Lessons from Classical Gravity about the Quantum Structure of Spacetime

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- *Alternatively, field equations can be derived by exremising a phenomenological entropy density of spacetime.*
- Gravitational sector is immune to zero-level of energy just like matter sector.
- Quantizing any classical gravitational field will be like quantizing equations of fluid dynamics or elasticity. Gravitons$\leftrightarrow$phonons.

PLAN OF THE TALK

- Horizon Temperature and Local Rindler Observers
- Lanczos-Lovelock models of Gravity
- Field equations on null surfaces
  - Field equations as theromynamic identity
  - Field equations as Navier-Stokes equation
- Surface Term in Action
- Horizon Entropy
  - Entropy from Surface Term
  - Entropy $\rightarrow$ Surface term $\rightarrow$ Full action
  - Entropy from Noether current
- Noether current and Dynamics
  - Equipartition in Lanczos-Lovelock models
  - Action and Free energy
  - Field equations as entropy balance
- Spacetime deformations and Dynamics
- Field equations from a thermodynamic extremum principle
- Conclusions

SPACETIMES, LIKE MATTER, CAN BE HOT

OBSERVERS WHO PERCEIVE A HORIZON
ATTRIBUTE A TEMPERATURE TO SPACETIME

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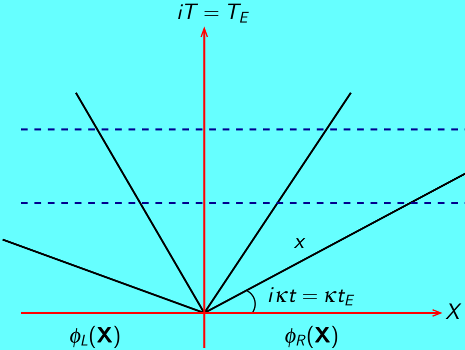

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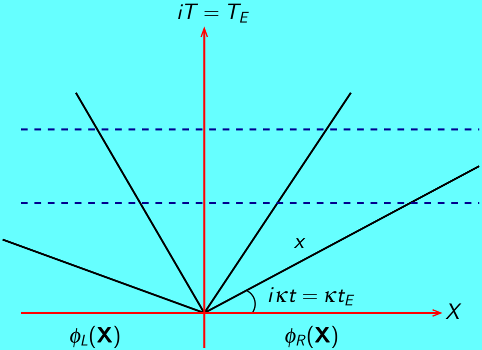

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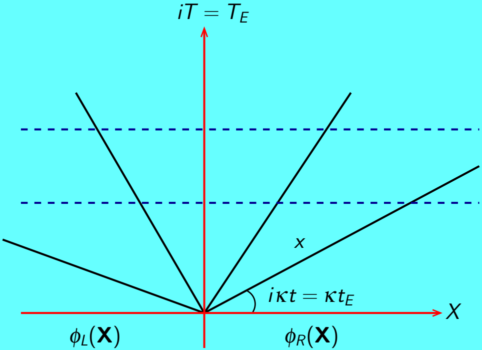

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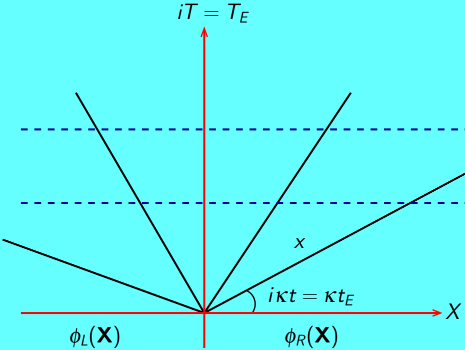

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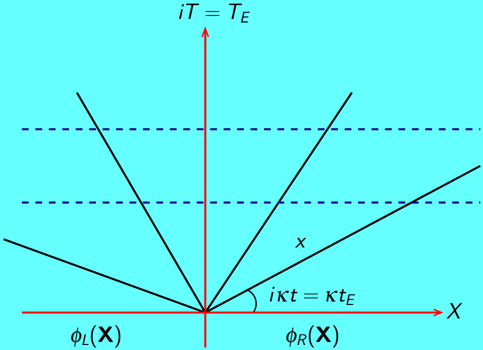

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- Tracing out ϕ_L gives a density matrix: (Lee, 1986)

$$\rho(\phi_R',\phi_R) = \int \mathscr{D}\phi_L \langle \mathrm{vac}|\phi_L,\phi_R'\rangle\langle \mathrm{vac}|\phi_L,\phi_R\rangle \propto \langle \phi_R'|e^{-(2\pi/\kappa)H_R}|\phi_R\rangle$$

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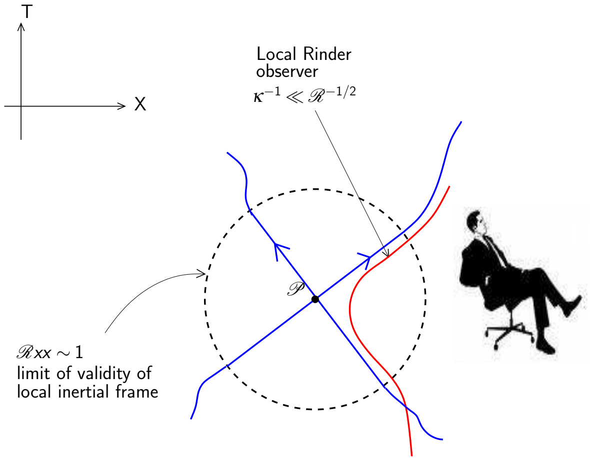
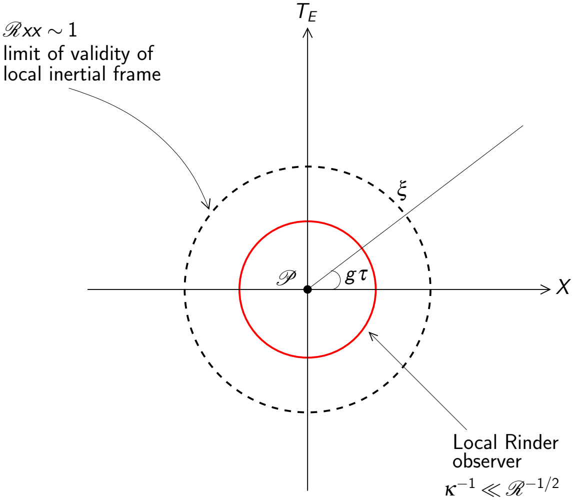
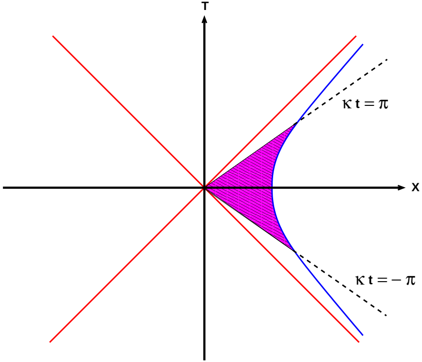
- A Local version of Rindler Horizon, approximate Killing vector, Temperature etc. exist when $\dot{\kappa}/\kappa^2 \ll 1$

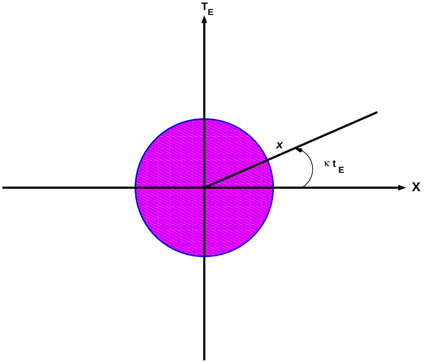
LOCAL RINDLER OBSERVERS




T_E
X
$\mathscr{R}xx \sim 1$
limit of validity of
local inertial frame
ξ
$\mathscr{P}$
$g\tau$
Local Rinder
observer
$\kappa^{-1} \ll \mathscr{R}^{-1/2}$

PERIODICITY IN EUCLIDEAN TIME





$$T = x \sinh \kappa t, \; X = x \cosh \kappa t \qquad T_E = x \sin \; \kappa t_E, \; X = x \cos \; \kappa t_E$$

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- A Local version of Rindler Horizon, approximate Killing vector, Temperature etc. exist when $\dot{\kappa}/\kappa^2 \ll 1$
- Large M limit of BH $\Leftrightarrow$ Local Rindler Frame.

- Action functional: *(Think beyond Einstein Gravity!)*

$$A = \int d^D x \sqrt{-g}\left[L_g(R^{ab}_{cd}, g^{ab}) + L_{matt}(g^{ab}, q_A)\right] \equiv A_g + A_{matt}$$

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- Variation of A_g gives [with $P^{abcd} \equiv (\partial L / \partial R_{abcd})$]

$$\delta A = \int_{\mathscr{V}} d^D x \sqrt{-g}\, \mathscr{G}_{ab} \delta g^{ab} + \int_{\mathscr{V}} d^D x \sqrt{-g}\, \nabla_j \delta v^j$$

with

$$\delta v^j \equiv [2P^{ibjd}(\nabla_b \delta g_{di}) - 2\delta g_{di}(\nabla_c P^{ijcd})]$$

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- Ignoring surface variation gives [with $\mathscr{R}_{ab} = P_a{}^{cde} R_{bcde}$] the field equation:

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- Alternatively, in the thermodynamic perspective, one could demand

$$\delta \int_{\mathscr{V}} d^D x \sqrt{-g}\, L = \int_{\partial \mathscr{V}} d^{D-1} x \sqrt{h}\, n_i \delta v^i$$

LANCZOS-LOVELOCK MODELS

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- These are Lanczos-Lovelock models of gravity.

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- The D-dimensional Lanczos-Lovelock Lagrangian is a polynomial in the curvature tensor:

$$
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\mathscr{L}^{(D)} &= Q_a{}^{bcd} R^a{}_{bcd} = \sum_{m=1}^{K} c_m \mathscr{L}_m^{(D)}; \\
\mathscr{L}_m^{(D)} &= \delta^{a_1 a_2 \ldots a_{2m}}_{b_1 b_2 \ldots b_{2m}} R^{b_1 b_2}_{a_1 a_2} \ldots . R^{b_{2m-1} b_{2m}}_{a_{2m-1} a_{2m}} \sim R \wedge R \wedge R \wedge \ldots ..
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- The $m = 1$ and $m = 2$ terms give Einstein-Hilbert and Gauss-Bonnet Lagrangians

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L_{EH} &= \delta^{13}_{24} R^{24}_{13} \propto R \\
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- BH solutions exist; entropy is NOT proportional to area:

$$S^{(m)}_{\text{wald}} \propto m \int_{\mathscr{H}} d^{D-2} x_{\perp} \sqrt{\sigma} L_{(m-1)}$$

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  - The null vectors used in the formalism are different.
- *The $TdS = dE + PdV$ works for a wide class of theories in a natural fashion.*

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TP, PRD 83, 044048 (2011) [arXiv:1012.0119]

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$$(\partial_0 + v^\beta \partial_\beta)\left(\frac{\omega_\alpha}{8\pi}\right) = \frac{1}{8\pi}\partial_\beta \sigma^\beta_\alpha - \frac{1}{16\pi}\partial_\alpha \theta - \partial_\alpha\left(\frac{\kappa}{8\pi}\right) - T_{m\alpha}\ell^m$$

with $\Theta^\alpha_\beta = \sigma^\alpha_\beta + (1/2)\delta^\alpha_\beta \theta$ and

$$\Theta_{\alpha\beta} = -\ell_m \Gamma^m_{\alpha\beta}, \quad \omega_0 = -\ell^m \Gamma^0_{m0}, \quad \omega_\alpha = -\ell^m \Gamma^0_{m\alpha}$$

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- The viscous tensor Θ^β_α etc scales as Γ^a_{bc} but $\partial\Theta$ scales as $\partial\Gamma \sim R$. "Dissipation without dissipation" !

THE SURFACE TERM

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- Counter term to cancel surface contribution is known for the Lanczos-Lovelock models (but not unique).

FOR THOSE WHO LIKE SUCH THINGS ....

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- The counter tem is:

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- Counter-terms do *NOT* cancel surface terms (even in GR) in general; only variations cancel if h_{ab} is held fixed.
- No simple derivation of physical understanding exists.

THE SURFACE TERM

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and is 'holographic':

$$[(D/2)-m]L_{sur} = -\partial_i\left[g_{ab}\frac{\partial L_{bulk}}{\partial(\partial_i g_{ab})} + \partial_j g_{ab}\frac{\partial L_{bulk}}{\partial(\partial_i\partial_j g_{ab})}\right]$$

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  is not generally covariant.
- But at linear order

$$\sqrt{-g}L_{sur} \approx \frac{1}{2\ell_P^2}\partial_a\partial_b[\eta^{ab}h^i_i - h^{ab}]$$

  is invariant under $h_{ab} \to h_{ab} + \partial_a\xi_b + \partial_b\xi_a$!

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$$\frac{1}{8\pi}\int_x dt d^2x_{\perp}\sqrt{h}K = \frac{1}{16\pi}\int_x dt d^2x_{\perp}\sqrt{-g}V^x = -t\left(\frac{\kappa A_{\perp}}{8\pi}\right)$$

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- The surface Hamiltonian is

$$H_{\mathrm{sur}} \equiv -\frac{\partial \mathscr{A}_{sur}}{\partial t} = \frac{1}{8\pi} \int_x d^2 x_{\perp} \sqrt{h} K = \left(\frac{\kappa A_{\perp}}{8\pi} \right) = TS$$

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- Exponential redshift near the horizon implies

$$F_m(t) \to \exp[-iC\exp(-\kappa t)].$$

So the relative probability for BH radiation changing its area by $\Delta A_\perp$ is given by $\exp[\Delta A_\perp/4]$.

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- In normal units $G^{-1} = (4\hbar/c^3) \times$ Entropy per unit area. Positive because Area and Entropy are positive.
- Gravity is intrinsically quantum mechanical, just like solids; no strict $\hbar \to 0$ limit exists in e.g.,

$$F = \left(\frac{c^3\mathscr{A}_P}{\hbar}\right)\left(\frac{m_1 m_2}{r^2}\right)$$

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$$S = \beta \int d^{D-2}\Sigma_{ab}\, J^{ab} = \frac{1}{4}\int_{\mathscr{H}} (32\pi\, P^{ab}_{cd})\varepsilon_{ab}\varepsilon^{dc}\, d\sigma$$

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- *The connection between* $x^a \to x^a + q^a(x)$ *and entropy is a mystery in the conventional approach.*

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- The Noether entropy of the surface term of action is the correct entropy:

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- The resulting Virasoro algebra has a central charge that leads to correct Lanczos-Lovelock entropy.

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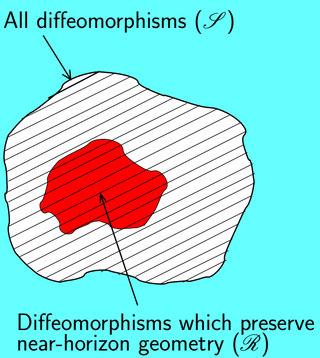

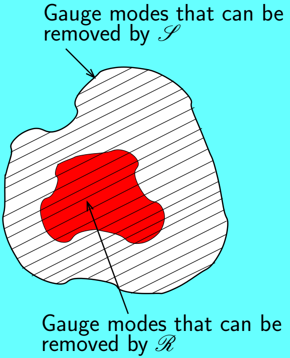

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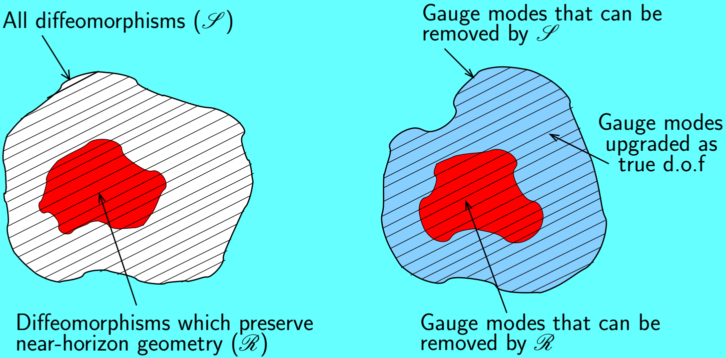

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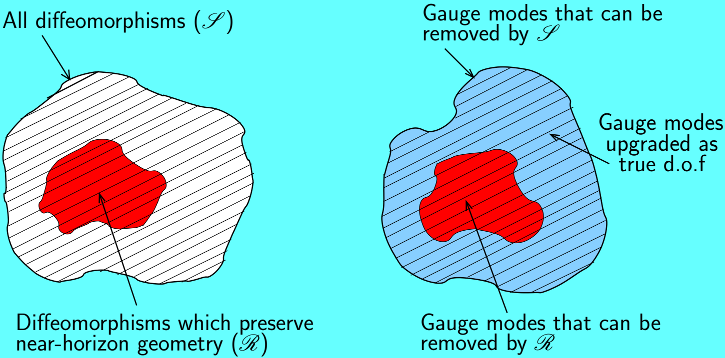

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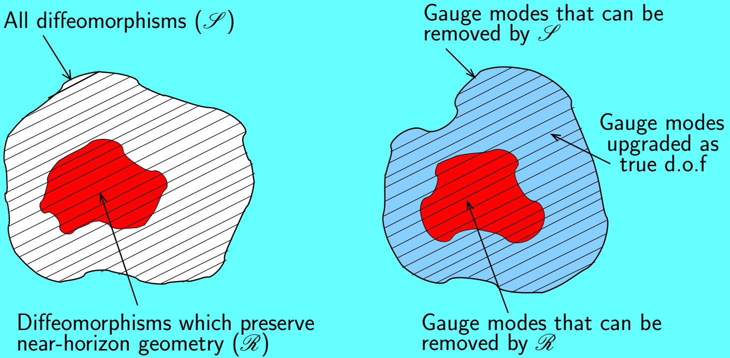

- Demanding the existence of a horizon reduces available diffeomorphisms; upgrades gauge dof to true dof in an observer dependent manner.
- "What are the dof responsible for BH entropy?" cannot have an observer-independent answer.

NOETHER CURRENT AND DYNAMICS

- In any static spacetime with Killing vector ξ^a and $q^a = \xi^a$ we have

$$D_\alpha(J^{b\alpha} u_b) = 2N\mathscr{R}_{ab} u^a u^b; \quad u^a = \xi^a/(-\xi^2)^{1/2}$$

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- Equipartition law:

$$E = \frac{1}{2} k_B \int_{\partial\mathscr{V}} dn T_{loc}; \qquad \frac{dn}{dA} = \frac{dn}{\sqrt{\sigma} d^{D-2}x} = 32\pi P^{ab}_{cd}\varepsilon_{ab}\varepsilon^{cd}$$

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- Also note

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- The on-shell action is the free-energy.

- Let $\mathscr{S}$ be set of spherically symmetric static metrics with a horizon at $r = a$.

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where

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- So even in this calculable, off-shell case, Euclidean action is free-energy.

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- The field equations $2\mathscr{G}^a_b = T^a_b$ imply the entropy balance $\delta S_{\rm grav} = \delta S_m$!

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- Conservation of J^a $\Leftrightarrow$ Associated vector field q^a $\Leftrightarrow$ Diffeomorphism $\mathscr{D}(q)$: $\bar{x}^i - x^i = q^i$ induced by q^a $\Leftrightarrow$ Invariance of certain scalars under $\mathscr{D}(q)$ $\Leftrightarrow$ Conservation of J^a.

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- In general,

$$2\mathscr{R}^a_b q^b = \nabla_d[\mathscr{S}^{ad} + \mathscr{F}^{ad}]$$

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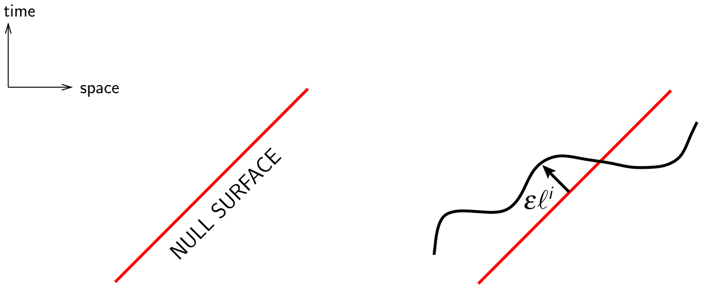

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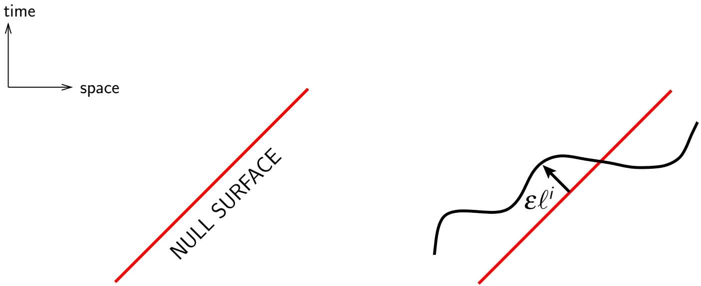

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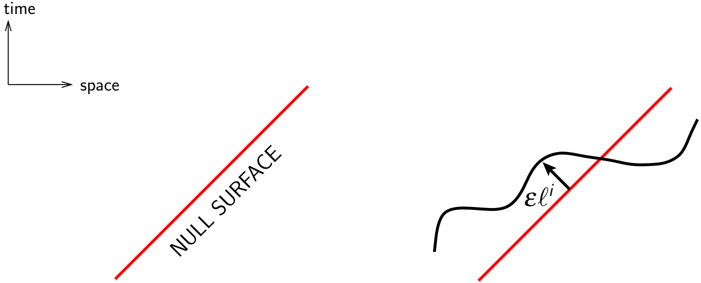

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$$\mathfrak{I} \sim A(\nabla \ell)^2 + B\ell^2$$

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- Associate with virtual displacements along null vectors ℓ^a a potential $\Im(\ell^a)$ which is quadratic in deformation field:

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- For null foliation with $\ell_a = \nabla_a\phi$,

$$\Im_{\rm grav}[\ell^a] \Rightarrow \ell^a v_a = \text{Surface density of entropy}$$

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- A new symmetry: Action and field equations are invariant under $T_{ab} \to T_{ab} + \rho_0 g_{ab}$. Gravity does *not* couple to bulk vacuum energy.

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&= q_a J^a[q_a] + q_a \mathscr{V}^a[q_a] + \text{ tot. div.}
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- In general, both metric deformation and Noether potential contribute

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- Two-dim system with $PA = TS$; i.e., $G = E$.
- Integral of this expression over the null surface is proportional to the entropy production rate:

$$\int \sqrt{\sigma} d^2x^A d\lambda [4P^{cd}_{ab}\nabla_c n^a \nabla_d n^b] = \frac{1}{8\pi}\int \sqrt{\sigma} d^2x^A d\lambda \, R_{ab}\ell^a\ell^b + \frac{1}{8\pi}\frac{dA_\perp}{d\lambda}\Big|_{\lambda_1}^{\lambda_2}$$

EMERGENT PARADIGM FITS NATURALLY WITH ...

- Why does the current related to $x^a \to x^a + q^a(x)$ have anything to do with a thermodynamical variable like entropy ?
- Why do Einstein's equations reduce to a thermodynamic identity on the horizons ? And, as Navier-Stokes equations on null surfaces?
- Why does Einstein-Hilbert action have several peculiar features ? (holographic surface/bulk terms, thermodynamic interpretation ....)
- Why does the surface term in the action give the horizon entropy ? And on-shell action reduces to the free energy ?
- Why does the microscopic degrees of freedom obey thermodynamic equipartition ?
- Why does a thermodynamic variational principle lead to the gravitational field equations?
- Why do all these work for a wide class of theories?

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