phenomenology
- Modern treatments: (Preskill, Bub, Kochen): very close to ours, except for language, primarily use of 'measurement' in MIQM.
- Many-worlds: geared to cosmology; unnecessarily complicated for a minimal theory.
- Consistent and decoherent histories: the 'static' theory is essentially our formulation. The 'dynamic theory' (multitime histories and consistency conditions) is also unnecessary (just as MW).
- These theories are not wrong: we claim our minimal formulation clarifies the language and eliminates excess baggage.
- Neo-classical theories: Bohmian or Spontaneous Collapse (GRW):

  These are neoclassical in the sense that they have a classical ontology, with nonlocal or stochastic dynamics to reproduce (some of) QM. These theories deny the primacy of Hilbert space. They are "not even wrong".

The Ten Commandments of Quantum Mechanics

- Quantum mechanics (QM) does not require an interpretation. It requires a clear and unambiguous formulation. Such a minimal formulation exists for classical mechanics (CM), in which states confer truth on properties in Euclidean space.
- Both classical and quantum mechanics are first formulated microscopically, for closed systems of arbitrary size (MICM and MIQM).
- Quantum mechanics replaces Euclidean space with Hilbert space, in which ☠ quantum incompatibility ☠ (noncommutativity of operators) is fundamental.
- Quantum states do not in general confer truth on properties. They confer the probability of being true to subsets of compatible properties, called frameworks (Exhaustive Sets of Exclusive Alternatives, ESEA).
- A microscopic measurement consists of (i) selecting a state, (ii) selecting (choosing) a framework (breaking framework symmetry) and (iii) selecting the outcome that is true with some probability.

- Conditioning on the selected (true) outcome adds information and thereby changes the state: information is physical. This is the microscopic ‘collapse of the wavefunction’.
- Subsystems of quantum systems are in general entangled: the state of the composite system is in general incompatible with the states of the subsystems.
- The secret to ‘solving the measurement problem’ is the fact that states are not observable. Only the truth of properties is observable.
- In this way the microscopic theory (MIQM) is fully formulated for closed systems of any size, without reference to external apparatus or external agents, and with no paradoxes. It is logically complete.
- Experimental tests of the predictions of MIQM are described by the macroscopic theory (MAQM), which involves classical measurement apparatus or preparation devices. MAQM is an application of the theory, involving no new principles. It provides a test and a consistency check on the theory.

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The future of Foundations of Quantum Mechanics

- Pierre's quixotic dream is that Foundations of QM should rest in peace alongside the Foundations of Classical Mechanics.

R.I.P.