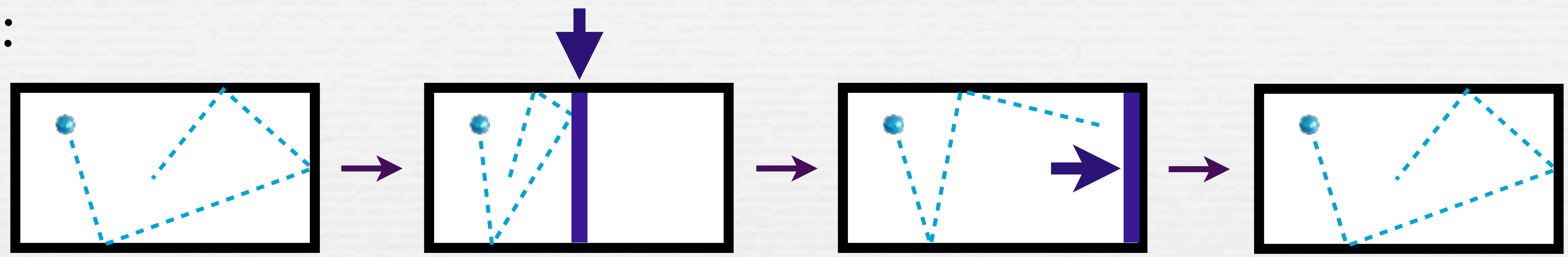
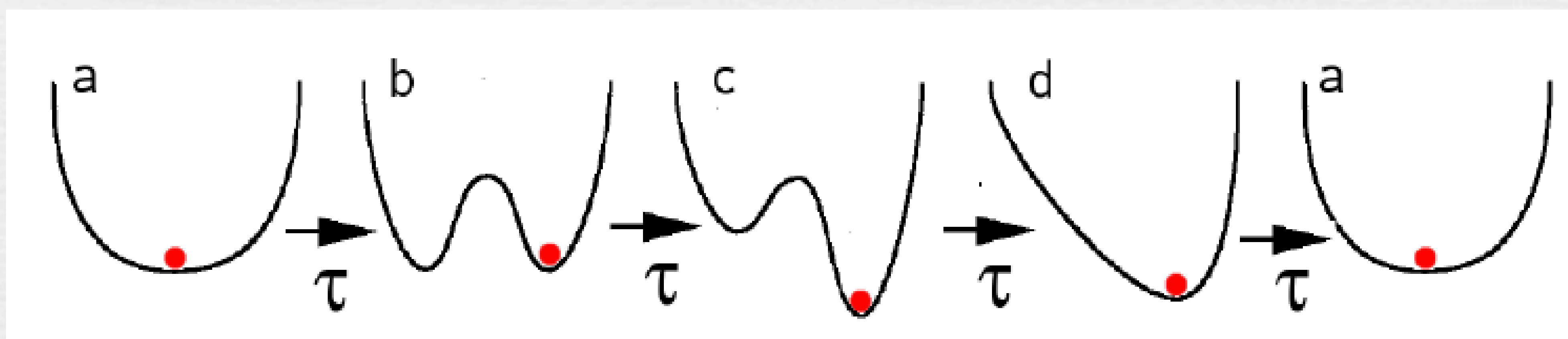
8. Creating information: symmetry breaking

- One particle gas:



- Brownian particle:

Kawai, JMRP, van den Broeck. PRL **98**, 080602 (2007).

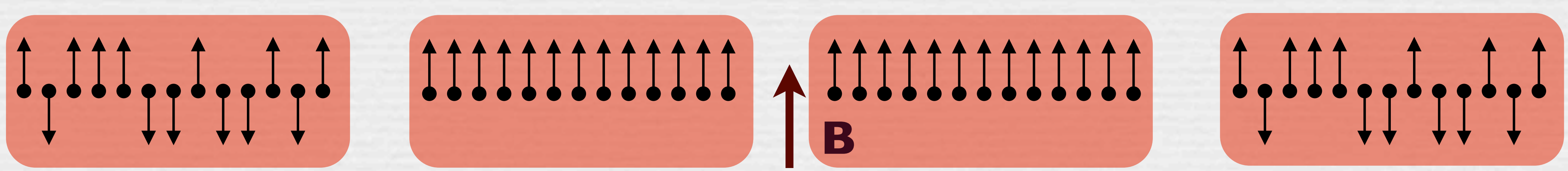


- Ising model:

JMRP. Chaos **11**, 725 (2001)

Coupling Field

(J, B)



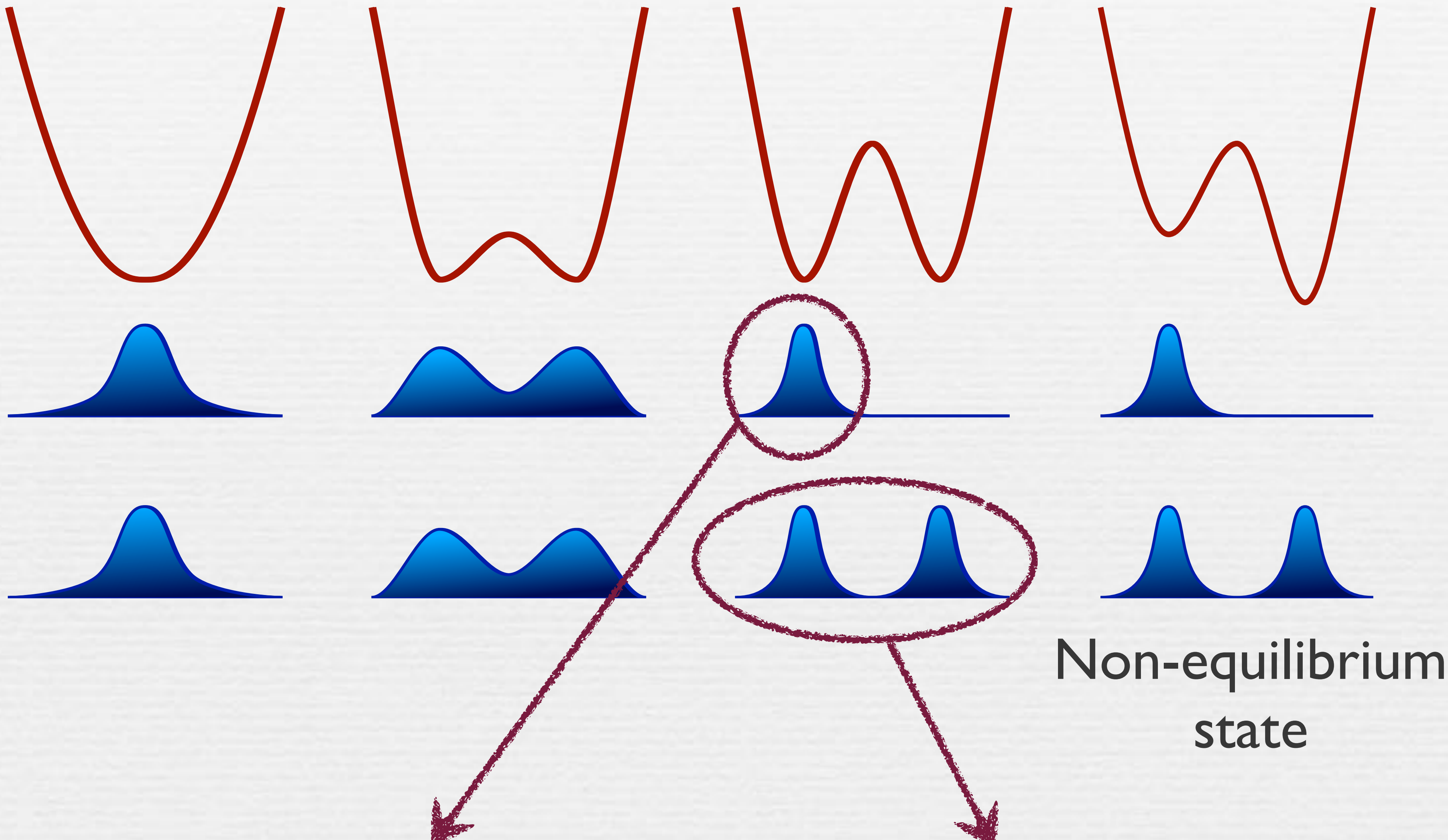
$(0,0) \rightarrow (J,0) \rightarrow (J,\pm B) \rightarrow (0,\pm B) \rightarrow (0,0)$

Measurement

Informational states

Single system:

Ensemble:



Conformational entropy/free energy

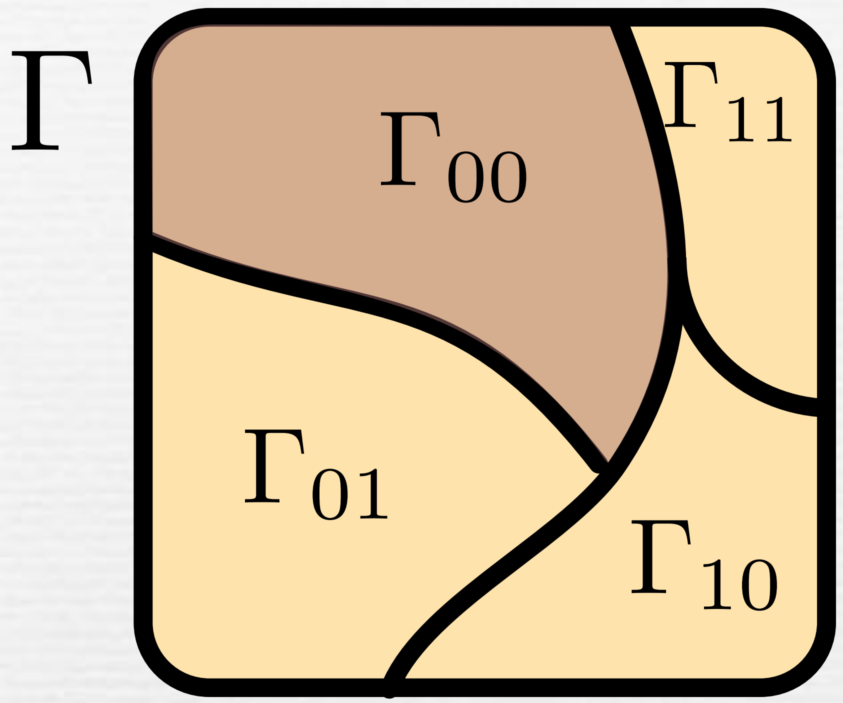
Equilibrium entropy/free energy

$$Z_m = \int_{\Gamma_m} dq dp\, e^{-\beta H(q,p)}$$

$$F_m = -kT \ln Z_m$$

$$F = -kT \ln \int_{\Gamma} dx e^{-\beta H(x)}$$

Informational states



Probability of state m: p_m

Partition function of state m (m= 00,01,10,11):

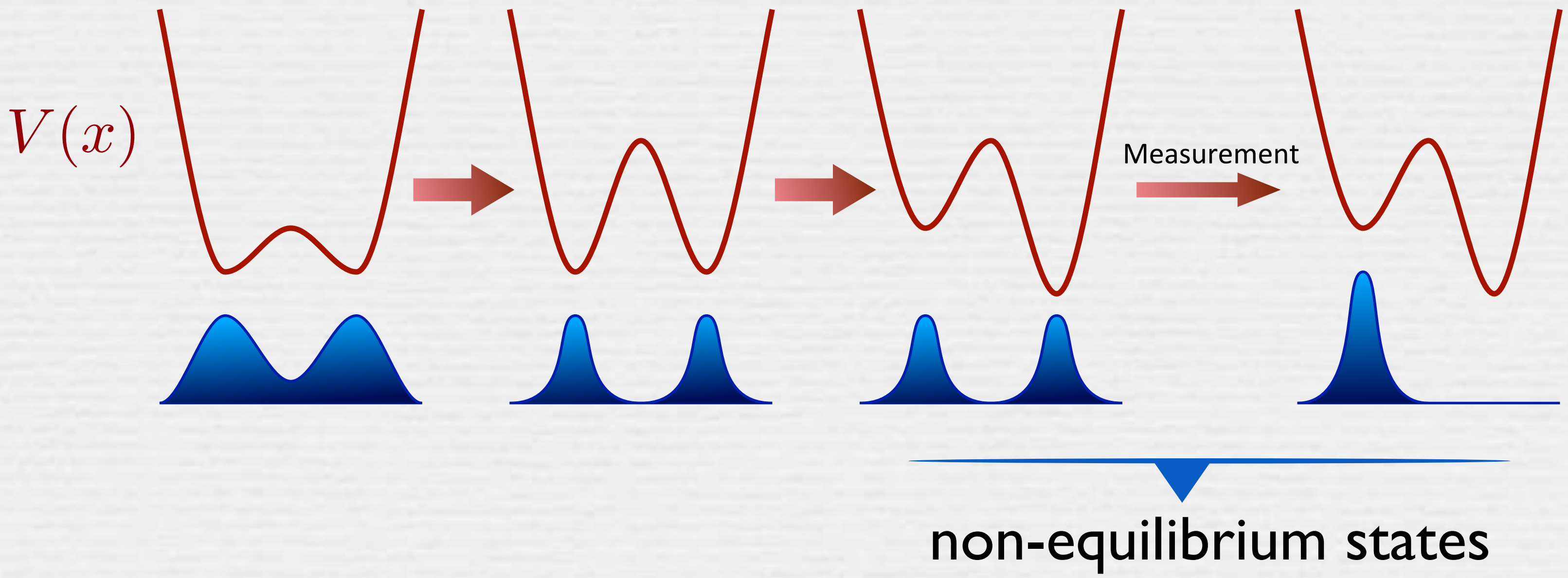
$$Z_m = \int_{\Gamma_m} e^{-\beta H(x)} dx$$

Free energy of state m:

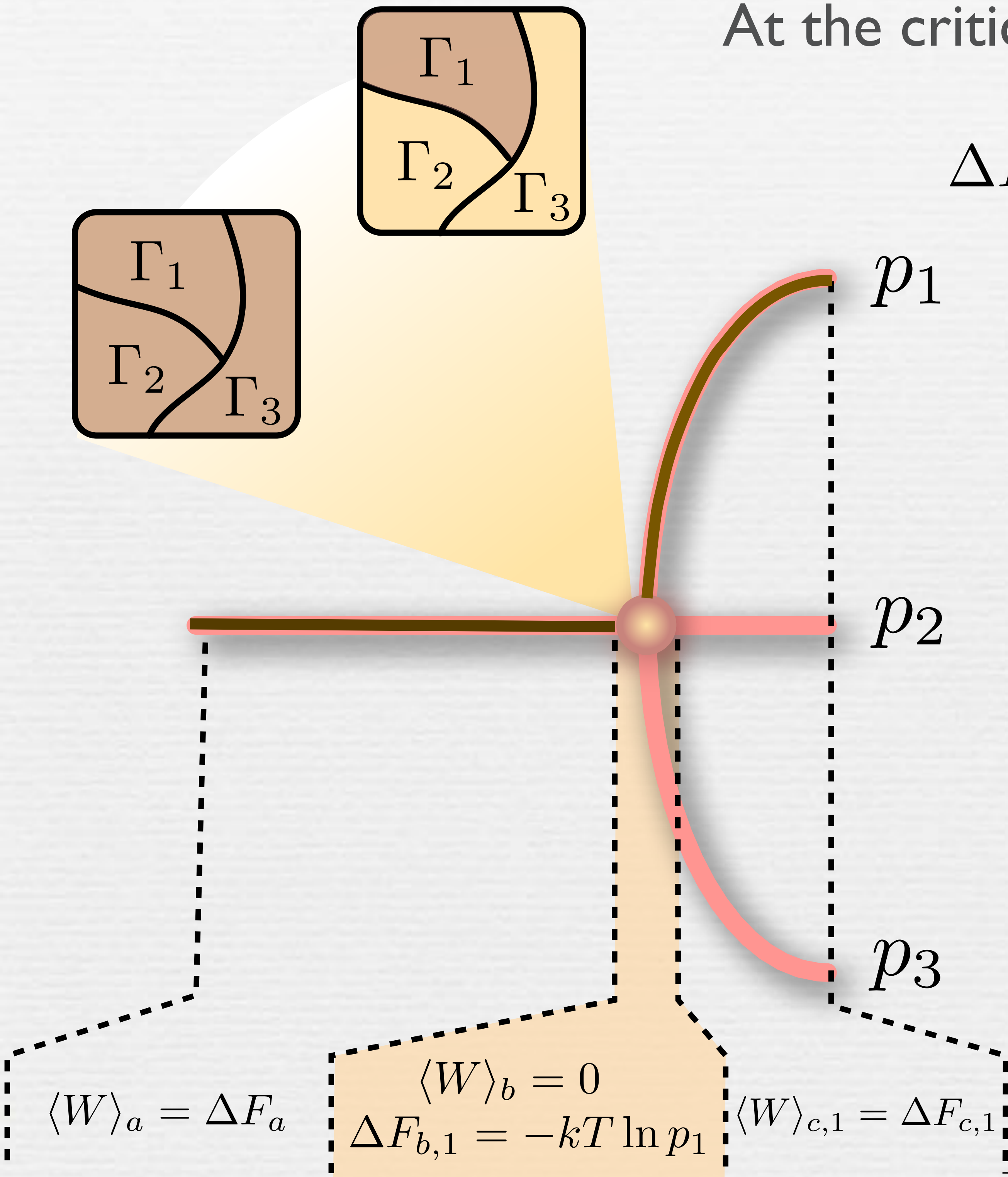
$$F_m = -kT \ln Z_m$$

Global equilibrium state:

$$p_m^{\text{eq}} = \frac{Z_m}{Z} = \frac{e^{-\beta F_m}}{Z}$$



Energetics of symmetry breaking



At the critical point the free energy changes as:

$$\Delta F_{b,1} = -kT \ln Z_1 + kT \ln Z = -kT \ln \frac{Z_1}{Z}$$

$$Z_1 = \int_{\Gamma_1} dx\, e^{-\beta H(x)}$$

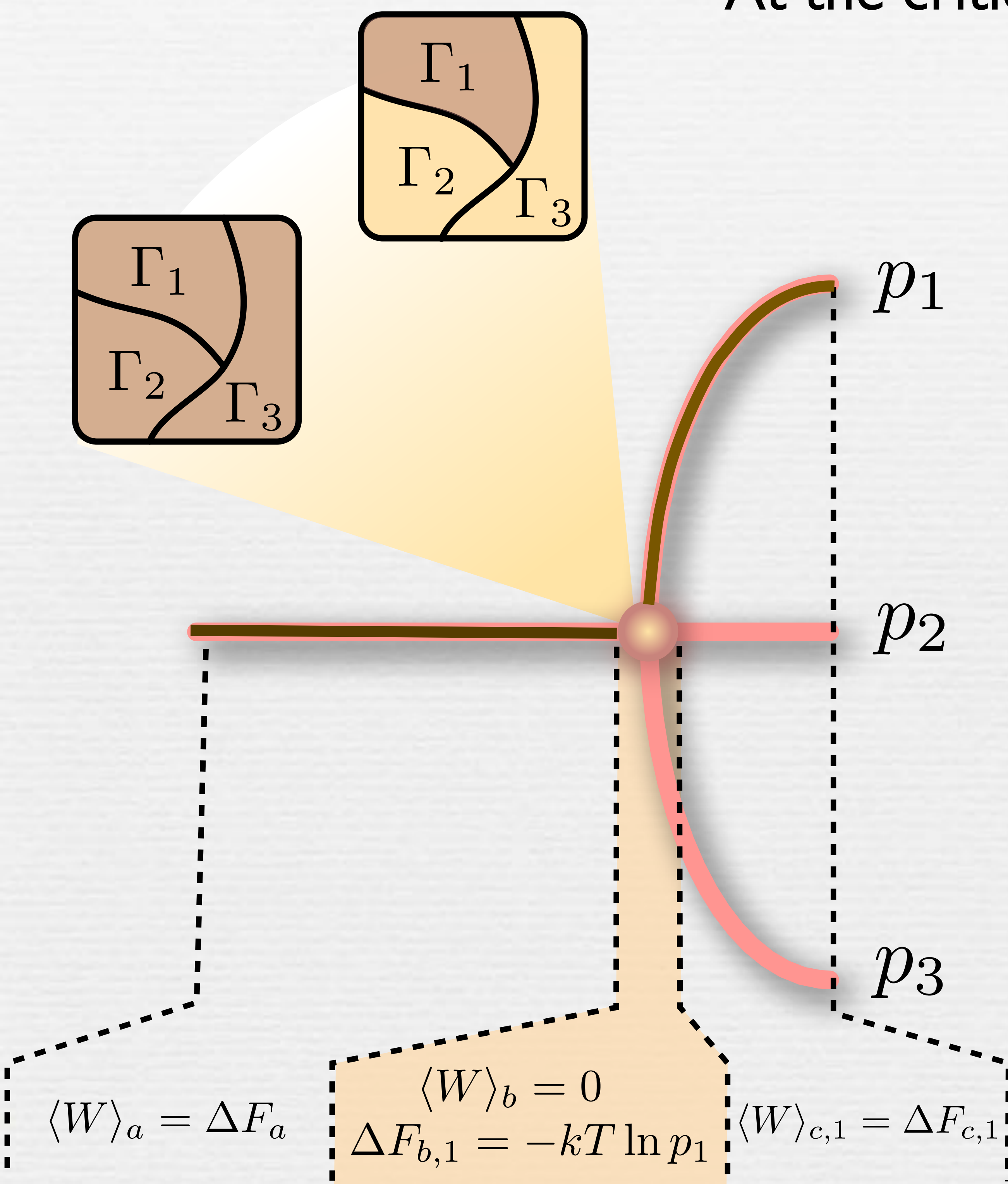
$$Z = \int_{\Gamma} dx\, e^{-\beta H(x)}$$

$$p_1 = \frac{Z_1}{Z}$$

$$\Delta F_{b,1} = -kT \ln p_1$$

No work needed!

Energetics of symmetry breaking



At the critical point the free energy changes as:

$$\Delta F_{b,1} = -kT \ln p_1$$

No work needed!

Along the whole process:

$$\langle W \rangle_i - \Delta F_i \geq -kT \ln p_i$$

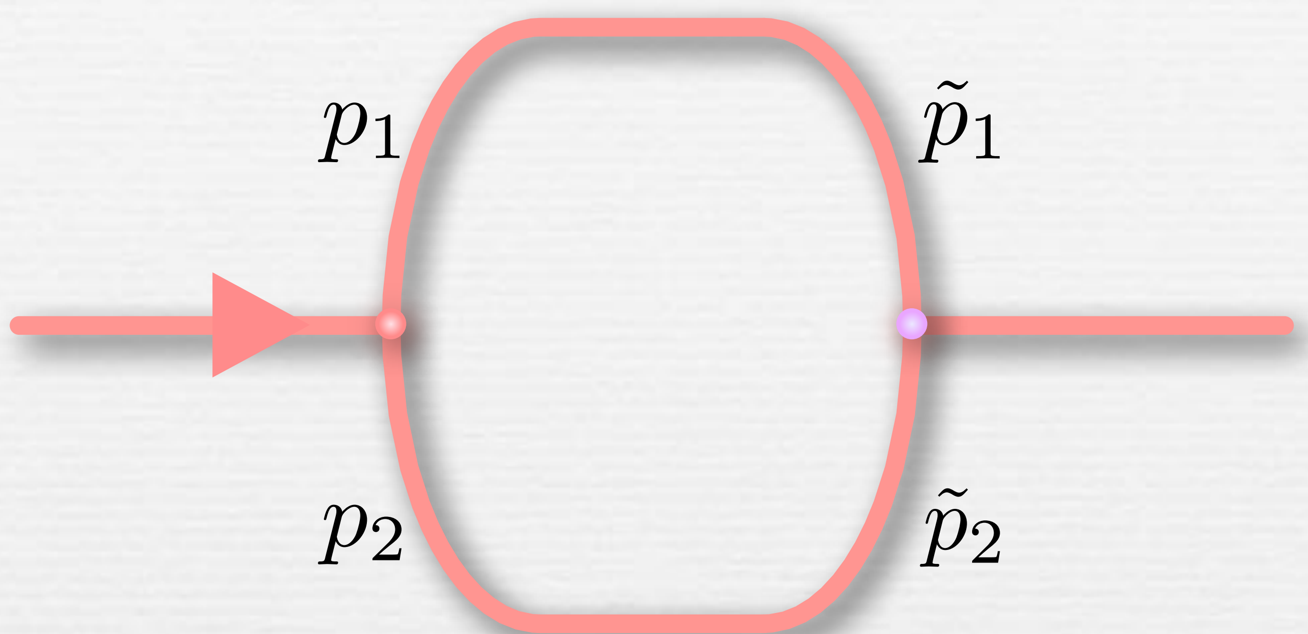
Work done when i is chosen

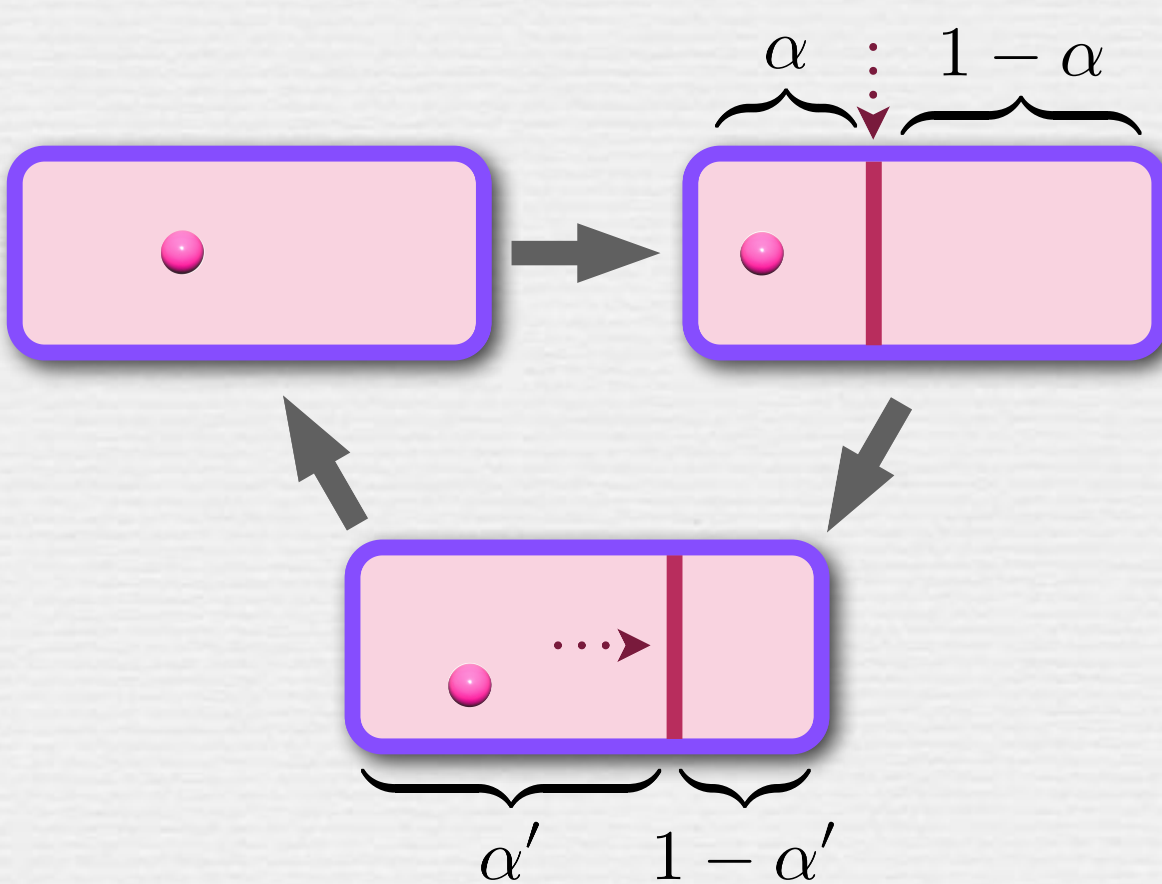
Probability that i is chosen

$$\Delta S_i \geq k \ln p_i$$

(<0!!) Not a proper entropy production.

Breaking and restoring symmetries





- Breaking the symmetry:

$$\langle W^{\rm br}\rangle_i - \Delta F_i^{\rm br} \geq -kT\ln p_i$$

- Restoring the symmetry:

$$\langle W^{\rm res}\rangle_i - \Delta F_i^{\rm res} \geq kT\ln \tilde{p}_i$$

Probability of choosing i in the backward process

In a cycle:

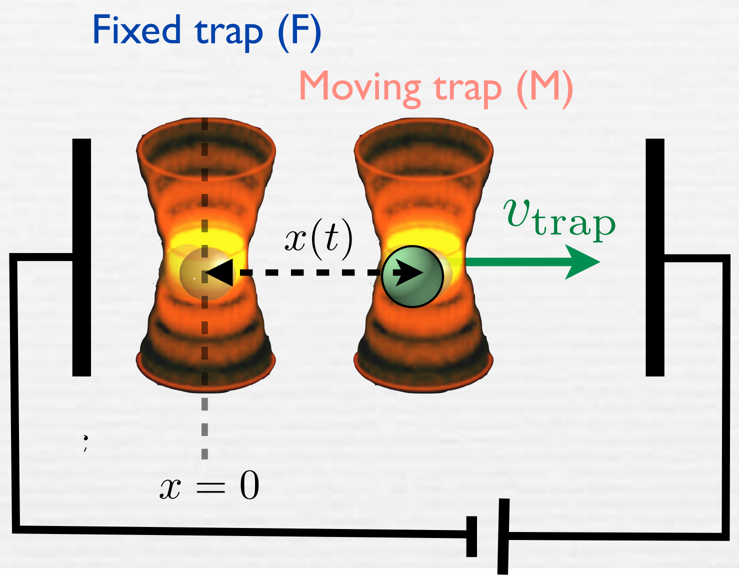
$$\langle W\rangle_i \geq kT\ln\frac{p_i}{\tilde{p}_i}$$

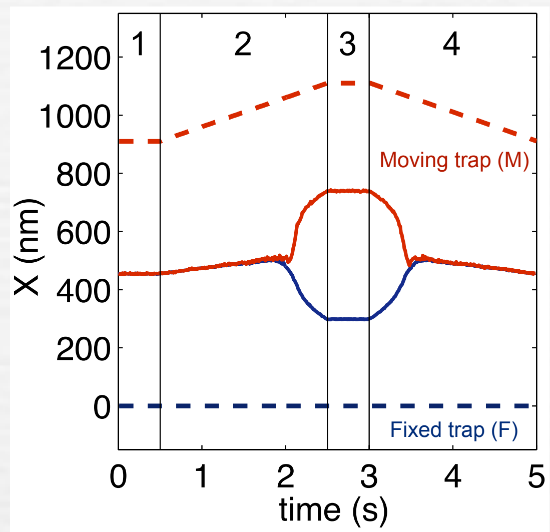
An example:

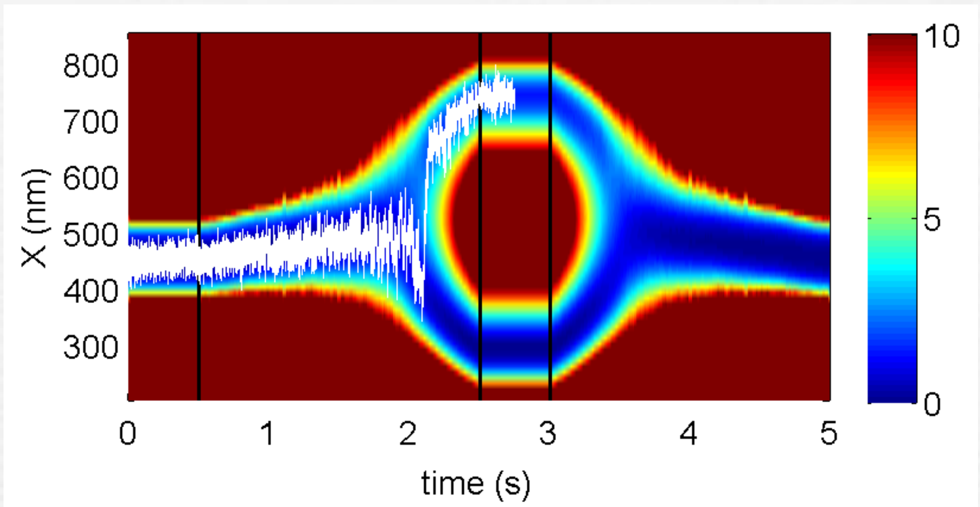
$$\langle W\rangle_L \geq kT\ln\frac{\alpha}{\alpha'}$$

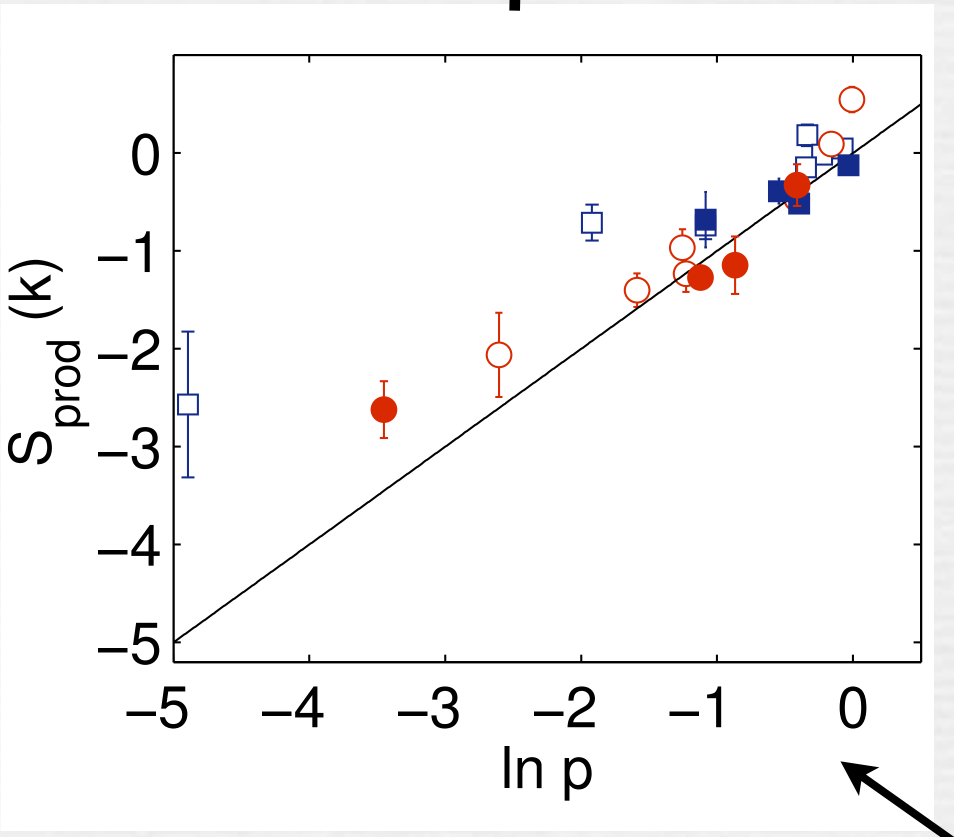
$$\langle W\rangle_R \geq kT\ln\frac{1-\alpha}{1-\alpha'}$$

An experiment (D. Petrov, ICFO)



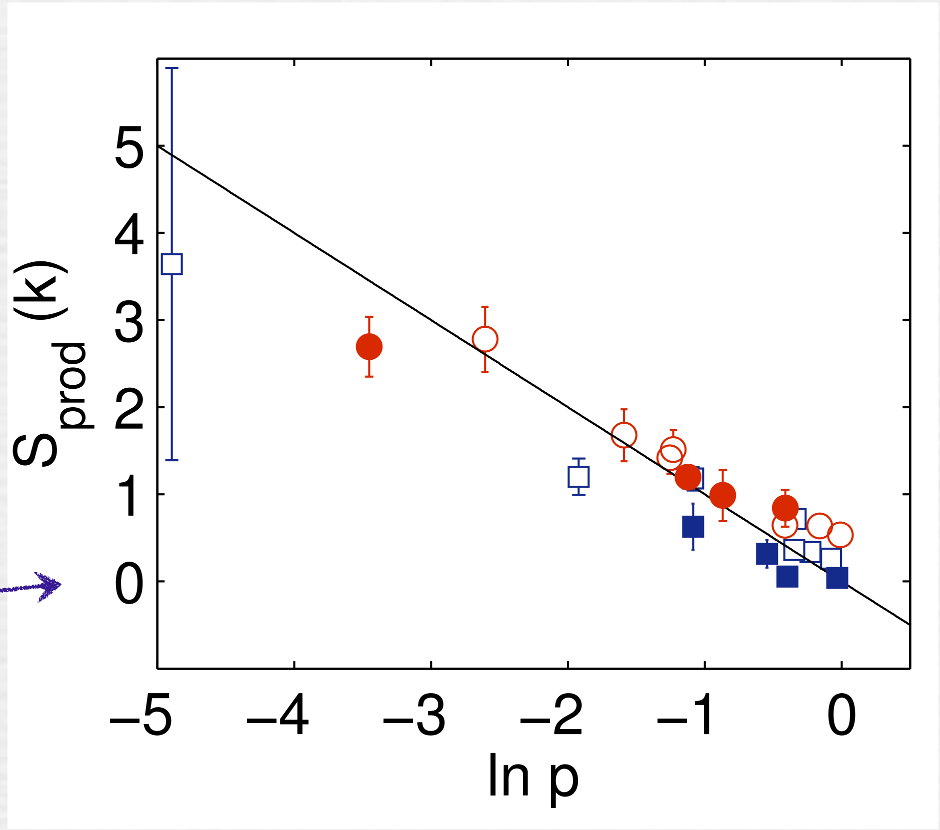






Symmetry breaking

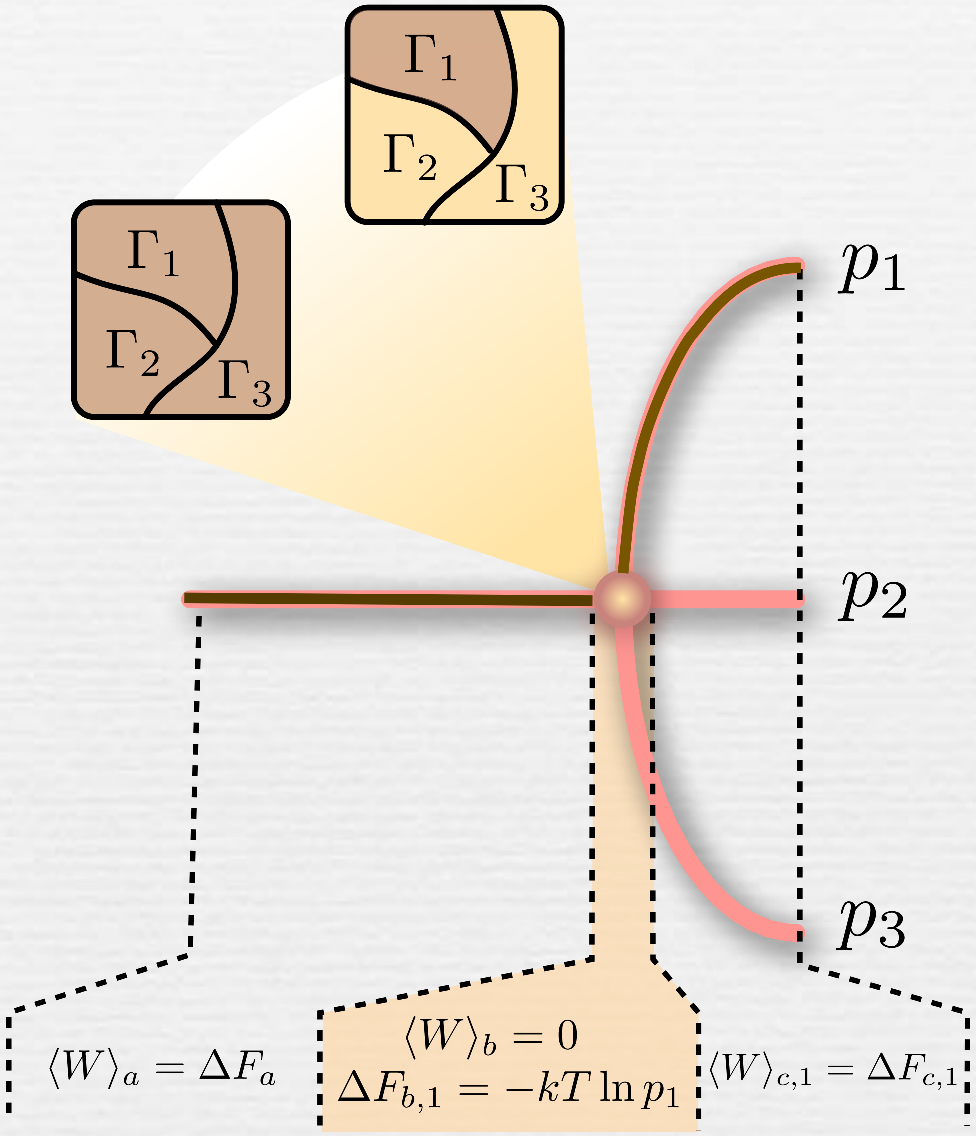
Symmetry restoration



$$\Delta S_i \geq k \ln p_i$$

$$\Delta S_i \geq -k \ln \tilde{p}_i$$

Does this matter?



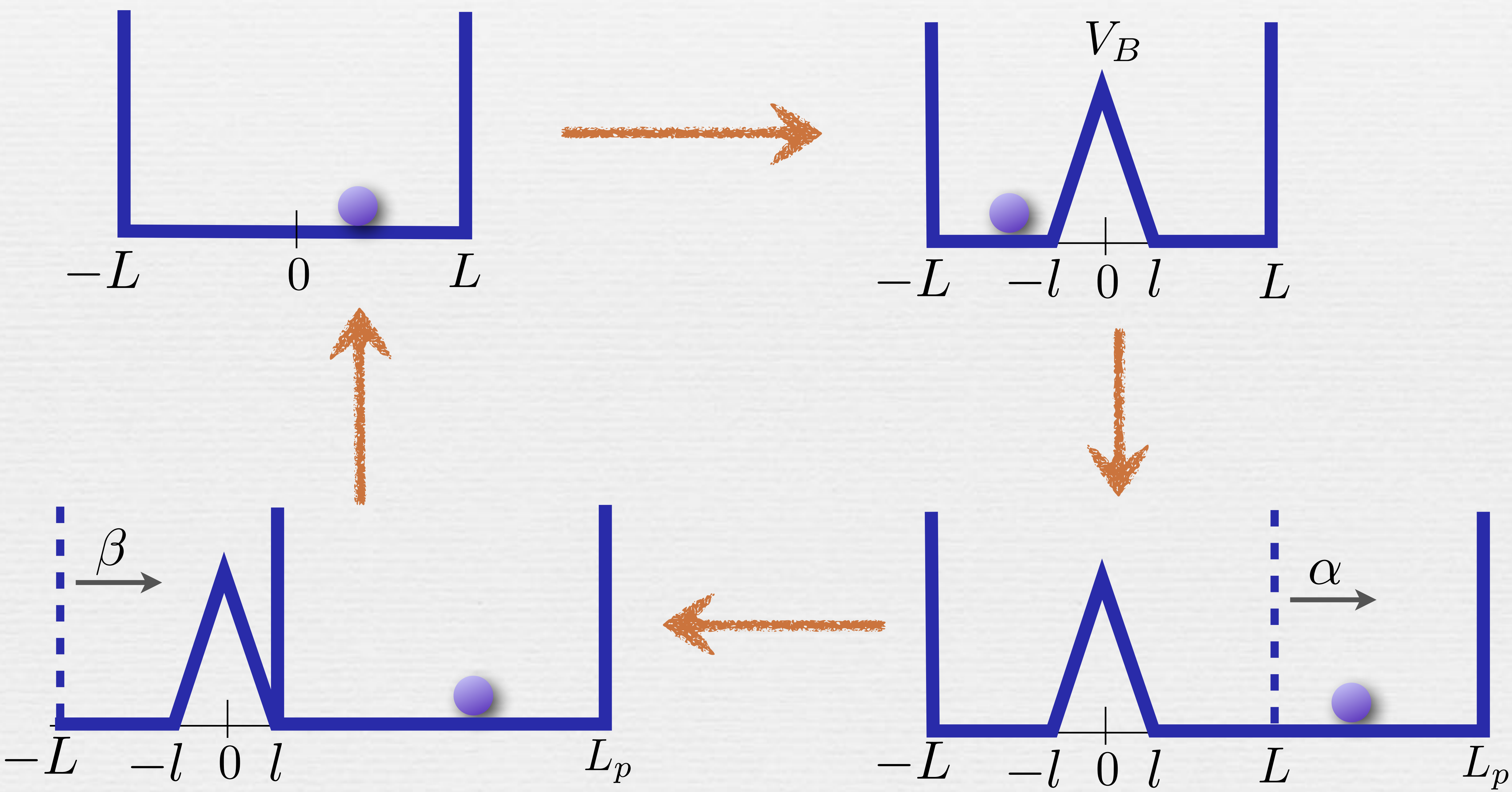
- Any meso- or macroscopic degree of freedom is the result of a symmetry/ergodicity breaking.

- Biological evolution: each DNA sequence is the result of a symmetry/ergodicity breaking.

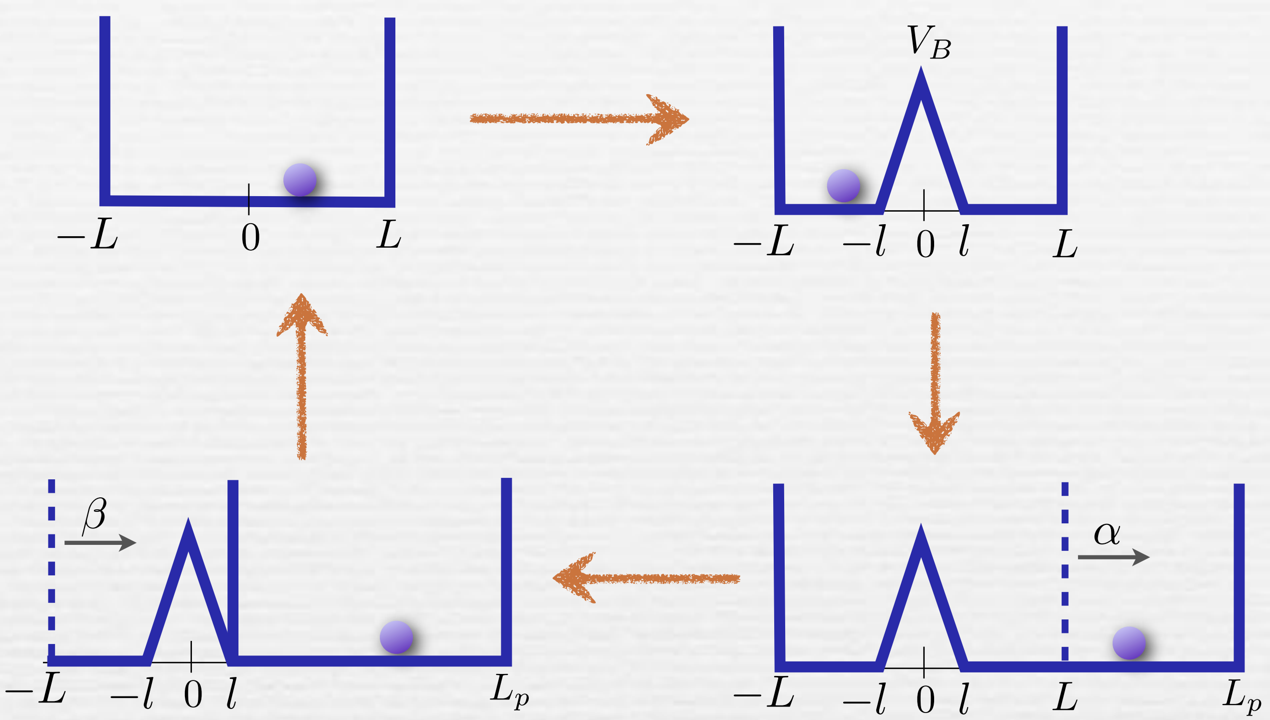
9.1. Microcanonical Szilard engines

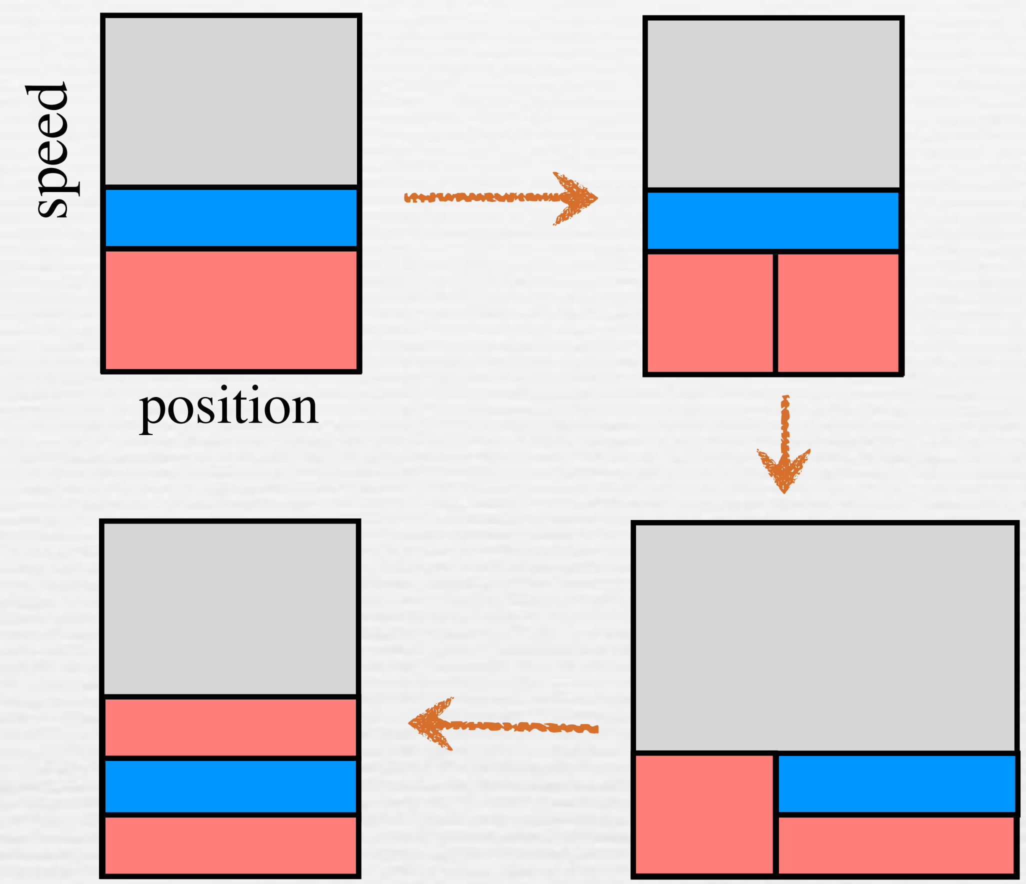
Marathe, JMRP, PRL 2010.

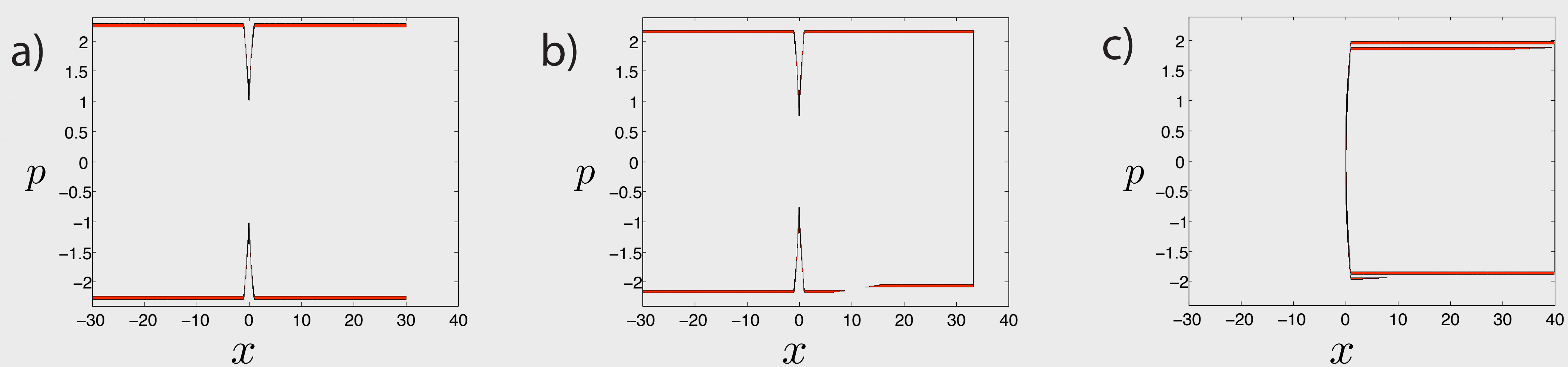
A single isolated particle obeying Newtonian dynamics



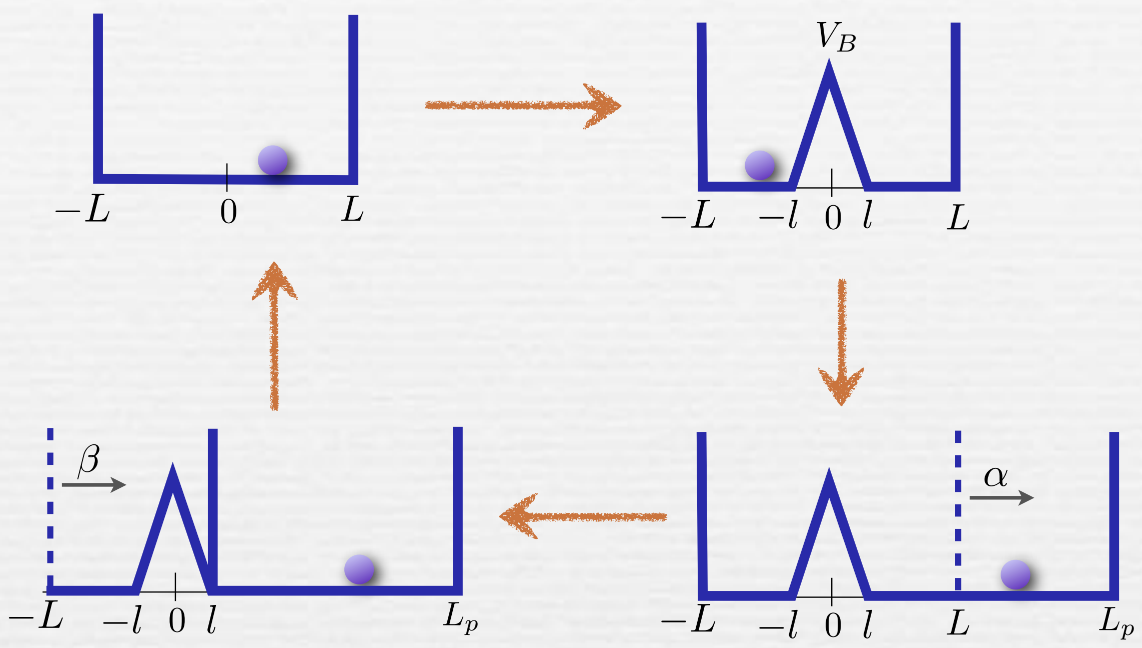
9.1. Microcanonical Szilard engines

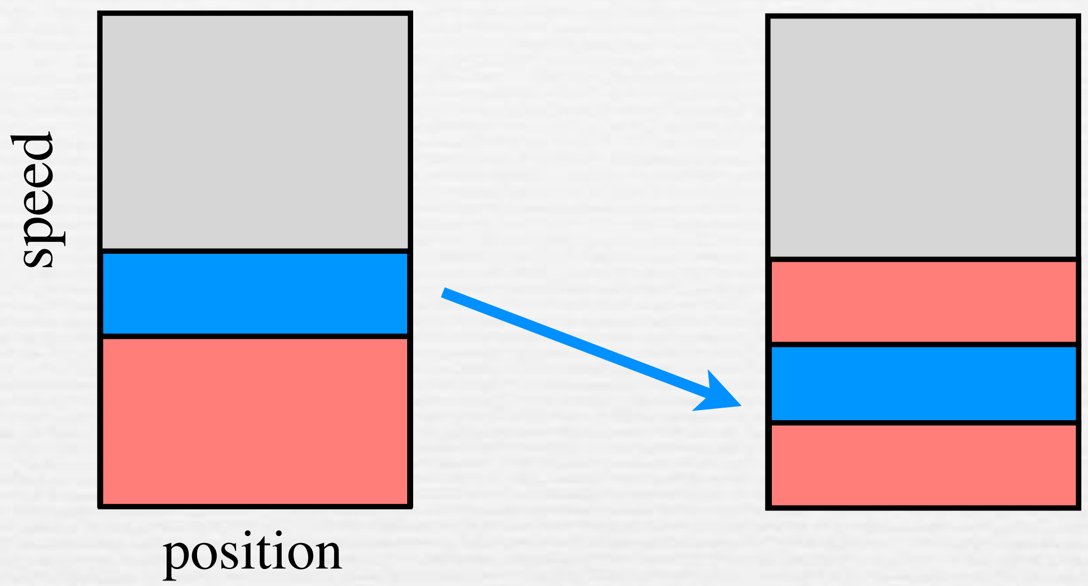




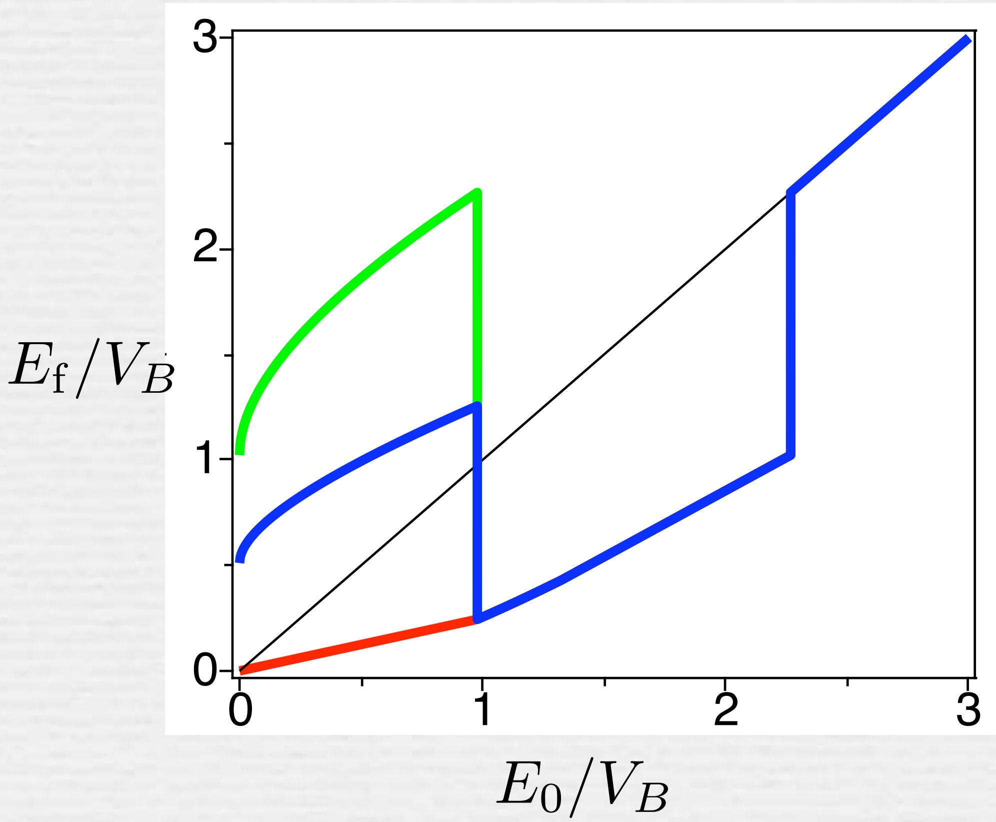


9.1. Microcanonical Szilard engines

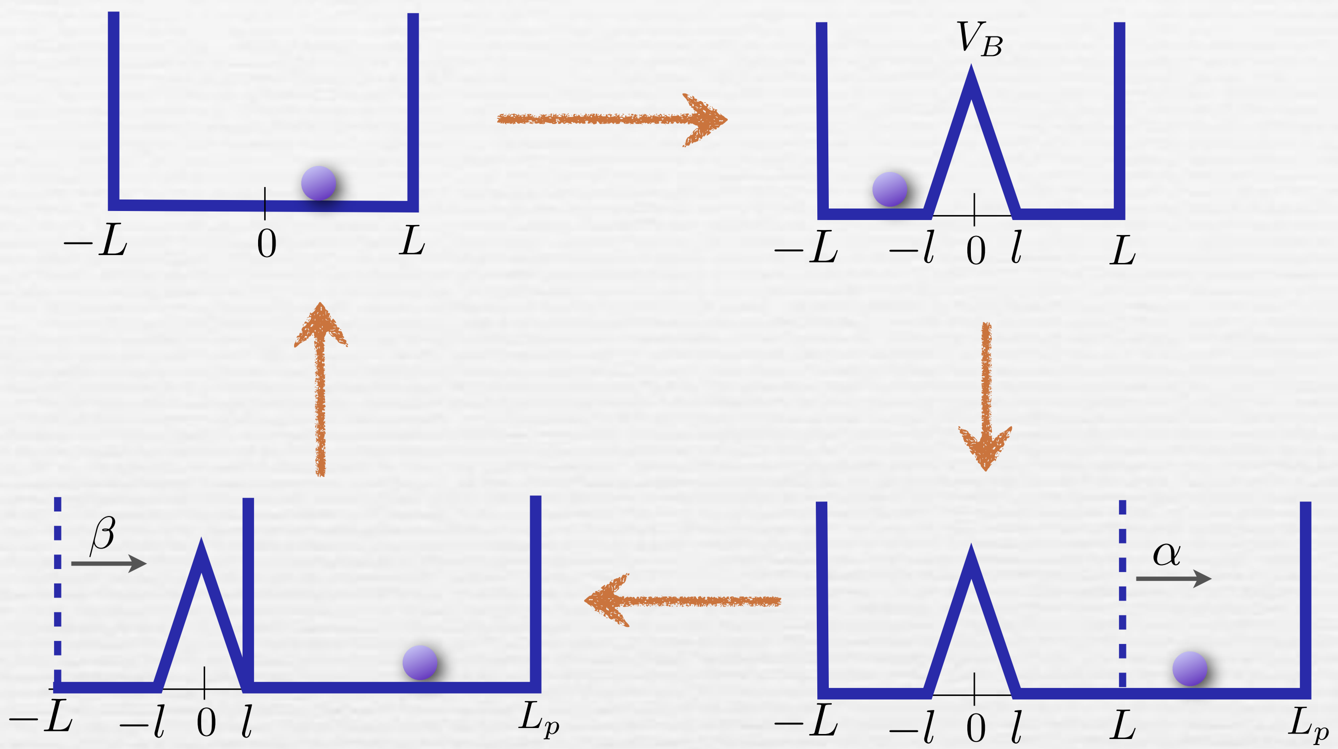


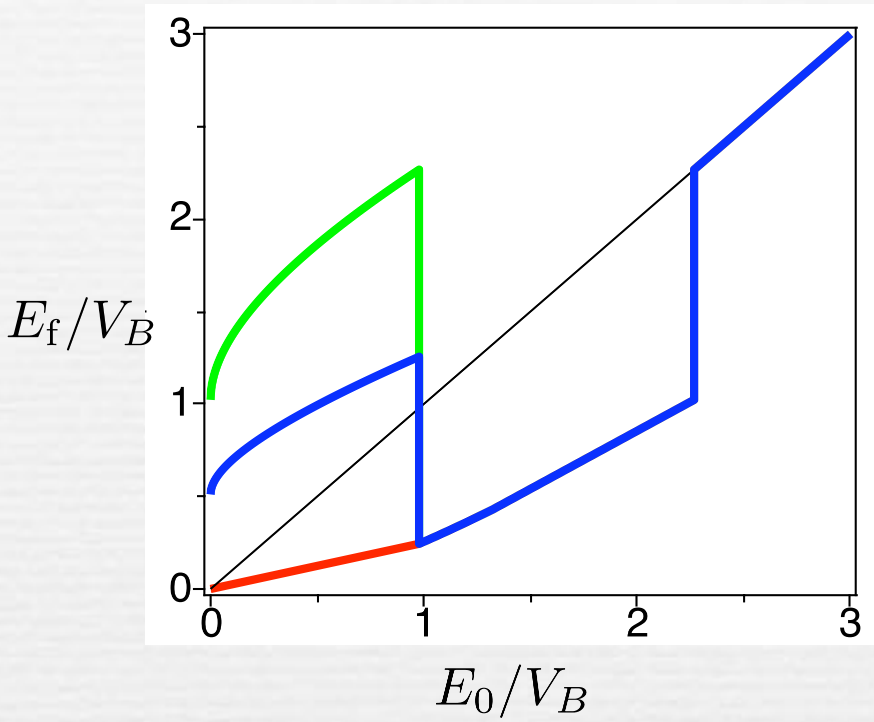


Final energy

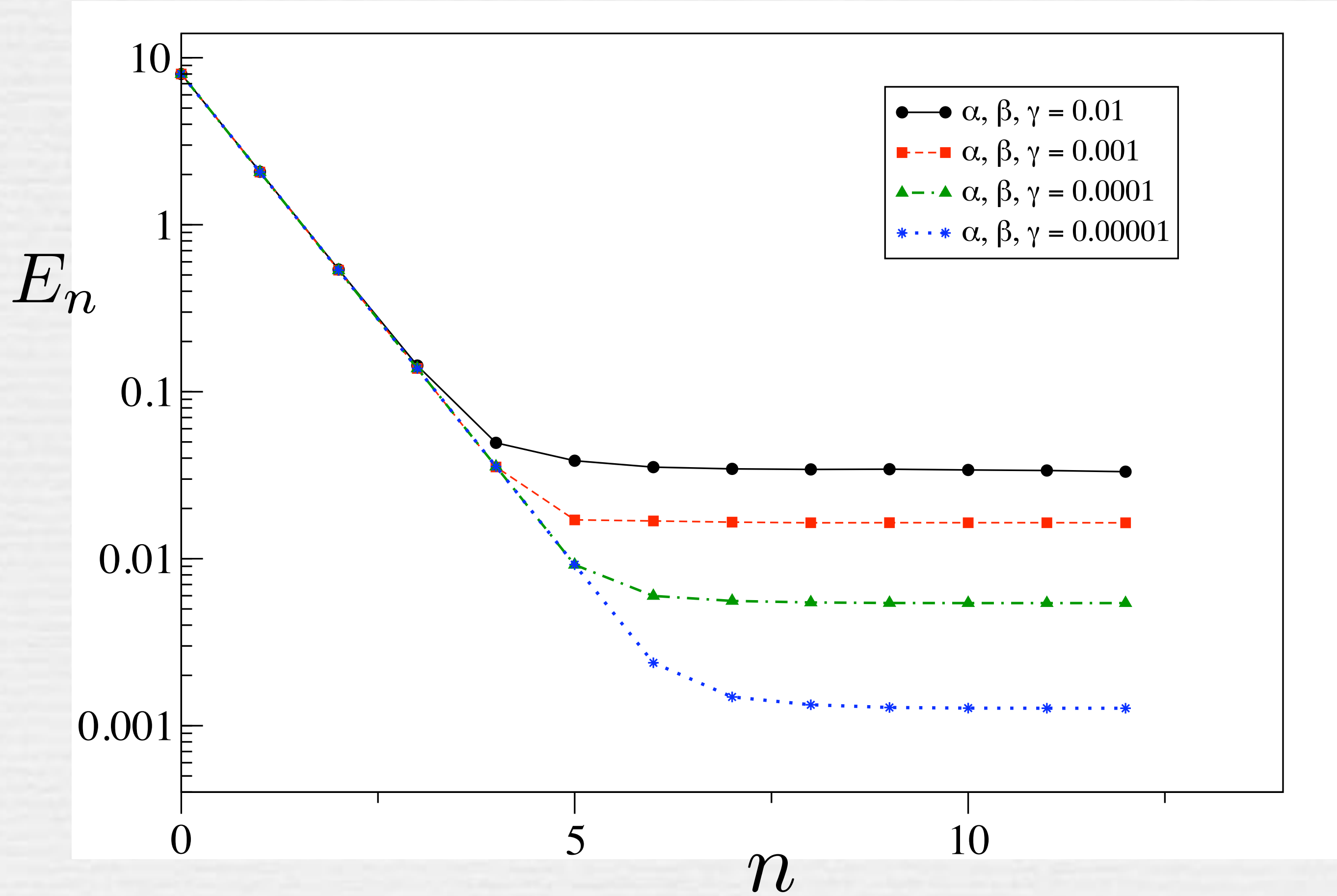


9.1. Microcanonical Szilard engines





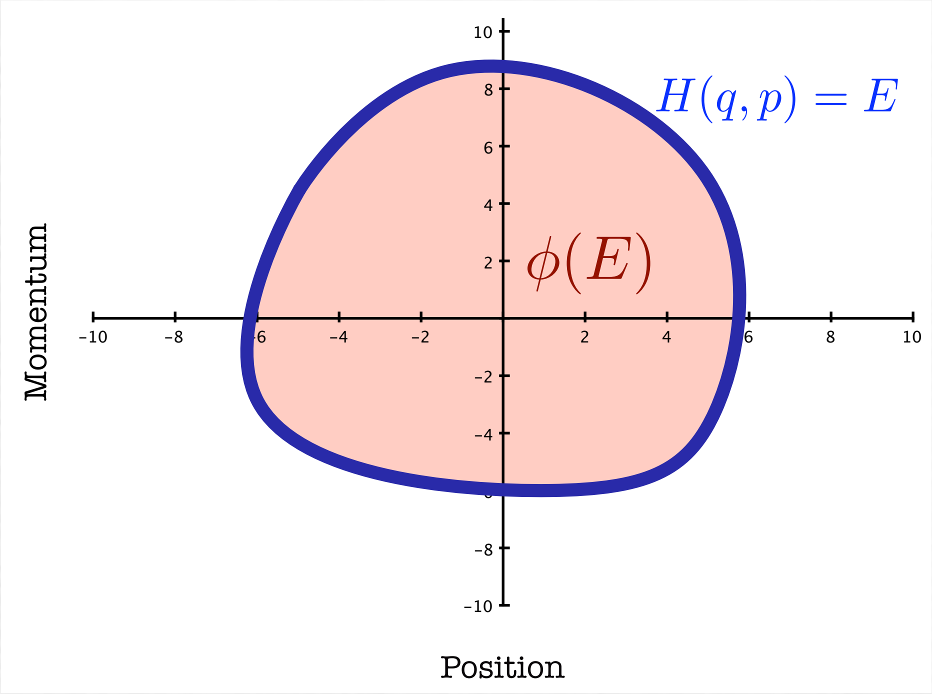
$V_B^{(n)} = \kappa^n E_0$, with $\kappa = [1/2 + l/(6L)]^2$



9.2. Maxwell demons in phase space

Two properties of Hamiltonian dynamics closely related with the second law:

1) Volume in phase space is invariant.
2) The volume enclosed by an energy shell is an **adiabatic** invariant:

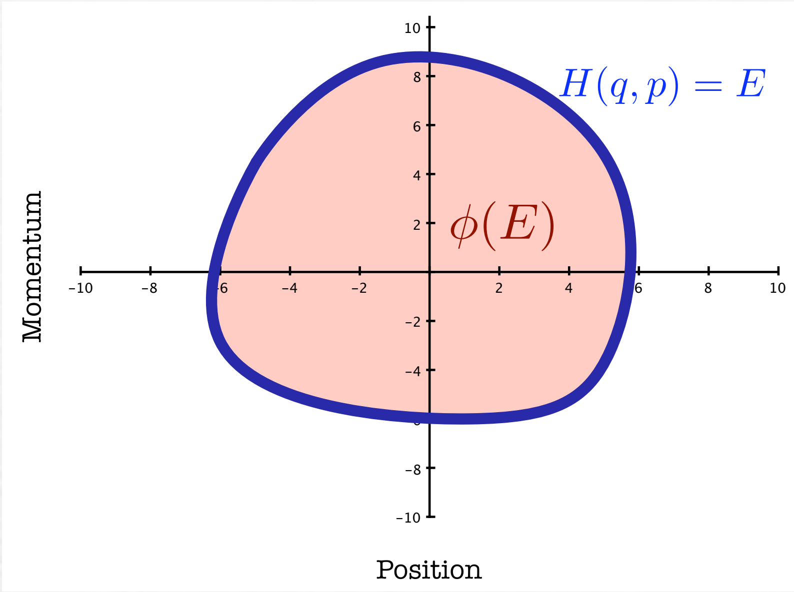


$$\phi(E) = \int_{H(q,p)<E} dq\,dp$$

Then, in a quasi-static cycle the final energy and the initial energy are the same, ie., **work is zero**.

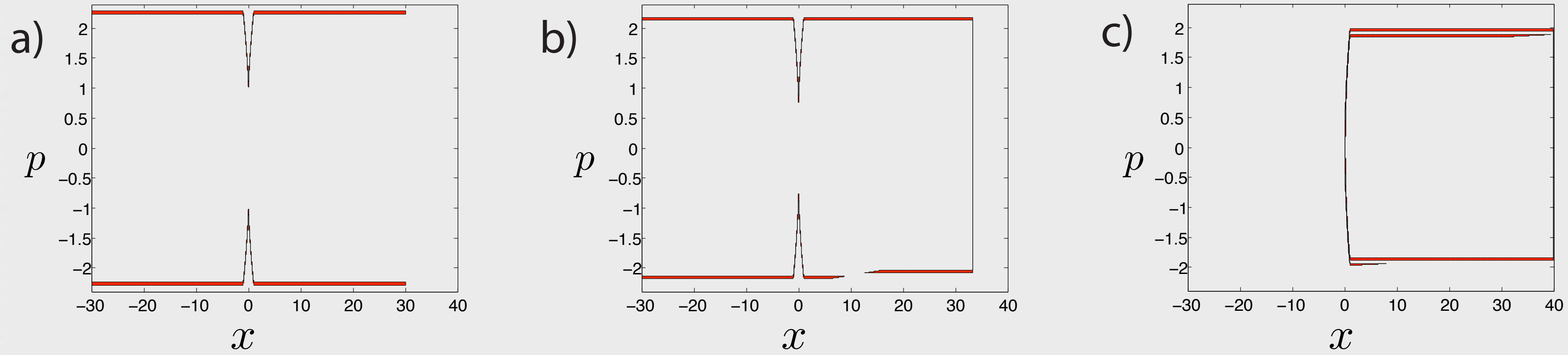
9.1. Microcanonical Szilard engines

The phase space volume is an adiabatic invariant:

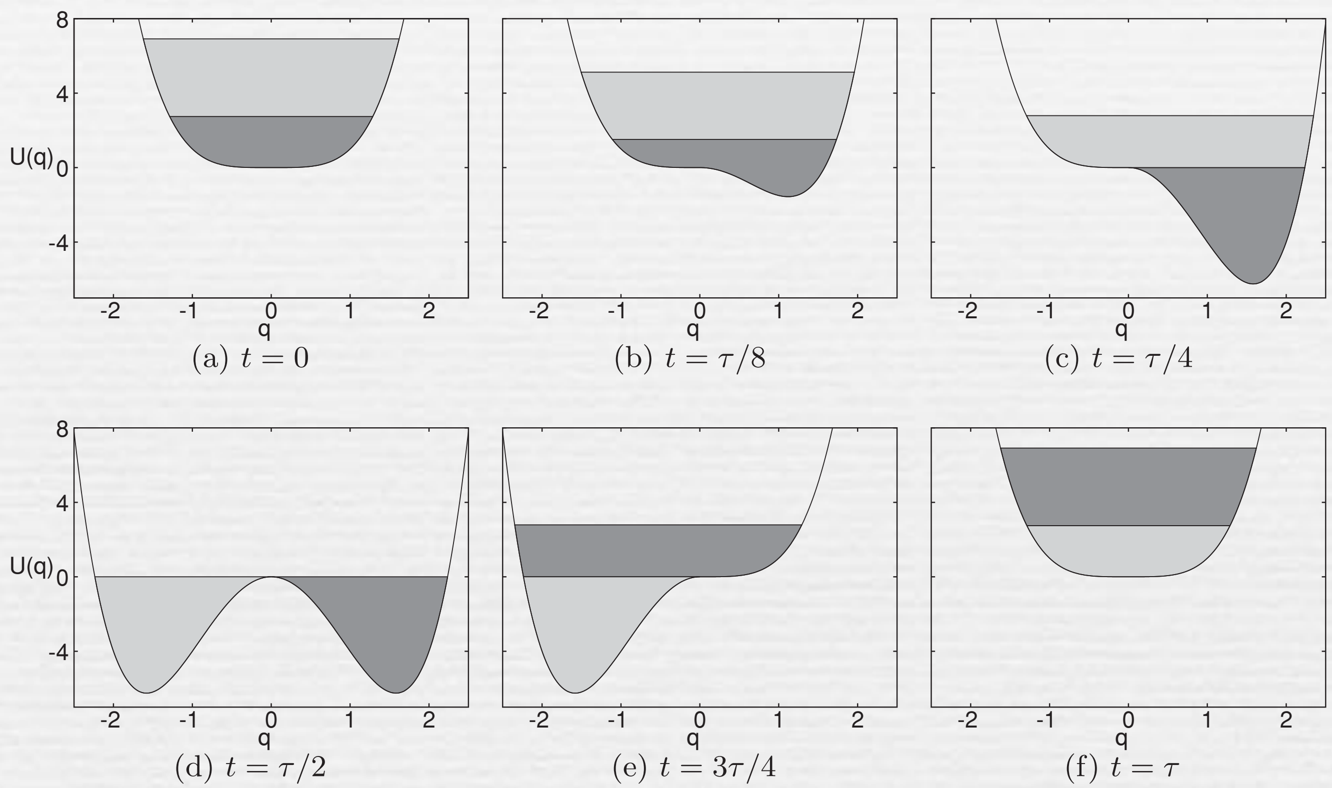


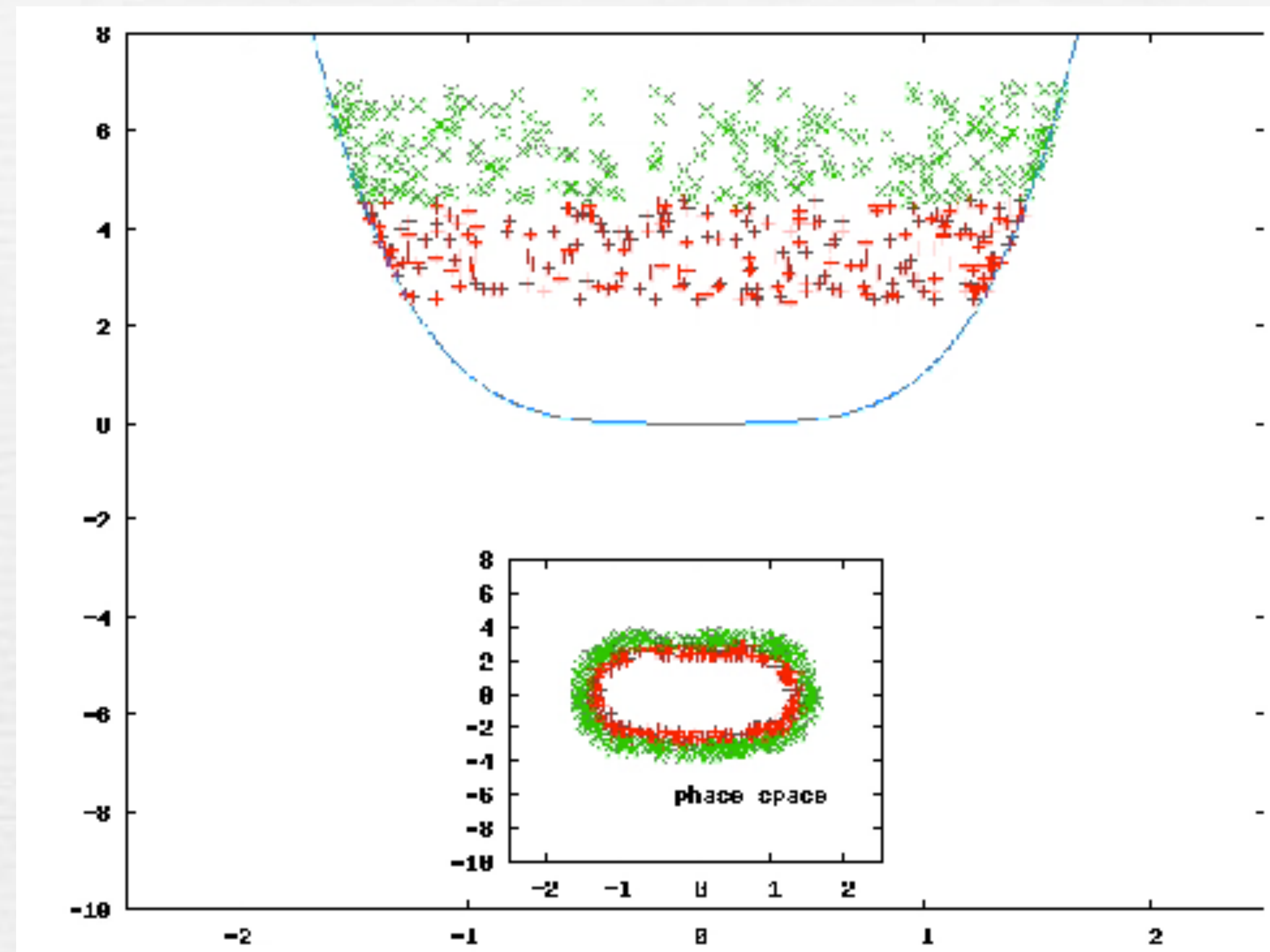
$$\phi(E) = \int_{H(q,p)<E} dqdp$$

Adiabatic invariance breaks down if orbits collapse or split



9.1. Microcanonical Szilard engines





Vaikuntanathan, Jarzynski, PRE (2011).

9.2. Maxwell demons in phase space

JMRP, Granger. Eur. Phys. J. (2015)

$$\mathcal{U}_i'(\phi_{\lambda_0}(E)) = \frac{p_i(E)}{\tilde{p}_i(E)}$$

$$\phi(E) = \int_{H(q,p)<E} dqdp$$

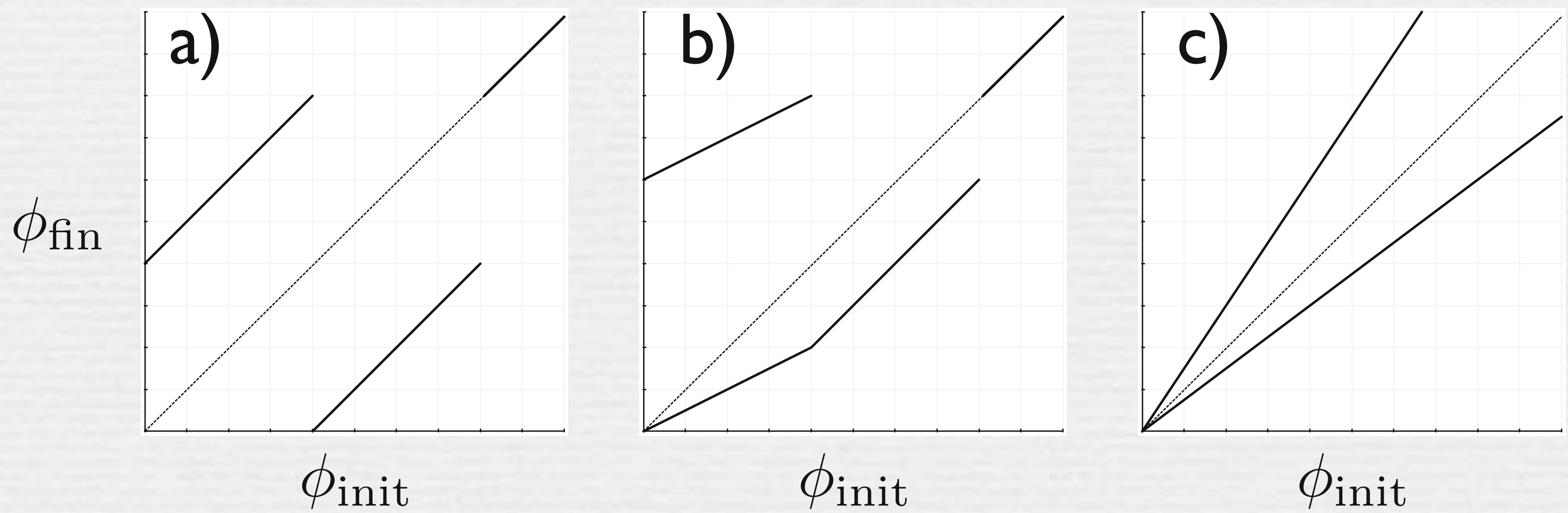
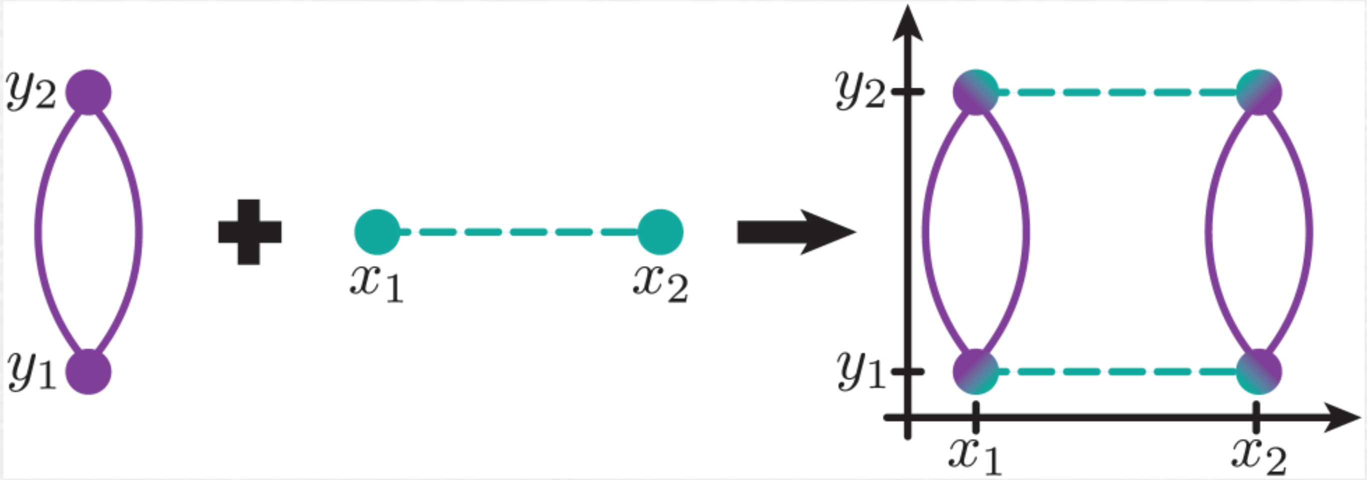


Fig. 3. Three examples of the transformation of the volume enclosed by energy shells. The initial volume $\phi_{\text{init}} \equiv \phi_{\lambda_0}(E)$ is mapped into $\phi_{\text{fin}} \equiv \phi_{\lambda_\tau}(E + W_i(E))$: a) corresponds to the microcanonical Szilard engine introduced by Vaikuntanathan and Jarzynski [13] with $p_i = \tilde{p}_i = 1$ (see Fig. 4); b) corresponds to the microcanonical Szilard engine introduced by Marathe and Parrondo [12] with $p_i = 1/2$ $(i = L, R)$ and $\tilde{p}_i = 1$ (see Fig. 5); c) corresponds to a Szilard engine in contact with a thermal bath at temperature T with $p_L = p_R = 1/2$ and $\tilde{p}_L = 2/3$, $\tilde{p}_R = 1/3$ (see Fig. 6). It is easy to check that in all cases the slopes verify Eq. (9). The diagonal is depicted in the three cases to guide the eye.

10. Information flows

Horowitz, Esposito, PRX 2014.

Bipartite systems:



$$\dot{S}^{X}_{\text{tot}} = \dot{S}(\rho(x)) + \dot{S}^{X}_{\text{res}} - k\dot{I}^{X} \geq 0$$

Changes due to x transitions

$$\dot{S}^{Y}_{\text{tot}} = \dot{S}(\rho(y)) + \dot{S}^{Y}_{\text{res}} - k\dot{I}^{Y} \geq 0$$

Changes due to y transitions

What is information?

- Metastable states, ergodicity breaking, large separation of time scales.
- Creation and annihilation of correlations