Thermodynamics of information

Juan MR Parrondo
Universidad Complutense de Madrid (Spain)
December 2016

Session 1

1. Consider the following random variables: $X =$ the sum of points obtained in tossing a pair of dice; $Y =$ the result of a lottery with eleven balls drawn, ranging from 2 to 12. Calculate the relative entropies $D(X|Y)$ and $D(Y|X)$. Which one is bigger? Could you argue why?

2. Calculate the mutual information of the measurement output of a binary variable X. The variable takes on two values, 0 and 1, with probability p and $1-p$ respectively, and the measurement has a symmetric error ϵ, i.e., the probability of a wrong outcome m is ϵ, independent of the true value x. What is the maximum work that we can extract from a Szilard engine in a container of volume V if our measurement has an error ϵ? Devise a protocol able to extract that amount of work.
Hint: We cannot perform the whole expansion from $V/2$ to V, because, if the measurement is wrong, we would compress the particle to zero volume (which takes an infinite amount of work). Consider then a partial expansion from $V/2$ to αV with $1/2 < \alpha < 1$.

3. Consider a particle of mass m in a harmonic potential of frequency ω_0, $V(x) = \frac{m\omega_0^2 x^2}{2}$, in contact with a thermal bath at temperature T. The particle is in equilibrium and the frequency is *suddenly* changed from the initial value ω_0 to a value $\omega_1 > \omega_0$.

a) Calculate the work done to change the frequency.

b) After the change, the particle relaxes to a new equilibrium state dissipating energy to the thermal bath. Calculate this dissipated heat.

c) Compare the work obtained in the first question with the variation of equilibrium free energy.

d) Calculate the entropy production along the whole process.

Session 2

4. Write the Horowitz-Vaikuntanathan fluctuation theorem for the optimal Szilard engine with error that you found in exercise 2.

Session 3

5. Following [Horowitz, Esposito, PRX **4**, 031015], Sec. IV, analyze the information flow in the optimal Szilard engine with error that you found in exercise 2.