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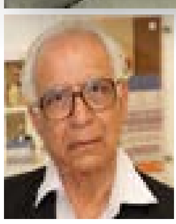
L → R: C. Musili, C.S. Seshadri, M.S. Narasimhan, M.S. Raghunathan and David Mumford at TIFR (1968)

Michael Atiyah lecturing at TIFR

Michael Atiyah during his visit to TIFR, Mumbai, in 1984.



MICHAEL ATIYAH

M.S. NARASIMHAN & C.S. SESHADRI

M.S. NARASIMHAN

Sir Michael Atiyah was a great mathematician who made path-breaking contributions to various fields in mathematics and played a crucial role in promoting the resurgence of contact between mathematics and theoretical physics in the latter half of the twentieth century.

He was the recipient of many honours, including the Fields Medal and the Abel Prize. His work combined creatively Analysis, Differential Geometry and Topology. His major contributions have been to Topological K-Theory (which proved to be a remarkably powerful tool in Topology), Index Theorem for linear elliptic operators and mathematical aspects of gauge theory arising in theoretical physics.

His best known result is the celebrated *Atiyah-Singer index theorem*, which is a vast generalisation of famous results like the Riemann-Roch theorem and which is intimately related to K-Theory. A major related work is the *Atiyah-Patodi-Singer index theorem*, which deals with elliptic operators on manifolds with boundary and non-local boundary conditions. (A version of the Hirzebruch signature theorem for manifolds with boundary)

Atiyah and Bott made an extensive study of the Yang-Mills equation on compact Riemann surfaces and its relationship to the moduli spaces of holomorphic vector bundles on the Riemann surface which have been studied earlier by Seshadri and myself. This work of Atiyah and Bott was very influential and popularised these moduli spaces among physicists.

He had an effervescent and inspiring personality. He valued collaboration in mathematics and loved intense mathematical discussions. He seemed to derive many of his insights and ideas in the process of talking.

I maintained a close and fruitful contact with him over the years and we had several conversations on moduli problems and some aspects of the index theorem. Moreover, many of my best students did their post-doctoral study with Atiyah, the best known example being Vijay K. Patodi. Patodi's stay in Princeton, working with Atiyah, resulted in his celebrated joint work with Atiyah, Bott and Singer. (Atiyah and I later edited the *Collected Papers of Patodi*. In the foreword Atiyah wrote that the work of Atiyah-Patodi-Singer *'was a great collaboration, exploiting the different talents of the participants.'*)

I will mention a few recollections of my mathematical interactions with him. The original proof of the Atiyah-Bott fixed point theorem used a result of Takeshi Kotake and myself on fractional powers of linear elliptic operators. Atiyah asked me to give a talk on this result. On another occasion, during an Arbeitstagung in Bonn, I told him about the canonical filtration which Harder and I had introduced on vector bundles on curves and suggested that it could be of use in the work on Yang-Mills equation on Riemann surfaces he was undertaking at that time with Bott.

His passing is a great loss for the mathematical community. □

C.S. SESHADRI

I had heard of Michael Atiyah as a brilliant student of Hodge when I was a graduate student at the Tata Institute of Fundamental Research (TIFR).

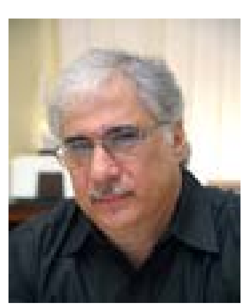
Then I came across his paper on the classification of vector bundles on Elliptic Curves. This attracted me since Grothendieck had classified vector bundles on the projective line and I noticed in my thesis that Grothendieck's result is a consequence of an early work of G.D. Birkoff.

In 1957, I proved that a finitely generated projective module over the ring of polynomials in two variables is free (the first non-trivial case of Serre's problem). I attended the International Congress held at Edinburgh in 1958 and remember conversing with Atiyah during a cruise trip at this Congress. He told me that a British mathematician (whose name I forget) also solved Serre's problem for two variables but found that I had already done this earlier. Atiyah was the star attraction at the International Conference on Differential Analysis in 1964 at TIFR. A few months later, the Narasimhan-Seshadri theorem (NS) was proved. M.S. Narasimhan gave a talk at this conference on an ingredient of NS. Atiyah was among the earliest to perceive the importance of NS and let his student Newstead to calculate the cohomology of certain *"moduli spaces"* figuring in NS.

Newstead got a position in Liverpool and Atiyah arranged for a visit to Liverpool by M. S. Narasimhan and me. Atiyah again perceived the importance of "moduli spaces of parabolic vector bundles" – a work I had done with Vikram Mehta (i.e. the geometry and physics of knots, by Atiyah). I met Atiyah several times during 1975–76, at the Institute for Advanced Study, Princeton, TIFR Colloquium on vector bundles, 1982, sometime in Bonn when he expressed his great regard for Patodi.

It is a great privilege for having known Atiyah and to have had his interest in my work. ■

M. S. Narasimhan and C. S. Seshadri are eminent mathematicians, well known for the famous Narasimhan-Seshadri theorem.They were among the main pillars of the famed School of Mathematics of TIFR, where they spent most of their professional life. Narasimhan later spent some years building mathematics at ICTP and Seshadri founded the Chennai School of Mathematics.



REMEMBERING MICHAEL FRANCIS ATIYAH (1929-2019)

SPENTA R. WADIA

Michael Francis Atiyah, one of the most significant mathematicians of the second half of the 20th century, passed away on 11th January 2019. His profound contributions to topology, geometry and mathematical physics earned him the highest mathematics honours, viz. the Fields Medal in 1966 and the Abel Prize in 2004, which he shared with Isadore Singer. His achievements are well summarised by the Abel Prize citation: "*For discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics.*"

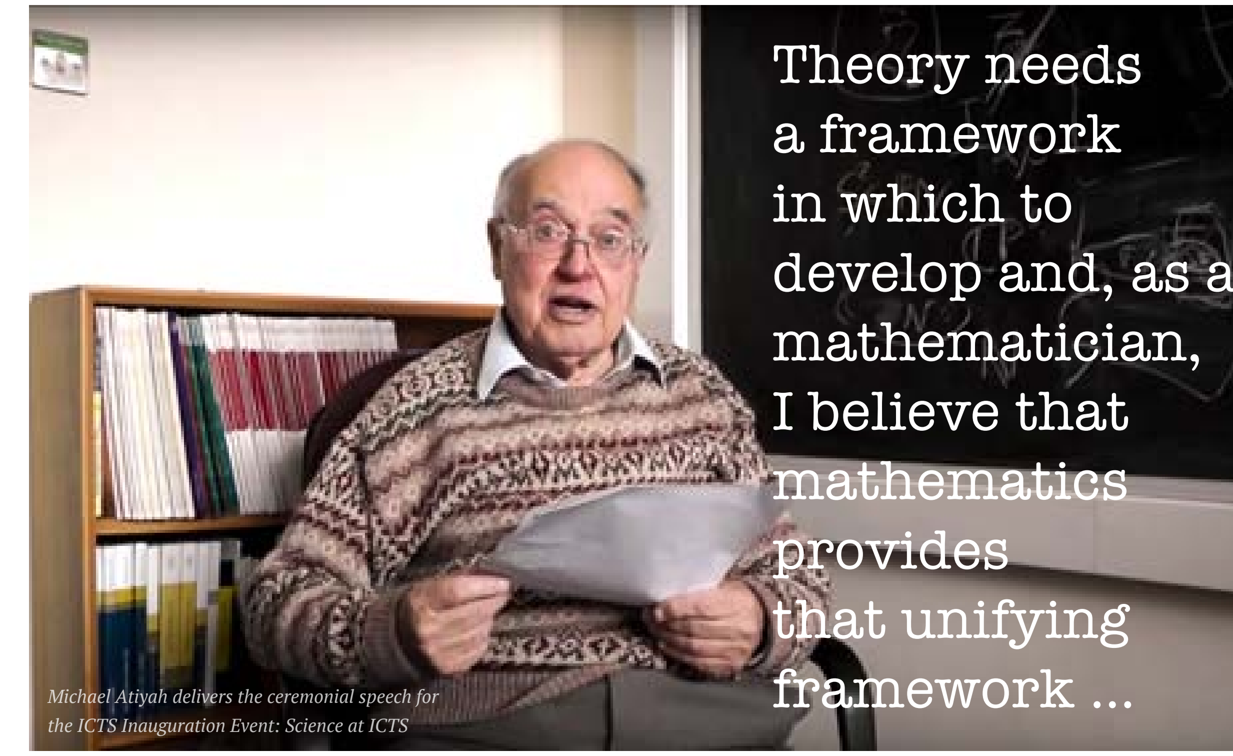
His enthusiasm to intertwine mathematics and physics and in particular quantum physics and emphasise their symbiotic existence was infectious. An example of this, based in part by suggestions from Michael, is Witten's application of Chern-Simons gauge theory to the theory of knots in 3—dimensional topology. String theory embodies this essential unity in a natural way.

During my tenure at the Enrico Fermi Institute, Michael visited the University of Chicago for a couple of months. Someone (I think Nambu) had mentioned about my interests to him. When I met him, he was surprised about my gender because my first name ends in a vowel. He also thought my last name was Arabic!

Michael was interested in my work on the unitary matrix models and the large N phase transition. We used to have several discussions. At some point, to my great surprise, he mentioned that he did not know what a Bessel function was! I found that unbelievable but told him that he did not need to know Bessel functions. I learnt a wonderful lesson here – that one does not need to know 'everything' to do outstanding work.

He was deeply interested in bringing the math and physics communities together to talk and share. His enthusiasm was infectious. He attributed his interest in physics to his collaborator and friend Isa Singer, who also visited UC during Michael's visit. He believed physicists were bubbling with ideas, flying in jet planes that would crash every now and then, while mathematicians were like sure footed old men with walking sticks … proof is the business of the mathematician, the important thing are the ideas that come from physics.

He suggested that I work on a problem of deriving the heat kernel for particle motion on the manifold of the unitary group U(N) using the Feynman path integral. By summing over all classical solutions, both stable and unstable, and including the linear oscillations (Hessian) with appropriate phases one can reproduce the exact answer that can be easily obtained using the method of free fermions. This



Michael Atiyah delivers the ceremonial speech for the ICTS Inauguration Event: Science at ICTS

was a nice problem that taught me that at times in a path integral it is necessary to sum over both stable and unstable classical solutions to obtain the correct answer. I did not publish this result as my PhD mentor Bunji Sakita had emphasized that one should write up only very substantive papers! In retrospect I think it would have been been valuable to publish this simple result.

Michael also played an important role in the establishment of the ICTS. The committee he chaired that reviewed Mathematics at the Tata Institute of Fundamental Research (TIFR) in 2005 made a strong recommendation for an interactive program for physicists and mathematicians. This reinforced the recommendation of the Theoretical Physics review committee in 2006 to establish ICTS in 2007.

He strongly supported the idea of creating the ICTS and was a member of its International Advisory Board from inception till his passing away. His active involvement and advice has been encouraging and invaluable. Commenting on the structure of the ICTS in a letter to me, dated 14 February 2009, he said,

"*I dislike rigid departmental boundaries. They have a habit of perpetuating old demarcations well beyond their 'sell-by date'.*

As you know the most active scientific frontiers are those which cross boundaries. I hope the structure of your Institute will be flexible and adaptable.

In your tentative list of subject areas you allocate 6 faculty in mathematics. Given the widespread application of mathematics in the sciences I find this number quite small. On the other hand I imagine many of the other areas (e.g. string theory) will contain mathematically oriented scientists. So the exact allocation is not important, so long as flexibility and quality are maintained.

Finally, I totally agree that your initial appointments should be made cautiously and not in haste."

He embraced the idea and purpose of the ICTS, which he put forth in his remarks for the Foundation Stone Ceremony of ICTS on 28 December 2009,

"Science has the noble aim of trying to understand the natural world in human terms: to make sense of what we see. This brief phrase encapsulates both theory and experiment. What we see, in the broad sense, covers experiment and making sense is the task of theory. As the great French mathematician Henri Poincare said, science is no more a collection of facts than a house is a collection of bricks: it requires theory to hold it together.

Theory needs a framework in which to develop and, as a mathematician, I believe that mathematics provides that unifying framework …

I am sure that mathematics, in all its various aspects, will play an important part in the future activities of this Center. In the complex modern world with the enormous challenges that we face, from climate change to energy, from poverty to water shortages, science provides the bedrock on which we can build our future. I am sure that this Center will play its part in guiding both India and the wider world in the years ahead."

Michael Atiyah will be remembered with fondness and gratitude by the institution he helped establish and strongly supported, and whose value and relevance he well understood. I end this note with Michael's last address to ICTS on the inauguration of its new campus on 20th June 2015.

"… The fact that I had physics friends is an indication that the frontiers between disciplines were breaking down, and this has become the main feature of our times. The old rigid disciplines of the past are giving way to a much more fluid scene, which is why the ICTS is the right body for the future. The future belongs to the young and the science that is now emerging will affect the lives of everyone on the planet. The ICTS has a noble task, that of providing the right atmosphere to inspire the next generation of scientists …" ■

Read the full text of the speeches here:
https://bit.ly/Atiyah_ScienceAtICTS_Speech
https://bit.ly/Atiyah_ScienceAtICTS_youtube
https://bit.ly/Atiyah_FoundationStoneRemarks

Spenta R. Wadia is a theoretical physicist and the Founding Director, Infosys Homi Bhabha Chair Professor and Professor Emeritus at ICTS–TIFR, Bengaluru

BETWEEN THE SCIENCE

ABHIRUP GHOSH' s submission towards the Augmenting Writing Skills for Articulating Research (AWSAR) Award 2018, organised by the Department of Science and Technology (DST), Government of India, has been selected among the top 100 entries**.**

ADHIP AGARWALA's PhD thesis titled 'Excursions in ill condensed quantum matter: from amorphous topological insulators to fractional spins' has been selected by Springer to be published under the Springer thesis series. Adhip is currently a postdoctoral fellow at ICTS-TIFR. He was a PhD student at IISc, Bangalore.

RUKMINI DEY has been awarded a Core Research Grant (CRG) of the SERB as Principal Investigator by the Department of Science and Technology, Govt. of India.

MANAS KULKARNI has been awarded the SERB Early Career Research award by the Department of Science and Technology, Govt. of India.

2004 Abel Laureates Isidore Singer and Michael Atiyah at the Norwegian Royal Palace, 2004

PHOTOGRAPHER - KNUT FALCH/SCANPIX | www.abelprize.no

THE ATIYAH-SINGER INDEX THEOREM

PRANAV PANDIT

The Atiyah-Singer index theorem is one of the most profound and beautiful mathematical discoveries of the second half of the 20th century. It provides a fundamental link between *analysis* and *topology*. The discovery of this link has led to a tremendous synthesis of ideas, and to synergistic activity bringing together ideas from algebraic topology, differential geometry, functional analysis and operator algebras. The purpose of this expository article is to convey something of the flavor of the mathematics of the index theorem and its ramifications, with a bare minimum of technical detail.

Topology is concerned with the study of those properties of 'shapes' that remain unchanged under continuous deformations. Analysis involves the study of operators, such as the operator of differentiation, that transform one function into another. At its core, the Atiyah-Singer index theorem is a statement relating *the analysis of elliptic partial differential operators to the topology of the space of invertible complex-valued matrices*. It states that the *analytic index* of an elliptic partial differential equation is equal to the *topological index* of its symbol. The analytic index is a quantity that measures the 'number of independent solutions' to a differential equation, while the topological index is a measure of the obstruction to deforming one shape into another. The index theorem allows us to 'count' the solutions to a complicated differential equation without actually finding the solutions themselves. The next few sections attempt to explain the meaning of these terms, and to elucidate the content of the index theorem.

The index of an operator

Consider a system of n linear equations in m unknowns:

$$\sum_{j=1}^{m} a_{ij}x_j = 0$$

$i = 1, \ldots, n$, with complex coefficients a_{ij}. A natural quantity of interest is the (maximal) number of linearly independent solutions to this system of equations, i.e., the dimension $\dim(U)$ of the space U of solutions. However, this quantity is not robust, in the following sense: small perturbations of the coefficients a_{ij} can lead to jumps in $\dim(U)$. We can get around this problem as follows. Let $A = (a_{ij})$ be the $n\times m$ matrix of coefficients, and let U^* be the orthogonal complement to the set of vectors $b = (b_1, \ldots, b_m)$ for which the equation $Ax = b$ has a solution. Alternatively, we could define U^* to be the space of solutions to the equation $A^*y = 0$, where A^* is the conjugate transpose of A. Then we can define the *index* of A to be the quantity

$$\operatorname{ind}(A) = \dim(U) - \dim(U^*)$$

The first term is a measure of the (failure of) *uniqueness* of solutions to the equation $Ax = b$, while the second term contains information about the *existence* of solutions to the same equation. One easily checks that the index does not change under perturbations of A. Indeed, by a linear change of variables $y_j = f_j(x_1, \ldots, x_m)$ (i.e., by performing elementary row and column operations on the matrix A), we can bring the system of equations into the standard form $y_k = 0$, $k = 1, \ldots, r$ for some $r \leq m$. From this one immediately sees that $\dim(U) = m - r$ and $\dim(U^*) = n - r$, and therefore $\operatorname{ind}(A) = m - n$. Thus, the index of A depends only on the number of equations and unknowns, and is completely independent of the coefficients a_{ij}.

In terms of abstract linear algebra, we can express all this as follows. The index of a linear operator $T : V \longrightarrow W$ between finite dimensional vector spaces is defined by

$$\operatorname{ind}(T) = \dim(\ker(T)) - \dim(\operatorname{coker}(T))$$

where $\ker(T)$ is the kernel of T and $\operatorname{coker}(T)$ is the cokernel of T. The matrix A defines a linear operator $T_A : V \longrightarrow W$, where $V = \mathbb{C}^m$ and $W = \mathbb{C}^n$. The index of A as defined above coincides with the index of T_A, and we have $\operatorname{ind}(T_A) = \dim(V) - \dim(W)$, which is independent of A.

The situation becomes much more interesting when there are infinitely many degrees of freedom, i.e., infinitely many equations in infinitely many variables. This means that V and W are infinite dimensional. Then the index is only defined for operators T for which $\ker(T)$ and $\operatorname{coker}(T)$ are finite dimensional. An operator is said to be *Fredholm* if it has this property. Unlike the finite dimensional case, it is clearly no longer possible to give a formula for the index in terms of the dimensions of V and W, and in fact the index does depend on the operator T. However, the robustness of the index still persists in the following form:

FACT: If a Fredholm operator T can be deformed into a Fredholm operator T', i.e., if there is a path connecting T to T' in the space of Fredholm operators, then the index of T equals the index of T'.

We can think of passing to the infinite dimensional case as allowing the matrix $A = (a_{ij})$ above to become an infinite matrix. There is a special class of infinite matrices called *Toeplitz* matrices. These are matrices whose entries are 'constant along diagonals', i.e., matrices of the form

$$A = \begin{bmatrix} c_0 & c_{-1} & c_{-2} & & \cdots \\ c_1 & c_0 & c_{-1} & c_{-2} & \ddots \\ c_2 & c_1 & c_0 & c_{-1} & \ddots \\ \vdots & \ddots & \ddots & \ddots & \end{bmatrix}$$

To a Toeplitz matrix, one can associate its *symbol* which is the function defined by the *Laurent* series

$$\sigma_A(z) = \sum_{n=-\infty}^{\infty} c_n z^n$$

We can view the symbol as a complex valued function on a circle $S^1 \subset \mathbb{C}$ enclosing the origin. It turns out that the operator defined by A is Fredholm if and only if the symbol never takes the value zero. Below, we will see that there is a manifestly topological formula for the index of the operator A in terms of the symbol.

From celestial mechanics to algebraic topology

Switching gears, we turn now to the topological side of the story. In the late 19th century, the French mathematician and physicist Henri Poincaré was interested in the question of the stability of planetary orbits. In order to capture the qualitative behavior of systems of non-linear ordinary differential equations, such as those that describe planetary motions, Poincaré introduced, in 1881, the notion of the *index* at a singular point of a system of two differential equations in two variables. A pair of differential equations

$$\dot{x}_1 = v_1(x_1, x_2)$$

$$\dot{x}_2 = v_2(x_1, x_2)$$

is specified by associating to every point $p = (x_1, x_2)$ in the plane a vector $\vec{v}(p) = (v_1(p), v_2(p)) \in \mathbb{R}^2$. A point p_0 is called a *singular point* of the system of differential equations if the vector $\vec{v}$ vanishes at p_0: $\vec{v}(p_0) = 0$. The index of $\vec{v}$ at p_0 is an integer that is defined as follows. Choose a small loop going once counterclockwise around p_0, such that: (i) $\vec{v}$ does not vanish anywhere on the loop, and (ii) the only point inside the loop where $\vec{v}$ vanishes is p_0. Associating to every point p on the loop the vector $f(p) = \vec{v}(p)$ at that point defines a loop in the space of non–zero vectors, i.e., a map

$$f : S^1 \longrightarrow \mathbb{C}^\times$$

from the circle S^1 to the space $\mathbb{R}^2 - \{0\}$ of non–zero vectors, which we have identified with the set $\mathbb{C}^\times$ of non-zero complex numbers. The index of $\vec{v}$ at p_0 is defined to be the *winding number* of the loop f: the number of times the loop winds counterclockwise around the origin in $\mathbb{C}$. For instance, if the loop winds 4 times counterclockwise and 7 times clockwise around the origin, then the winding number is –3. If the vector $\vec{v}$ represents a magnetic field, then the winding number would be the net number of counterclockwise rotations performed by a compass needle when the compass is taken once around a small loop enclosing p_0.

Poincaré's seminal work "*Analysis Situs*", which appeared in 1895, marks the beginning of modern algebraic topology. There he studies the question of when curves in an arbitrary 'space' can be deformed into one another, introducing what we today call the fundamental group of a topological space. He also introduced the main ideas of homology theory. The simplest of the results in Analysis Situs clarifies the notion of 'winding number' used in Poincaré's previous work on the local index of a differential equation, and can be summarized as follows:

(i) There is an integer $\deg(f)$, called the *winding number* or the degree of f, which measures the "number of times f winds around the origin".

(ii) The integer $\deg(f)$ is stable under small perturbations of f: it does not change if we deform f in a continuous manner.

(iii) Loops f and g in $\mathbb{C}^\times$ can be deformed into each other continuously if and only if $\deg(f) = \deg(g)$. In other words, the degree completely determines the homotopy class of f.

(iv) For any integer n, there is a loop f with $\deg(f) = n$.

A remarkable feature of the degree is that there are a multitude of different ways of describing and computing it:

GEOMETRIC DESCRIPTION
We can give two equivalent descriptions:

- Draw a ray from the origin that intersects f transversally and does not pass through a point of self-intersection of f. Then count, with sign,the number of times the curve f intersects the ray. To assign a sign to each intersection point, we use the "*traffic rule of the right of way*": a point of intersection gets a positive sign if the ray has the right of way and a minus sign if the loop f has the right of way (following Indian traffic conventions).
- Alternatively, replace f by the loop $g := f/|f|$; in other words we project the loop f onto the unit circle $S^1 \subset \mathbb{C}^\times$ by rescaling every vector in $\mathbb{C}^\times$ so that it becomes a unit vector. Approximate g by a nearby differentiable map h. Now fix a point q in the circle such that the derivative of h does not vanish at any point p that is mapped to q by h. Finally, count with sign the number of points that are mapped to q by h. A point p contributes with a positive or negative sign depending on the sign of the derivative of h at p.

COMBINATORIAL DESCRIPTION
Approximate f by a piecewise linear path g. Then we modify g by adding or deleting edges that bound triangles that do not contain $\{0\}$ until g is reduced to a simple form where the winding number is 'obvious'. We will not discuss the details here.

DIFFERENTIAL GEOMETRIC DESCRIPTION
Approximate f by a differentiable function g and then define

$$\deg(f) = \frac{1}{2\pi\sqrt{-1}}\int_{S^1} \frac{dg}{g}$$

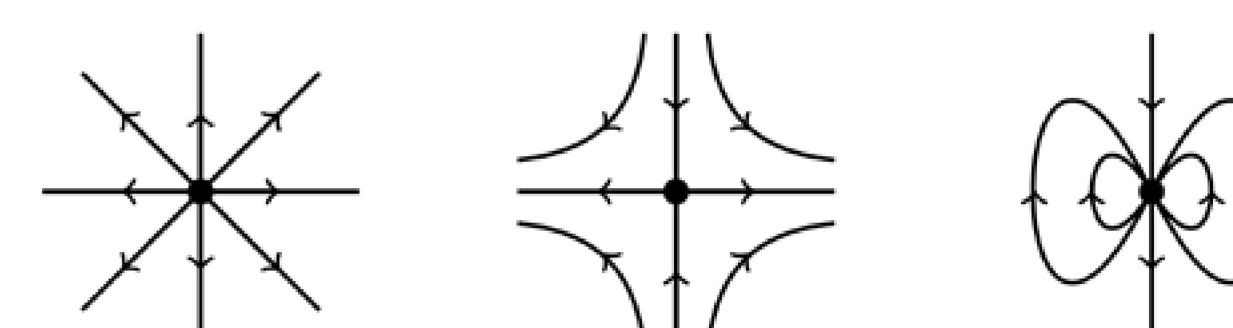


Figure 1: *(From left to right)* Vector fields with index 1, –1, and 2.

ALGEBRAIC DESCRIPTION

Approximate f with a function g whose Fourier series is finite so that we can write

$$g(z) = \sum_{k=-n}^{n} a_k z^k$$

for $z \in S^1 \subset \mathbb{C}$. This formula defines a meromorphic function on the unit disc $\{ z \in \mathbb{C} \mid |z| \leq 1\}$. Then the degree of f is given by the formula

$$\deg(f) = \#\{\text{Zeroes of } g\} - \#\{\text{Poles of } g\}$$

where the zeroes and poles that are counted are those that are contained in the unit disc.

Recall the definition of a Toeplitz operator from the first section. We are now in a position to state our first index theorem, describing the index of a Toeplitz matrix in terms of the winding number of its symbol:

THEOREM (Toeplitz index theorem)

If T is a Toeplitz operator with symbol $\sigma : S^1 \longrightarrow \mathbb{C}$ that is nowhere zero, then T is Fredholm and $\mathrm{ind}(T) = -\deg(\sigma)$.

The Atiyah-Singer index theorem is formally analogous to the Toeplitz index theorem, although it is much deeper. The role of the Toeplitz operator is played by an elliptic partial differential operator. Roughly speaking, the symbol of an elliptic partial differential operator is a higher dimensional analogue of the Toeplitz symbol, with the S^1 being replaced by on odd-dimensional sphere, and $\mathbb{C}^\times$ being replaced by the space of invertible $N \times N$ matrices. The topological index of the symbol of an elliptic differential operator is a higher analogue of the winding number.

The index of elliptic partial differential equations

Interesting examples of Fredholm operators arise from the study of linear partial differential equations. Consider a system of linear partial differential equations on a d-dimensional space X:

$$\sum_{|\alpha| \leq r} a_\alpha (x_1, \dots, x_d) \frac{\partial^\alpha}{\partial x_1^{\alpha_1} \dots \partial x_d^{\alpha_d}} \phi = 0$$

where the sum is taken over multi-indices $\alpha = (\alpha_1, \dots, \alpha_d)$ such that $\sum \alpha_i \leq r$. We may allow $\phi = (\phi_1, \dots, \phi_m)$ to take values in $\mathbb{C}^m$. More generally, we may allow to take values in a vector space E_x that varies with the point $x \in X$. Then we can think of ϕ as a map $\phi : X \longrightarrow E := \coprod_{x \in X} E_x$ such that $\phi(x)$ lies in E_x. We will further assume that for every point x in X there is a neighborhood U with such that $E_U := \cup_{x \in U} E_x$ is identified with $U \times \mathbb{C}^m$ via a map $\psi_U : E_U \longrightarrow U \times \mathbb{C}^m$ compatible with the projections to X; we say that E is trivialized over U by ψ_U. If E is equipped with a collection of trivializations as above, we say that E is a *vector bundle* of rank m on X, and that ϕ is a *section* of E.

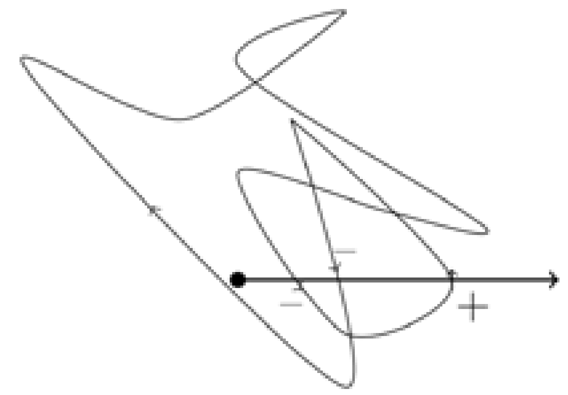


Figure 2: Computing the winding number using the geometric description. This curve has winding number -1 = 1 -1 -1.

A point e in the fiber E_x over x of the vector bundle E is described by co-ordinates $\theta_U \in \mathbb{C}^m$ given by $\psi_U(e) = (x, \theta_U(e))$, for all x in the neighborhood U. The data of the vector bundle E is equivalent to the data of transition functions $g_{UV}(x)$, taking values in $m \times m$ invertible complex valued matrices, which tell us how to change from the 'ψ_V coordinate system' on E_x to the 'ψ_U coordinate system', for $x \in U \cap V$.

If X is a manifold, then an example of a vector bundle on X is obtained by taking $E_x = T_x X$, the space of tangent vectors to X at the point x. This is called the tangent bundle, and denoted TX, and a section of this bundle is called a vector field. Another example is the bundle $\Lambda^k T^\vee X$ of differential k-forms on X, which is obtained by taking E_x to be space of multilinear mappings $(T_x X)^{\times k} \longrightarrow \mathbb{C}$.

If X is equipped with a notion of volume (e.g., if X is a Riemannian manifold), and the vector spaces E_x are equipped with hermitian inner products $\langle -, - \rangle_x$ varying continuously with x, then we can define

$$\langle \phi_1, \phi_2 \rangle := \int_X \langle \phi_1(x), \phi_2(x) \rangle_x \, vol_X$$

The collection of sections ϕ for which $\langle \phi, \phi \rangle$ is finite form an infinite dimensional inner product space (a Hilbert space), denoted $L^2(X, E)$. Given two vector bundles E and F, of ranks m and n respectively, an operator

$$T : L^2(X, E) \longrightarrow L^2(X, F)$$

is a *differential operator of degree r* if

$$T(\phi) = \sum_{|\alpha| \leq r} a_\alpha \frac{\partial^\alpha}{\partial x_1^{\alpha_1} \dots \partial x_d^{\alpha_d}} \phi$$

for all sufficiently *differentiable* sections ϕ, and for some matrix valued functions a_α (strictly speaking, $a_\alpha(x)$ takes values in the vector space $\mathrm{Hom}(E_x, F_x)$ of linear operators from E_x to F_x; in local coordinates, such an operator can be represented by an $n \times m$ matrix). We require that at least one of the a_α with $|\alpha| = r$ is not identically zero.

We would like to 'count' the number of linearly independent solutions to the partial differential equation $T(\phi) = 0$. In general, the space of solutions can be infinite dimensional, so in order for this counting to make sense, we need to impose additional conditions on our equation. This is where *ellipticity* enters the game. To explain this notion, we need to introduce the *symbol* $\sigma_T(x, \xi)$ of the differential operator T, which is the homogeneous polynomialof degree r in d variables given by the formula

$$\sigma_T(x, \xi) = \sum_{|\alpha| = r} a_\alpha \xi^\alpha$$

We say that the differential operator T is *elliptic* if whenever $\xi = (\xi_1, \dots, \xi_d) \neq 0$, the linear transformation $\sigma_T(x, \xi) : E_x \longrightarrow F_x$ is *invertible*. Examples of elliptic operators include the Laplacian Δ acting on functions or differential forms, the Dirac operator (a 'square root' of the Laplacian, which plays an important role in quantum mechanics), and the operator $d + d^*$ acting on differential forms. Here d is the exterior derivative.

Let us further assume that X is compact (it does not have punctures or ends that go off to infinity). The relevance of ellipticity is explained by the following facts:

FACT: If T is elliptic, then it is Fredholm: if U is the space of solutions to $T(\phi) = 0$ and U^* is the space of solution to $T^*(\phi) = 0$, then both U and U^* are finite dimensional, and so the index of T is well-defined.

Since the index of a Fredholm operator is not sensitive to deformations, it is natural to expect that the index has an expression in purely topological terms. This led Israel Gelfand, in 1960, to pose the problem of finding a topological formula for the index. It is this problem that was solved by Michael Atiyah and Isadore Singer with their proof of the index theorem.

The topology of the group of invertible matrices

Returning now to topology, there are several conceivable ways in which we might imagine trying to generalize the ideas about the winding number to higher dimensions. For instance, we might observe that $\mathbb{C}^\times$ can be deformed continuously to S^1, and so the winding number is concerned with the problemof classifying maps $f : S^1 \longrightarrow S^1$ upto continuous deformation. One natural analogue of this in higher dimensions would be the problem of classifying continuous maps $f : S^n \longrightarrow S^m$ between spheres of dimensions n and m, respectively. This is the problem of determining the *homotopy groups of spheres*. This remains one of the deepest open problems in mathematics. Its pursuit has brought into existence entire new disciplines within topology, and led to the discovery of beautiful new mathematical structures over the last several decades.

On the other hand, we could view $\mathbb{C}^\times$ as the group $\mathrm{GL}(1, \mathbb{C})$ of 1×1 invertible matrices with complex entries. From this point of view, a natural generalization would be the study of the space continiuous maps

$$f : S^{n-1} \longrightarrow \mathrm{GL}(N, \mathbb{C})$$

from the $(n-1)$-sphere to the space of $N \times N$ invertible matrices with complex entries. Given how difficult it is to understand the space of continuous maps between spheres, it is quite surprising that this question was almost completely settled by Raoul Bott as early as the 1950's.

THEOREM (Raoul Bott, 1958)

Suppse $2N \geq n$. If n is odd, then each map f as above, from the $(n-1)$-sphere S^{n-1} to the space $\mathrm{GL}(N, \mathbb{C})$ of $N \times N$ invertible complex-valued matrices, can be deformed to a constant map. For n even, there is an integer $deg(f)$ associated with any such map f, such that

– f can be deformed to g if and only if $deg(f) = deg(g)$

– For any integer k there is a map f with $deg(f) = k$.

Just as in the case of the winding number, there are many ways of describing and computing $deg(f)$, drawing on ideas from the different branches of topology.

Now suppose we are given an elliptic partial differential equation given by a differential operator acting on sections of some vector bundle E and taking values in the sections of a vector bundle F. Its symbol defines a map $\sigma(x, \xi) : E_x \longrightarrow F_x$ for each x in X and ξ in the tangent space $T_x X$ (for the experts: we have implicitly identified the tangent bundle with the cotangent bundle by choosing a Riemannian metric). In other words, the symbol can be viewed as a map $\sigma : E' \longrightarrow F'$ between vector bundles living on the tangent bundle TX. Here $E'_{(x,\xi)} = E_x$ and $F'_{(x,\xi)} = F_x$. The ellipticity of the equation is the statement that this map is invertible away from the locus where $\xi = 0$.

If we choose an embedding of our space X inside some standard coordinate space $\mathbb{R}^n$, then we get an induced embedding of tangent bundles $TX \subset T\mathbb{R}^n = \mathbb{R}^{2n} \subset S^{2n} = \mathbb{R}^{2n} \cup \{\infty\}$. Using this, and ideas of Bott and Thom, one can use the data of the symbol $\sigma : E' \longrightarrow F'$ to construct a vector bundle of a certain rank N on S^{2n}. Any vector bundle on a sphere is trivial over the upper hemisphere and over the lower hemisphere, because each of these spaces can be deformed to a point. So any such vector bundle is determined by the transition function $g_{UL} : S^{2n-1} \longrightarrow \mathrm{GL}(N, \mathbb{C})$ that tells us how the coordinates transform on the bundle as we move from the upper hemisphere to the lower hemisphere. Here the S^{2n-1} is the equator of the $2n$-dimensional sphere. Using Bott's theorem on the classication of maps $f : S^{2n-1} \longrightarrow \mathrm{GL}(N, \mathbb{C})$ discussed above, we can assign an integer to this data. This integer is called the *topological index* of the elliptic partial differential equation.

A precise discussion of the topological index involves the notion of *K-theory*. The K-theory $K(X)$ of a space bears to the collection $\mathrm{Vect}(X)$ of vector bundles on X the same relation that integers bear to natural numbers. Just as integers can be viewed as formal dierences $n - m$ of natural numbers, elements of $K(X)$ are formal differences $[E] - [F]$ of vector bundles. K-theory is a vast subject in its own right, and the mathematics of the index theorem was one of the main original motivations for its development. For a more detailed discussion of these ideas, the reader is referred to [Ati67, Ati68].

The index theorem and its applications

We can now state the Atiyah-Singer index theorem:

THEOREM (Atiyah-Singer, 1963)

The analytic index of an elliptic partial differential operator on a compact manifold X is equal to the topological index of its symbol.

As an illustration of the power of this theorem, we mention two important mathematical results that are special cases of the index theorem:

THE GAUSS-BONNET-CHERN THEOREM

This is a fundamental theorem in differential geometry. Suppose a surface is cut into triangles, so that there are V vertices, E edges and F faces. Euler observed that the quantity $V–E+F$ is independent of how we cut the surface into triangles, and does not change under continuous deformations of the surface. This quantity is called the *Euler characteristic* of the surface. The theorem of Gauss-Bonnet expresses the Euler characteristic of a surface as an integral of the curvature of the surface, thus relating the global topological properties of the surface to local geometric features. The Gauss-Bonnet-Chern theorem is a generalization of this theorem to higher dimensions. It can be obtained by applying the Atiyah-Singer index theorem to the elliptic operator

$$d + d^* : L^2 \Omega^{\mathrm{odd}}(X) \longrightarrow L^2 \Omega^{\mathrm{even}}(X)$$

Here d is the exterior derivative, d^* its adjoint, and $\Omega^{\mathrm{odd}}(X)$ the bundle of differential forms of odd degree. The analytic index is easily shown to be equal to the Euler characteristic, and computing the topological index using its differential geometric incarnation (see the discussion on winding numbers above) gives a formula in terms of the curvature of the space.

THE HIRZEBRUCH-RIEMANN-ROCH THEOREM

Algebraic geometry is the study of spaces that arise as solutions to polynomial equations. The Hirzebruch-Riemann-Roch theorem is a cornerstone of modern algebraic geometry. It is a higher dimensional analogue of the Riemann-Roch theorem, which 'counts' the number of analytic functions with a prescribed behavior of zeroes and poles in terms of a topological quantity involving the number of holes in the surface. The relevant elliptic operator here is $\bar{\partial} + \bar{\partial}^*$, where $\bar{\partial}$ is the Cauchy-Riemann operator $\partial/\partial\bar{z}$ acting on a certain bundle of differential forms.

There are several other important applications of the index theorem that we do not have space to discuss here, including the index theorem for Dirac operators which is important in physics, and the Hirzebruch signature theorem, which plays an important role in cobordism theory (which in turn is currently being used to classify phases of matter!).

The richness of the mathematics underlying the index theorem is also reflected in the multitude of proofs that have been given for it. The original proof [AS63] relied on the *cobordism theory* of manifolds. Soon after that, Atiyah and Singer gave their K-theoretic proof, which is closest in spirit to the discussion in this article. Later on, other proofs were discovered that relied on a study of the heat equation, the equation that describes the flow of heat. Yet another argument is based on ideas from quantum field theory. For a detailed discussion of all this and more, we refer the reader to the book [BB85], and the references therein.

Conclusion

The Atiyah-Singer index theorem reveals fundamental connections between analysis, topology, algebra and physics. Drawing on a diverse and eclectic set of ideas, yet underpinned by simple unifying principles, it draws our attention to the underlying unity of mathematics and science. It has provided the impetus for cross-fertilization between subjects and the creation of entire disciplines, radically transforming the mathematical landscape in the process. In all of these respects, it epitomizes much of Atiyah's work. The deep ideas underlying the index theorem will undoubtedly continue to play an important role in the mathematical explorations of the 21st century. ■

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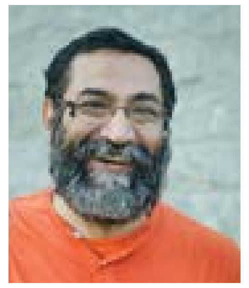


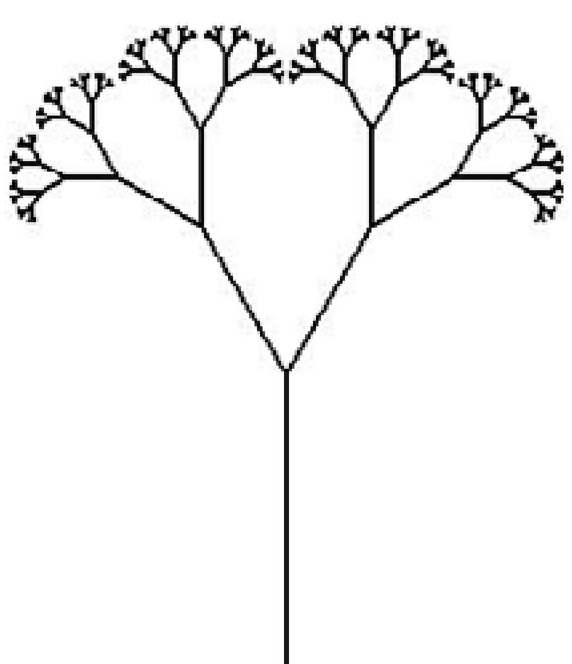
HYPERBOLIC GEOMETRY AND CHAOS IN THE COMPLEX PLANE

MAHAN MJ

Hyperbolic Geometry in Nature

Perhaps the quickest way to introduce hyperbolic geometry is to say that it is the geometry underlying trees. Here is a picture of a tree with the foliage removed to expose the underlying structure. (Somewhat unkindly to the tree, the geometer treats foliage only as clutter.)

Let us move from this picture to a more formal mathematical representation where the underlying geometry becomes clearer:

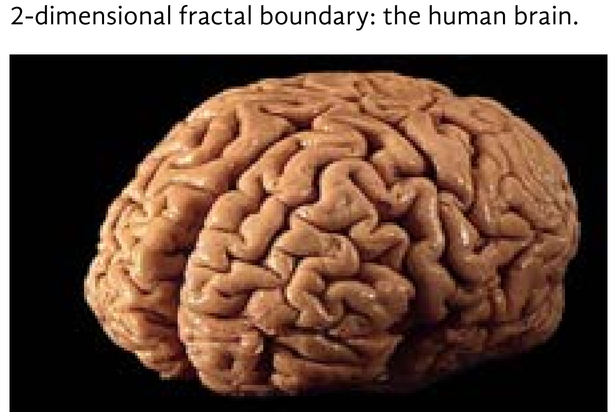


Note that in order to draw the above tree in the plane we are forced to draw the branches shorter and shorter as we go further and further away from the root. This is not an accident, nor our inefficiency in drawing. It illustrates the fact that the intrinsic geometry of the tree is at variance with the geometry of the flat Euclidean space in which it is drawn.

Note also that the branches become shorter and shorter and finally converge to a dust, which has a self-similar structure at all scales. Such self-similar objects are called fractals. The specific structure at the boundary of the tree is called a Cantor set and is one of the simplest fractals that we can think of. It is 0-dimensional, in the sense that its connected components are points. Thus we see that when we try to force the hyperbolic geometry of a tree into flat Euclidean space, a fractal in one dimension less naturally arises at its boundary.

The tree is locally a one dimensional object and so its boundary is 0-dimensional. However, this phenomenon is not restricted in terms of dimension. Here is a 2-dimensional object with a 1-dimensional fractal boundary.

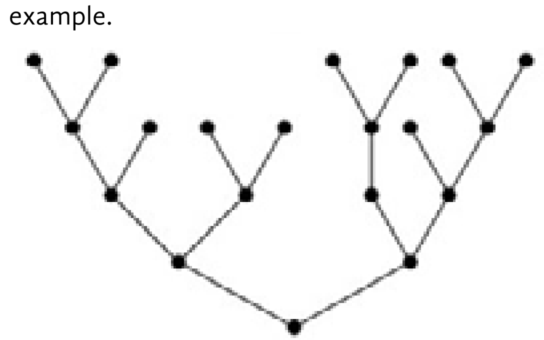
And finally a 3-dimensional geometric object with a 2-dimensional fractal boundary: the human brain.



There is a natural phenomenon that underlies all the above examples. The tree is trying to maximize its surface area of contact with light. The sponge/sea-anemone pictured above is trying to maximize its surface area of contact with the surrounding water so as to have the best chance of getting food. Hence it is trying to divide/replicate itself as fast as possible. The brain too is trying to develop fast neuronal connections and hence trying to solve a 'maximization of surface area' problem. Whenever there is such an instance of trying to maximize the area of the boundary inside Euclidean space, hyperbolic geometry becomes the natural geometry of the object. And a tell-tale signature is a fractal of one lower dimension at the boundary. The reader is probably familiar with the opposite problem of trying to minimize surface area where spherical geometry naturally results.

Hyperbolic Metric Spaces and Groups

Having illustrated the occurrence of hyperbolic geometry in nature, we shall now furnish a more formal mathematical approach. First, a metric graph is a collection of vertices and unit length edges connecting them. A tree is a connected graph without any closed loops. See below for an example.



Graphs (finite or infinite) are examples of geodesic metric spaces, where any two points can be joined by a distance minimizing path. The length of the shortest path for a graph is simply the smallest number of edges one needs to cross to go from the initial to the final vertex. The shortest paths between two points are called geodesics and are the analogs of straight lines in Euclidean geometry. If there are three points a, b, c and we draw the geodesics [a, b], [b, c], [c, a] we obtain the triangle $\Delta(a, b, c)$.

Trees are special amongst these. What do triangles $\Delta(a, b, c)$ look like in trees? Staring at the above tree and picking 3 vertices a, b, c on it, we see that triangles $\Delta(a, b, c)$ look like a Y , possibly after moving it around in Euclidean space, where a, b, c form the extremities of the Y . Thus for any triangle $\Delta(a, b, c)$ in a tree, any side [a, b] is contained in the union of the other two sides. This provides us the motivation to say abstractly what a hyperbolic metric space is. Gromov defined a hyperbolic metric space to be one where triangles are thin or skinny, i.e. they are tree-like with a bounded amount of error.

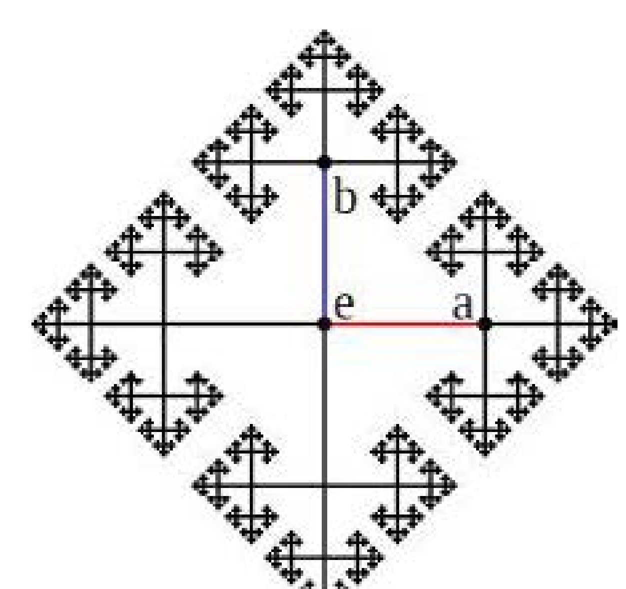
Trees give us the skinniest triangles possible and are examples of 0–hyperbolic geodesic metric spaces. We now introduce the next player in the game: symmetries. The mathematical structure we shall be using to formalize symmetries will be the notion of discrete groups. Imagine finitely many symmetries of a space and all their permutations and combinations. Then the collection of all the resulting symmetries is called a group and the finitely many we started with are called generators. There might be relations between the generators, too. This gives rise to a formal description

$G = \langle a_1, \cdots, a_n | r_1, \cdots, r_m \rangle$,

of a finitely presented group, where ai are the generators and ri the relations.

Hyperbolic Geometry and Fractals

From this purely algebraic structure, a geodesic metric space (a geometric object) can be constructed quite naturally as follows. The Cayley graph ΓG of G with respect to the finite generating set $\{a_1, \ldots, a_n\}$ has vertex set $\nu = \{g | g \in G\}$ and edge set ε consisting of pairs $\{(g, h)\}$ differing by a generator. Note that each element in the group comes with its bete noire, the symmetry that reverses its action. The latter is called the inverse of the original element. As a first example, let us consider the Cayley graph of the free group F_2 on 2 generators–n = 2 and s = 0. Note here that F_2 consists of formal words in the two generators a_1, a_2 and their inverses with the only pro viso that a generator is not followed by its inverse. Thus we avoid acting by a symmetry and then immediately undoing it. The following diagram describes the Cayley graph of F_2. Note that it is a tree where every vertex has 4 edges incident on it.



Thus ΓF_2 is a tree with respect to the above generating set. In particular, the Cayley graph ΓF_2 is hyperbolic. Gromov [Gro85]) defined a finitely generated group $G = \langle a_1, \cdots, a_n | r_1, \cdots, r_m \rangle$ to be a hyperbolic group if its Cayley graph with respect to some finite generating set is a hyperbolic metric space.

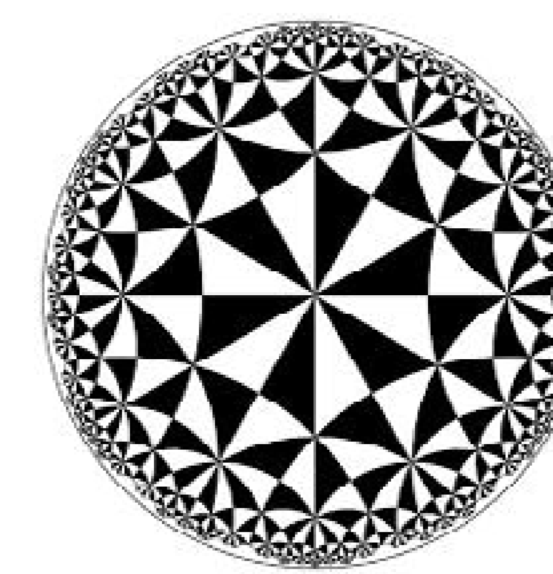
The basic fact that makes the theory take off is that if a group is hyperbolic with respect to some finite generating set, it is hyperbolic with respect to any finite generating set. Thus hyperbolicity is a property of the group, not the particular generating set we choose to construct its Cayley graph.

Any such hyperbolic group has a boundary, just as the boundary of a tree is a Cantor set. Further, boundaries of hyperbolic groups have a natural fractal structure, which is largely independent of the generating set.

We furnish another, more picturesque, example:

$G = \langle a, b, c | a^2, b^2, c^2, (ab)^2, (bc)^4, (ca)^6 \rangle$

These symmetries were used by Escher as the backbone of a number of his paintings.



G is generated by reflections in a hyperbolic triangle with angles $\pi/2$, $\pi/4$, $\pi/6$ in the standard disk model of the hyperbolic plane with metric on the open unit disk given by

$$ds^2 = \frac{4}{(1-r^2)^2} \, ds^2$$

where ds^2 is the Euclidean metric.

The Relative Problem

We now come to the relative version of the above theory. Any group comes with natural subcollections of symmetries called subgroups. Let $H \subset G$ be a hyperbolic subgroup of a hyperbolic group. Thus H is a sub-object of G algebraically (being a subgroup) and geometrically (being hyperbolic). Then there is a natural inclusion $i : \Gamma H \to \Gamma G$ of Cayley graphs. Thus i gives a natural inclusion of both the algebraic structure (since H is a subgroup of G) and the geometric structure (ΓH is a subgraph of ΓG). What happens at infinity, at the boundary? How do the fractal boundaries interact? We make this question precise as follows?

Question 3.1

Does i extend to a continuous map between the fractal boundaries ∂H and ∂G?

The answer to the above question is 'No' in this generality and a counterexample was found recently by Baker and Riley [BR13]. But an analogous (and much more classical) problem arises when a hyperbolic group G acts by symmetries (isometries) on H^3, the 3 dimensional hyperbolic space given by $H^3 = \{(x, y, z) : z > 0\}$ equipped with metric. The boundary of H^3 is $\mathbb{C} \cup \{\infty\}$, i.e. it is the complex plane (plus a point at infinity).

The group of (orientation-preserving) isometries of H^3 agrees exactly with the Möbius group Mob $(\hat{\mathbb{C}})$ given by the group of fractional linear transformations $z \to$ of the Riemann sphere $\hat{\mathbb{C}}$. Thus we are looking at a discrete subgroup G of

$Mob(\hat{\mathbb{C}}) = PSL^2(\mathbb{C}) = Isom+(H^3)$.

For the quotient hyperbolic manifold $M^3 = H^3/G$, the group G appears as its fundamental group. The sphere $S^2 = \hat{\mathbb{C}}$ appears as the 'ideal' boundary of H^3. Thus we have two different ways of studying discrete subgroups G of $Mob(\hat{\mathbb{C}})$:

1) Dynamically, in terms of the action of G on $\mathbb{C}$,

2) Geometrically, in terms of the geometry of the quotient hyperbolic 3-manifold M.

In a prescient paper [Thu82], Thurston conjectured the existence of an exact dictionary between

1) The dynamics of G on $S^2 = \hat{\mathbb{C}}$ (the Fractal side of the picture).

2) The geometry of M^3 (the Hyperbolic Geometry side of the picture).

The combined work of a number of authors [Thu80, Min02, BCM04, Mj14a, Mj14b] has established Thurston's conjecture so that we can say now:

Chaotic dynamics on the boundary Riemann sphere determines and is determined by the geometry of M. ■

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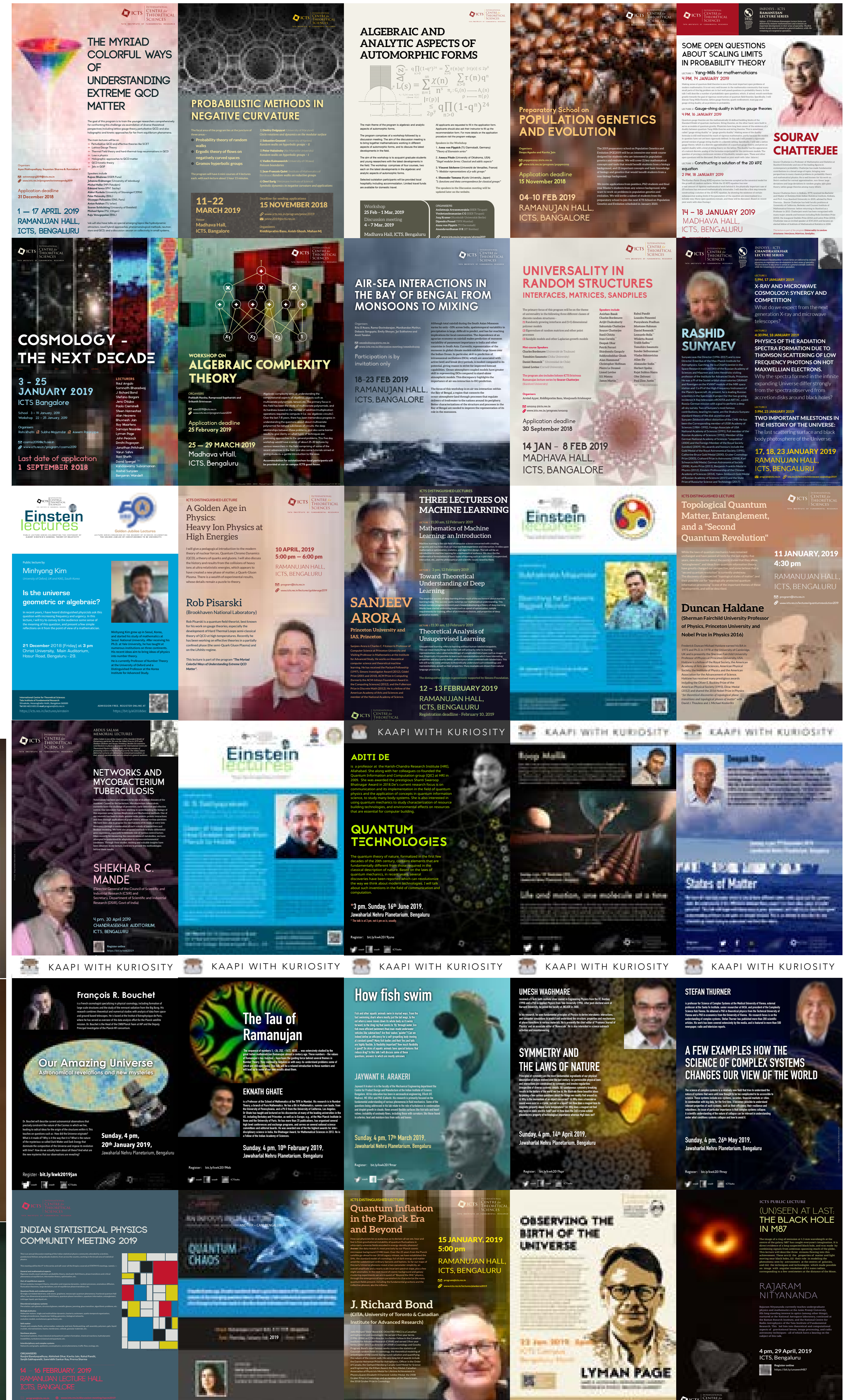

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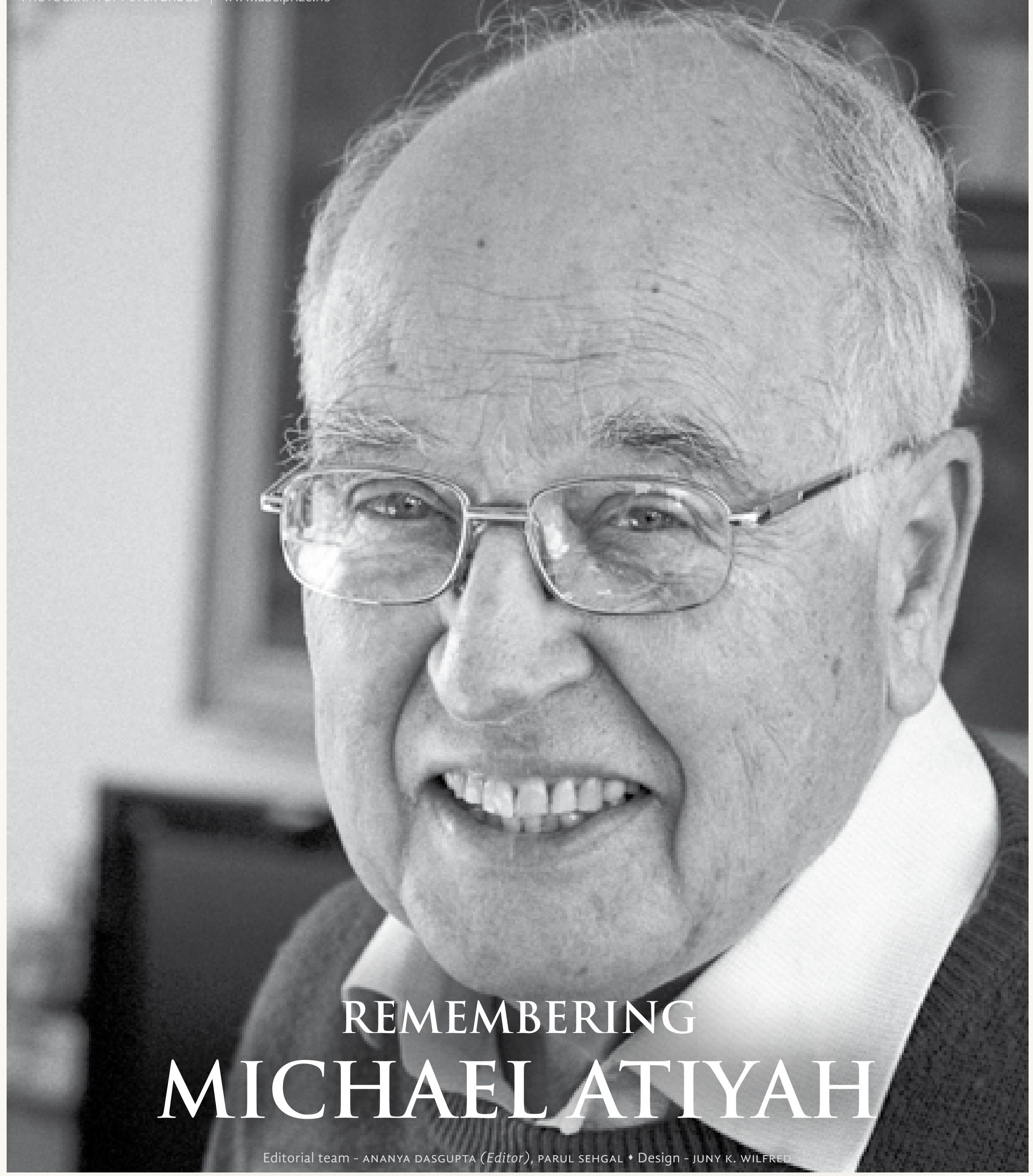

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