Topology, correlations and superconductivity in magic angle graphene

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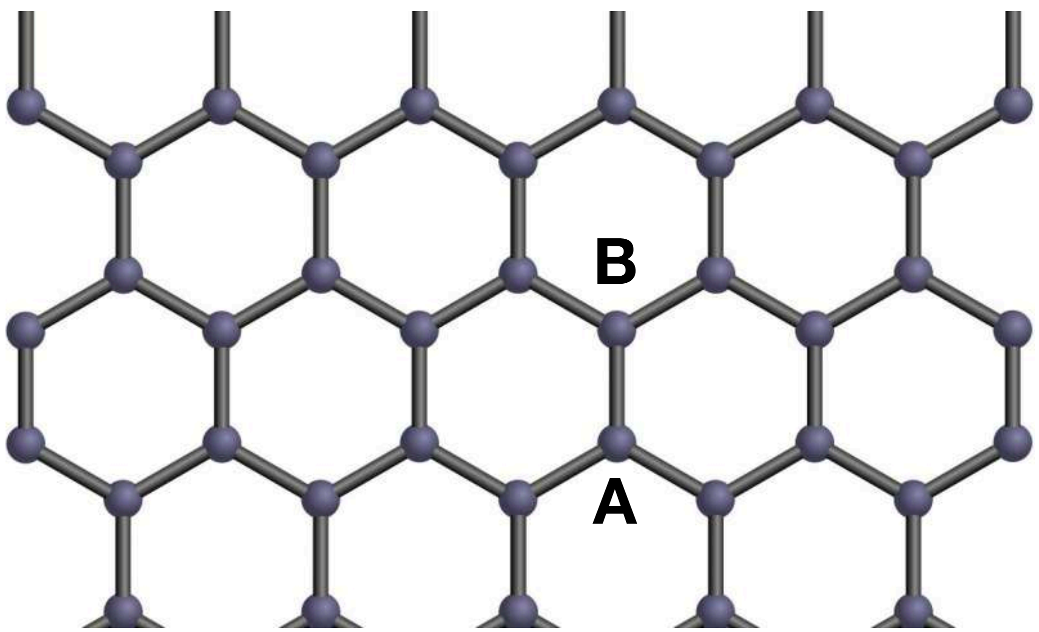
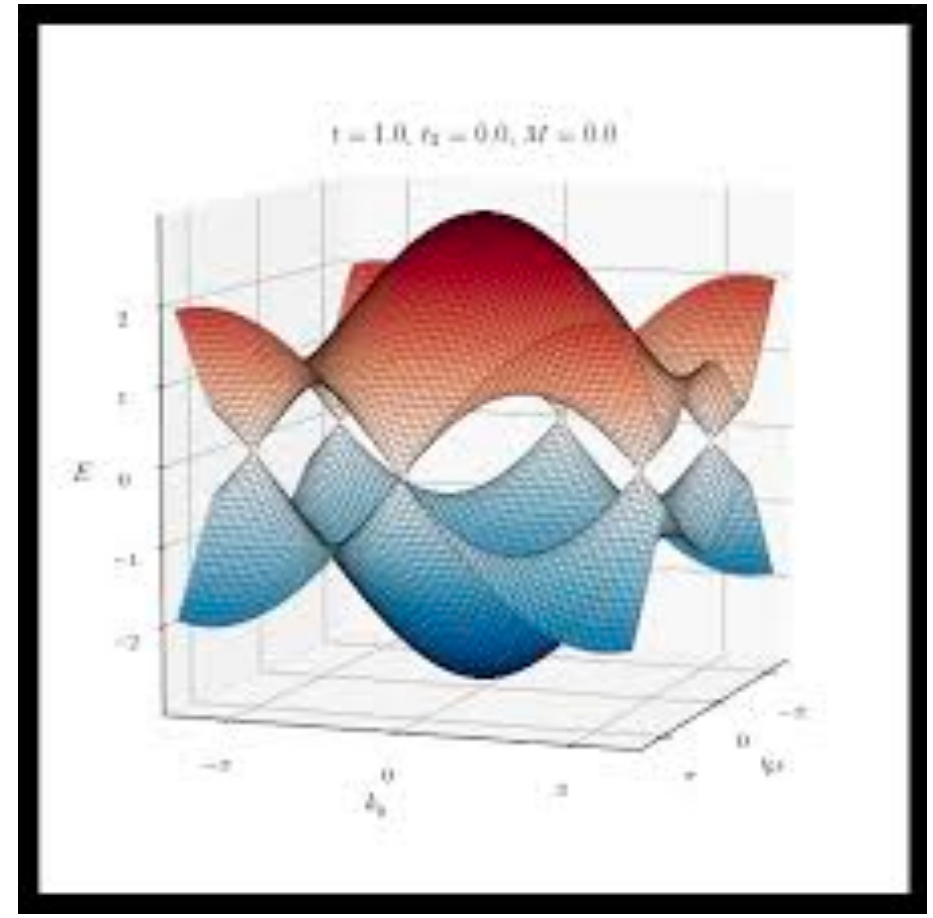
Graphene and Dirac Points





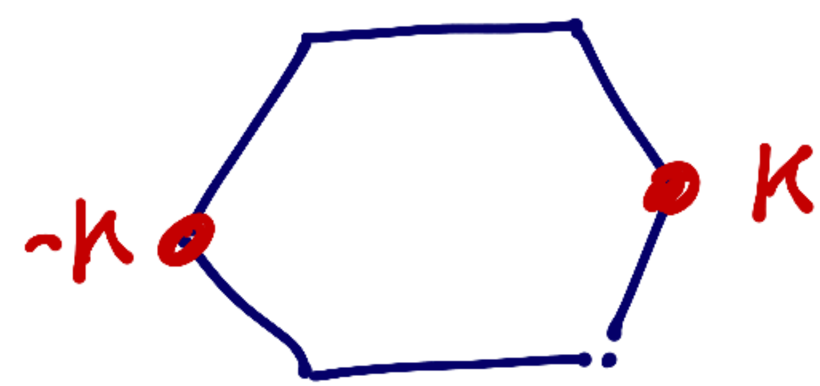
Dirac Points - vortices in d vector

$$H_k = \vec{d}_k \cdot \vec{\sigma}$$

$$d_x = -t\left(\cos k \cdot a_1 + \cos k \cdot a_2 + \cos k \cdot a_3\right)$$

$$d_y = -t\left(\sin k \cdot a_1 + \sin k \cdot a_2 + \sin k \cdot a_3\right)$$

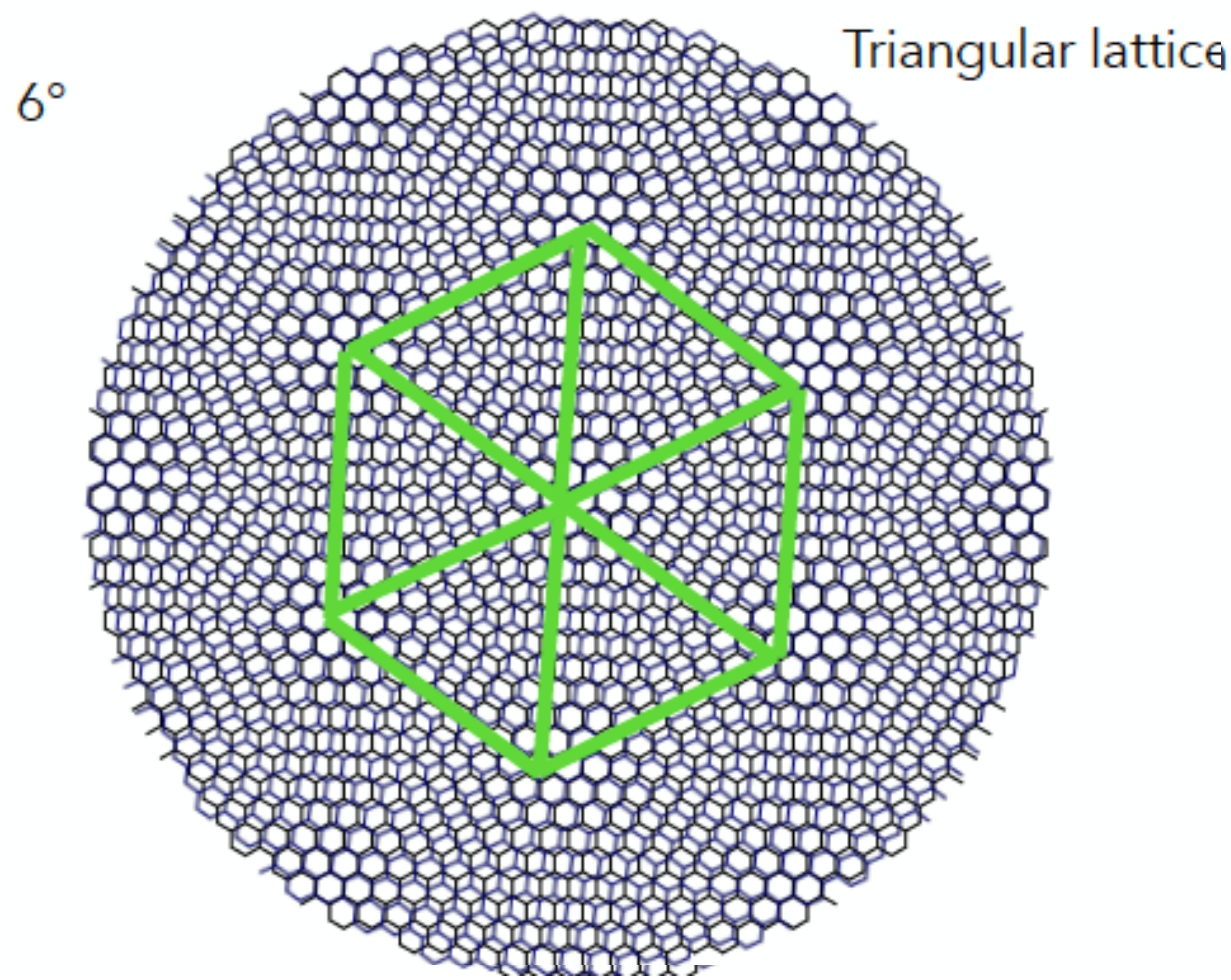
$$d_z = 0$$

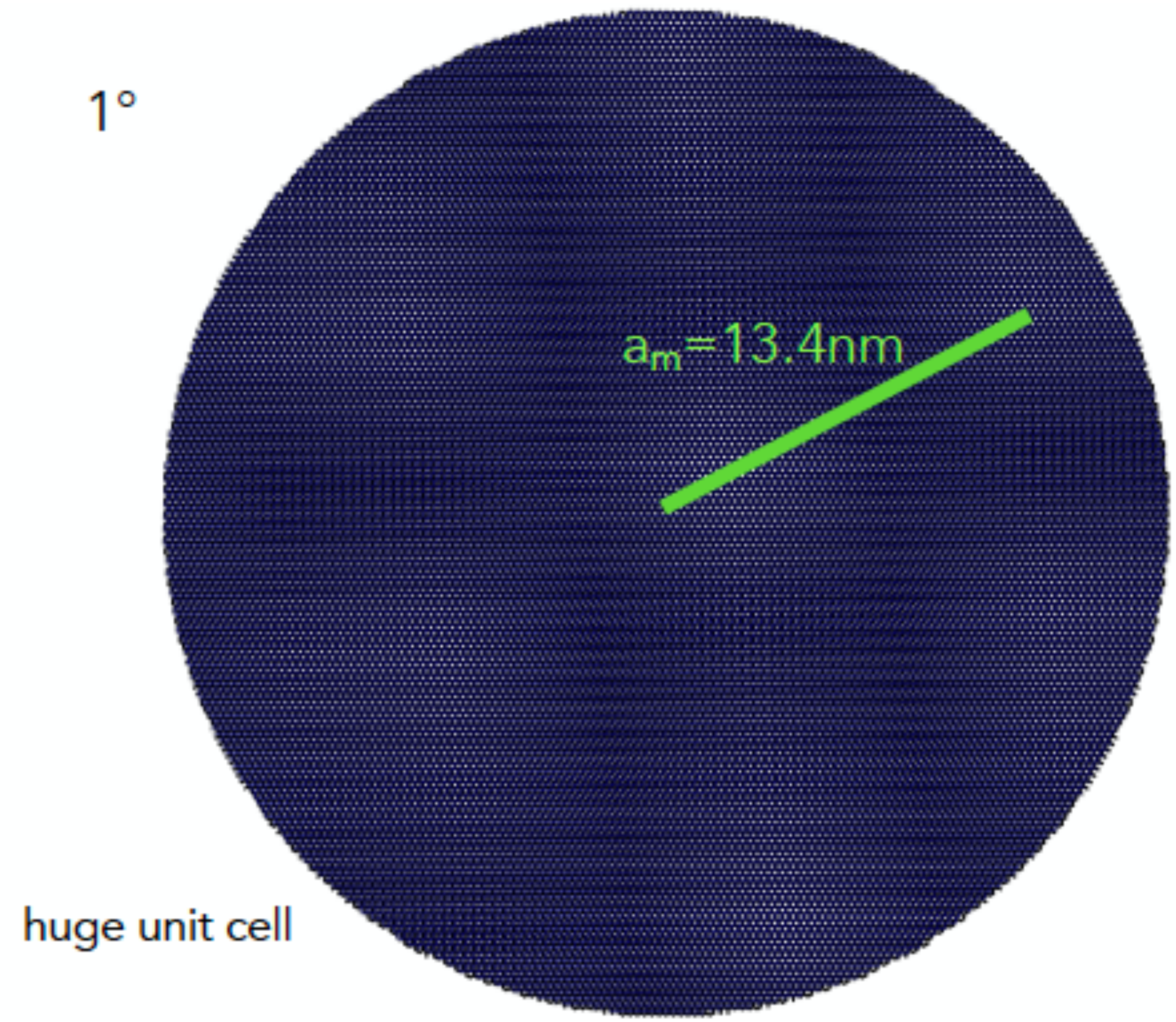


$$\left\{\sigma_z, H_k\right\} = 0$$

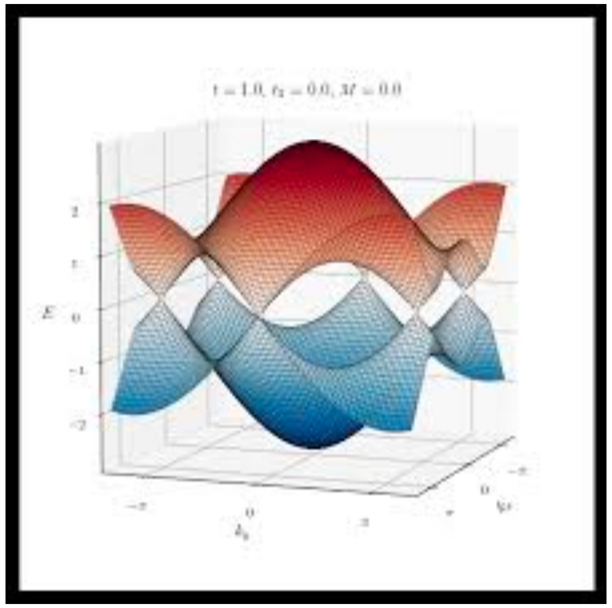
"Chiral Symmetry"

Visualizing Twisted Bilayer Graphene





Graphene and Dirac Points



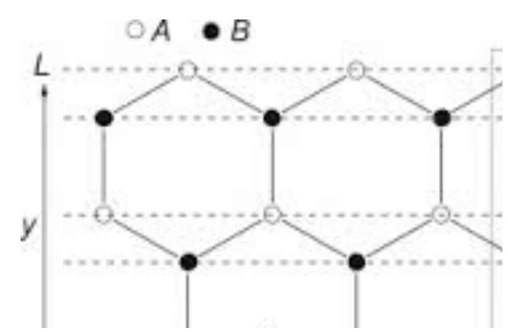
Three ways to remove Dirac Cones:

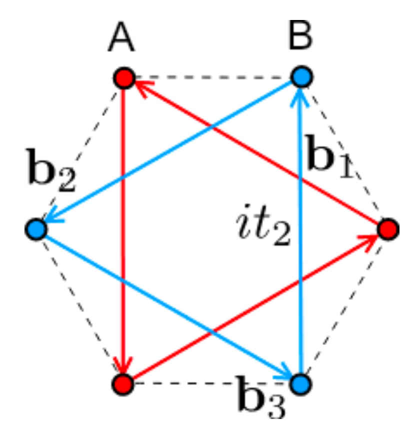
1. Break P (r—> -r) symmetry - staggered potential (B-N)

$$\Delta H = m\sigma_z$$

2. Break T symmetry - Haldane term

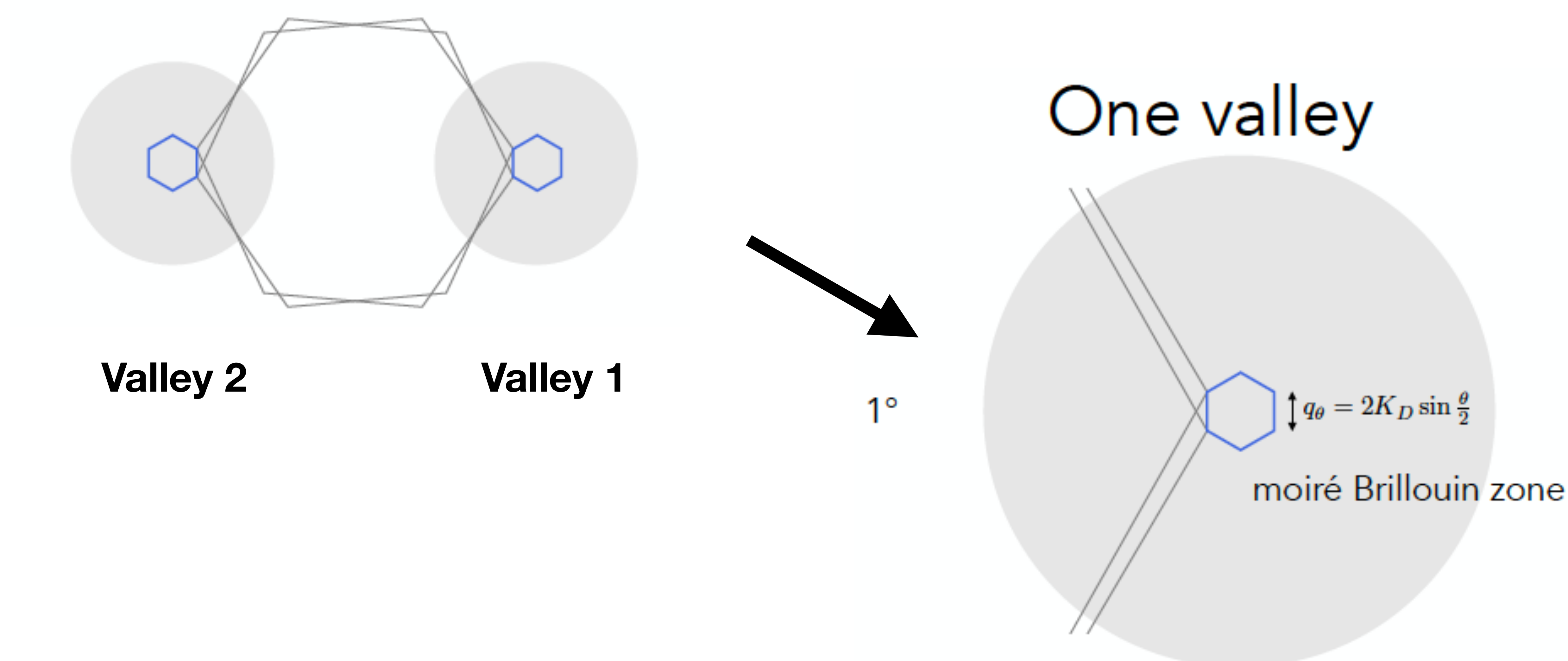
3. Break C3 rotation and annihilate
Dirac points (needs finite strength)





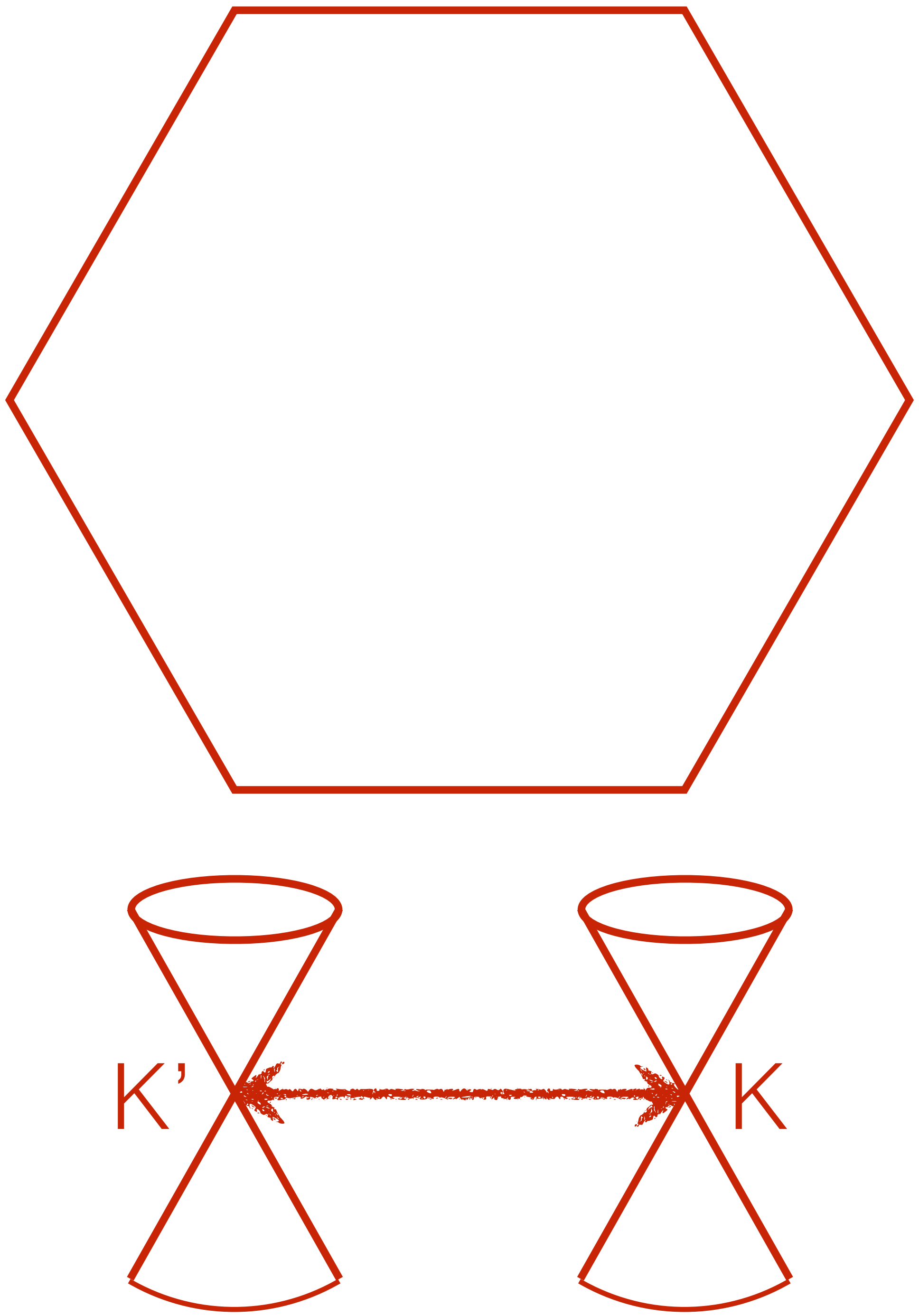
Twisted Bilayer Graphene Symmetries

Continuum Approximation - each layer Dirac points.



“Magic”

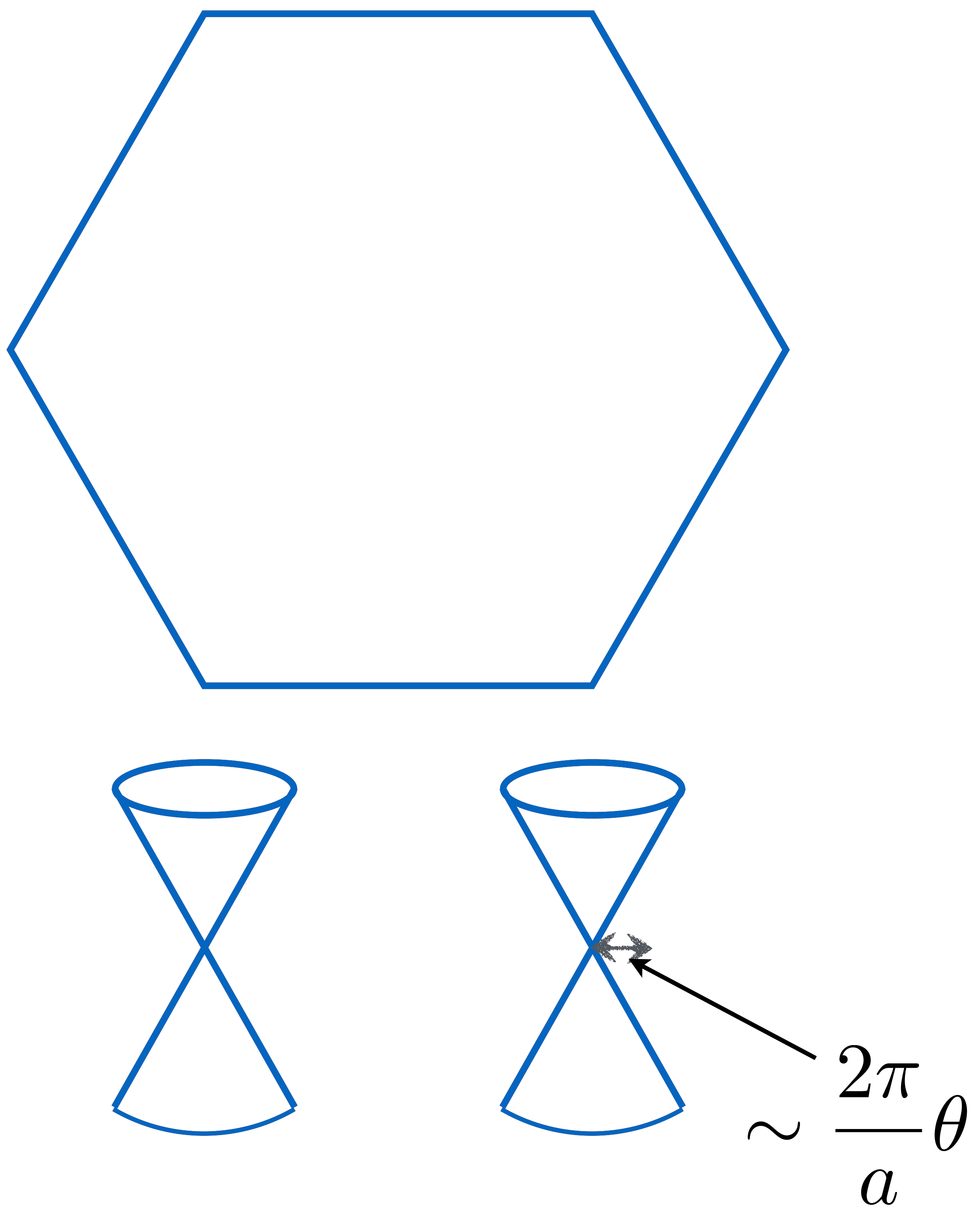
Monolayer Brillouin zone



$$\hbar v_F \frac{2\pi}{a} \simeq 16\text{eV}$$

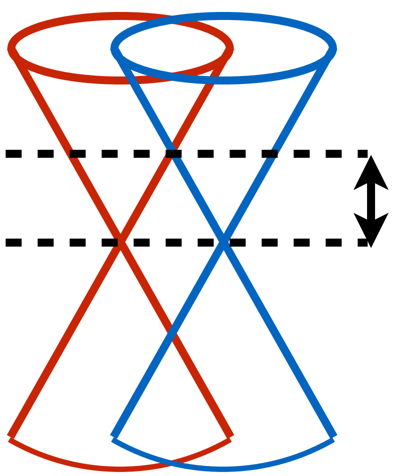
“Magic”

Twisted bilayer Brillouin zone



“Magic”

- Coupling between top and bottom layers: ~0.3 eV
- Dirac energy scale:



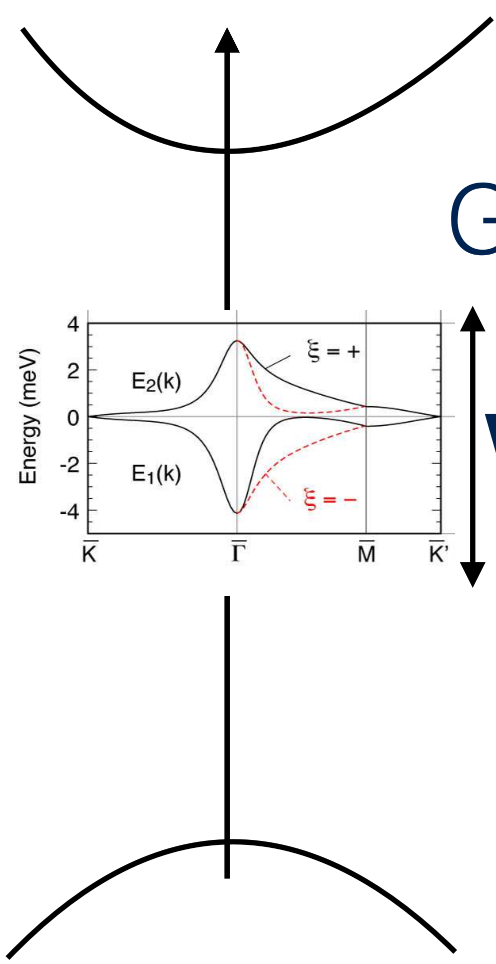
$$\sim \hbar v_F \frac{2\pi}{a}\theta = 16\,\theta \ \text{(eV)}$$

- For the other layer to matter…

$$\theta \simeq \frac{0.3}{16}\,\text{rad.} = 1.07^\circ$$

Band Structure of Twisted Bilayer Graphene

- Can produce a nearly flat band by carefully tuning the twist angle (Bistreitzer-MacDonald, Santos, Castro Neto et al.,).
- `Magic Angle' - $\sim 1.05^o$
- Then, **interactions** can dominate over kinetic energy -



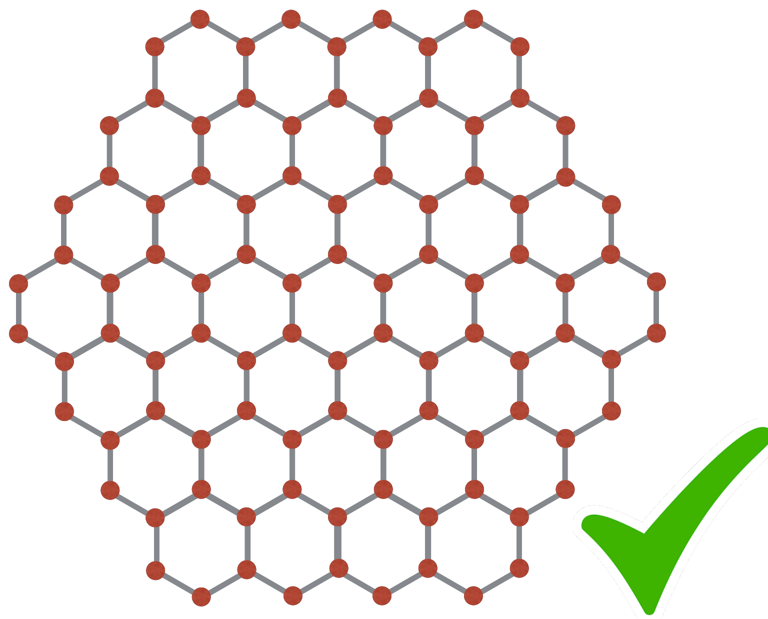
Gap ~ 30 meV

W ~ 8 meV

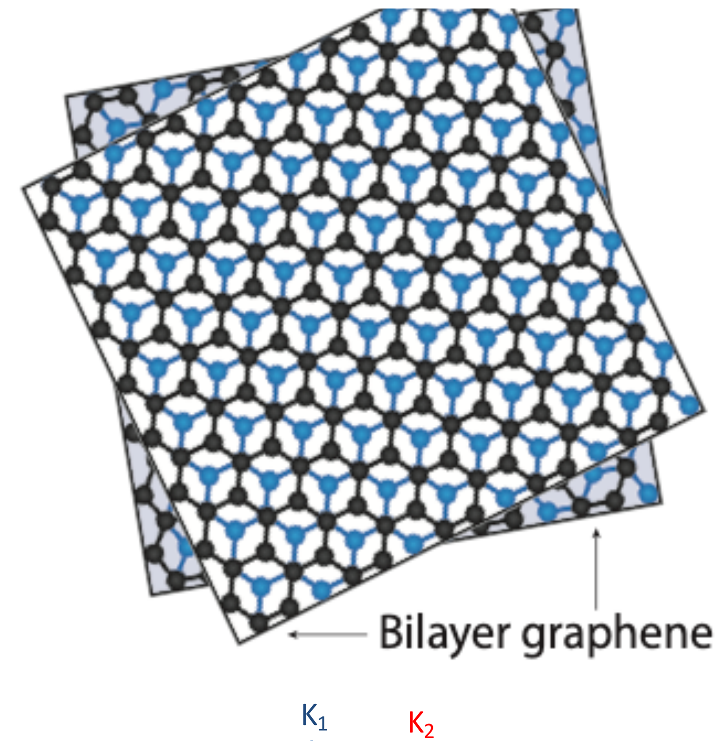
V~30meV

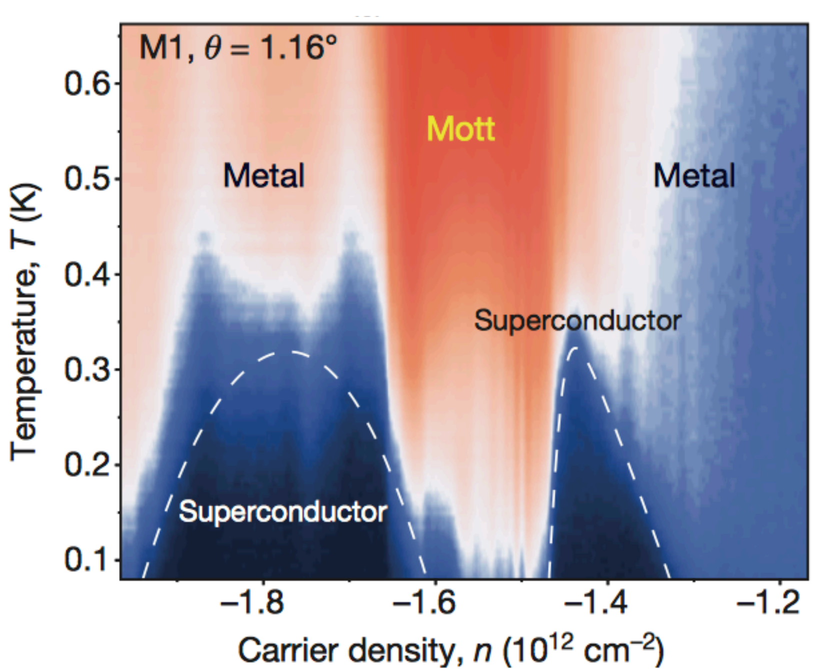
Koshino et al

Moire Materials

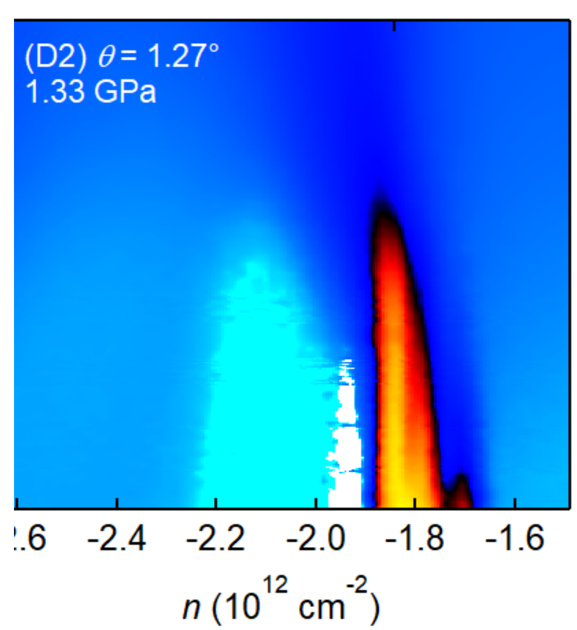


C_2
Symmetry





Cao, et al, Nature **556**, 43 (2018)



Yankowitz et al. 1808.07865
+ Dmitry Efetov group

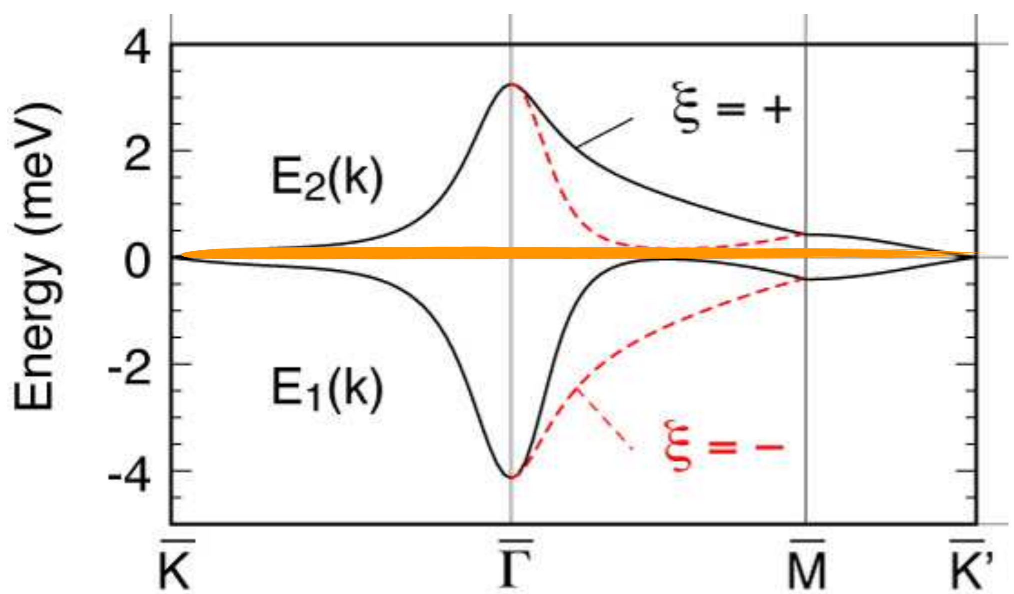
Other Systems:

(i) ABC Trilayer + hBN + D-Field (Feng Wang Lab)

(ii) Twisted Bilayer-Bilayer + D-Field (Philip Kim Lab, Pablo H.J. Lab)

(iii) Mono-Mono + hBN (Goldhaber-Gordon Lab)

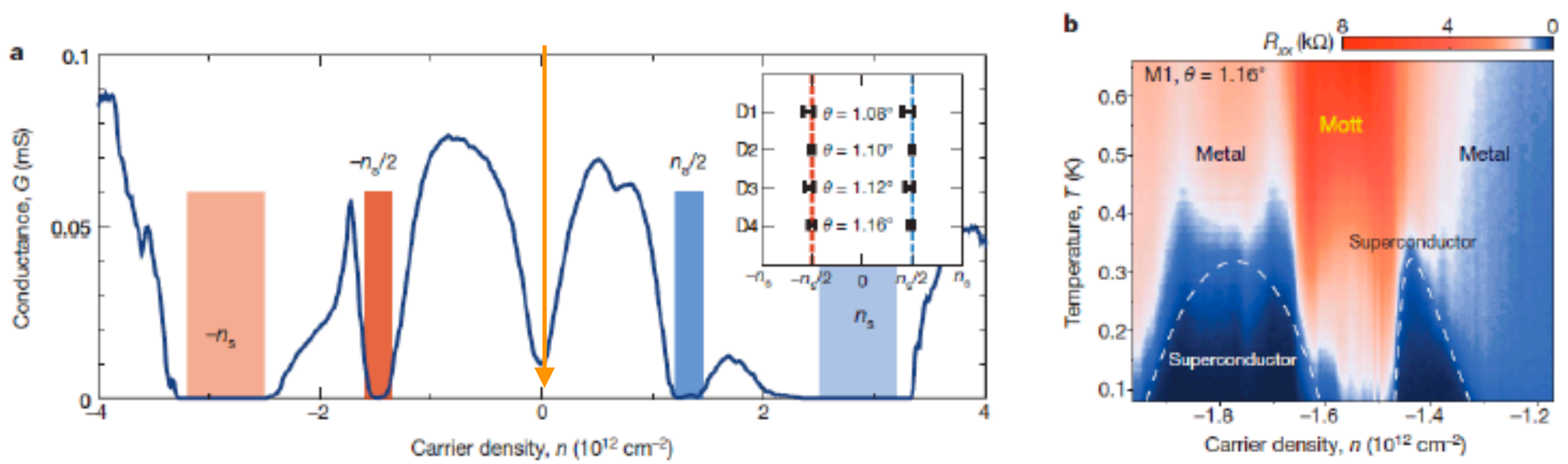
Review of Experiments



$\nu = +4$

Charge Neutrality

$\nu = -4$



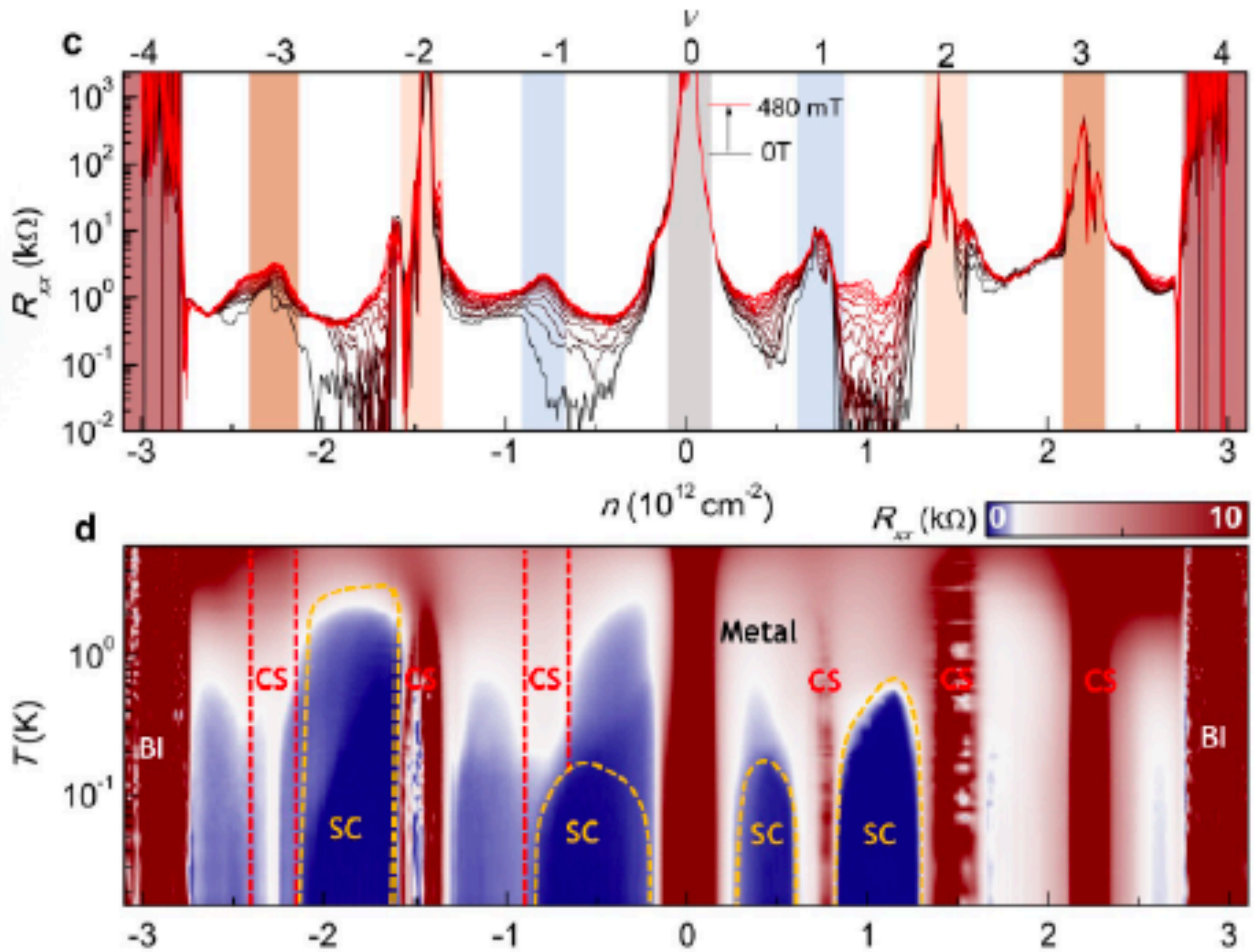
Pablo Jarillo-Herrero's group (MIT)

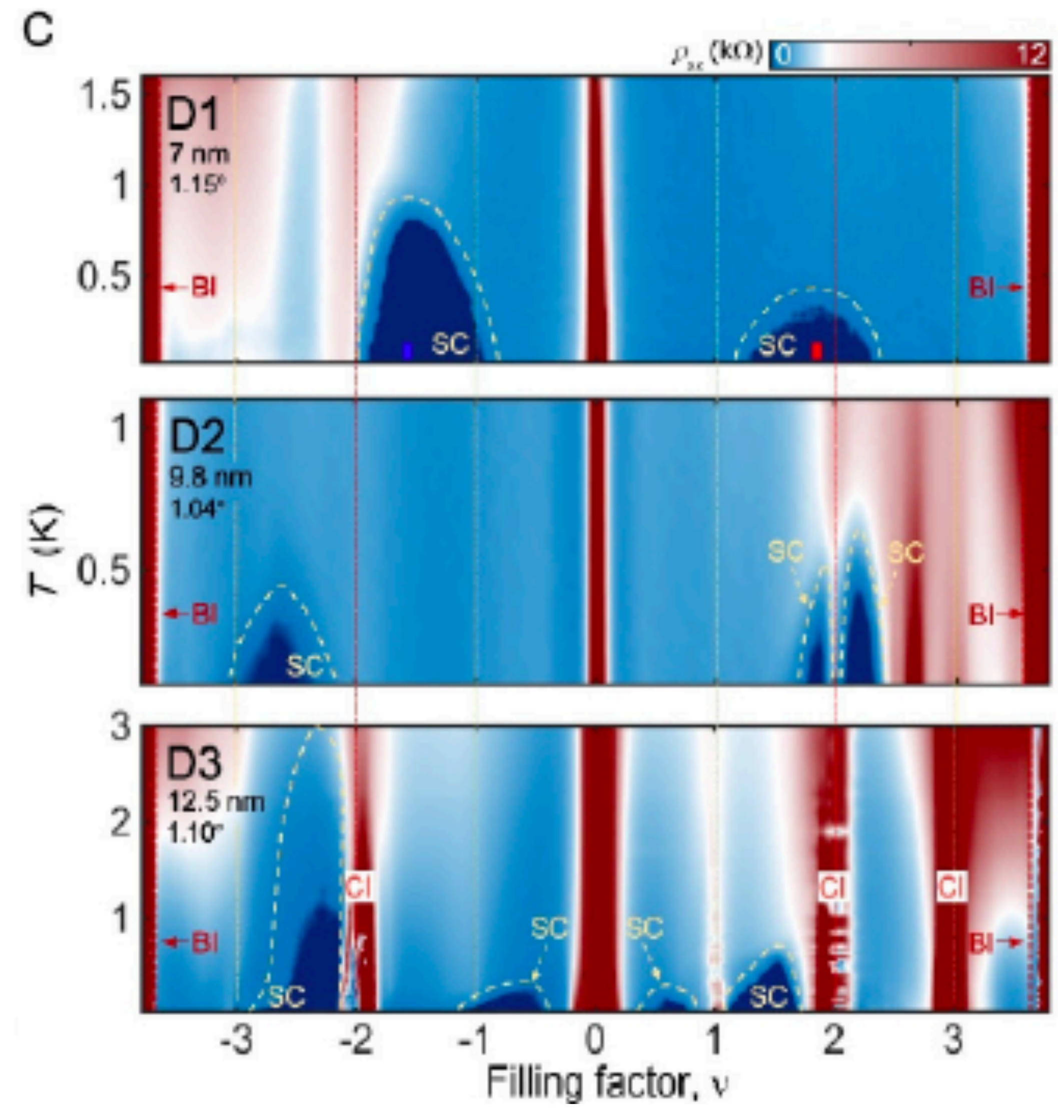
Cao *et al.* Nature 556, 80 (2018)

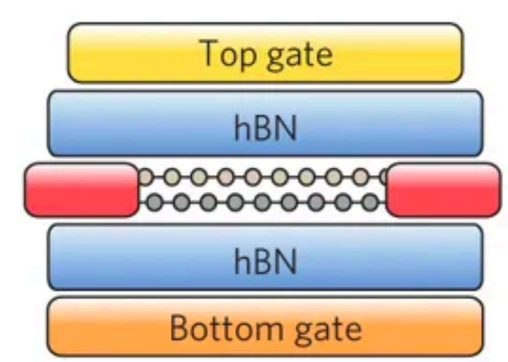
Cao *et al.* Nature 556, 43 (2018)

Review of Experiments

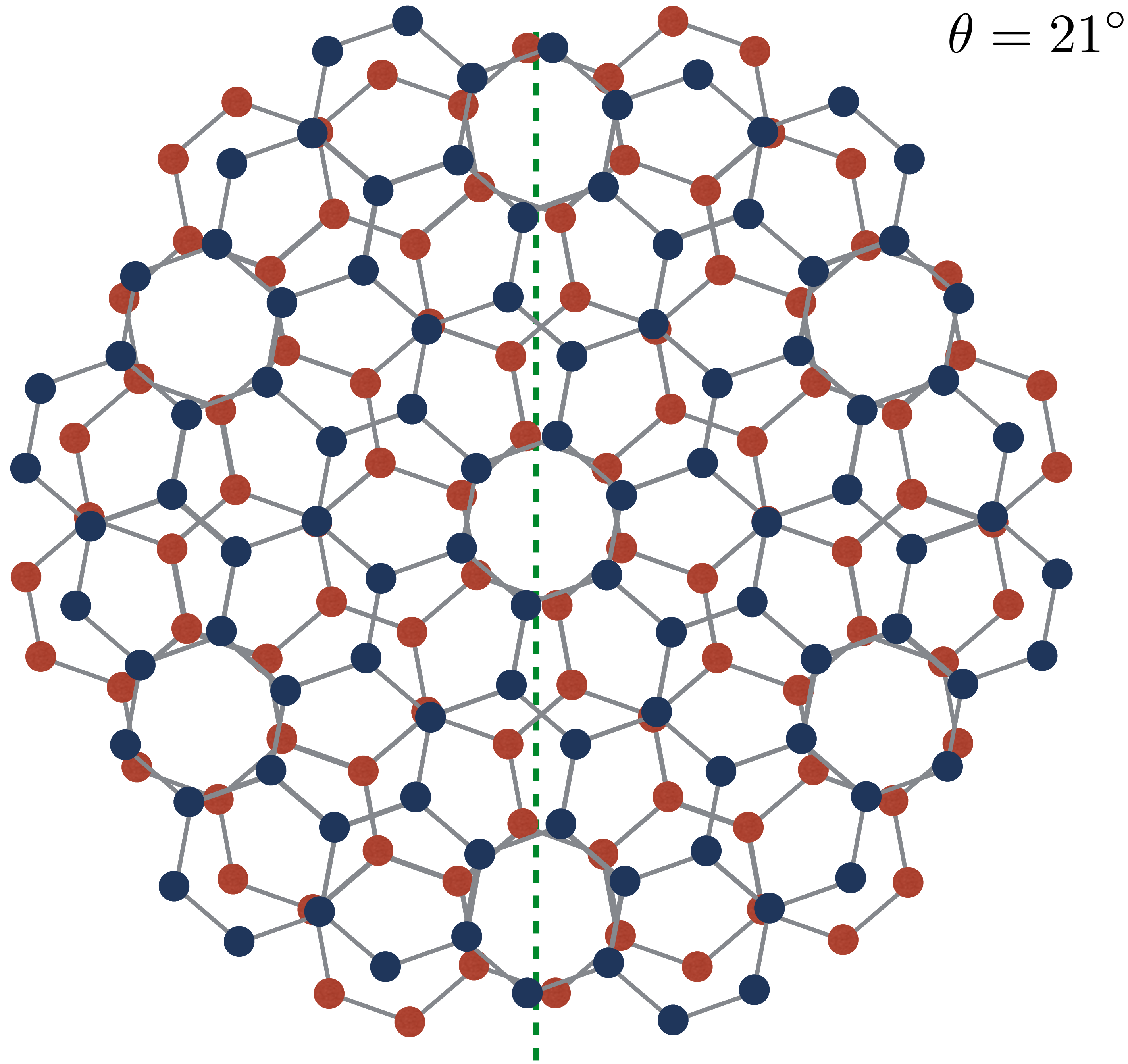
- More uniform samples: Lu *et al.* Nature 574, 653–657 (2019), Stepanos *et al.* arxiv:1911.09198 (2019)





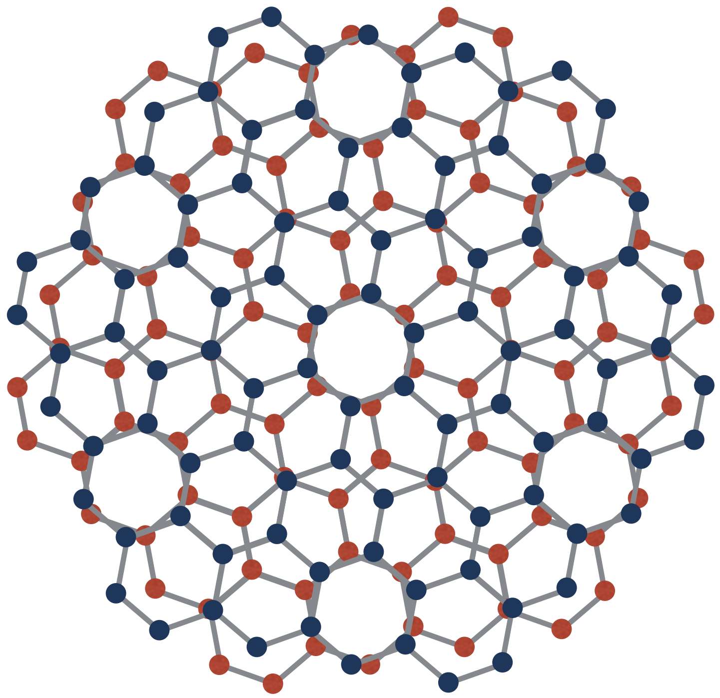


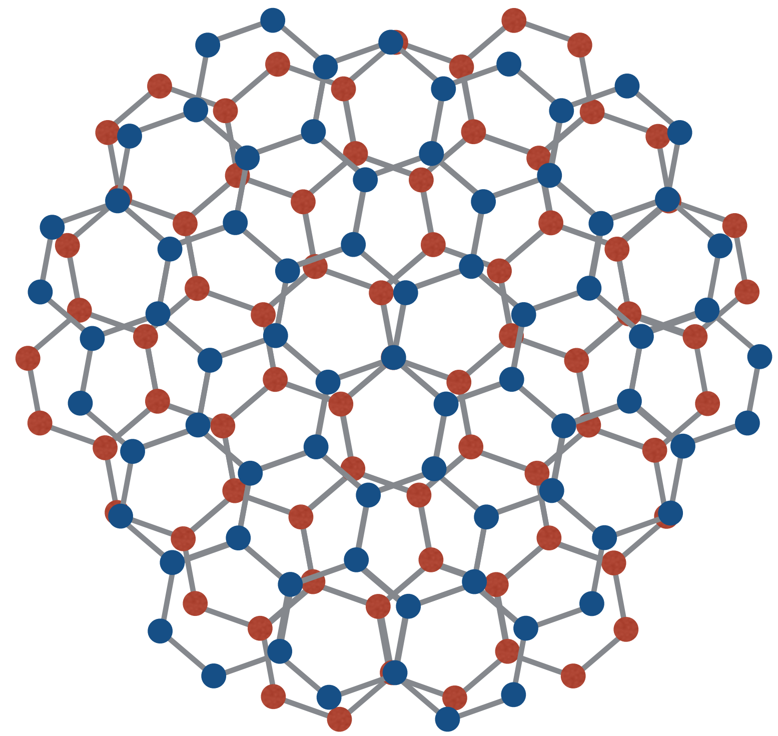
Symmetries of Twisted Bilayer Graphene



Symmetries: **C6** and Mirror **Mx** (interchanges layers) = **D6**

Symmetries of Twisted Bilayer Graphene





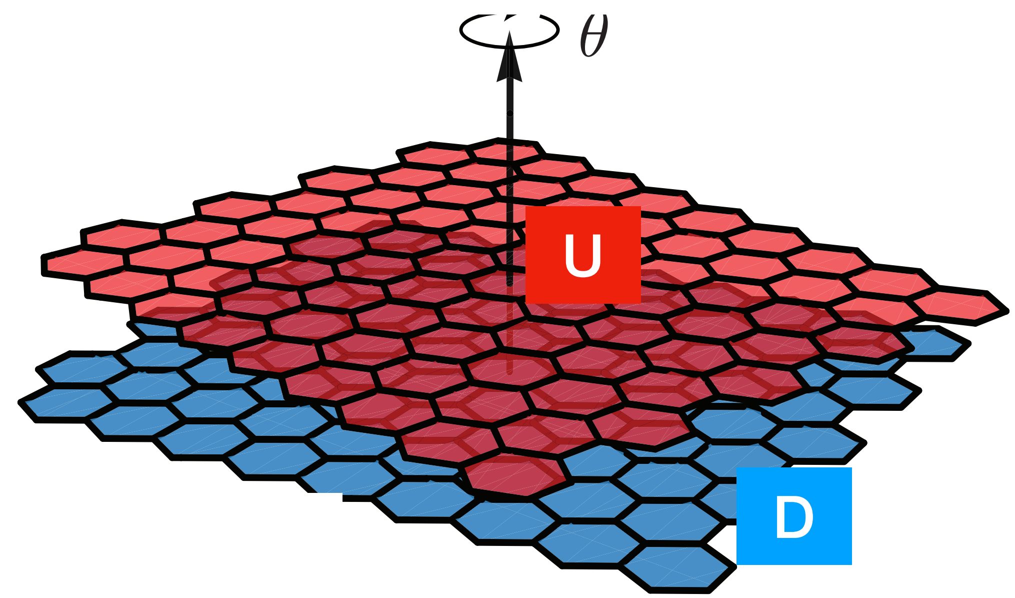
- Left structure has D6. Right has D3. Even less symmetry possible.

- However at *small angles* distinction is irrelevant - emergent C6 symmetry. Valley U(1) symmetry.

- Commensurate vs incommensurate also irrelevant.

The Continuum Model

Two layers of graphene with
a relative twist



U D A B

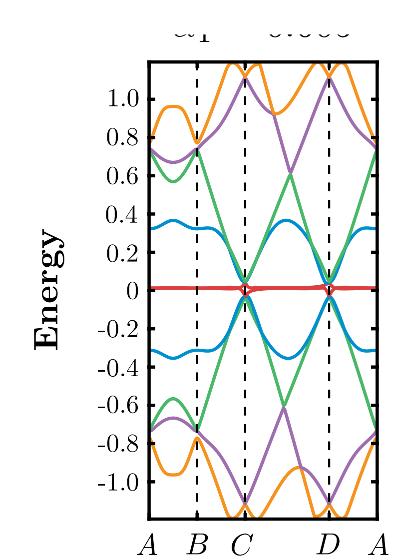
$$H = \begin{pmatrix} -iv_0 \boldsymbol{\sigma}_{\theta/2}\nabla & T(\mathbf{r}) \\ T^{\dagger}(\mathbf{r}) & -iv_0\boldsymbol{\sigma}_{-\theta/2}\nabla \end{pmatrix} \quad T(\boldsymbol{r}) = \begin{pmatrix} w_0 U_0(\boldsymbol{r}) & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & w_0 U_0(\boldsymbol{r}) \end{pmatrix}$$

A B

$$\alpha = \frac{w_{0,1}}{2 v_F k_D \sin\theta/2}$$

Dimensionless ratio

$\alpha \sim 0.6$ **“Magic angle”**

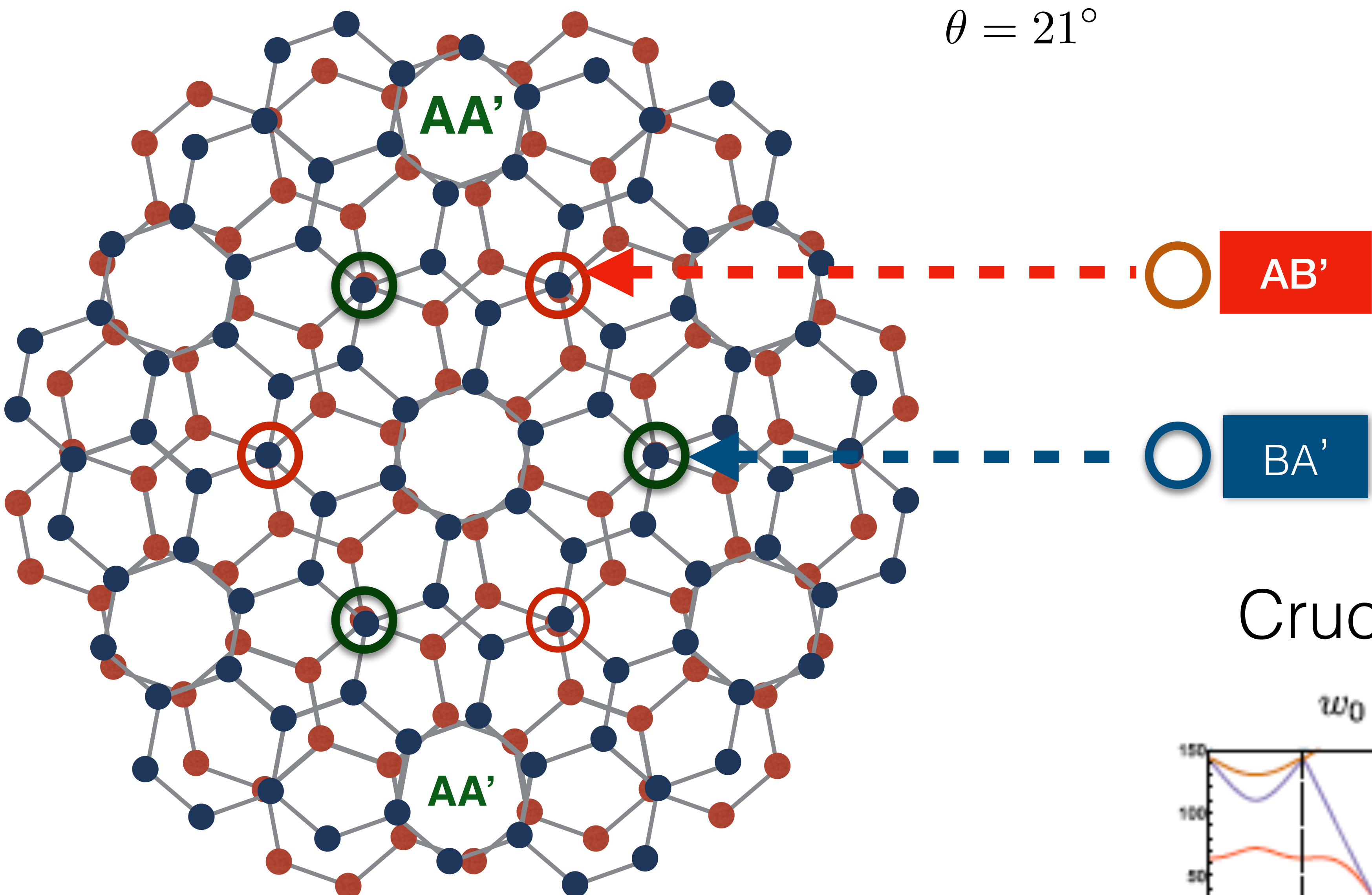


Bistritzer-MacDonald

Importance of Lattice Relaxation

Lattice Relaxation reduces AA coupling vs AB coupling

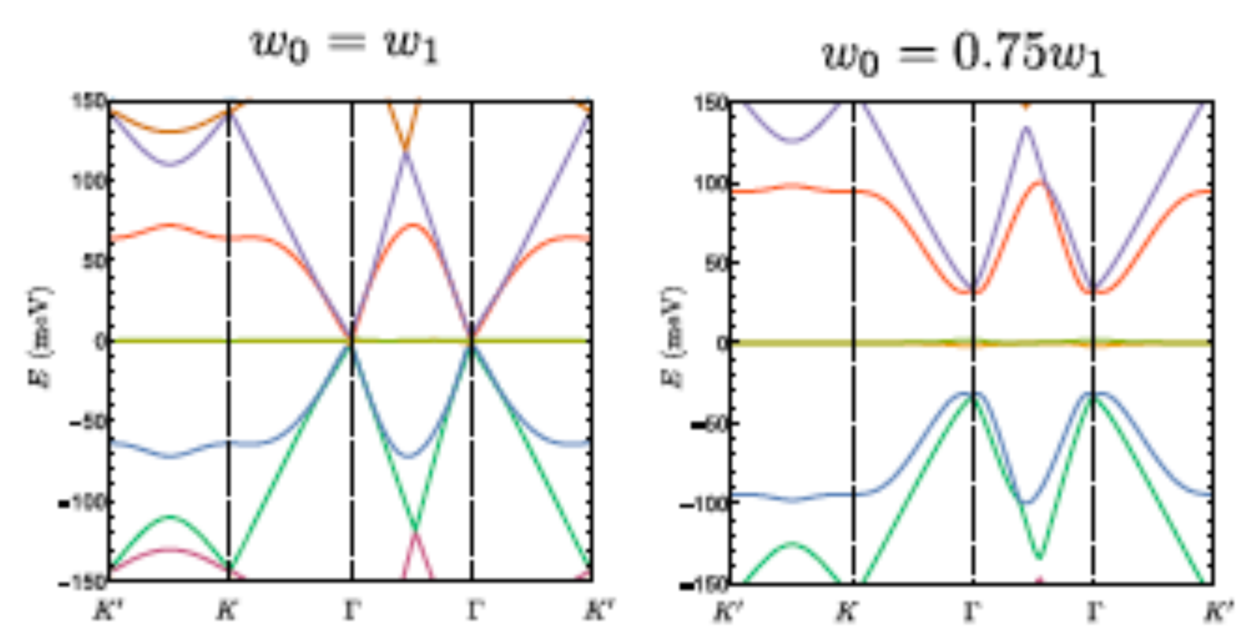
$\theta = 21^\circ$



$$T(\boldsymbol{r}) = \begin{pmatrix} w_0 U_0(\boldsymbol{r}) & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & w_0 U_0(\boldsymbol{r}) \end{pmatrix}$$

Carr 19, Nam and Koshino 18)

Crucial to obtaining gaps.



Chiral Model

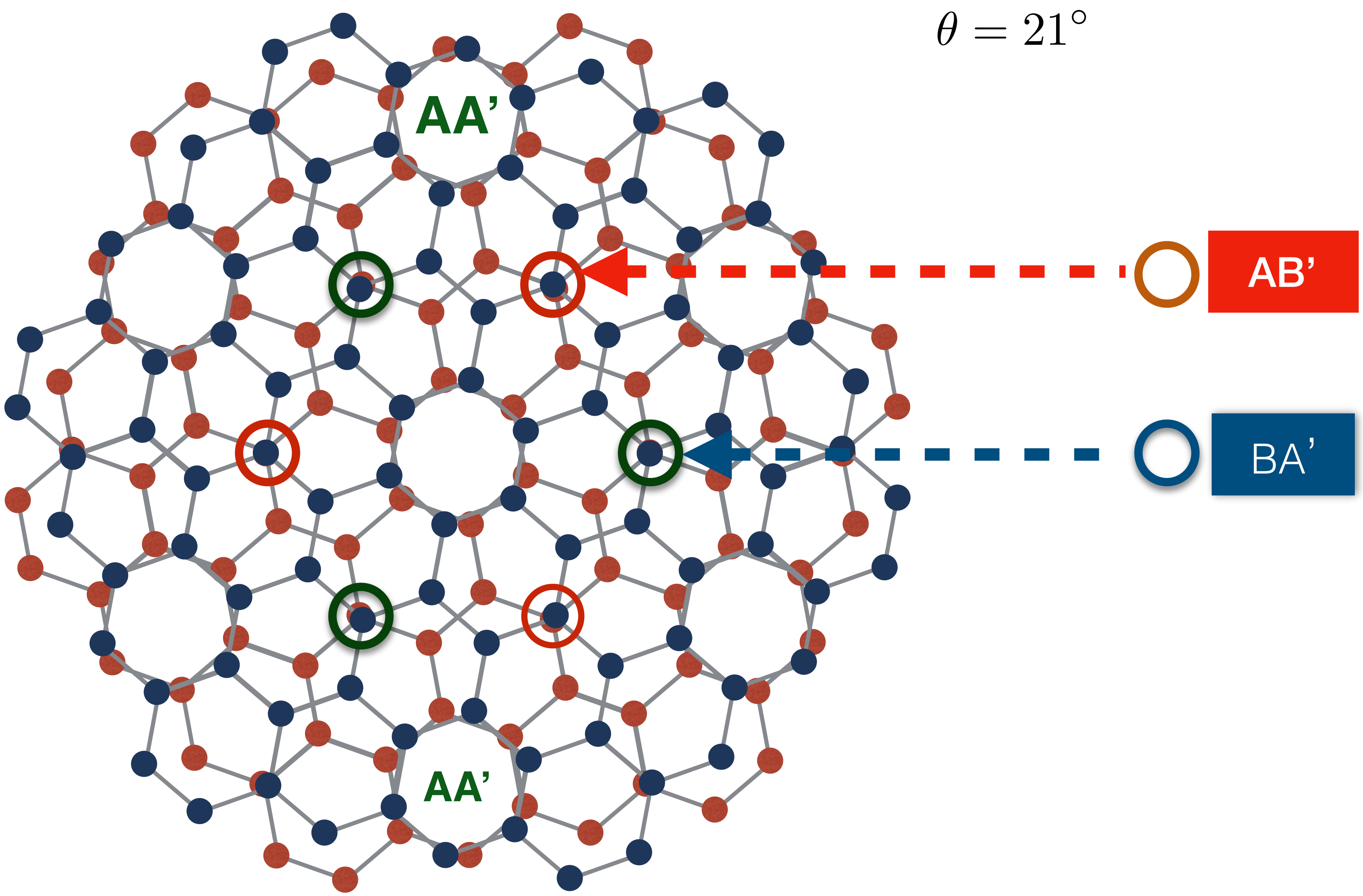
Tarnopolski, Kruchkov, AV PRL 2019

Switch off AA coupling. Only AB coupling

$$T(\boldsymbol{r}) = \begin{pmatrix} w_0 U_0(\boldsymbol{r}) & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & w_0 U_0(\boldsymbol{r}) \end{pmatrix}$$

Grisha Tarnopolski
Harvard

$\theta = 21^\circ$



Chiral Symmetry

$$\{\sigma_z \otimes 1, \mathcal{H}\} = 0$$

Chiral Model

Tarnopolski, Kruchkov, AV PRL 2019

Switch off AA coupling. Only AB coupling

A B U D

$$\mathcal{H} = \begin{pmatrix} 0 & \mathcal{D}^*(-\mathbf{r}) \\ \mathcal{D}(\mathbf{r}) & 0 \end{pmatrix}, \quad \mathcal{D}(\mathbf{r}) = \begin{pmatrix} -2i\bar{\partial} & \alpha U(\mathbf{r}) \\ \alpha U(-\mathbf{r}) & -2i\bar{\partial} \end{pmatrix}$$

U
D

$$\bar{\partial} = \tfrac{1}{2}(\partial_x + i\partial_y) \qquad \alpha = w_1/(2v_0 k_D \sin(\theta/2))$$

Can be viewed as Dirac fermions in a non-abelian SU(2) gauge field

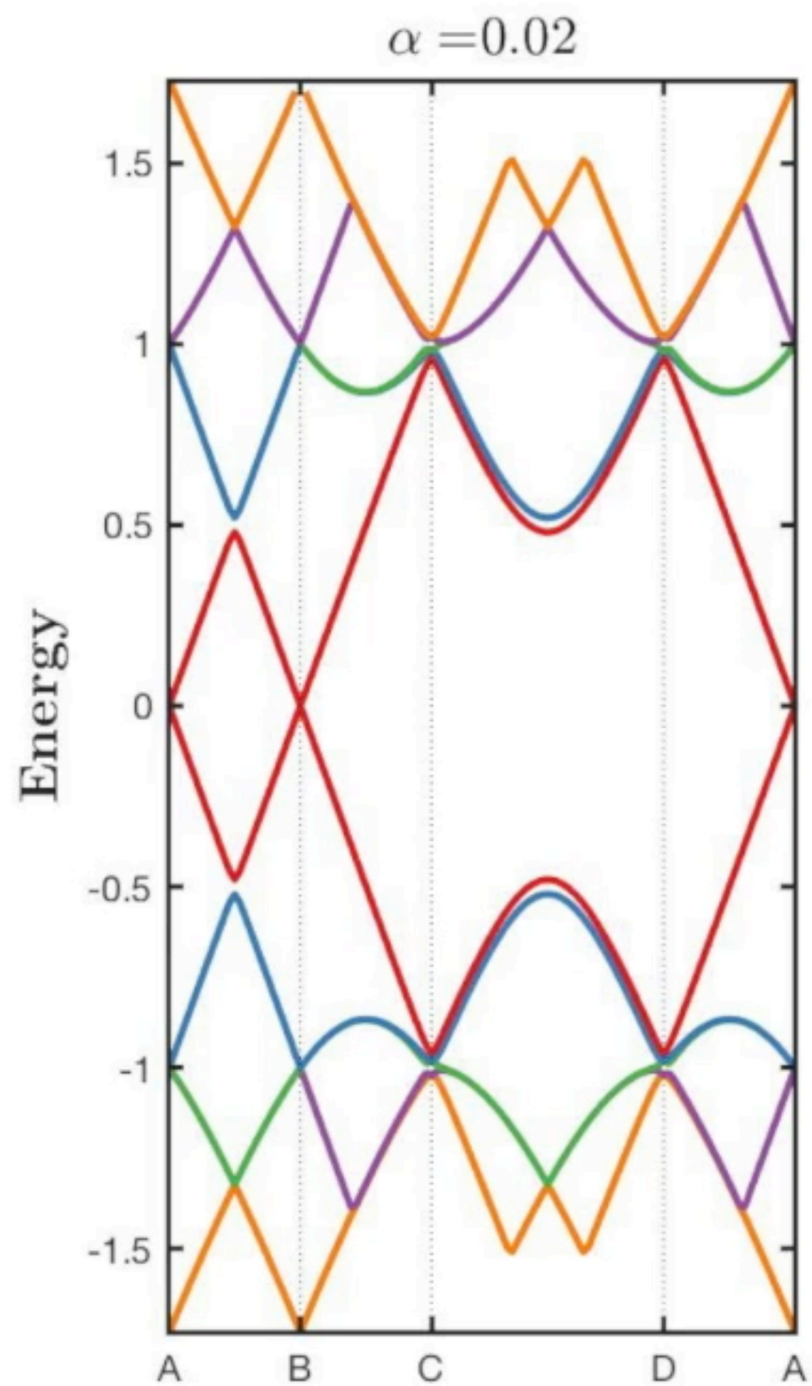
$$H = \vec{\sigma} \cdot \left(-i\vec{\nabla} - \vec{A}_a \tau^a\right)$$

Layer

$$\bar{A} = \frac{U(r) + U(-r)}{2}\tau^x + i\frac{U(r) - U(-r)}{2}\tau^y$$

P. San-Jose, J. Gonz´alez, and F. Guinea

Perfectly Flat Bands in the Chiral Model

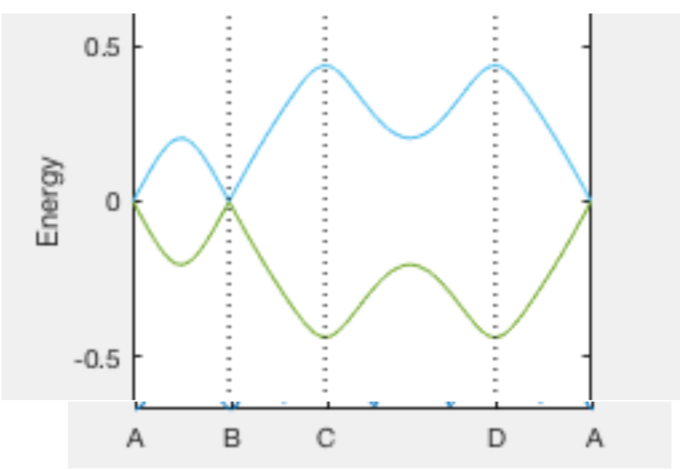


Exactly Flat Bands

Look for *exactly* zero energy states:

$$\mathscr{D}(r)\,\psi(r) = 0$$

For all angles there exists a zero-mode solution at K point $\mathcal{D}(\mathbf{r})\psi_K(\mathbf{r}) = 0$



$$\begin{pmatrix} -2i\bar{\partial} & \alpha U(\mathbf{r}) \\ \alpha U(-\mathbf{r}) & -2i\bar{\partial} \end{pmatrix} \begin{pmatrix} \psi_{K1} \\ \psi_{K2} \end{pmatrix} = 0$$

Generate new zero modes
- flat band?

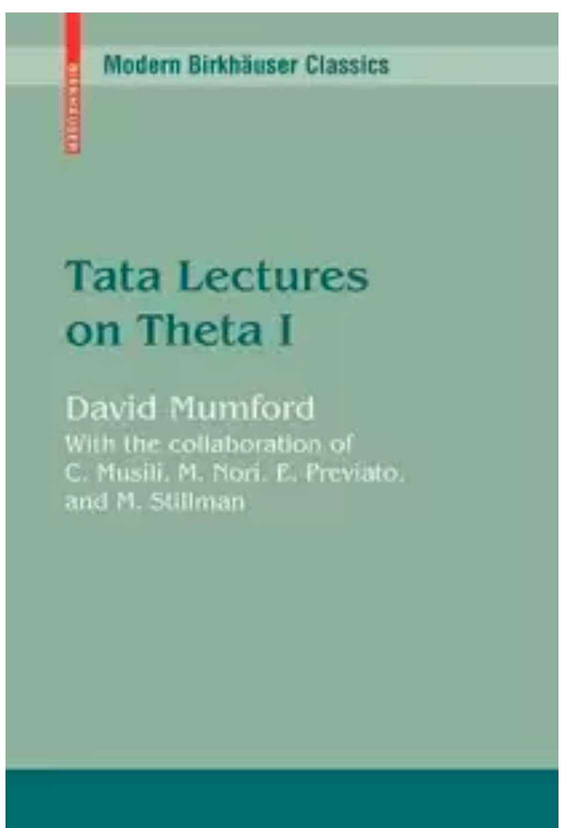
?? $$\begin{pmatrix} \psi_{k1} \\ \psi_{k2} \end{pmatrix} = f(z) \begin{pmatrix} \psi_{K1} \\ \psi_{K2} \end{pmatrix}$$ **??**

Theory of Exactly Flat Band

$$\begin{pmatrix} \psi_{k1} \\ \psi_{k2} \end{pmatrix} = f(z) \begin{pmatrix} \psi_{K1} \\ \psi_{K2} \end{pmatrix}$$

another Zero mode? -
but needs correct periodicity

$$f(z) \sim \frac{\theta_{a',b'}(z|\tau)}{\theta_{a,b}(z|\tau)}$$



Theta functions

$$\theta_{a=\frac{1}{2},\, b=\frac{1}{2}}(z|\tau) = -\theta_1(z|\tau)$$

$$\vartheta_1(u|\tau) = -i \sum_{n=-\infty}^{\infty} (-1)^n e^{\pi i \tau (n+\frac{1}{2})^2 + \pi i (2n+1) u}$$

ROOTS $\theta_1(u) = 0 : \quad u = n + m\tau,$

Theory of Exactly Flat Band

$$\begin{pmatrix}\psi_{k1}\\ \psi_{k2}\end{pmatrix} = f(z)\begin{pmatrix}\psi_{K1}\\ \psi_{K2}\end{pmatrix}$$

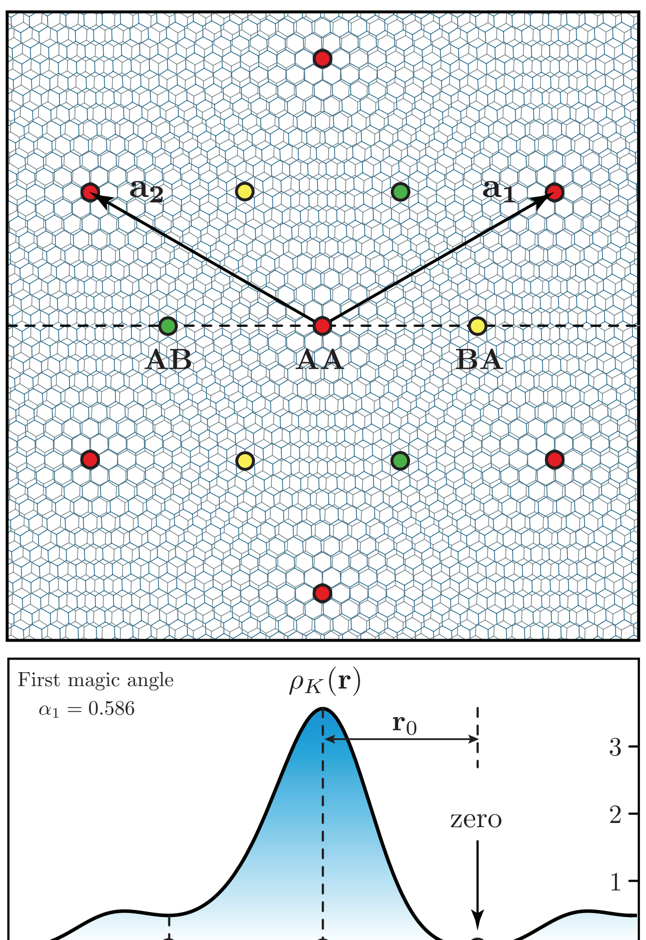
Theta functions

Ratio of theta functions gives correct periodicity

$$f(z) \sim \frac{\theta_{a',b'}(z|\tau)}{\theta_{a,b}(z|\tau)}$$

BUT requires wave-function zero
to cancel pole in denominator

At special (magic) angles,
the spinor wfn.
vanishes at **points** in the unit cell



Flat band wave functions

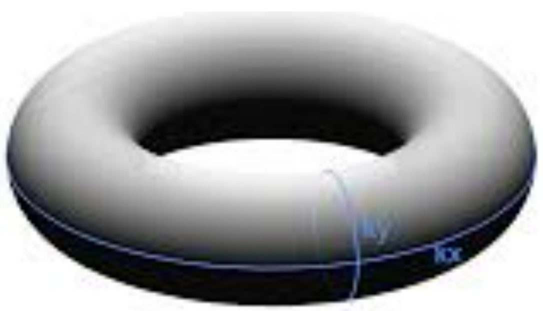
$$u_{\mathbf{k}}(r) = e^{-2\pi i r_2 k/b_2}\,\frac{\vartheta_1\left(\frac{z-z_0}{a_1}-\frac{k}{b_2}\Big|\omega\right)}{\vartheta_1\left(\frac{z-z_0}{a_1}\Big|\omega\right)}\psi_K(\mathbf{r}).$$

Tarnopolski, Kruchkov, AV 1808.05250

Exactly Flat Bands and Landau Levels

Related to quantum Landau Level Wins on Torus

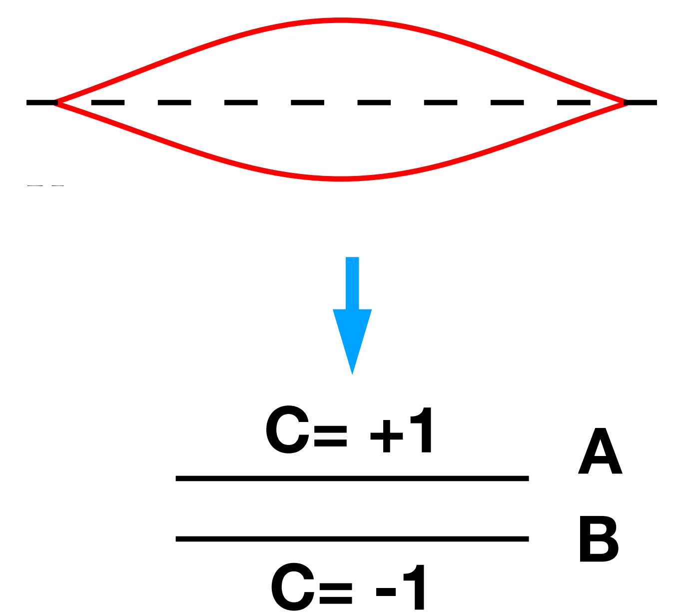
$$\psi_{\mathrm{LLL}} = e^{\imath\pi\tau N_s y^2} f\left(z\right)$$



Sublattice polarization like
Graphene in B field
BUT
Here single valley

Chiral Zero Modes in TBG

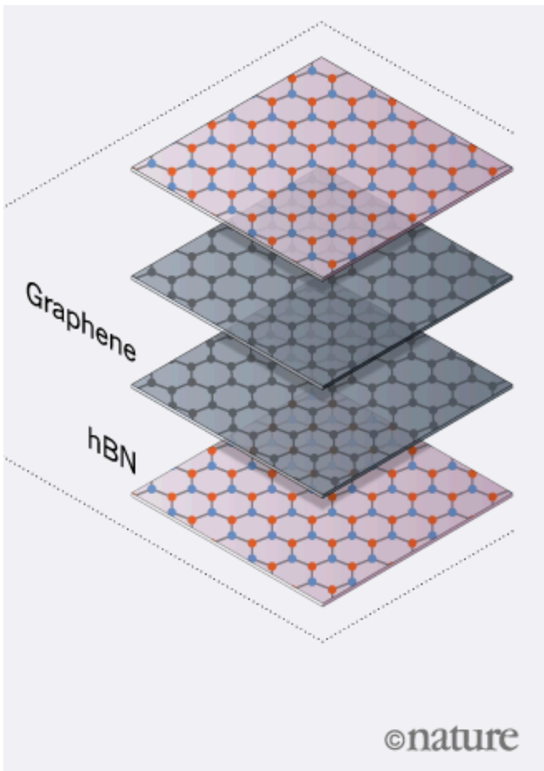
$$\psi(\mathbf{r}) = \frac{f(z)}{\vartheta_1((z-z_0)/a_1|\omega)}\psi_K(\mathbf{r}),$$



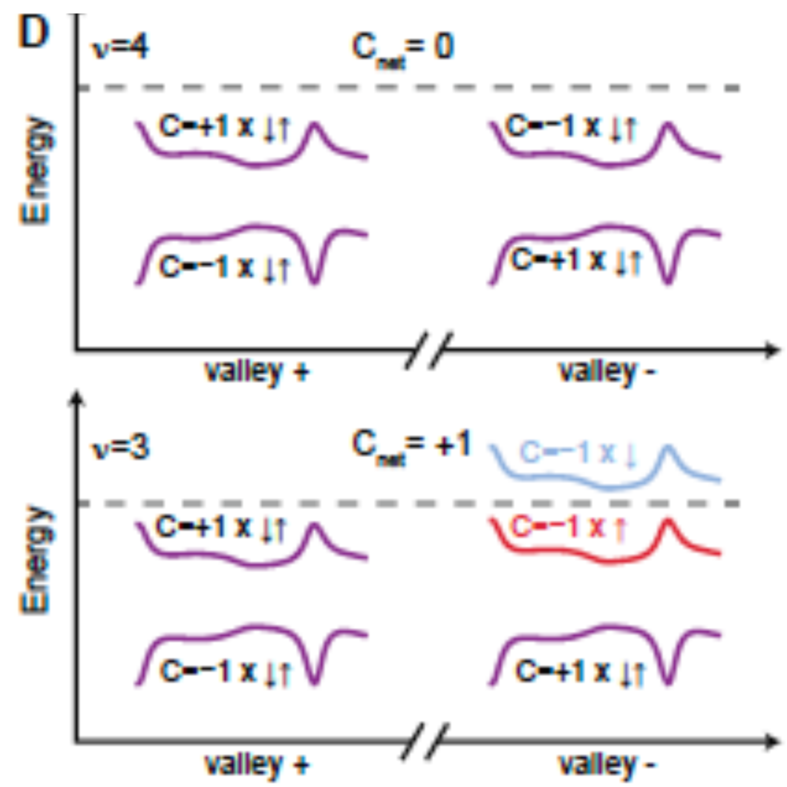
IDEA: add a sub lattice potential - should get Chern insulator (if spin and valley are polarized and unit filling)

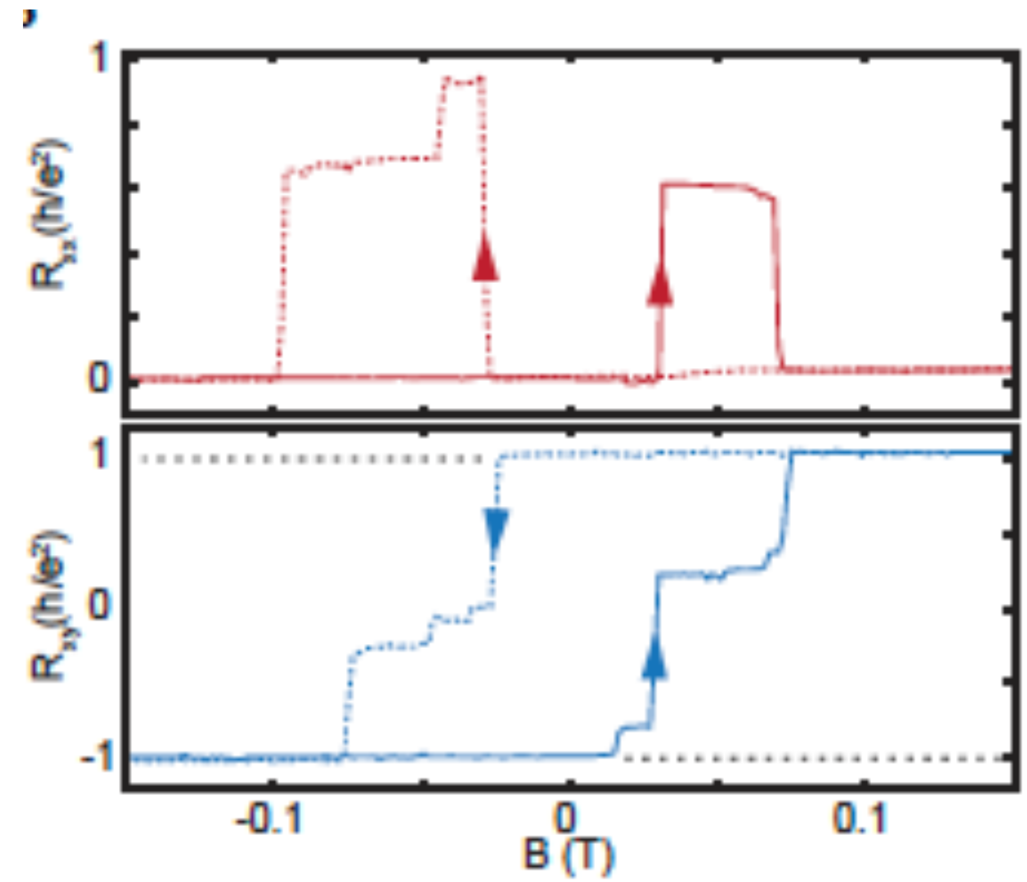
Intrinsic quantized anomalous Hall effect in a moiré heterostructure

M. Serlin,[1,*] C. L. Tschirhart,[1,*] H. Polshyn,[1,*] Y. Zhang,[1] J. Zhu,[1] K. Watanabe,[2] T. Taniguchi,[2] L. Balents,[3] and A. F. Young[1,†]



Aligned h-BN substrate





Fractional Filling - Fractional Chern?

Fractional Chern Insulator States in Twisted Bilayer Graphene: An Analytical Approach

Patrick J. Ledwith, Grigory Tarnopolsky, Eslam Khalaf and Ashvin Vishwanath
Department of Physics, Harvard University, Cambridge, MA 02138, USA
(Dated: December 23, 2019)

Quantum Metric of flat bands:

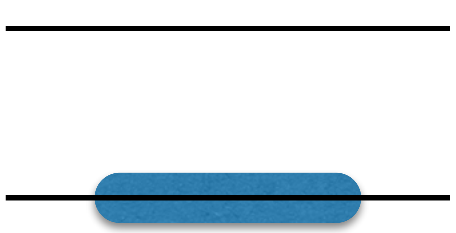
$$\eta(k) = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \Omega(k)$$

Interaction Effects in Pristine Magic Angle Graphene

Two Paradigms for Correlated Electrons

- Interactions energy exceeds kinetic energy (U>t)

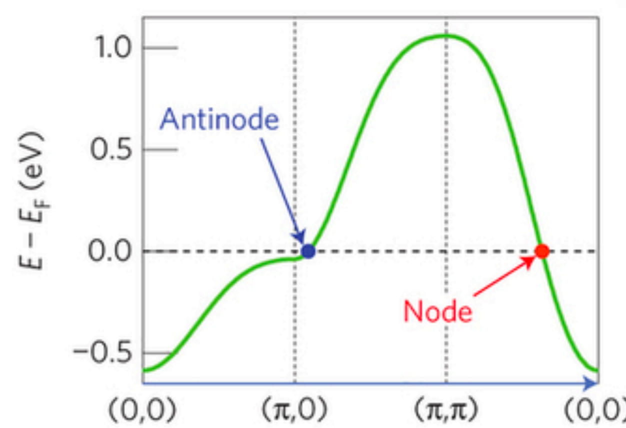
Quantum Hall

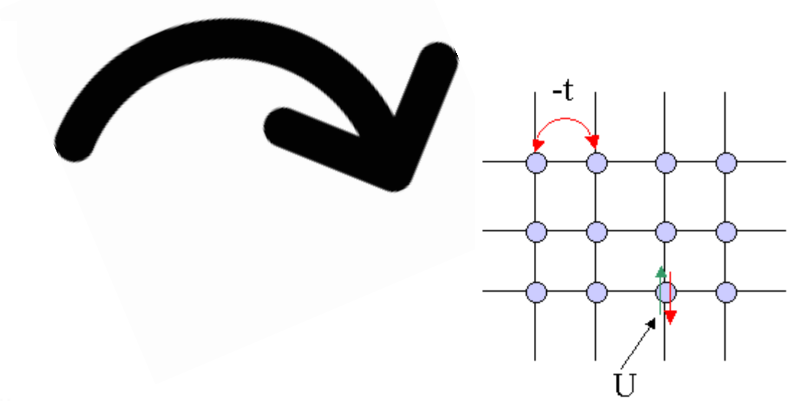


Landau Levels

$$\psi_n = z^n e^{-\frac{|z|^2}{4}}$$

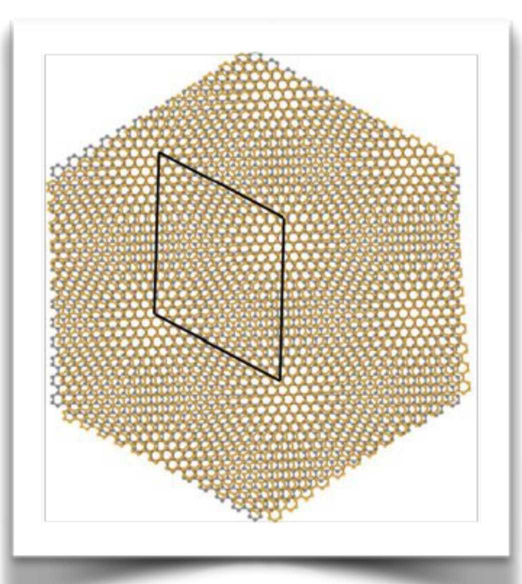
Correlated Solids eg. Cuprates

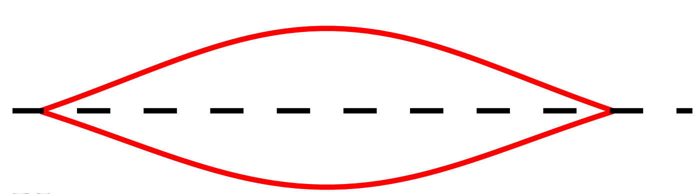




Wannier Functions

Hubbard Model





Magic angle graphene?

<u>Topology:</u>

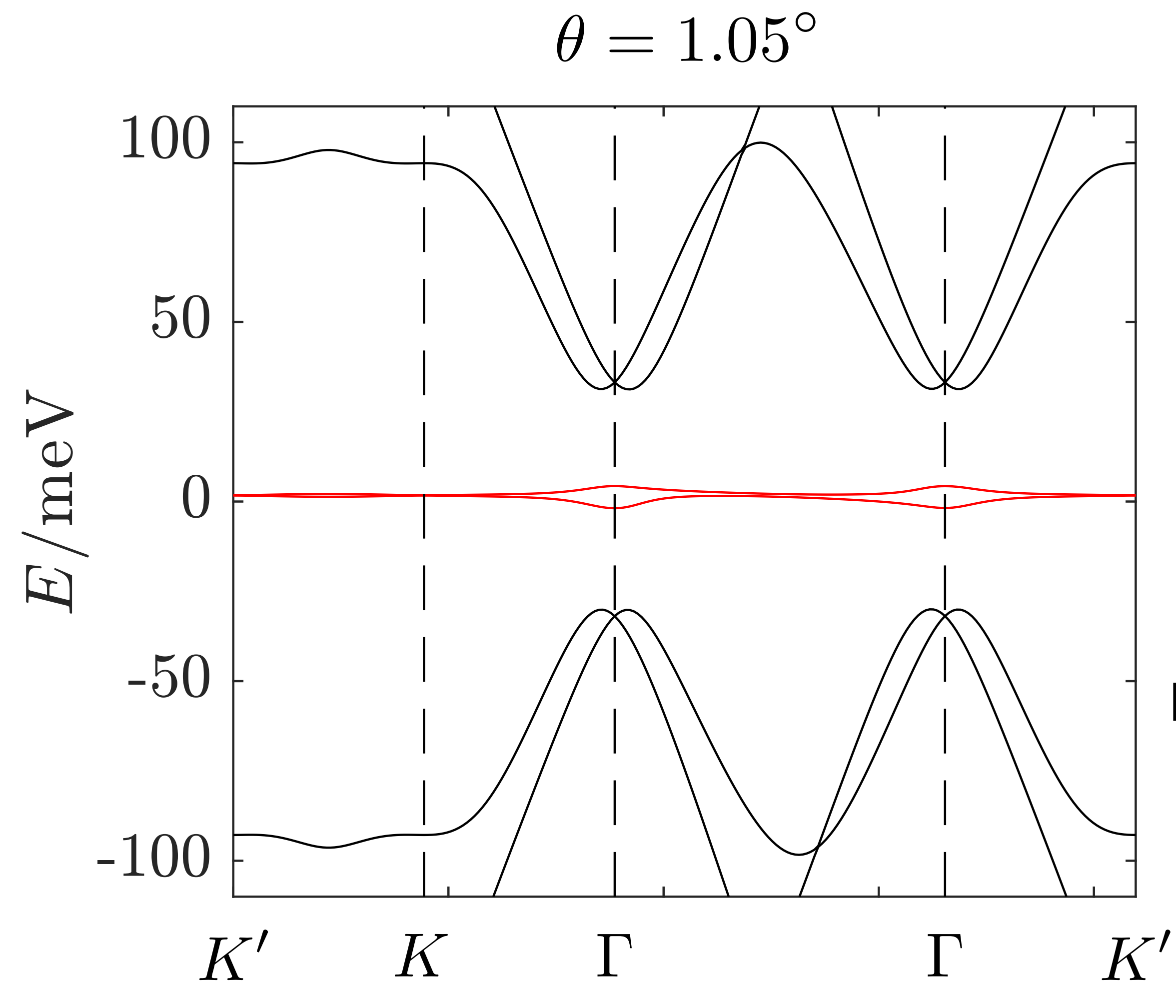
(i) Same chirality nodes

(ii) Landau **+** uɐpuɐ˥

<u>BUT</u>

admits an extended Hubbard model

Magic Angle Twisted Bilayer Graphene @ CNP




Eslam Khalaf


Shang Liu

Liu, Khalaf, Lee, AV, 2019.

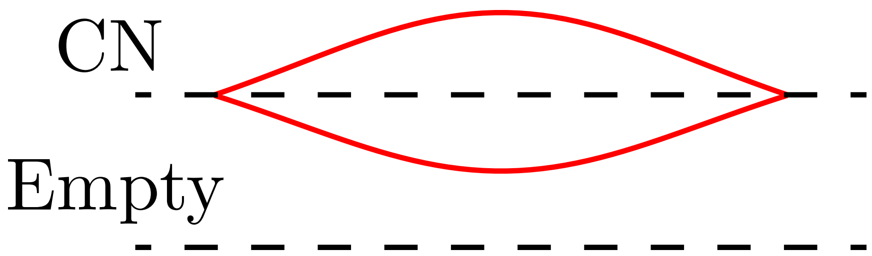
Bultinick, Khalaf, Liu, Chatterjee, AV, Zaletel

arXiv:1911.02045

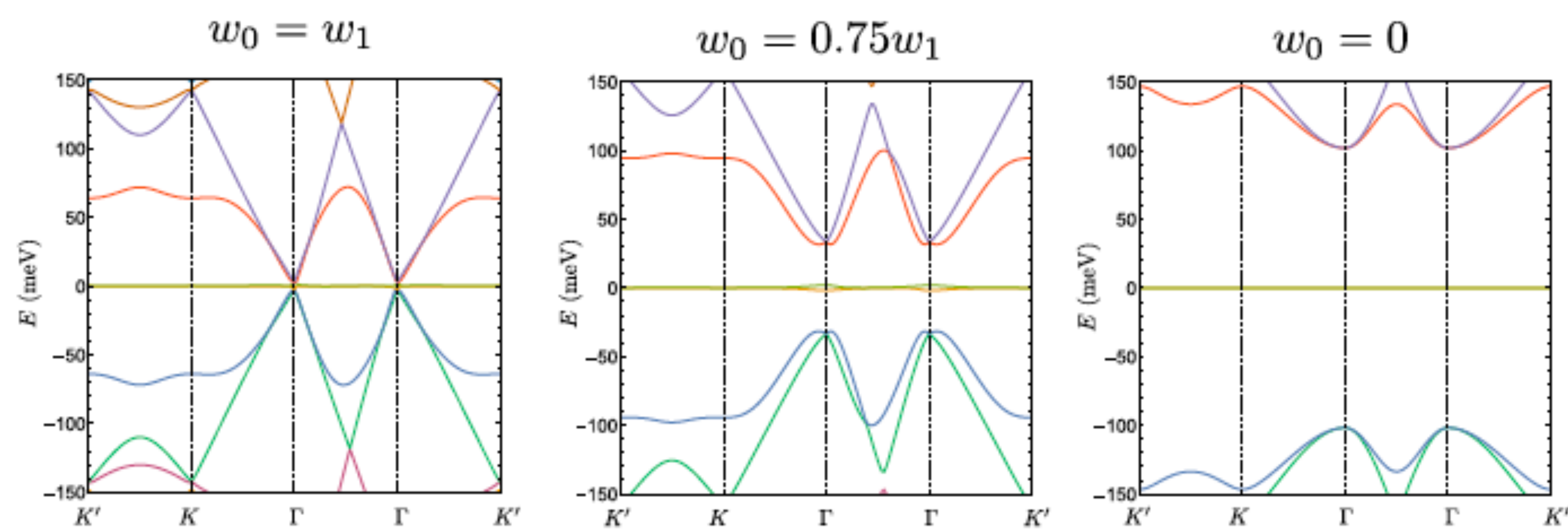
- The C_2T symmetry protects the Dirac points (same chirality).

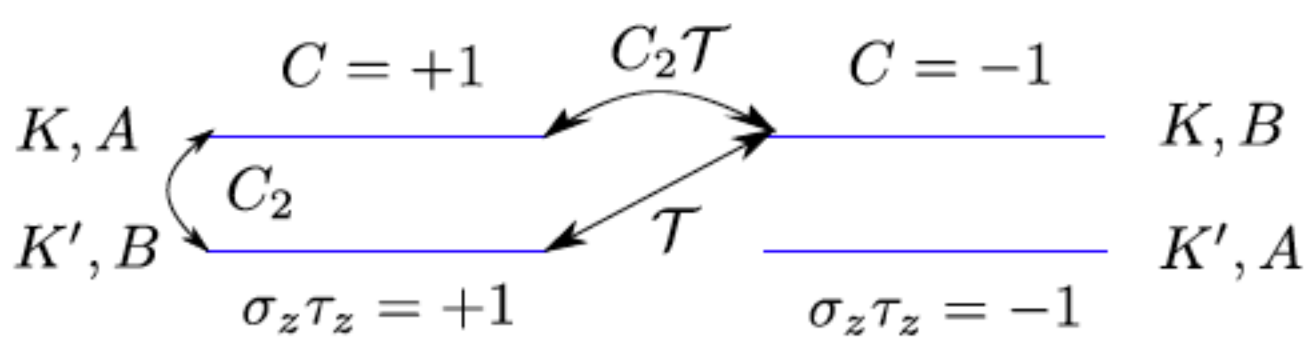
Bistritzer, MacDonald, 2011.
Po, Zou, AV, Senthil, 2018.

- Focus on the two flat bands at CNP. Experiments - insulator/



Simplified Model:
Chiral Limit; Spinless Fermions





Fill two of the four states -but which two?
To be continued…

Conclusions

- 1 particle physics of twisted bilayer graphene is nontrivial, topology & symmetry important to model building.
  - Origin of flat bands - intriguing connection to topology.

- Nature of the Mott insulator and superconductor?
  - Opportunity to understand central questions in solid state physics - ferromagnetism vs anti ferromagnetism, novel superconductors …

Ground State- “Kramers” IVC