Topology and Entanglement in Quantum Matter

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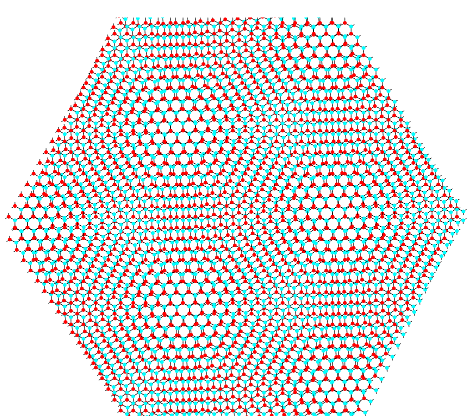
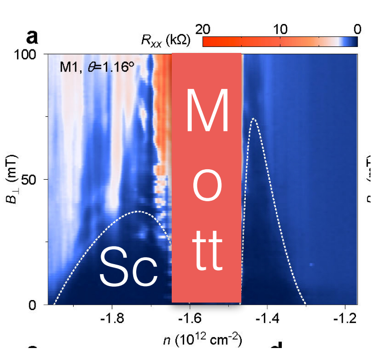
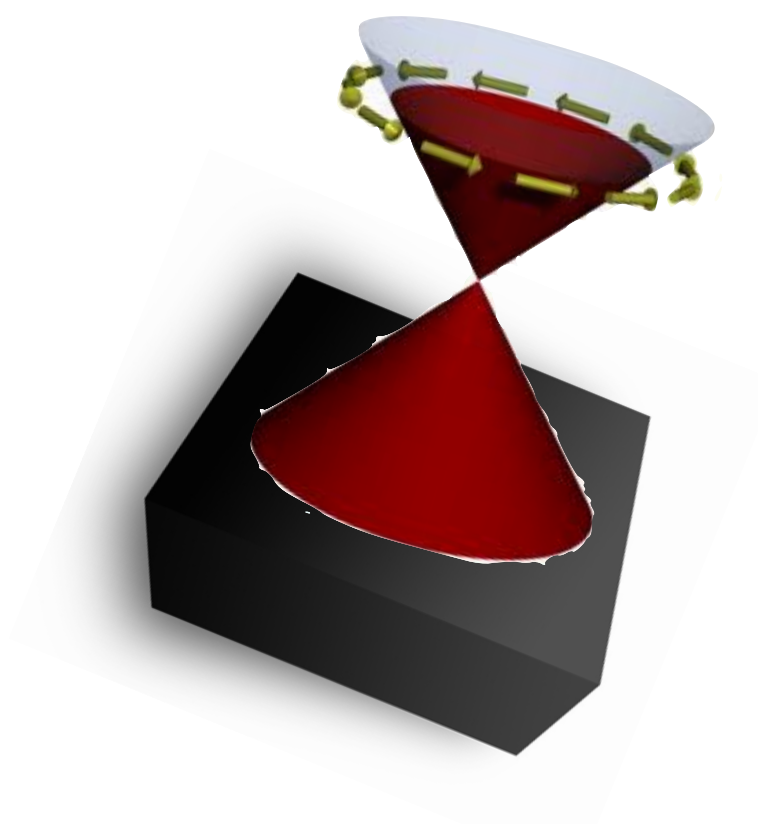
Overview

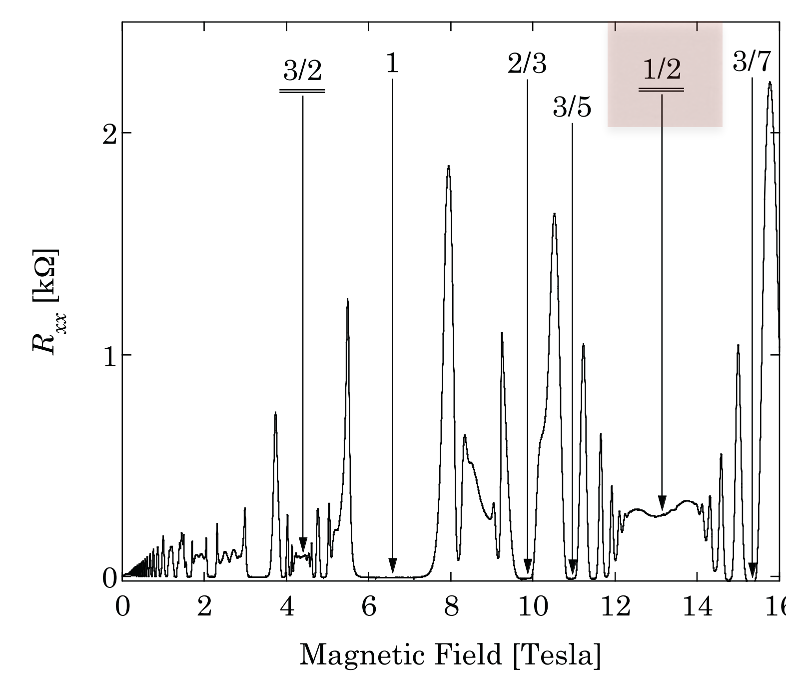
TOPOLOGY → Classify States of "*Quantum*" Matter ← ENTANGLEMENT

Applications

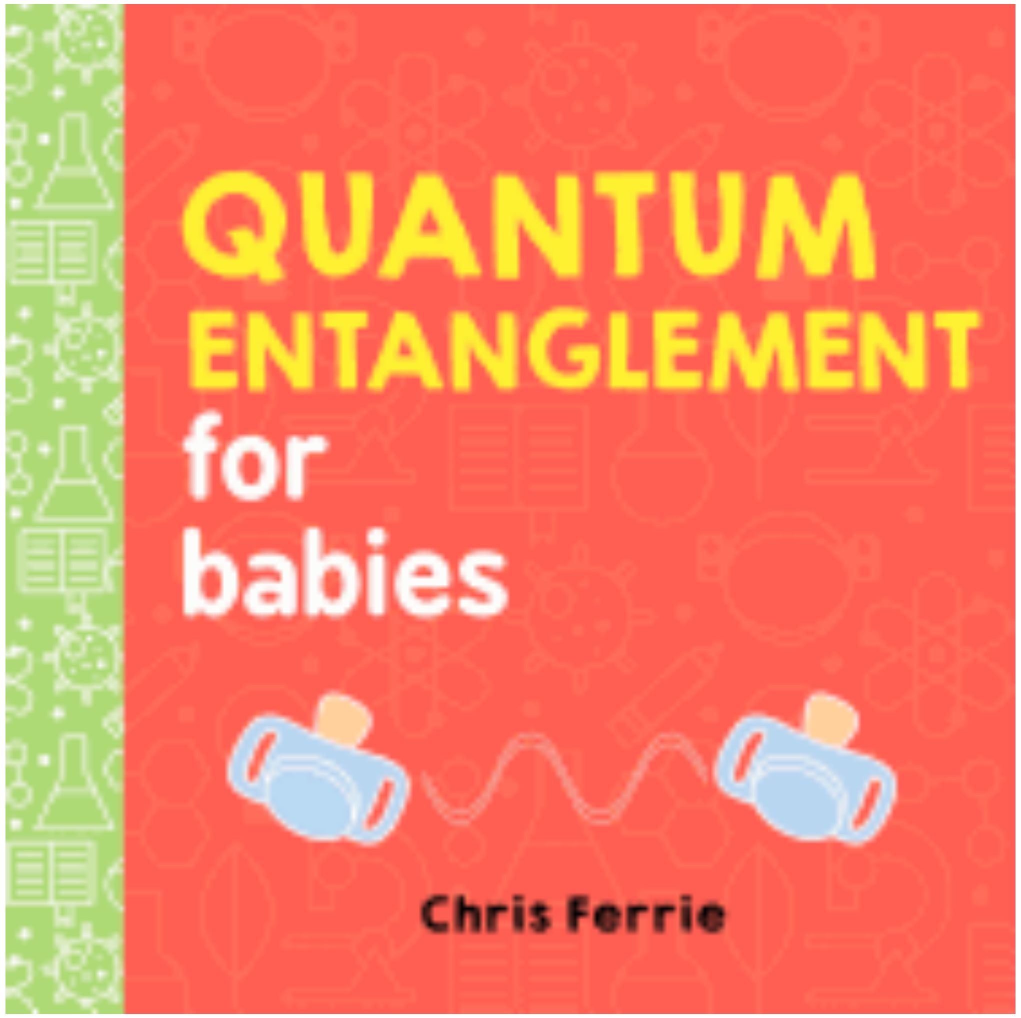






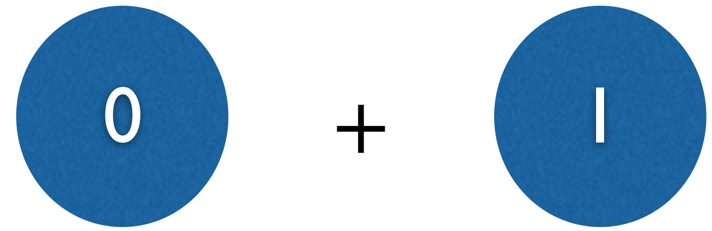


Quantum Entanglement



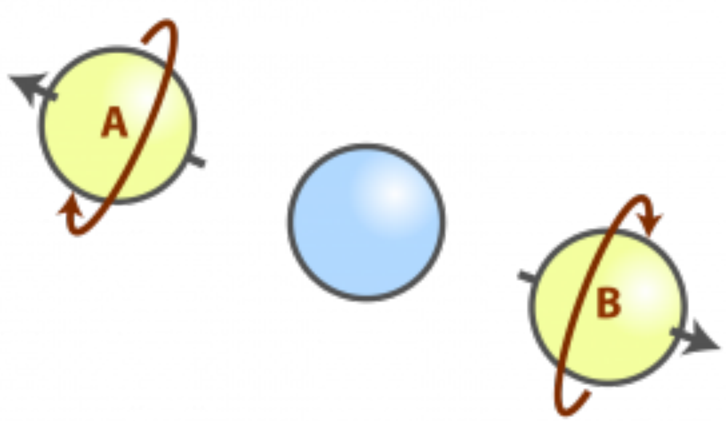
Quantum Superposition and Entanglement

Quantum Superposition:

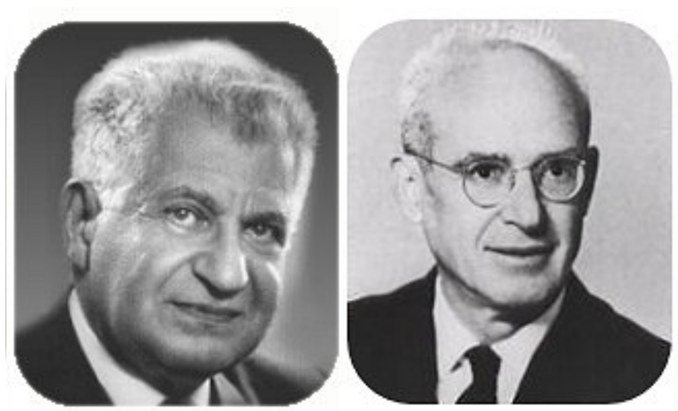


$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Quantum Entanglement:



$$\frac{1}{\sqrt{2}}\left[|0\rangle|0\rangle + |1\rangle|1\rangle\right]$$



B. Podolsky N. Rosen

Measure one particle: 50-50

BUT the other particle state is fixed.

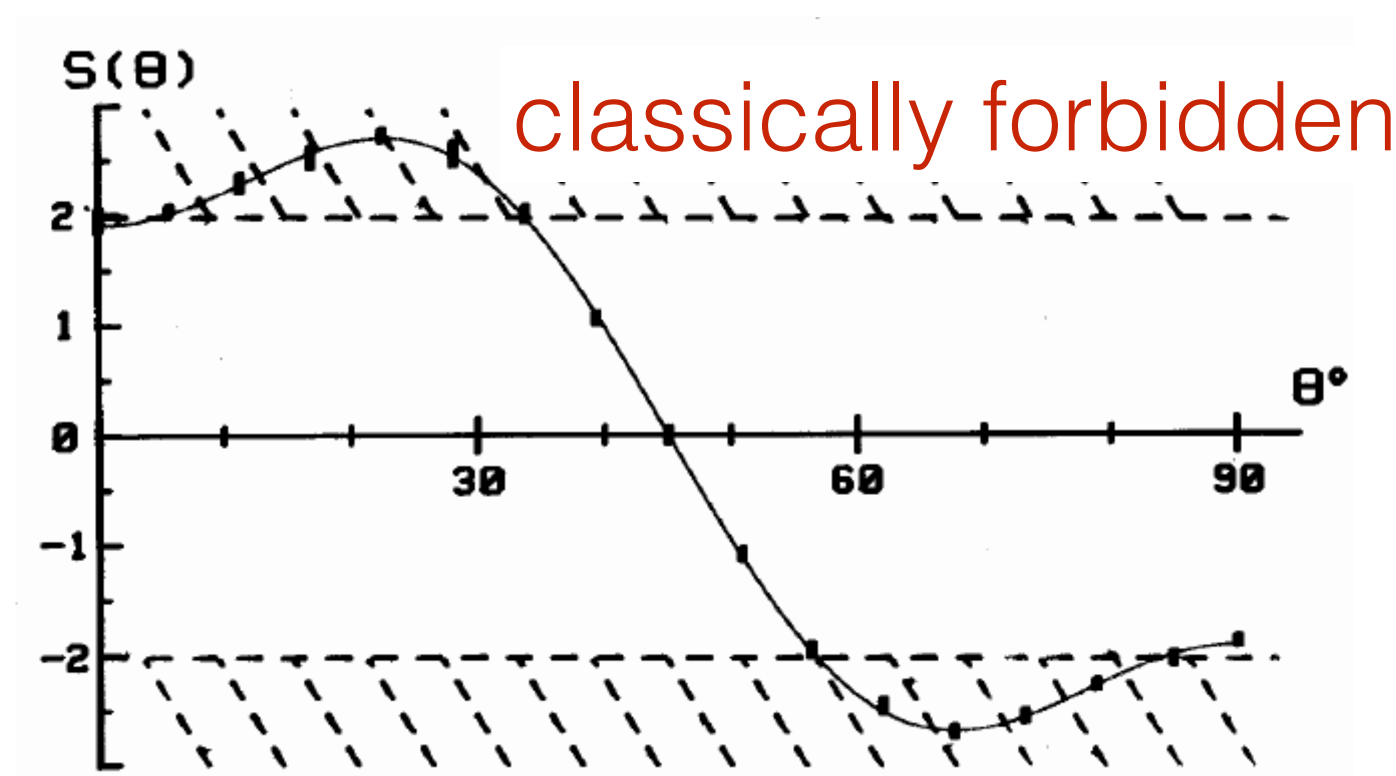
Quantum NonLocality

Classical Hidden variables?

Bell Inequality

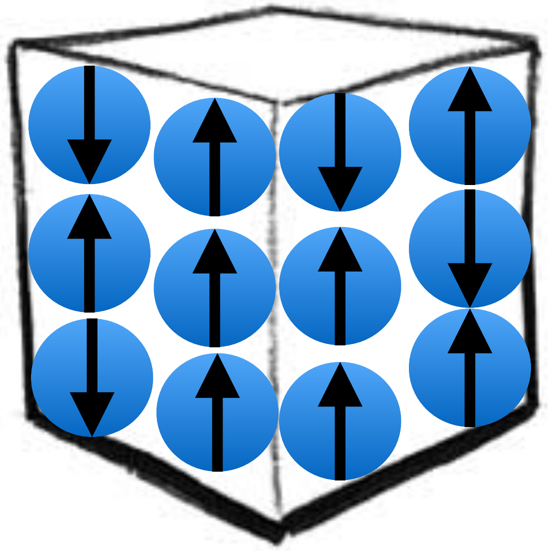
No local
`reality' can reproduce
measured correlation.

Quantum 

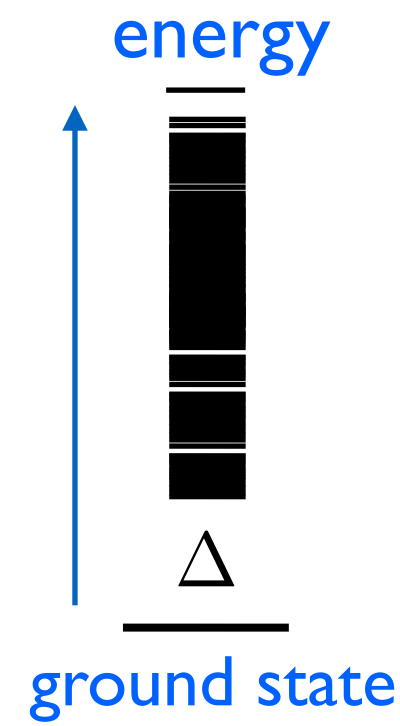


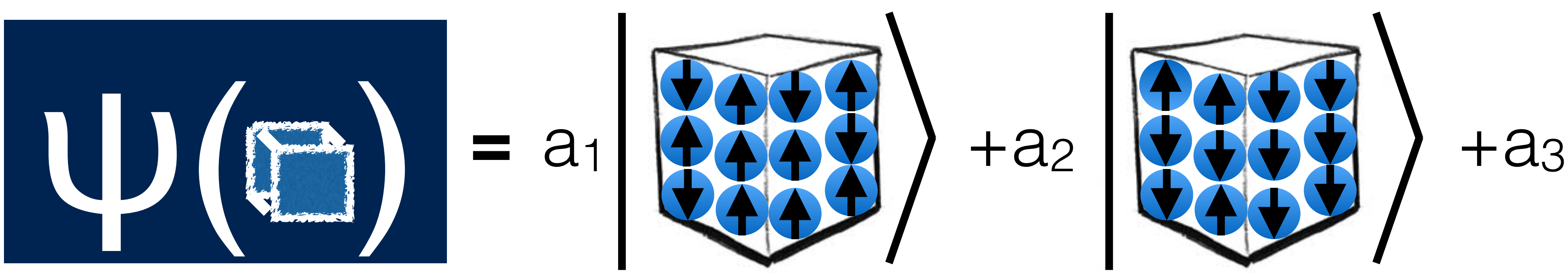
Aspect et al. PRL 1982

Quantum Mechanics *of Many Particles*



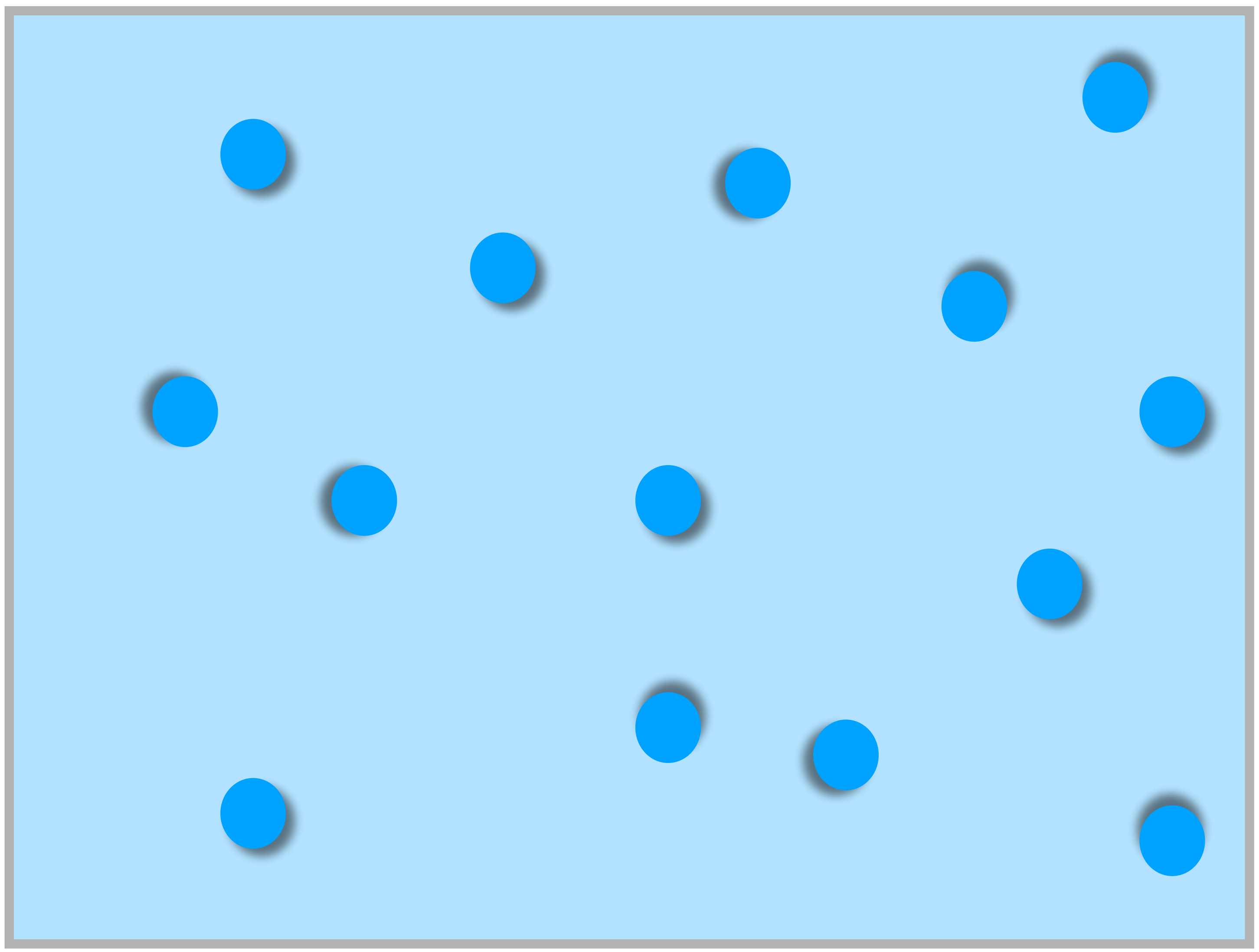
number of Spins: 10^{20}





- complex number for each configuration. $2^{10^{20}}$!
- How do we approach this complexity?

one, two, three … Infinity



Emergent properties of quantum systems?

<u>Emergent Properties:</u>

Only appear in the limit of *many* particles

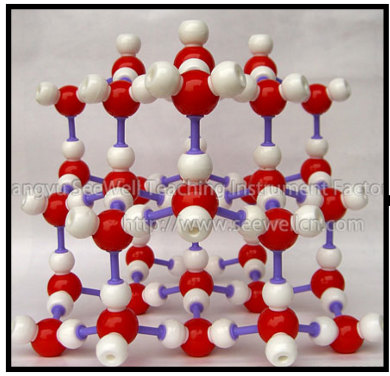
N -> ∞

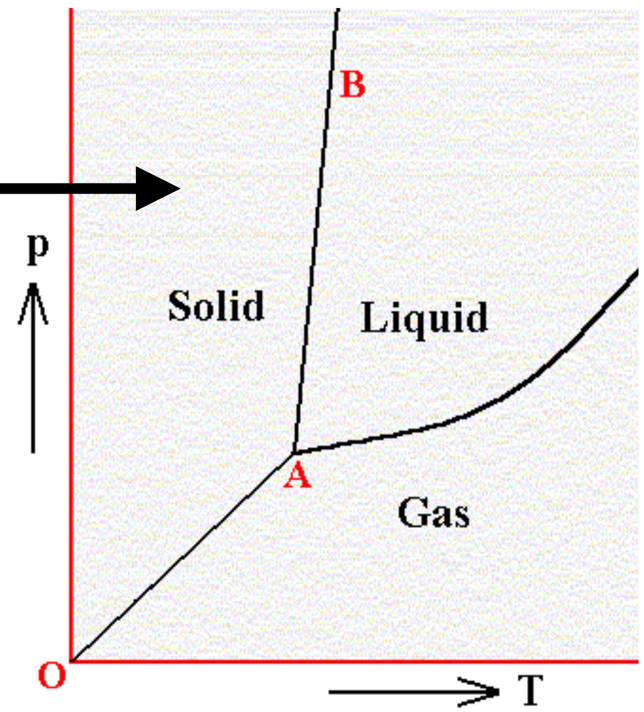
<u>Simple examples:</u>

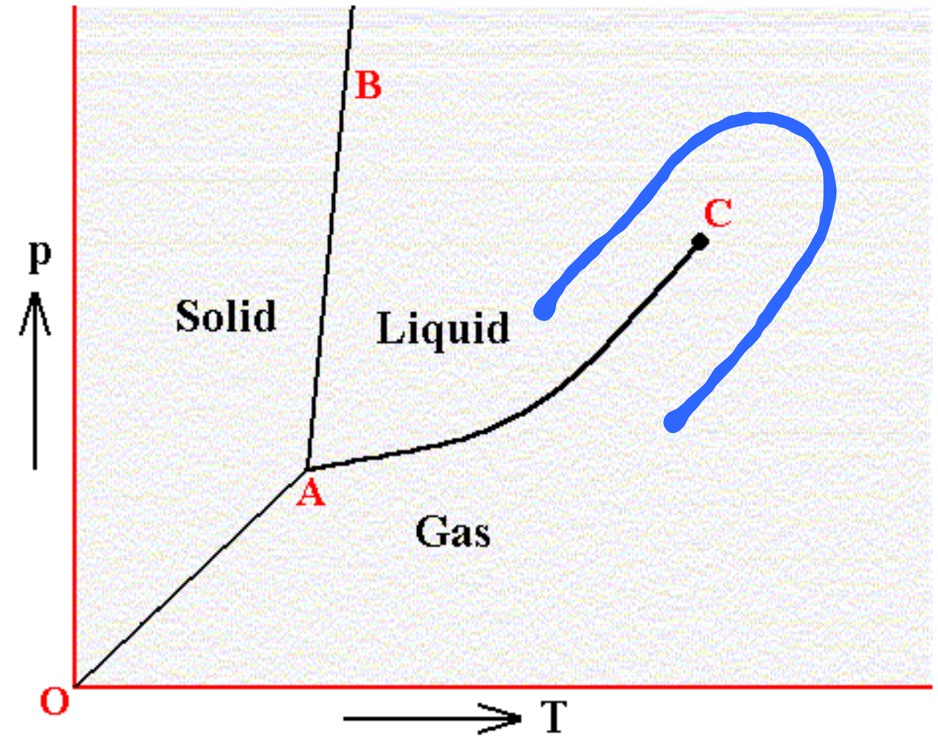
Temperature

Pressure

Classifying Phases of Matter



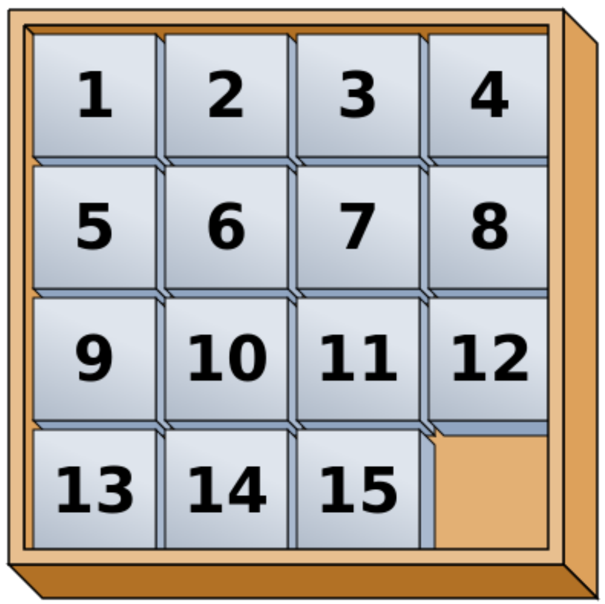


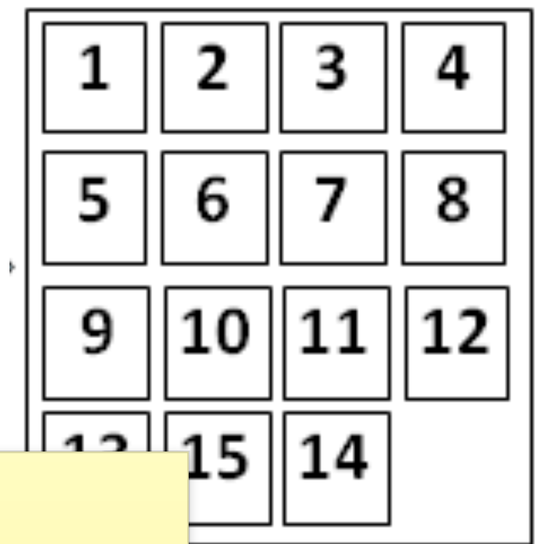


Many particles - new properties.

- Phases of Matter. N=>∞
- Separated by phase transition

Classifying Phases - An Analogy





You can show that the following quantity is an invariant:

[Parity of the sequence of numbers + row of the blank square] (mod 2)
Proof:
Moving the blank square horizontally does not change anything, but moving it vertically potentially changes parity of 3 pairs. (parity is number of wrong order pairs). Always switches parity.

- Can they be
- NO!
  - parity of pe… ved by moves.
  - Two configurations have opposite parity.

Classifying Phases of Matter

Crystal

Ferromagnet

Superfluid

- Phases of Matter *from* spontaneous symmetry breaking. *Classical* order parameter.
- Classify phases - different ways to break symmetry (230 types of crystals. *All* realized in nature!).
- Until experiments in 1980…

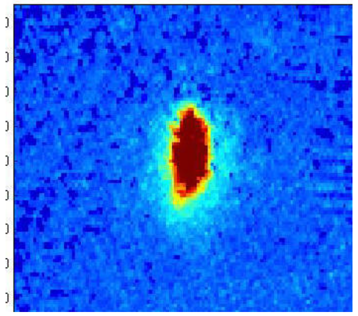
** Some exceptions eg. metals*

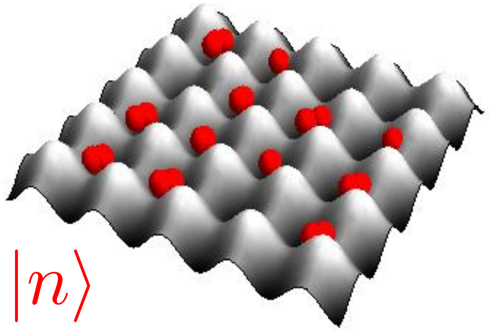
Entanglement of Ordered Phases

- With many particles - many patterns of entanglement possible.
- However ordered phases - *essentially* simple product states. Quantum entanglement inessential.

Ferromagnet

Superfluid

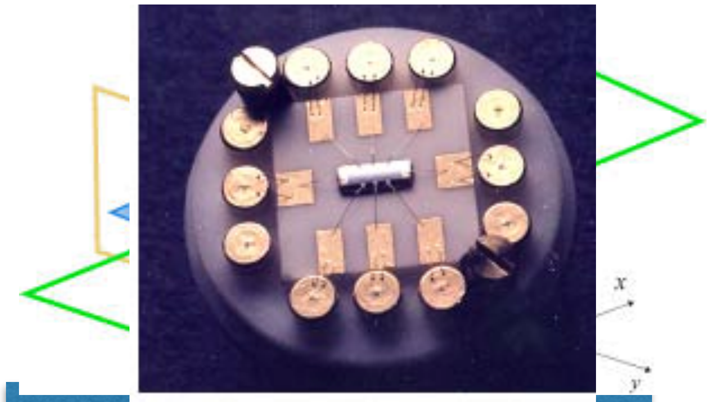




$$| \uparrow \rangle \otimes | \uparrow \rangle \otimes | \uparrow \rangle \otimes \ldots$$

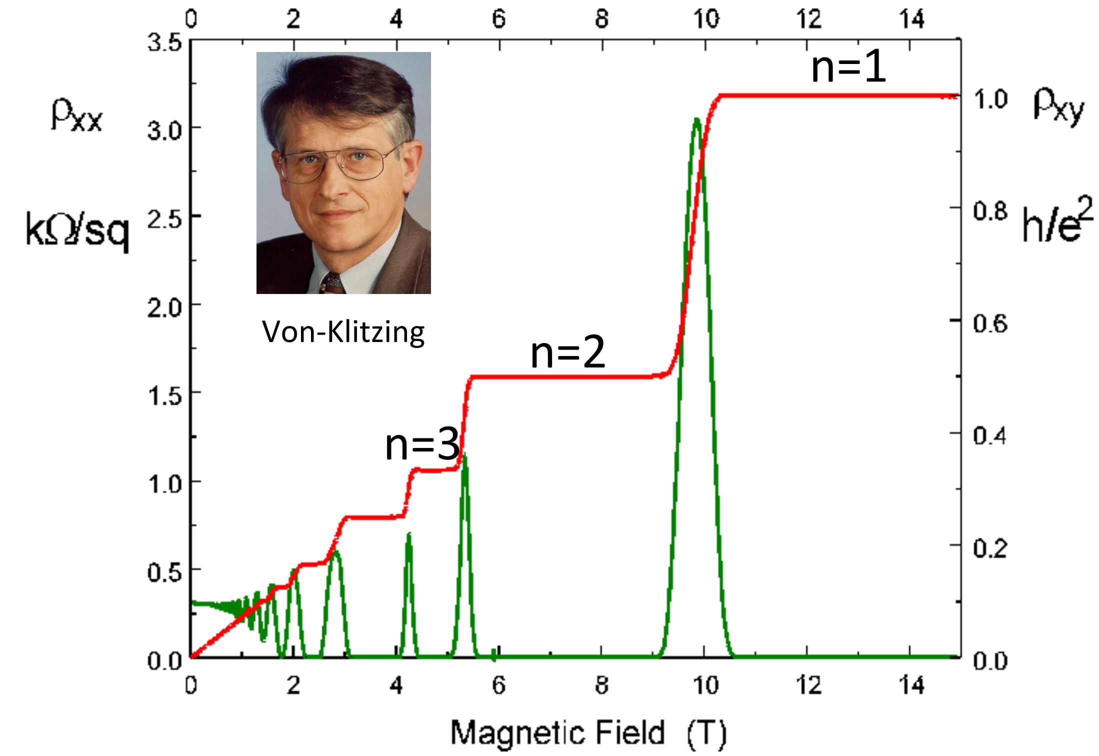
$$(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \otimes \ldots$$

Beyond Classical Orders?



Electrons in 2D in a magnetic field.

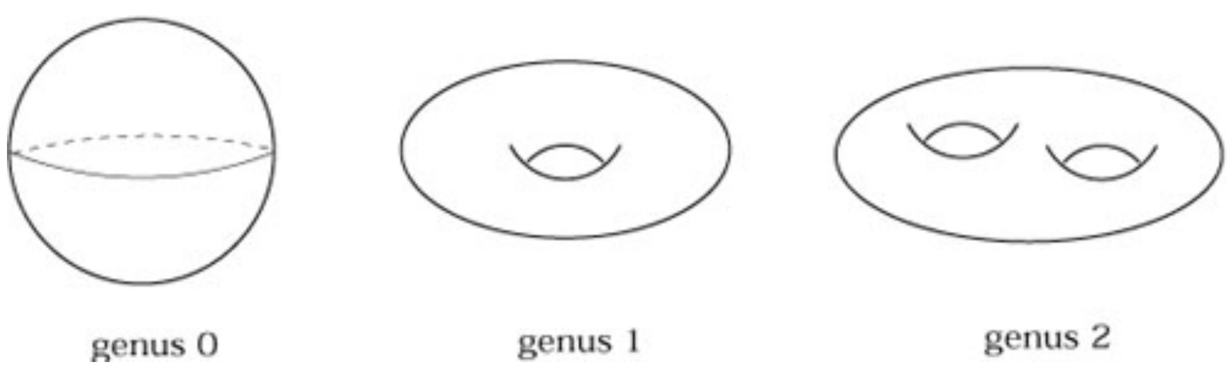
$$\rho_{xy} = \frac{h}{e^2}\frac{1}{n}$$



- Integer Quantum Hall States - (1980)
  - Different Integers `n' - different phases.
  - Same symmetry - topological distinction.
  - Accurate to 1 part in 10^9!

Topology and Phases of Matter

- Topology: Robust aspects of shapes



$$\oint K dS = 4\pi(1-g)$$

Gauss Formula

"Remarkable Theorem"

Topology of Surfaces:

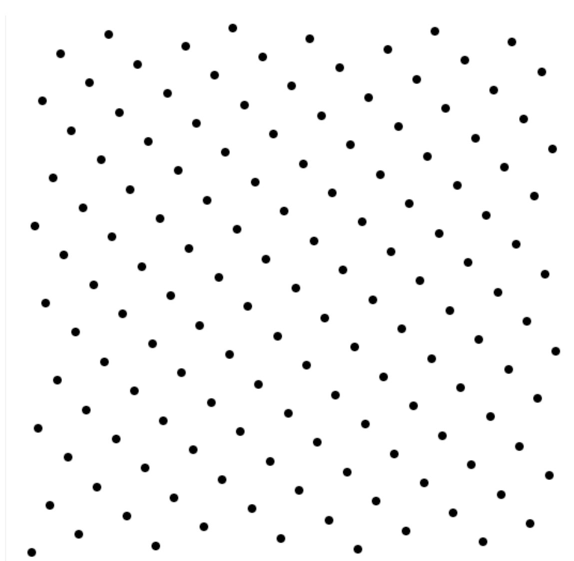
Genus = # of holes

Topology and Phases of Matter

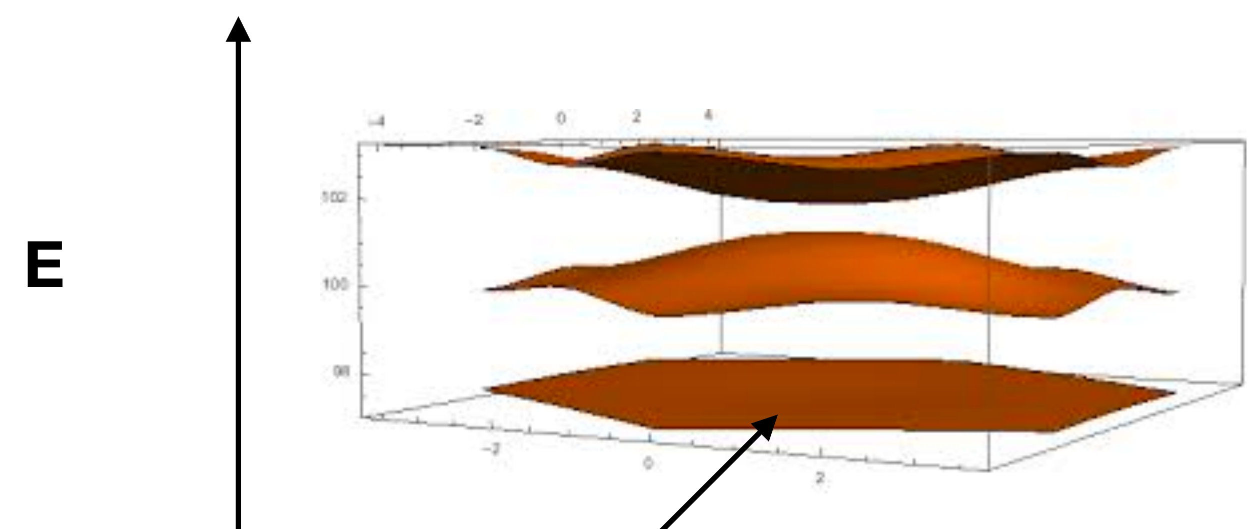
- Hall conductance as a topological invariant.

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,[a] M. P. Nightingale, and M. den Nijs
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 30 April 1982)



periodic solid



Bands with Chern Number = C

Integer Quantum Hall and Chern

$$\oint K \, dS = 4\pi(1-g)$$

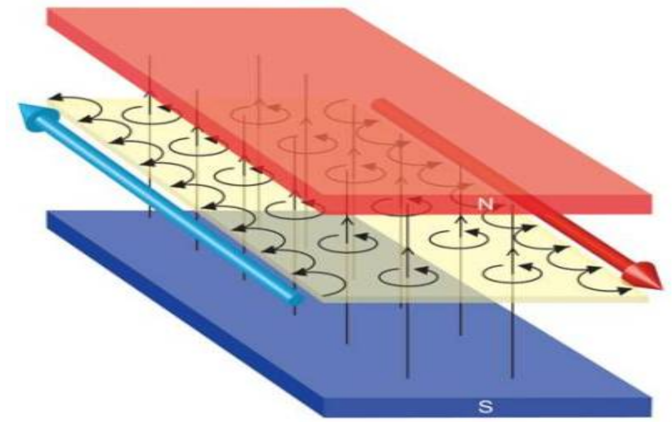
Gauss Formula

Integrate Berry Curvature

$$\oint \tilde{B} \, d^2k = 2\pi n$$

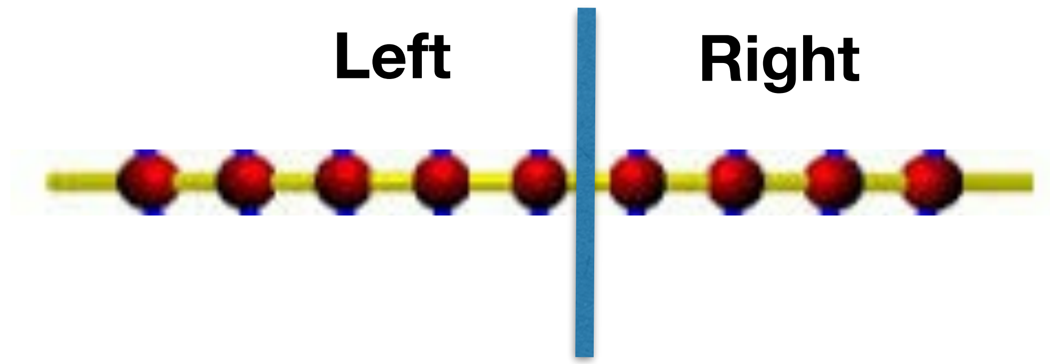
Chern Number
= # of Edge States
= Hall Conductance

Signature in
Quantum Entanglement?



Quantifying Entanglement

Schmidt Decomposition/SVD



$$|\Psi\rangle = \sum_i \sqrt{P}_i |R\rangle_i |L\rangle_i$$

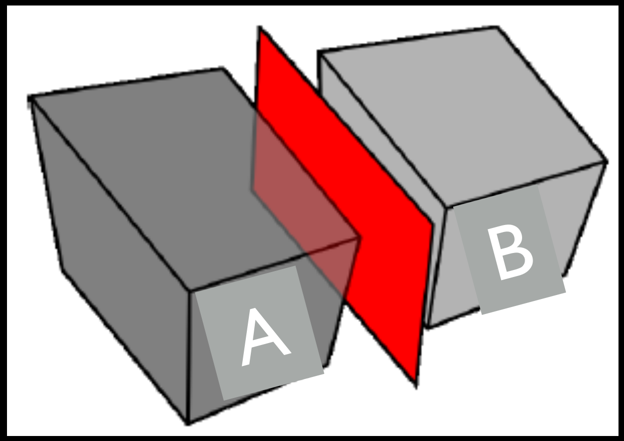
Generalization of:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{1}{\sqrt{2}}|1\rangle|1\rangle$$

$$\mathrm{Entanglement\,Entropy}: \quad S = -\sum_i P_i \log P_i$$

Signature in Entanglement *Spectrum* $P_i = e^{-E_i}$ **`psuedo' energy**

Quantifying Entanglement



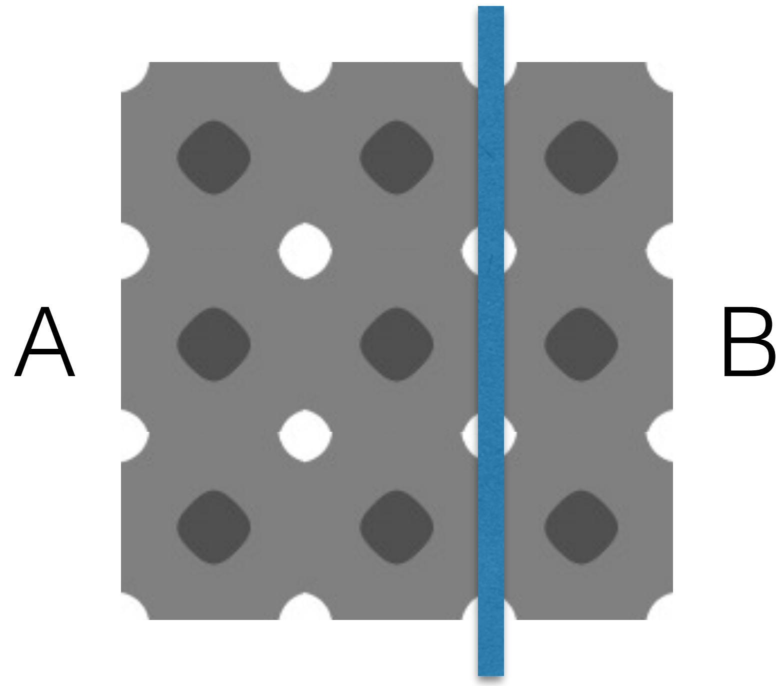
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle|0\rangle + |1\rangle|1\rangle\right]$$

$$\rho_A = \mathrm{Tr}_B\left\{|\Psi\rangle\langle\Psi|\right\}$$

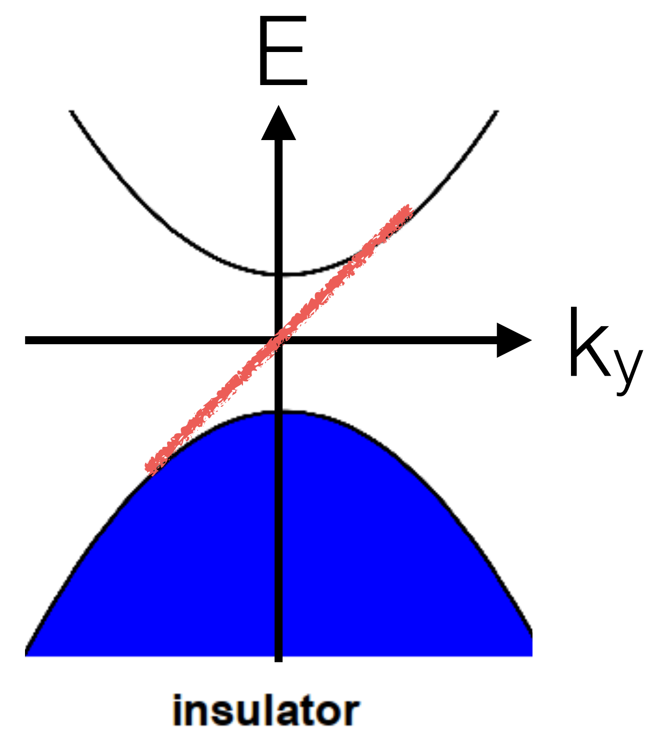
$$\rho_A = e^{-\mathcal{H}_A^e}$$

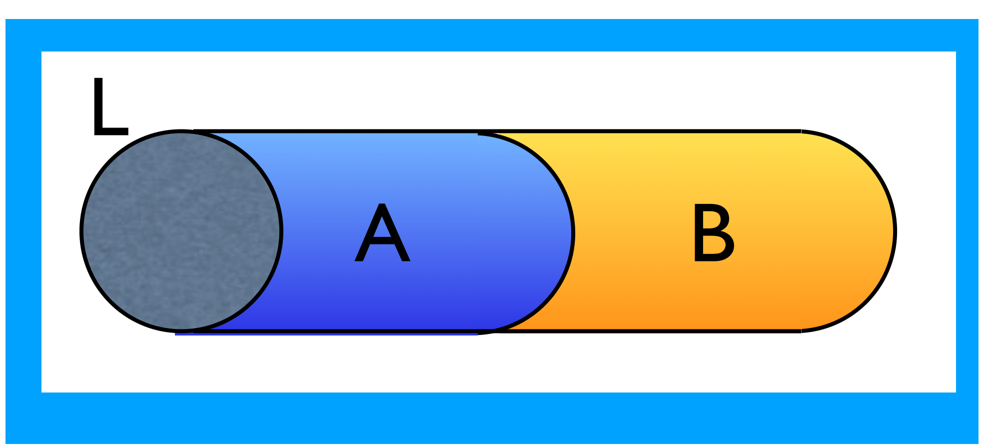
- Ground state restricted to region `A'
  - Defines an `entanglement Hamiltonian' . (Li&Haldane)

Chern insulator and Entanglement

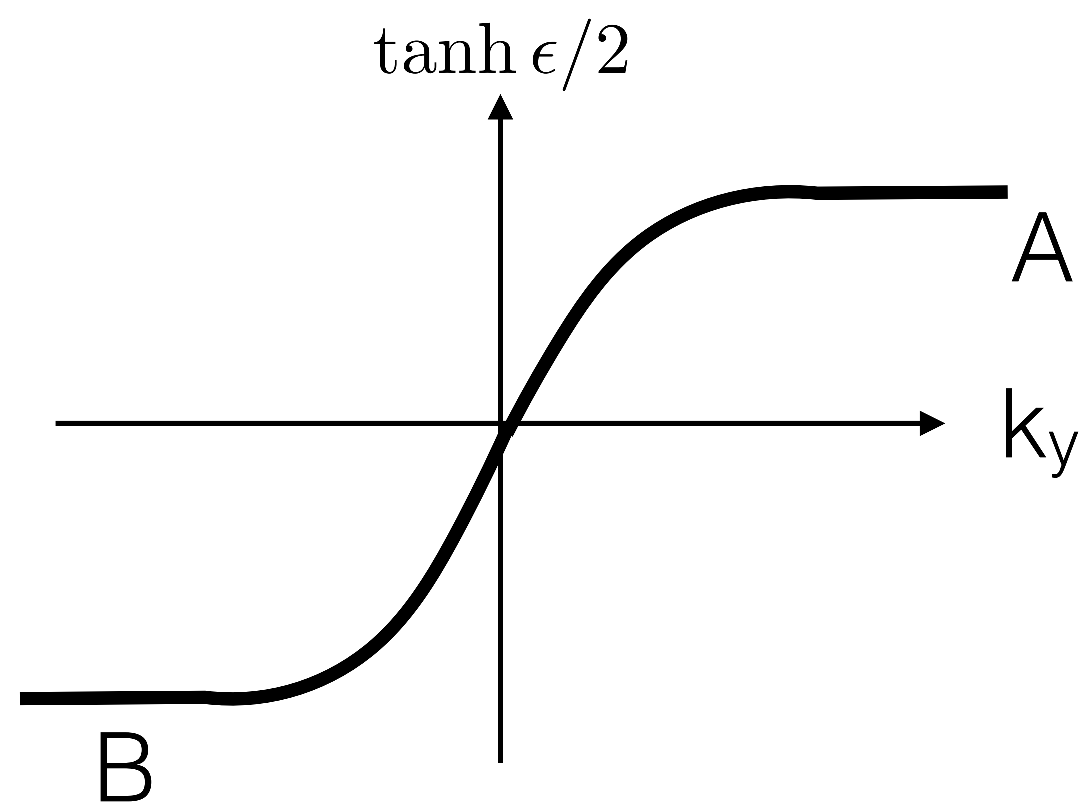


Physical Edge and Physical Spectrum

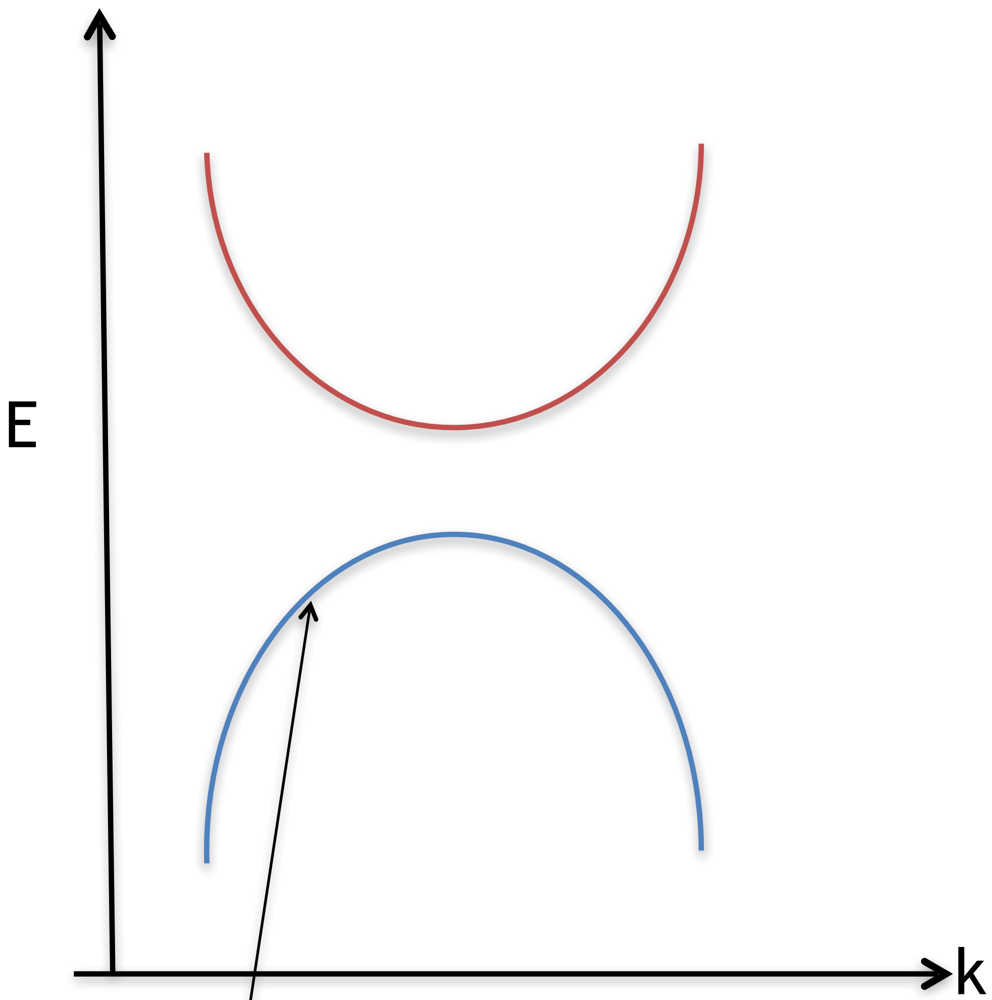




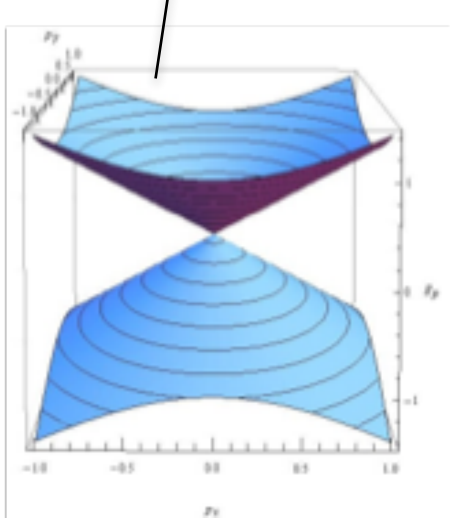
Entanglement Cut
and Entanglement Spectrum

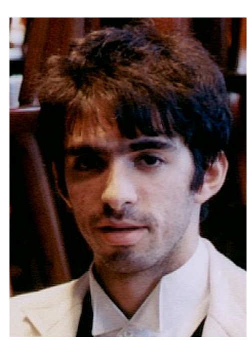


3D (Magnetic) Weyl Semimetals



- A pair of 3D bands cross (nondegenerate bands).
- Entanglement arguments - *must* be gapless (with inversion).



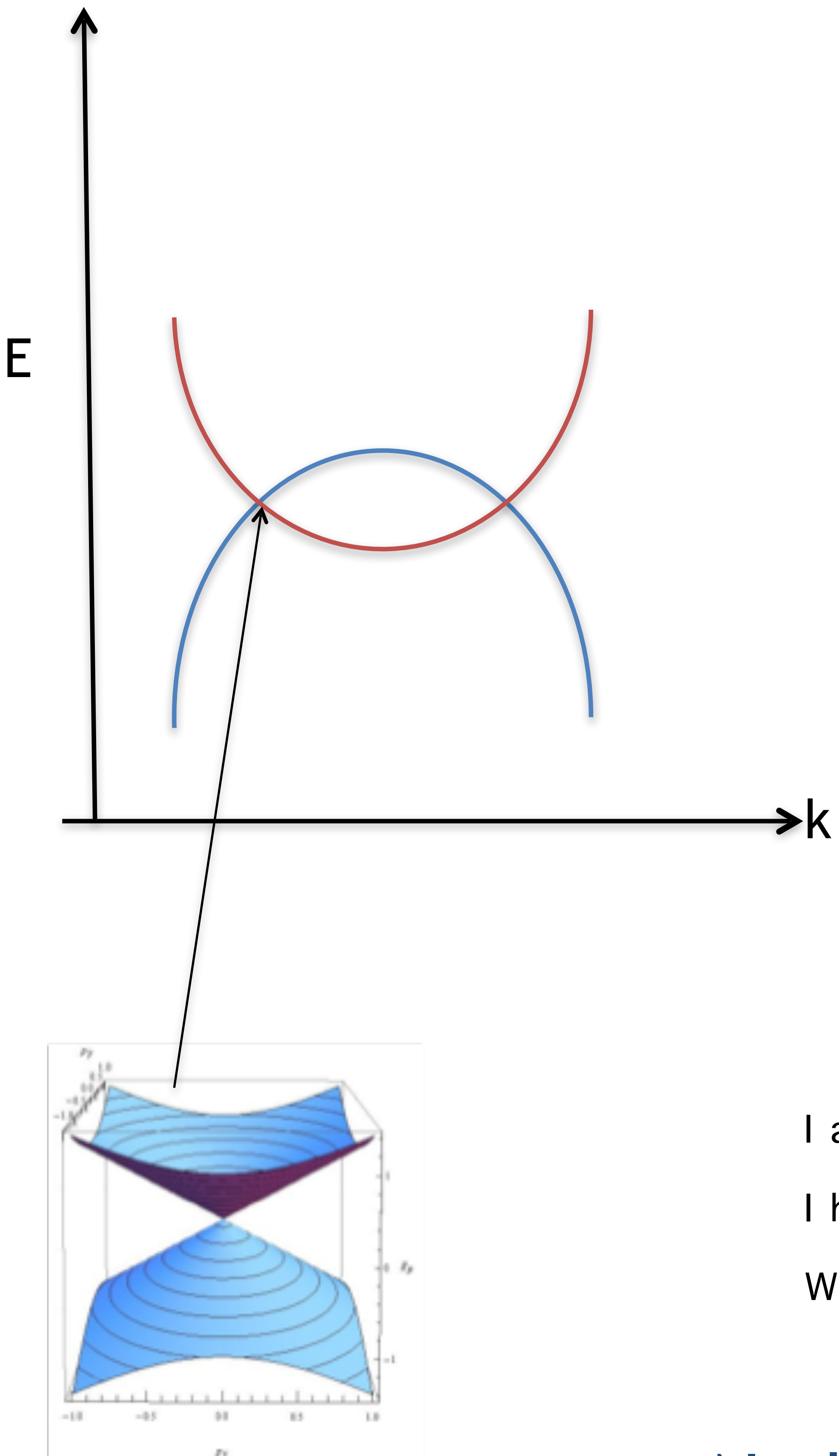


Ari Tuner

- Excitations near the touching points described by Weyl Eqn. [Weyl, Herring, Volovik]

- Novel surface states - Fermi arc. [Wan, Turner, AV, Savrasov]

3D (Magnetic) Weyl Semimetals



- The Weyl Eqn.

$$H_{\mathrm{Weyl}} = (p_x\sigma_x + p_y\sigma_y + p_z\sigma_z)$$

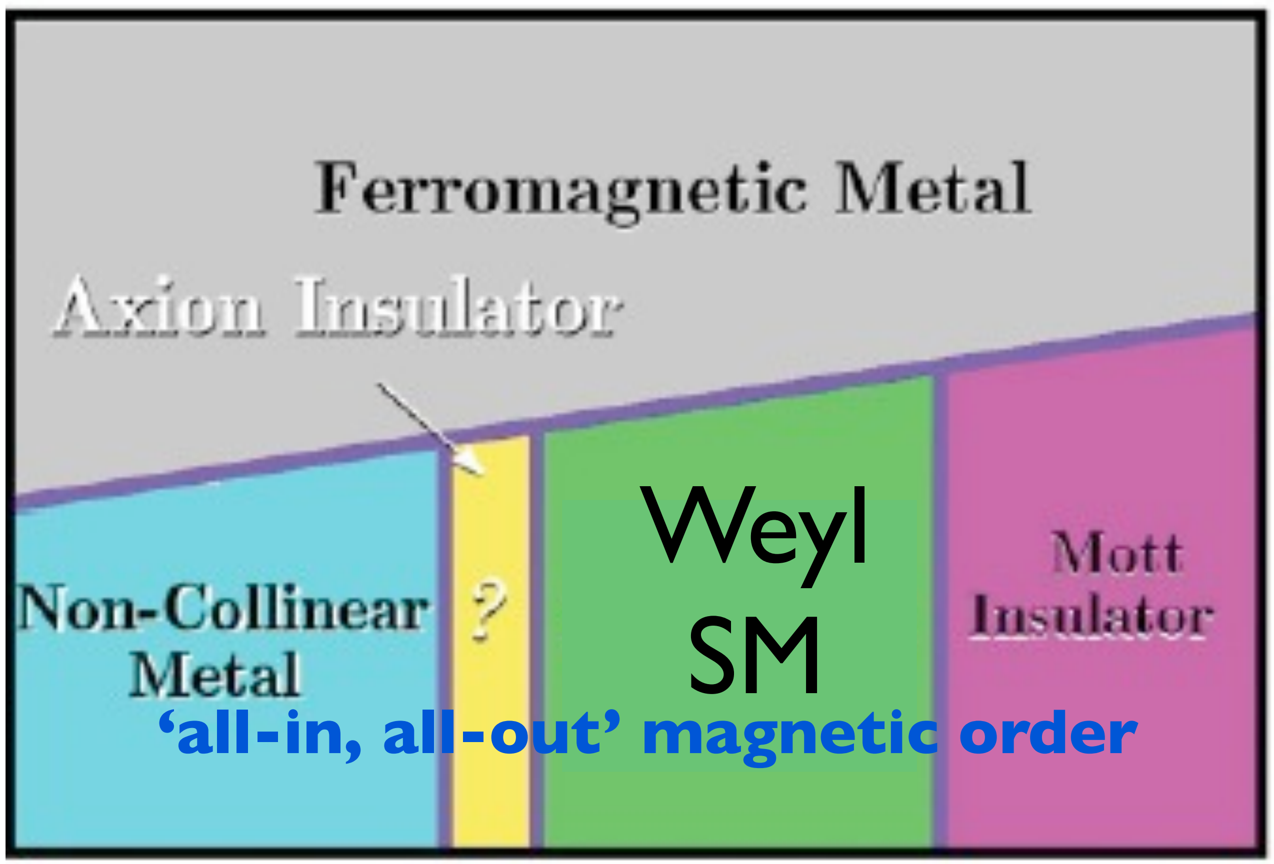
Half of Dirac's 4 component equation.
No mass term allowed.

I always tried to unite the truth with the beautiful, but when I had to choose, I usually chose the beautiful. - Hermann Weyl (1885-1955)

H. Weyl

No known Weyl fermion….. Realize in solids?

Weyl Semimetal Candidates?



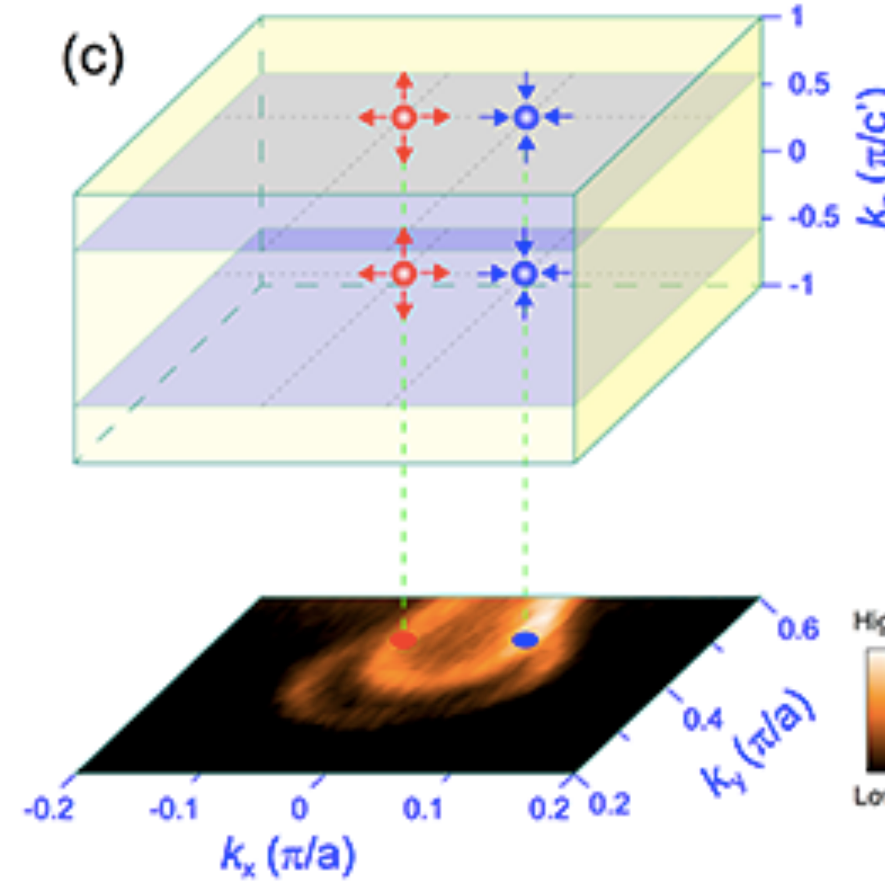
B_{ext}

0 $U \longrightarrow$ 2.5 eV

Early material candidate:
Pyrochlore Iridates
All-in All-out magnetic order

Wan, Turner, AV, Savrasov

Weyl Semimetal in **TaAs:**



S-M Huang et al., (Hasan Group)

B. Q. Lv, (Ding Group)

Photoemission Confirmation

REVIEWS OF MODERN PHYSICS

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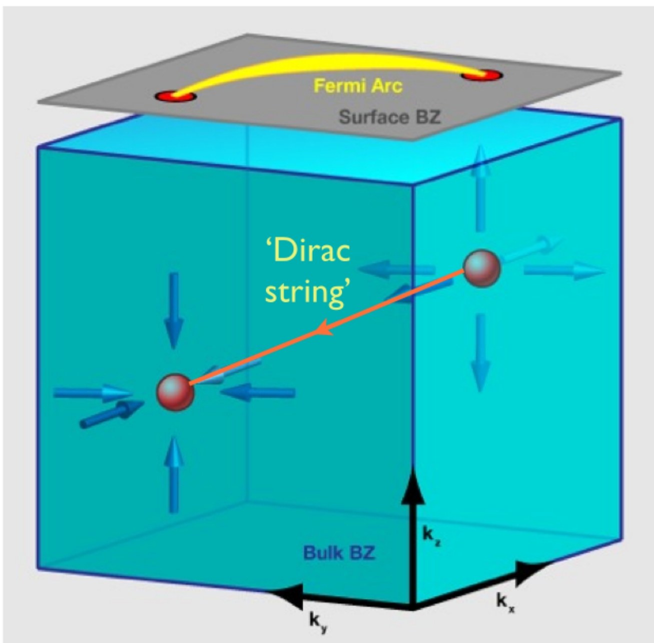
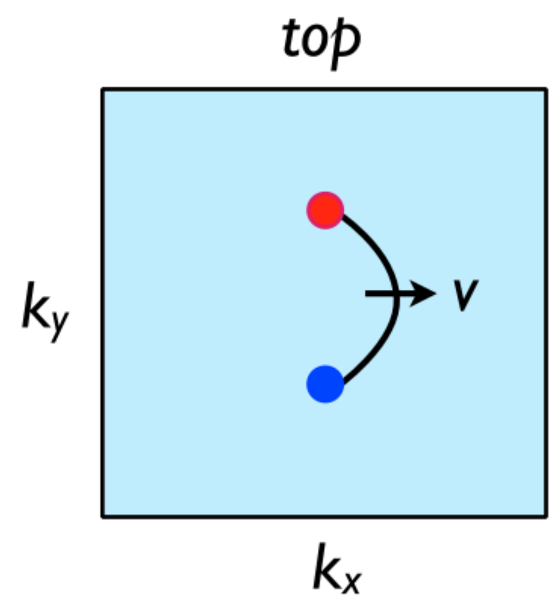
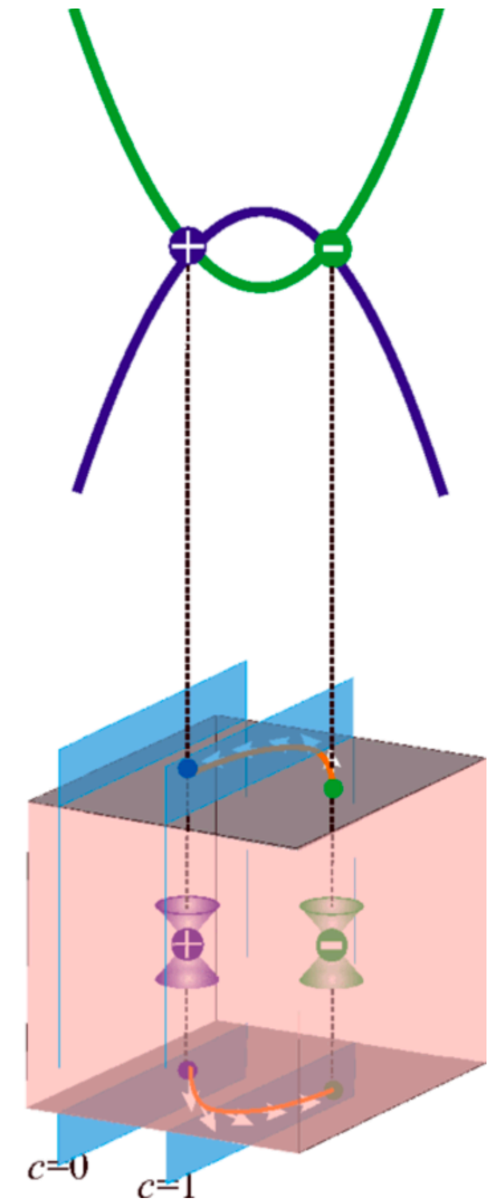
Weyl and Dirac semimetals in three-dimensional solids

N. P. Armitage, E. J. Mele, and Ashvin Vishwanath
Rev. Mod. Phys. **90**, 015001 – Published 22 January 2018

Topological Properties I - Surface States

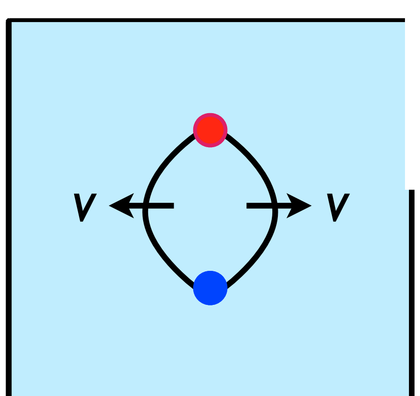






Surface states at the band touching energy => Fermi arc.

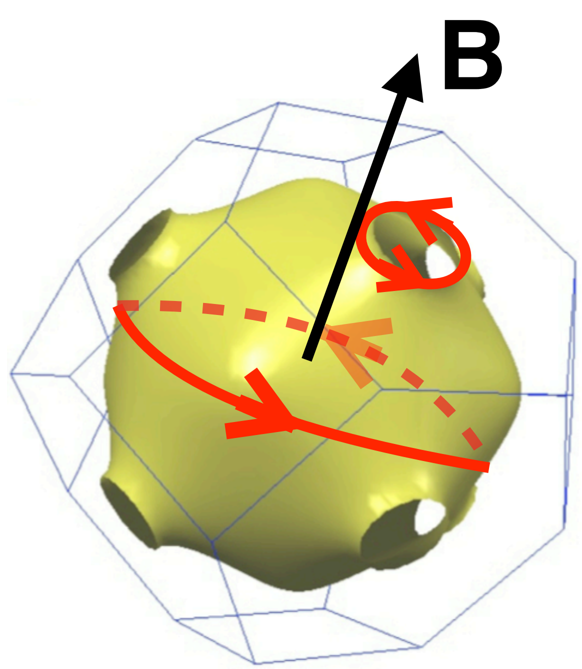
In a purely 2D system, Fermi surfaces are closed contours.

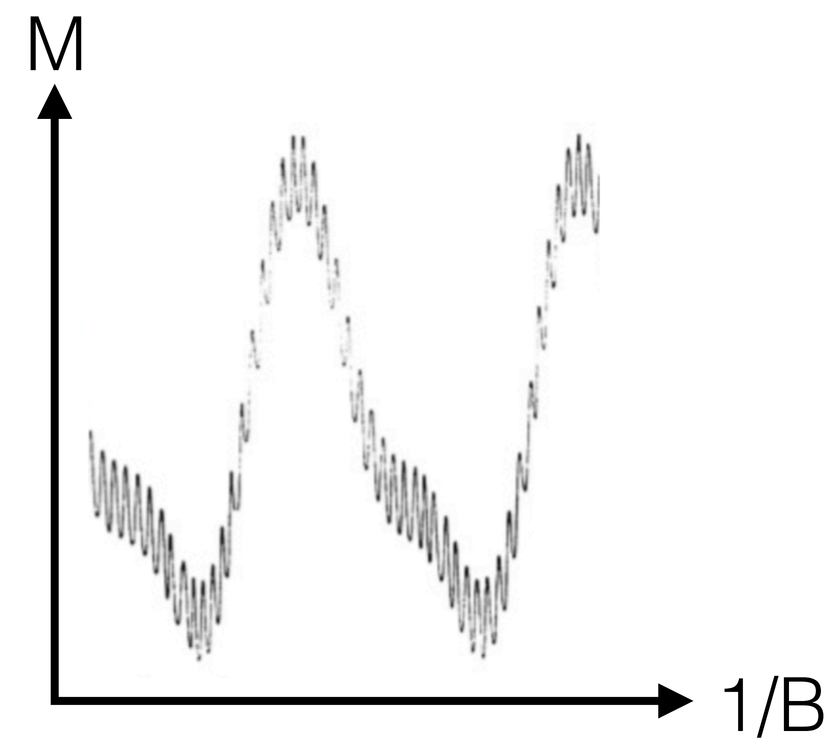


top+bottom = 'legal' 2D Fermi surface

Quantum Oscillations - from Fermi arcs?
Canonical signature of a Fermi surface

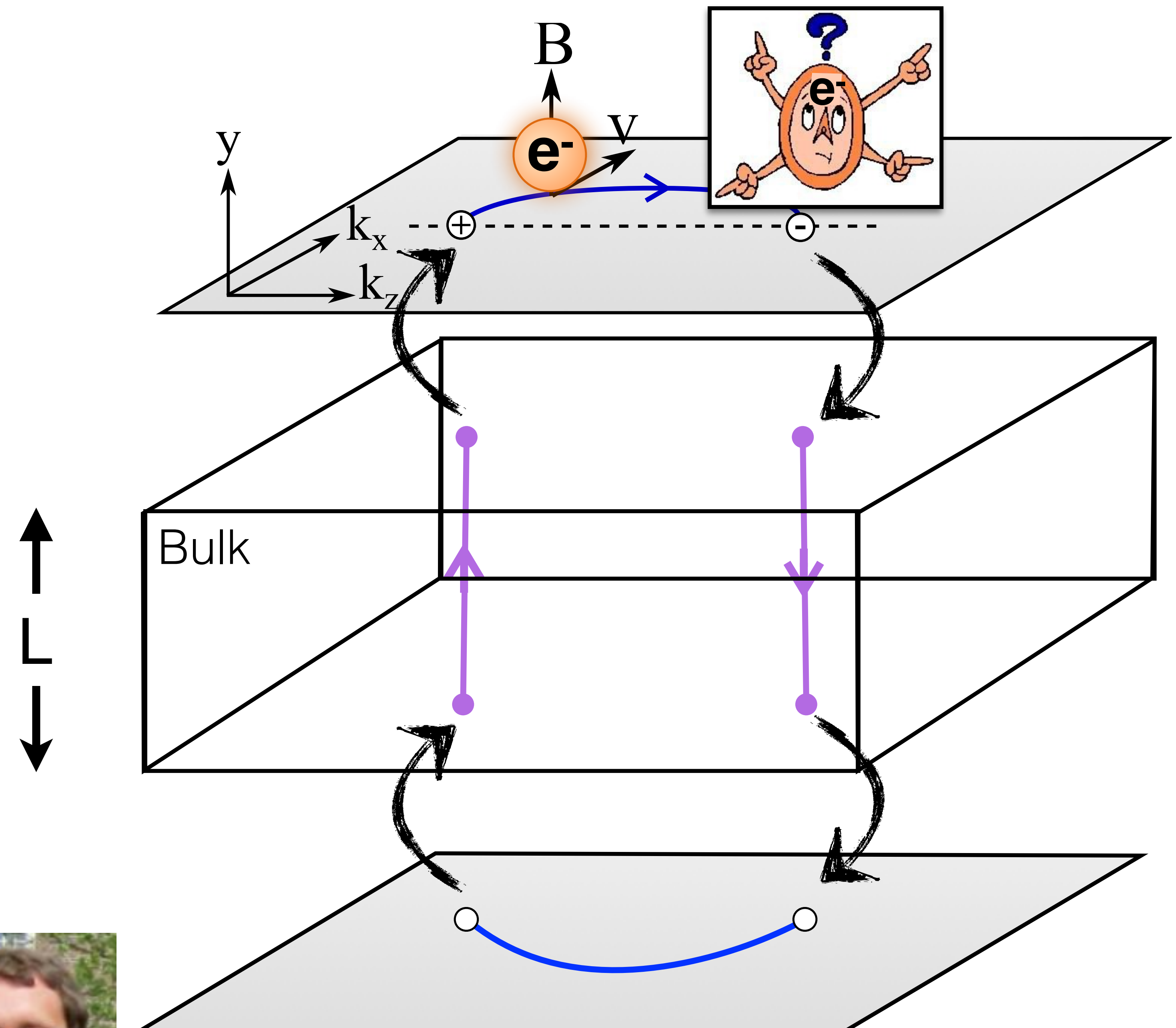
Classical Example: Copper

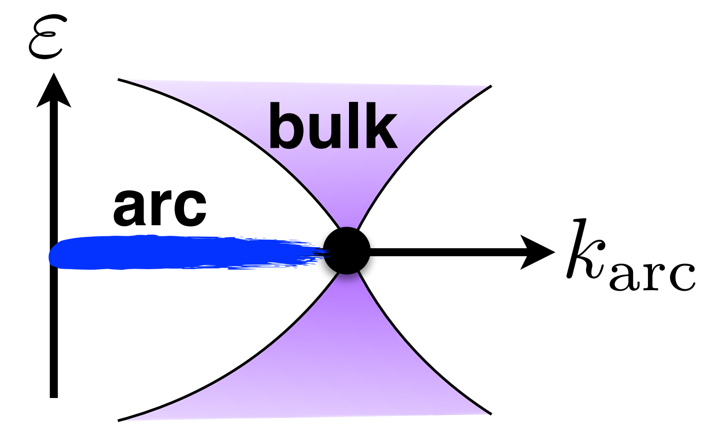




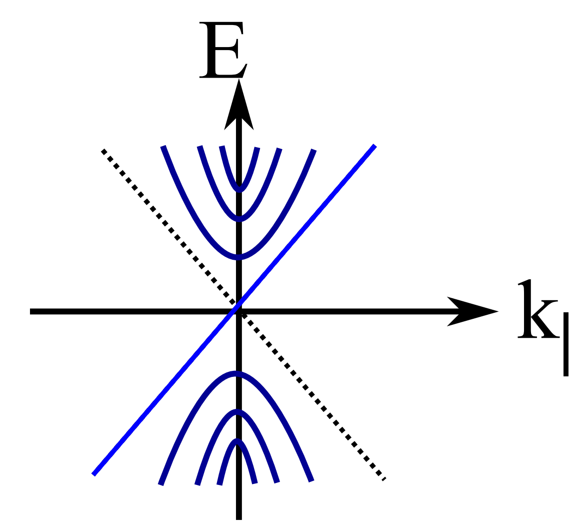
$$\dot{\mathbf{k}} = e\mathbf{v} \times \mathbf{B}$$

Quantum Oscillations from Fermi Arcs?





Surface-to-Bulk Transfer



Bulk Landau Levels

Drew Potter (Berkeley)

*Andrew **Potter**, I.Kimchi, A. V, Nature Communications (2014)*

Oscillations that are sensitive to thickness.

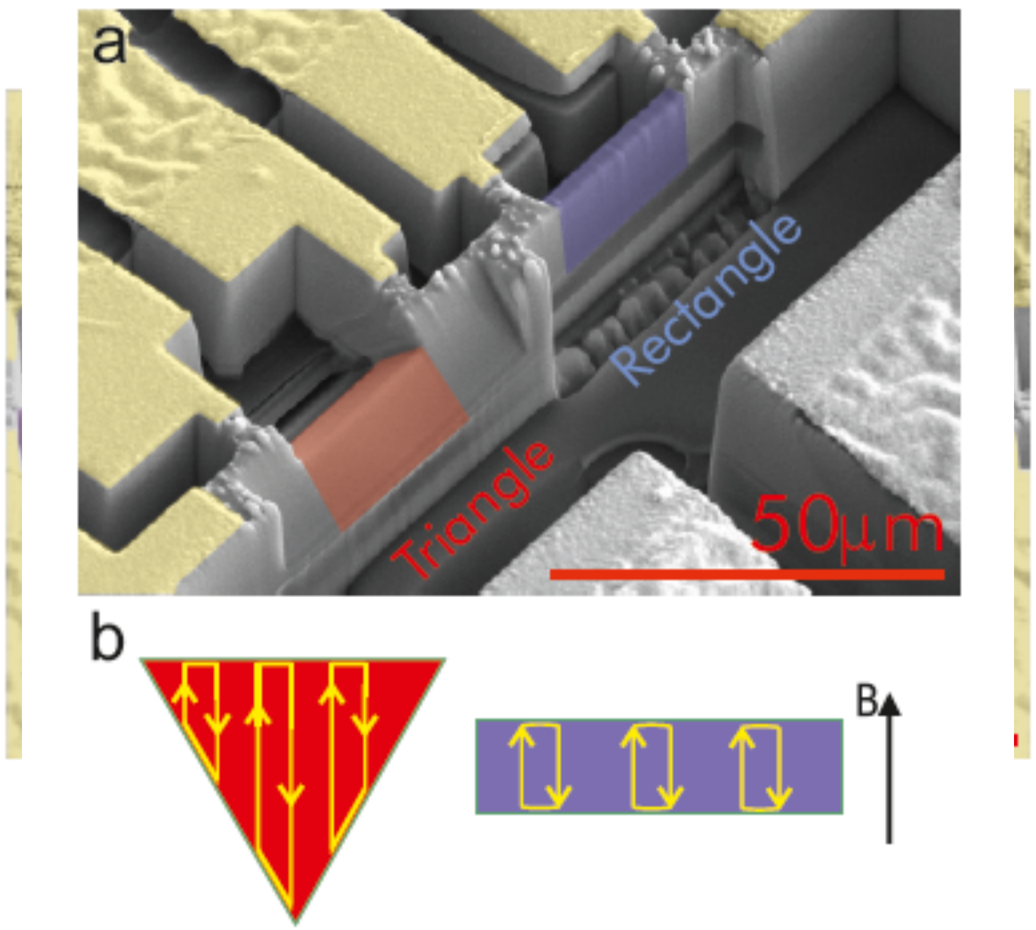
Experimental search for Dirac surface arcs

Cd3As2 - Dirac Semimetal

- *Moll, Nair, Helm, Potter, Kimchi, AV and James G. Analytis Nature (1505.02817)*

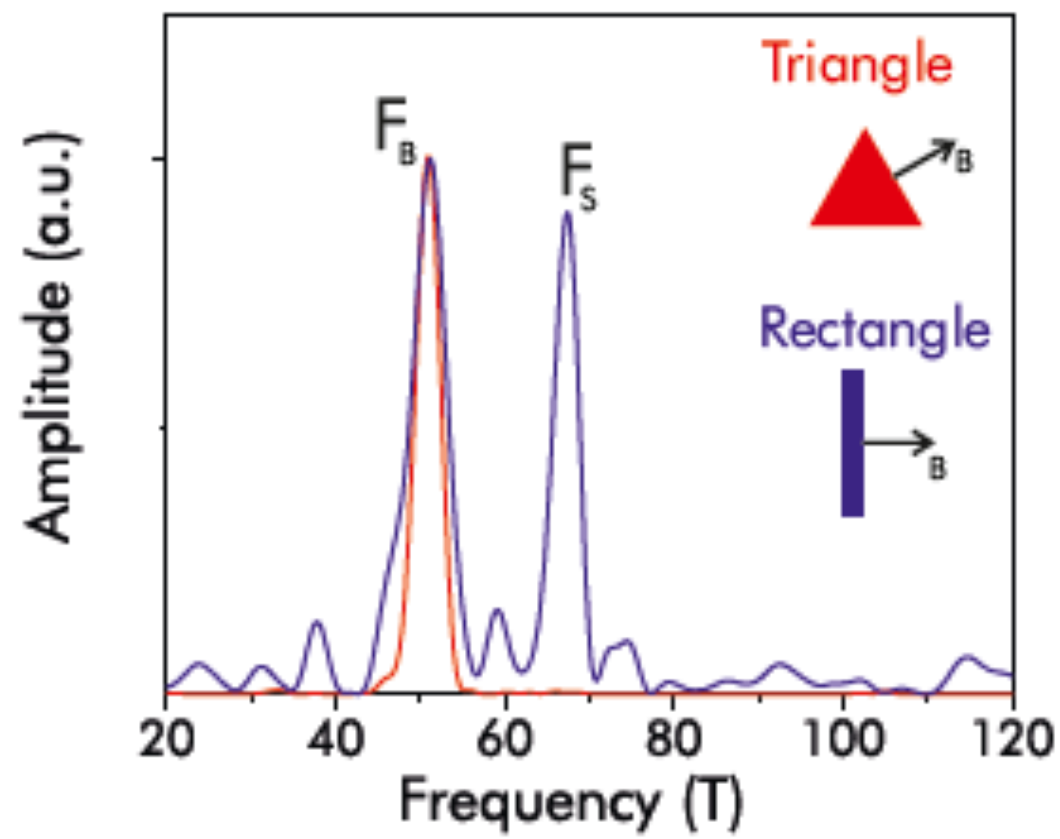
See oscillations from a surface state that is sensitive to thickness.

Focused Ion Beam Fabrication

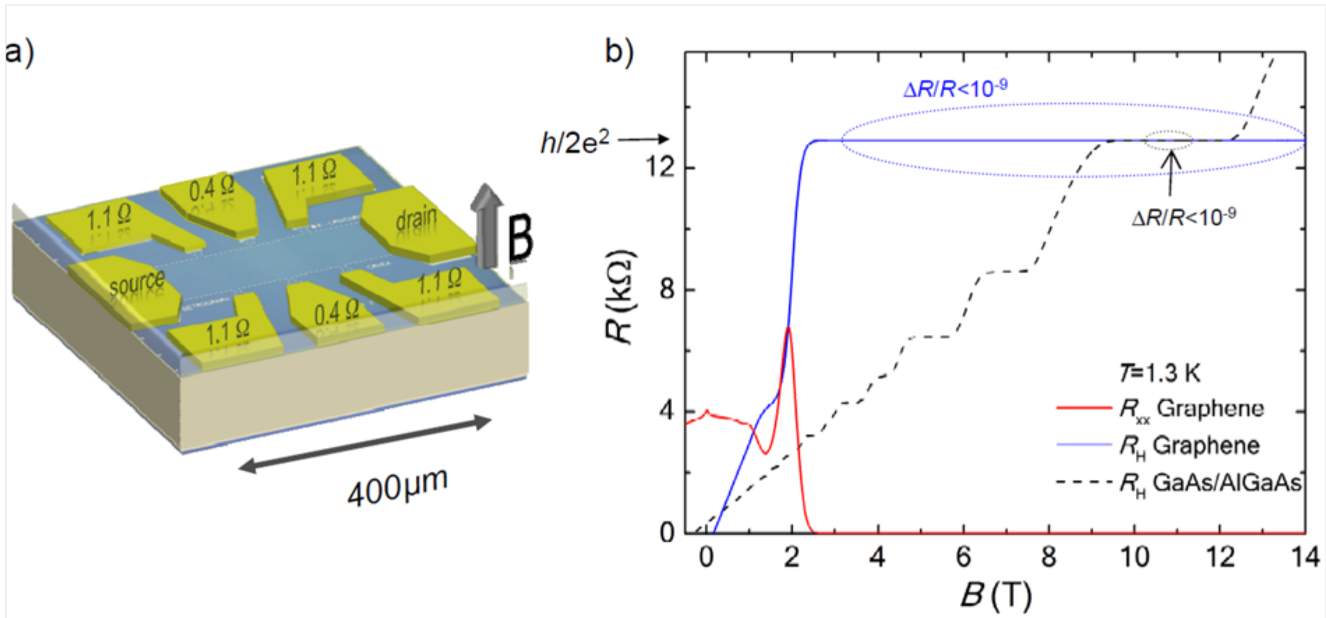


- Precisely tune thickness down to ~100nm
- Ultra-clean surface

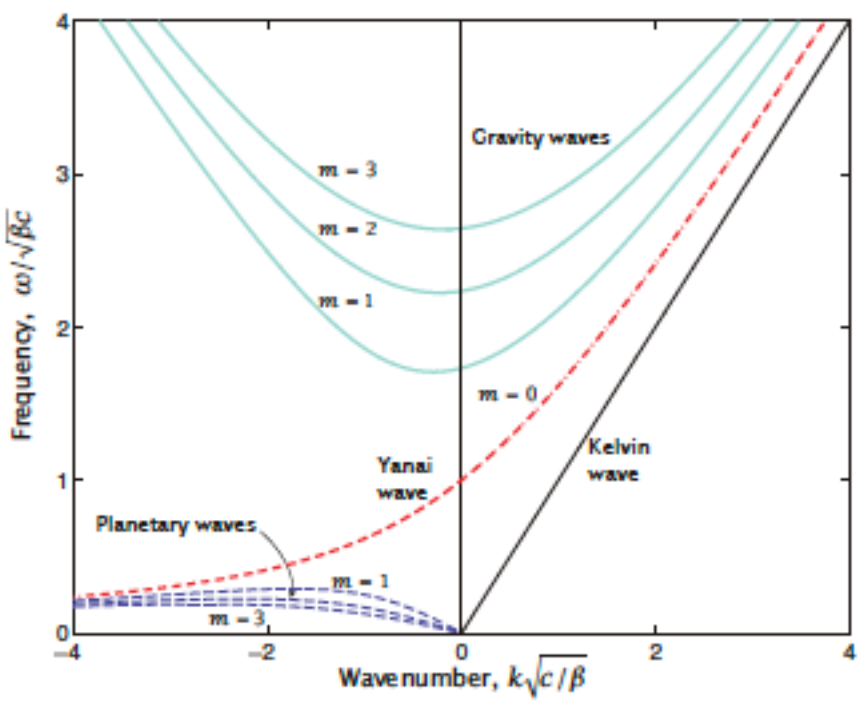
Surface + Bulk Quantum Oscillations

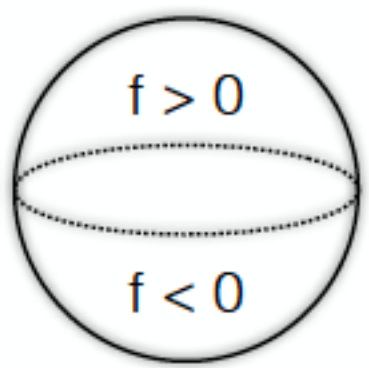


Applications



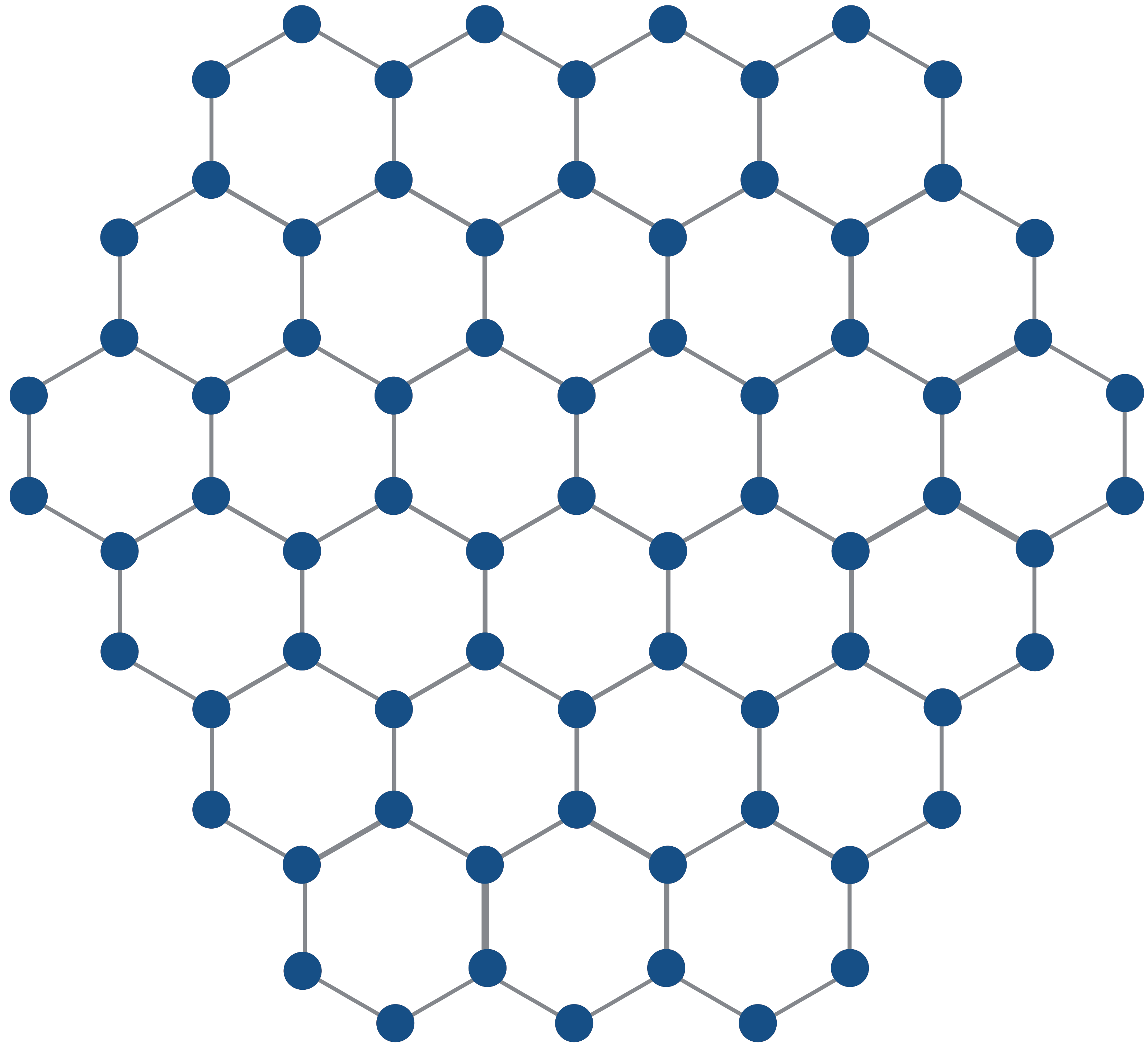
1. Quantum Hall used as a resistance standard - Kg defined in terms of R_H





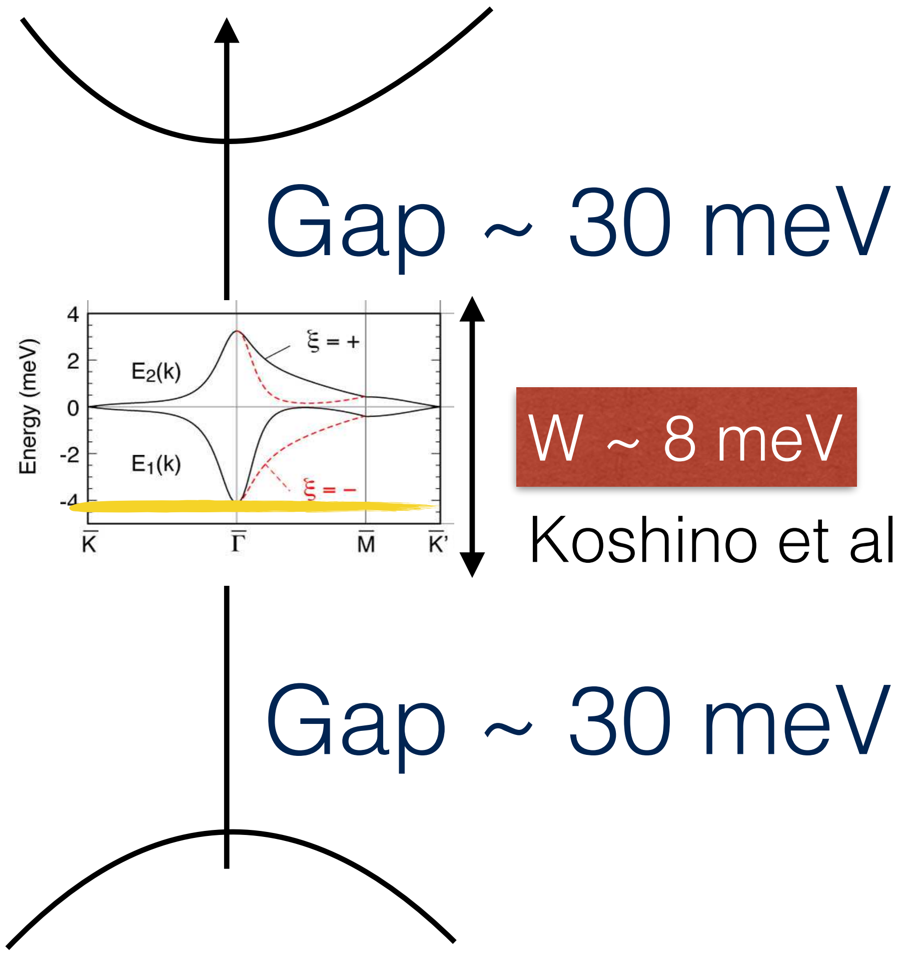
2. Classical Analogs: One way traveling Kelvin waves relevant to climate! [Marston et al. Science 2017]

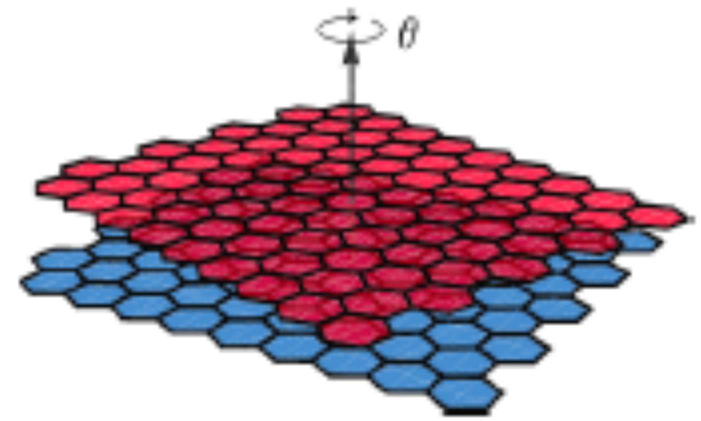
Application 3: Magic Angle Graphene



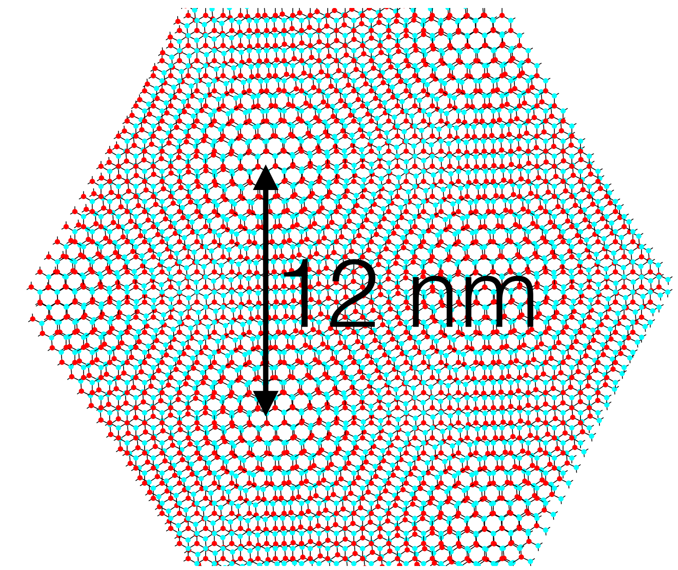
Twisted Bilayer
Monolayer
Bilayer

Correlation Effects in Twisted Bilayer Graphene



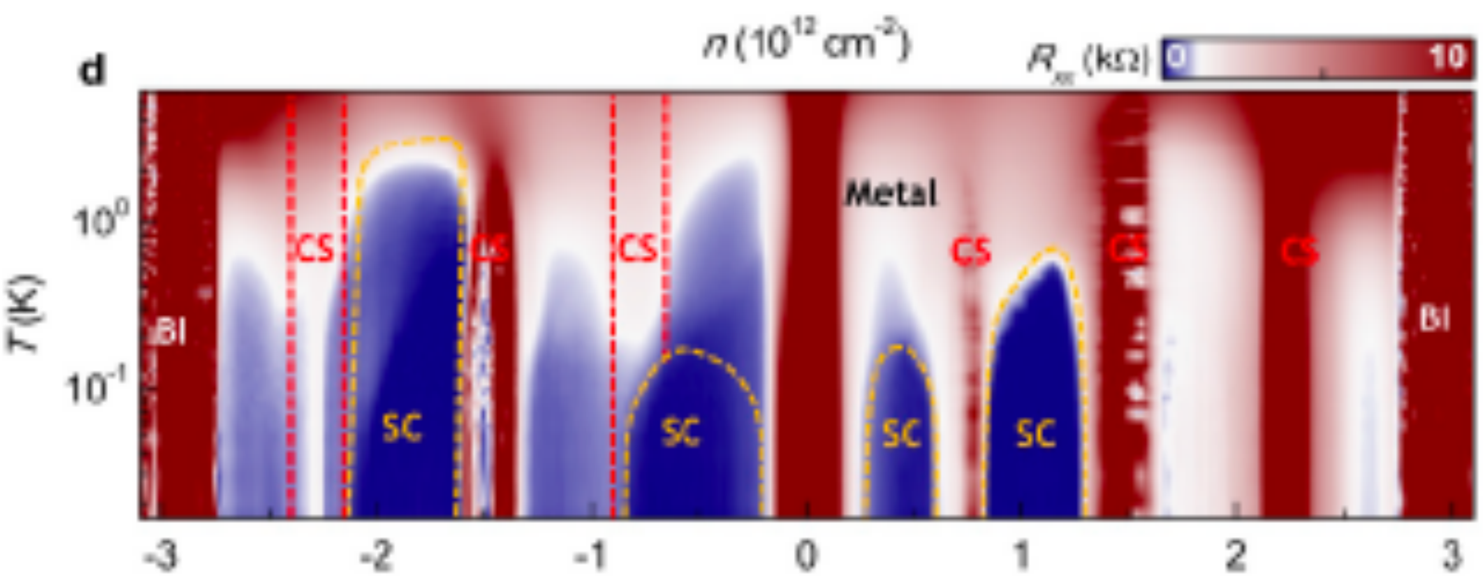


MAGIC ANGLE ~1.1º: Tunneling time = Lattice Moire time

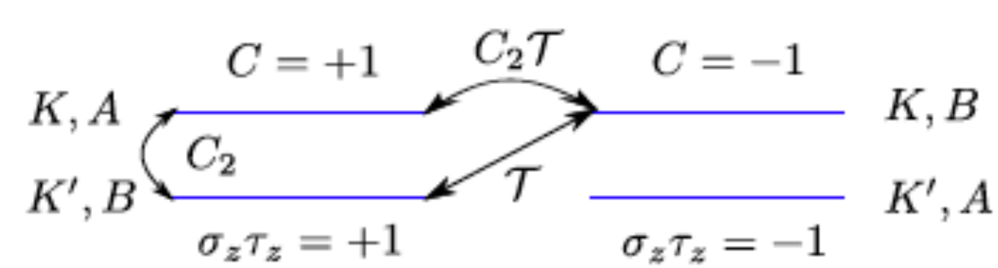


V~30meV

$\theta \sim 1/60$ radians



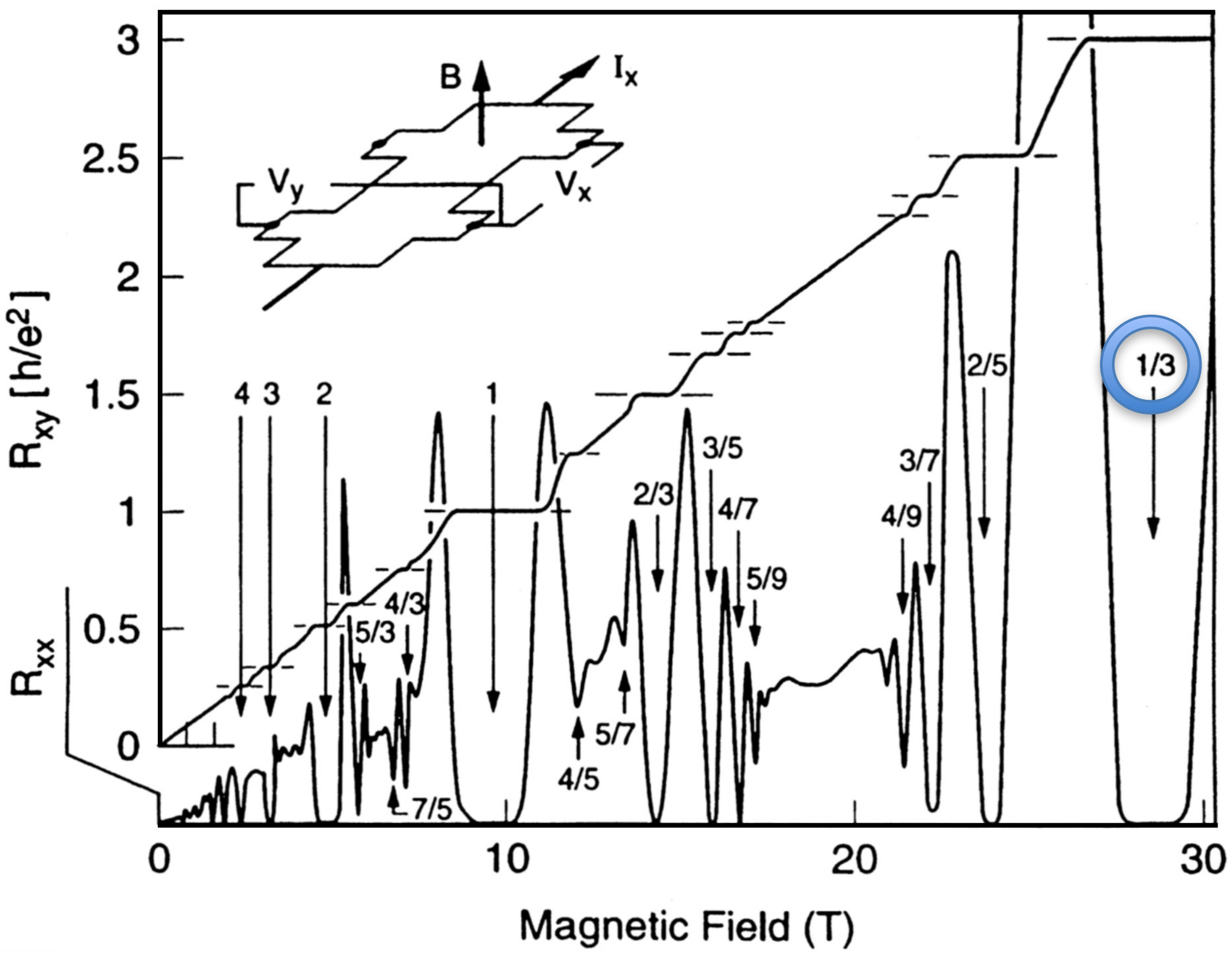
Lu et al. (Efetov group). Cao et al. (Pablo H. J. Group…)

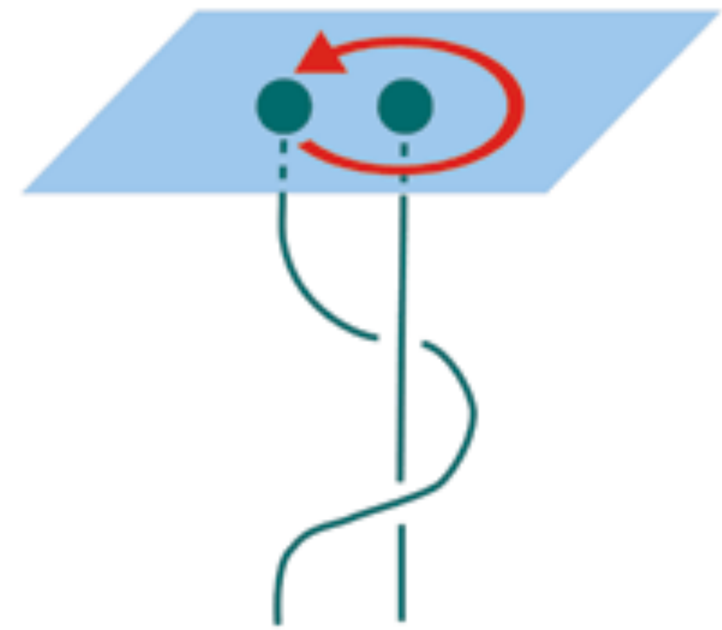


Decompose flat bands into Chern bands

Topology with Strong Interactions

Topology with Strong Interactions





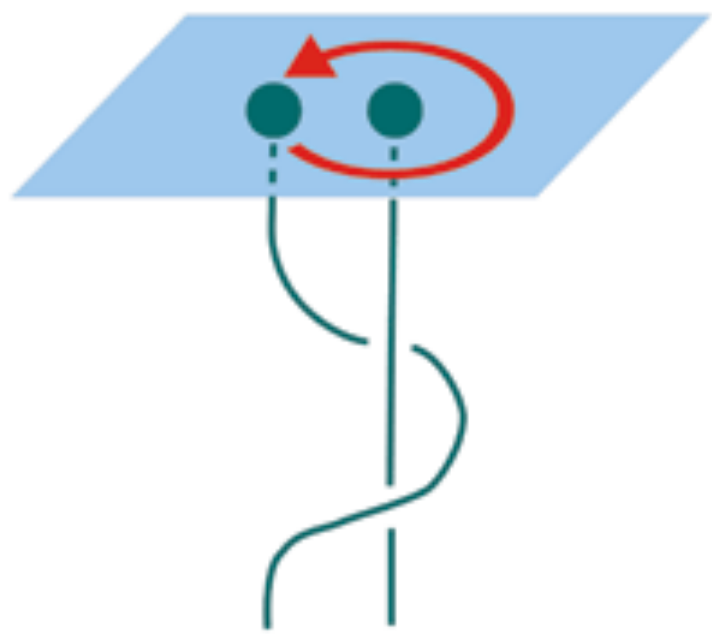
$$e^{i\phi} \neq 1$$

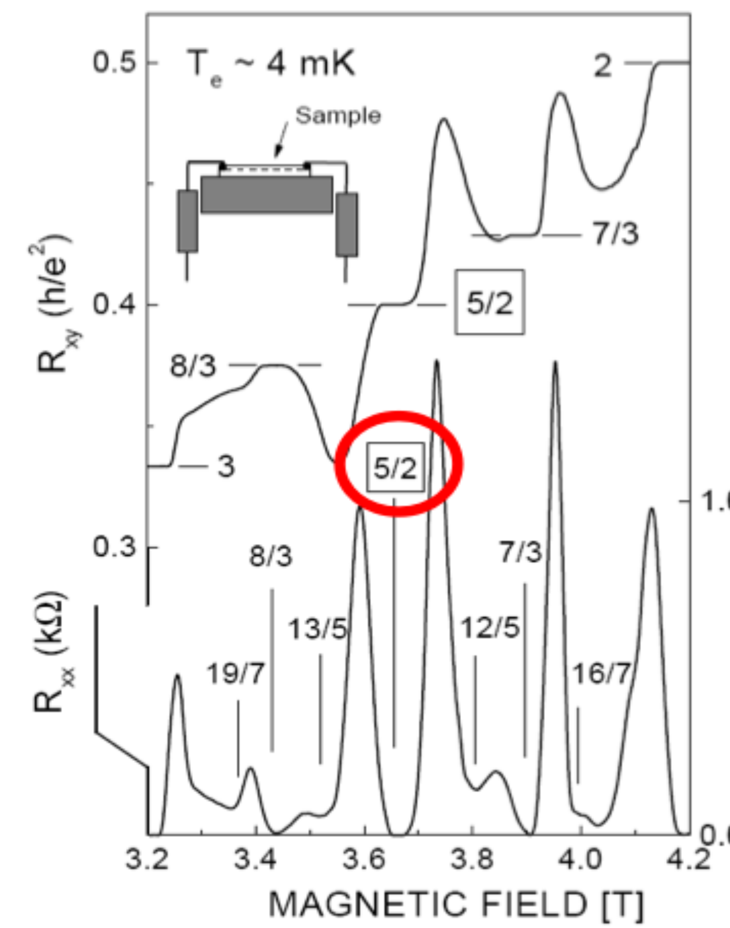
anyon -

unlike boson/fermion
possible in 2D

- Fractional Quantum Hall:
    - excitations (quasiparticles) carry fractional charge (eg. e/3!) and neither fermion nor boson “anyons”.

Non Abelian Quantum Hall





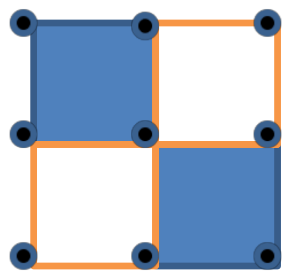
Pan et al. PRL 83, (1999)

Non-Abelian quasiparticles:
Excitations creates new ground states.

$$e^{i\phi} \rightarrow U_{12}$$

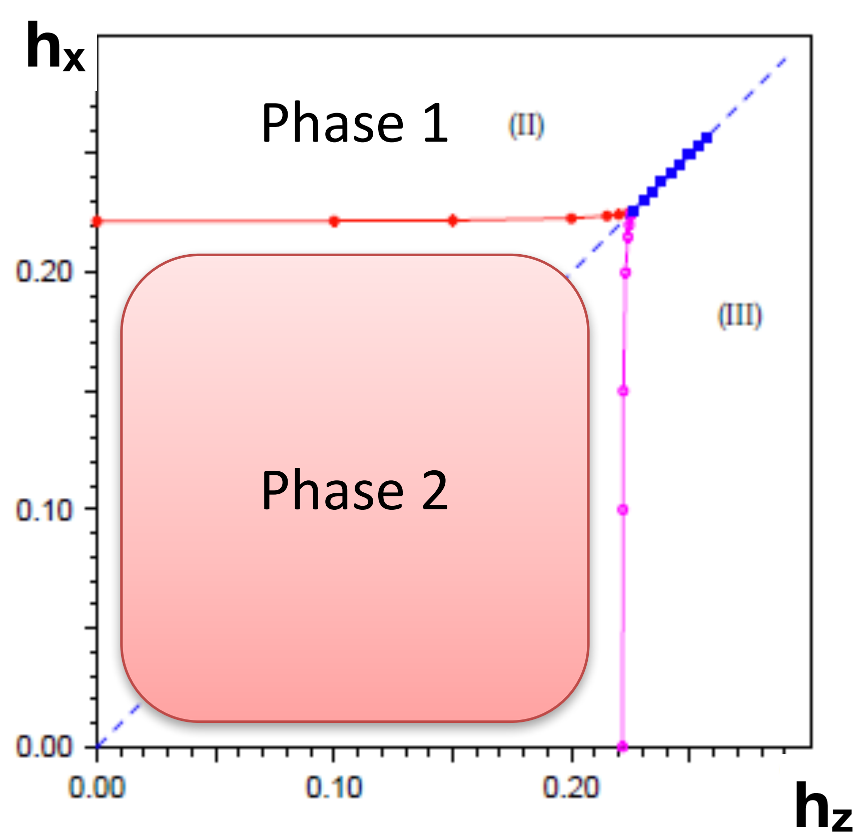
- Some may even carry non-Abelian statistics $\nu = 5/2$ Pfaffian state - particle exchange is a unitary matrix U. [Read, Moore, Wilczek, Wen, Greiter]

Topological Order in a Spin Model



$$H = -\sum_{\square} \sigma^z\sigma^z\sigma^z\sigma^z - \sum_{\square} \sigma^x\sigma^x\sigma^x\sigma^x - h_x \sum \sigma^x - h_z \sum \sigma^z$$

A spin model with no spin symmetry



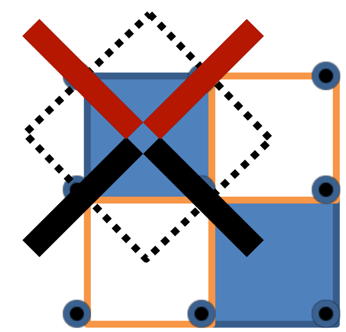
But two phases!
How to distinguish?

Kitaev; Tyupitsin et al.;
Fradkin and Shenker.

Z_2 Gauge Theory

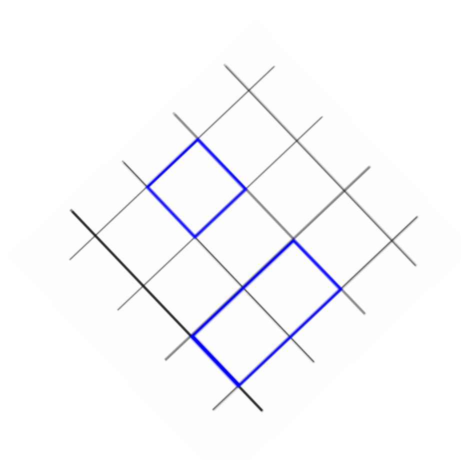
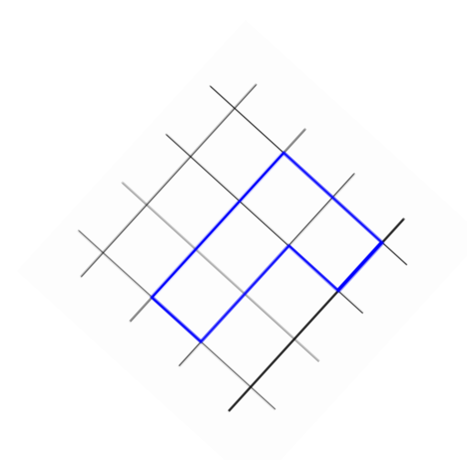
- Special point: $$H = -\sum_{\blacksquare} \sigma^z \sigma^z \sigma^z \sigma^z - \sum_{\square} \sigma^x \sigma^x \sigma^x \sigma^x$$

- Model of closed loops. (Z_2 gauge theory.
  $\nabla \cdot E = 0 \ (mod\ 2)$ Gauss law.



$\sigma^z = -1$ even

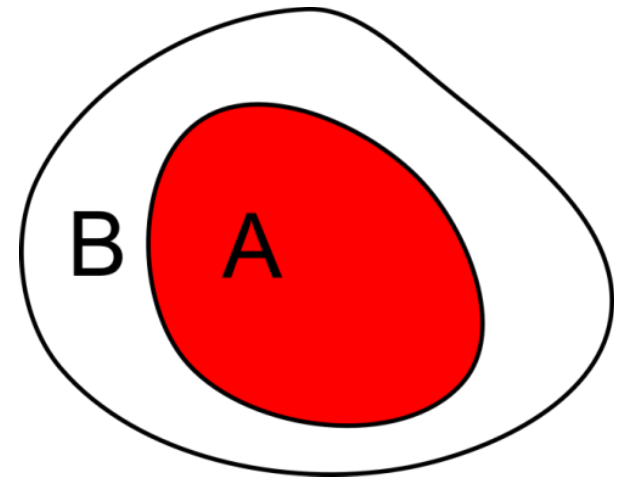
- Deconfined **Versus a confined phase**

$\Psi =$  $+$  $+$

Generically, gauge structure `emerges'.

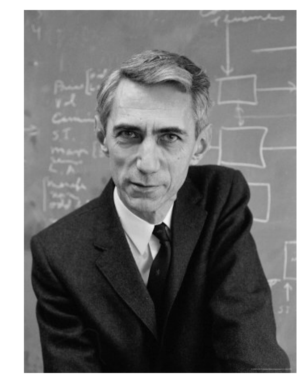
Entanglement Characterization of Topological Order

$$S_A = -\mathrm{Trace}_\mathrm{A}\, \rho_A \log \rho_A$$



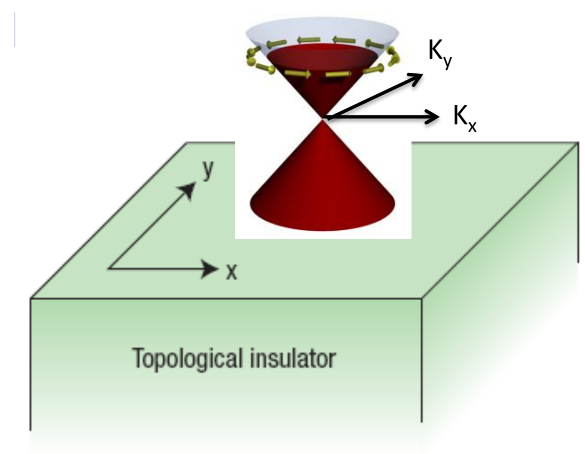
For a gapped phase, boundary law. $S_A \propto L_A$

Information obtained on measuring A
(C. Shannon)



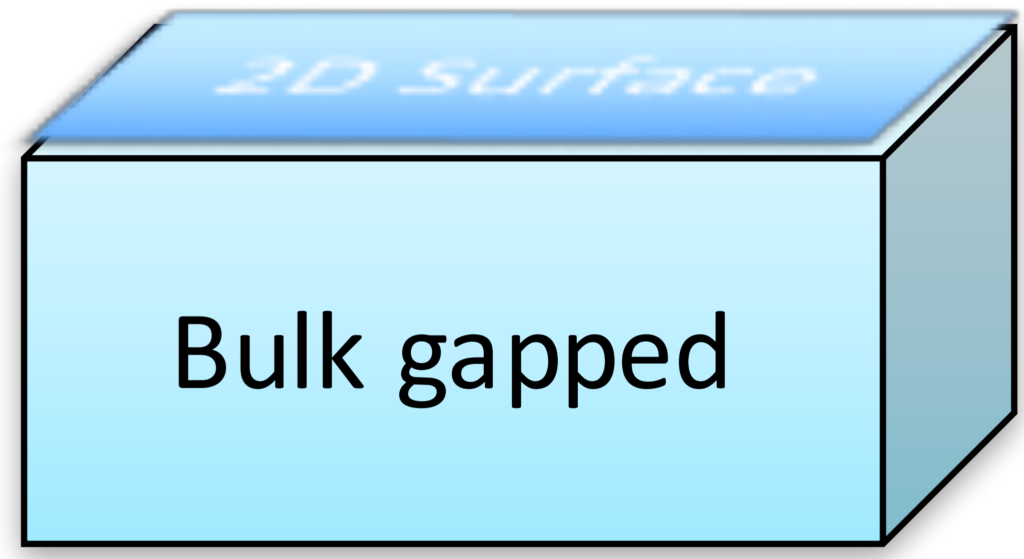
Topological Entanglement Entropy
(Levin-Wen;Kitaev-Preskill)

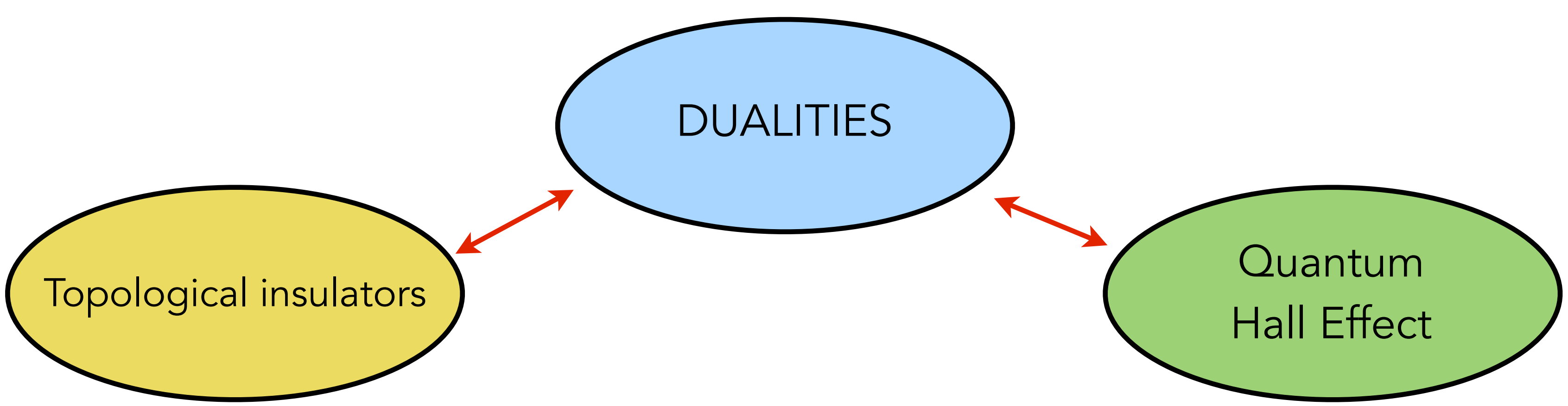
Surface Topological Order - A Remarkable Connection between two kinds of topology

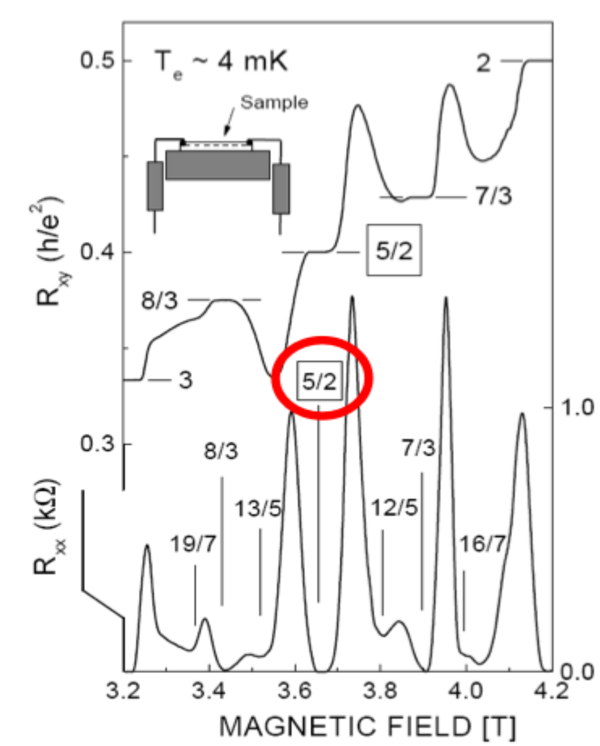


Surface - 2D Dirac dispersion.

T-Pfaffian State
Or
P-H Pfaffian State







Pan et al. PRL 83, (1999)

Conclusion

- Quantum mechanics of many particles is strange, beautiful and hard.
- This is closely linked to quantum entanglement which also give quantum communication & computing their power.

exotic quantum phase

"Today's reality is the ~~Utopia~~ of yesterday,
and
the ~~Utopia~~ of today
is nothing but
the reality of tomorrow.
»
Le Cobusier