Test models for filtering of moisture-coupled tropical waves

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- When Gaussianity and linearity are assumed, one obtains the famous linear Kalman filter.
- For high dimensional nonlinear dynamics with multiscale structure, the computational burden is in the **forecast** step.

Why the tropical waves?

- A recent article in the bulletin of the World Meteorological Organization [Moncrieff et al 2007] reported that the difficulties in improving weather and climate predictions from days to years is essentially due to the limited representation of the tropical convection and its multiscale organization in the contemporary convection parameterization.

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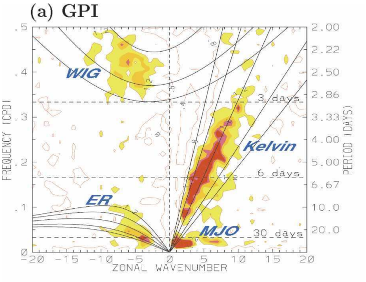
- We have a relatively accurate prediction for midlatitude weather dynamics, what so difficult about the tropics?

midlatitude	tropics
geostrophic balance	coriolis vanishes, convection
Rossby and inertio-gravity waves	Kelvin, MRG, ER, IG waves
denser measurement	sparse velocity measurements

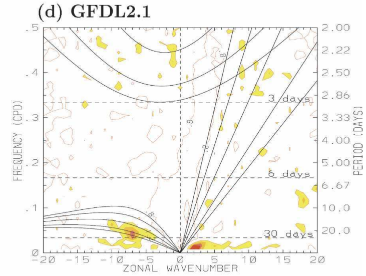
Wheeler-Kiladis space-time spectra

Multi-scale clouds and waves in the tropics

Observations



General Circulation Model (GCM)



from Lin et al. (2006)

MJO & CCEWs are **not** *currently part of GCM's intrinsic variability*

⟶ Implications for MJO prediction

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- In particular, we ask whether we can obtain accurate filtering skill by committing judicious model errors with a surrogate prior statistics $\tilde{P}$. We approximate the following filtering problem

  $$P(u|v) \propto P(u)P(v|u)$$

  with

  $$P(u,\lambda|v) \propto \tilde{P}(u,\lambda)P(v|u,\lambda).$$

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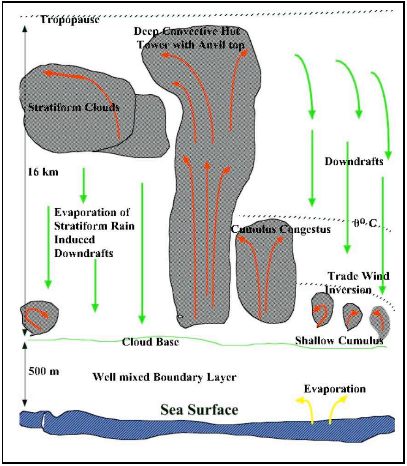
- The main questions are: How to choose $\tilde{P}$? How to parameterize λ? How do you justify your choice of $\tilde{P}$ are optimal? We need a reasonable test model for the tropical atmospheric dynamics, need to choose $\tilde{P}$ that is data-driven.

Three-cloud model [Khouider and Majda 2006, 2007]



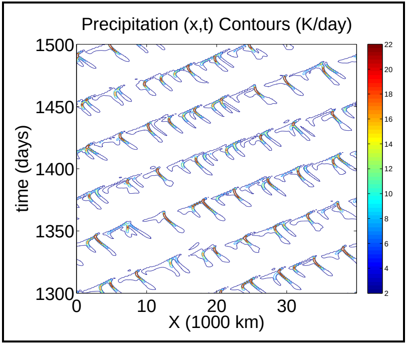
Three-cloud mechanism: (1) lower troposphere moistening through cumulus clouds triggers the (2) deep convection heating, and finally (3) trailing decks of stratiform precipitation.

The three-cloud mechanism has been observed on the eastward propagating convectively coupled Kelvin waves [Wheeler and Kiladis 2005], on the westward two-days waves [Haertl and Kiladis 2004], and on the MJO [Kiladis et al. 2005].

The three-cloud model has a nonlinear switch that triggers the deep convection heating during the moist episode and cumulus cloud heating through downdraft cooling during the dry episode.

An MJO-analogue parameter regime

[Majda-Stechmann-Khouider 2007]



5m/s phase speed westerly low frequency planetary scale envelope with intermittent mesoscales westward propagating deep convection event

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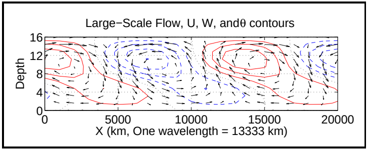
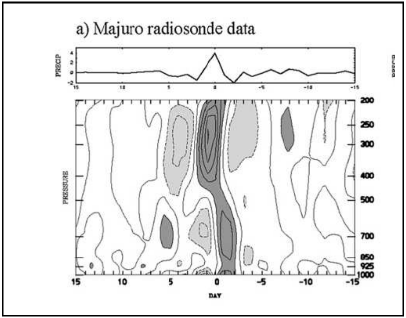
Moving average at 5m/s of the velocity fields and heating contour.



Vertical tilting profile from [Straub et al. 2010]



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on a periodic domain (for simplicity). Motivated by turbulent closure approaches [DelSole 2004, Majda and Timofeyev 2004, etc], we consider the following **Mean Stochastic Model** [MH2012, Ch 12],

$$\mathcal{N}(u) \to \sum_{k=0}^{J-1} \left(-\lambda_k u_k + \sigma_k \dot{W}_k \right) e^{2\pi i k j/J},$$

then parameterize λ_k, σ_k by fitting to the exact equilibrium statistical solutions of the linear reduced stochastic models, such as to the energy spectrum and correlation time.

Mean Stochastic Model for the three-cloud model

In our case, $\mathbf{\Psi} = (u_1, u_2, \theta_1, \theta_2, \theta_{eb}, q, H_s, H_c)^T \in \mathbb{R}^8$. Consider the linearized multicloud model about the RCE [KM2006]:

$$\frac{\partial \mathbf{\Psi}'}{\partial t} = \mathcal{P}(\partial_x)\mathbf{\Psi}',$$

where $\mathbf{\Psi}'$ denotes the perturbation field about the RCE and $\mathcal{P}$ denotes the linearized differential operator of the multicloud model at RCE.

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Consider eigenvalue decomposition $\mathrm{i}\omega(k)\mathbf{Z}_k = \mathbf{Z}_k\mathbf{\Lambda}_k$ and define $\hat{\mathbf{\Phi}}_k = \mathbf{Z}_k^{-1}\hat{\mathbf{\Psi}}_k$ such that

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$$\frac{d\hat{\mathbf{\Phi}}_k}{dt} = \mathbf{\Lambda}_k\hat{\mathbf{\Phi}}_k + \textit{damping} + \textit{forcing} + \textit{noise}$$

Parameterization for the MSM

We'd like to parameterize

$$d\hat{\mathbf{\Phi}}_k = \Big[(-\mathbf{\Gamma}_k + \mathrm{i}\mathbf{\Omega}_k)\hat{\mathbf{\Phi}}_k + \mathbf{f}_k\Big]dt + \mathbf{\Sigma}_k dW_k,$$

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Our discrete-time Kalman filtering problem with MSM as the prior model is defined on each horizontal wavenumber k:

$$\begin{aligned}
\hat{\mathbf{\Psi}}_{k,m} &= \mathcal{F}_k(\Delta t)\hat{\mathbf{\Psi}}_{k,m-1} + \mathbf{g}_{k,m} + \eta_{k,m}, \\
\mathbf{G}\hat{\mathbf{\Psi}}^o_{k,m} &= \mathbf{G}\hat{\mathbf{\Psi}}_{k,m} + \mathbf{G}\hat{\sigma}_{k,m},
\end{aligned}$$

where $\eta_{k,m} \sim \mathcal{N}(0, \mathbf{Q}_k)$ and

$$\mathbf{Q}_k = \frac{1}{2}\mathbf{Z}_k\mathbf{\Sigma}_k^2\mathbf{\Gamma}_k^{-1}(\mathbf{I} - |\mathcal{F}_k(\Delta t)|^2)\mathbf{Z}_k^*.$$

Analog of 3DVAR

We also consider a version of 3D-VAR by simply setting the prior error covariance statistics to be independent of time

$$\mathbf{B}_k \equiv \lim_{\Delta t \to \infty} \mathbf{R}^b_{k,m} = \lim_{\Delta t \to \infty} \mathbf{Q}_{k,m} = \frac{1}{2} \mathbf{Z}_k \boldsymbol{\Sigma}_k^2 \boldsymbol{\Gamma}_k^{-1} \mathbf{Z}_k^*.$$

Here, moist and dry background can be chosen appropriately based on the parameterization of $\boldsymbol{\Sigma}_k$.

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Dry and Cold model: We fit the model to $(u_1, u_2, \theta_1, \theta_2, \theta_{eb})$. Mathematically, we construct dry and cold eigenmode,

$$\hat{\mathbf{\Phi}}_k^{dc} = \mathbf{Z}_k^{-1}\begin{bmatrix} \mathbf{I}_{5\times 5} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\hat{\mathbf{\Psi}}_k.$$

to parameterize $\mathbf{\Gamma}_k, \mathbf{\Omega}_k, \mathbf{\Sigma}_k$.

Observation networks

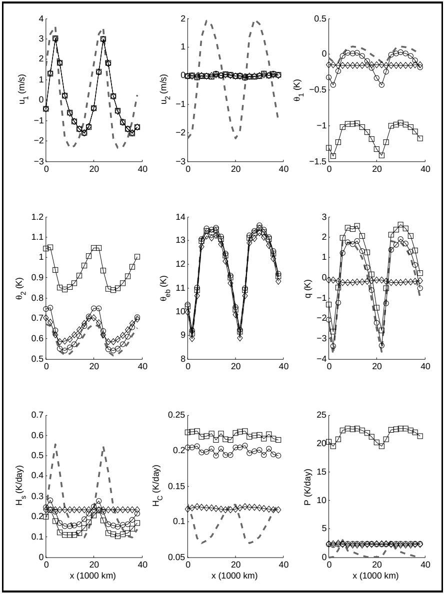
We define our observation model as follows:

$$\mathbf{G\Psi}^o_{j,m} = \mathbf{G\Psi}_{j,m} + \mathbf{Gs}_{j,m}, \quad \sigma_{j,m} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}^o),$$

at every $x_j = 2000$km and consider various observation networks:

- Surface Observations (SO): wind, potential temperature at surface height $z_s = 100m$ and θ_{eb}.
- Surface Observations + Middle Troposphere Temperature (SO+MT): add temperature at 8km.
- Surface Observations + Middle Troposphere Temperature & Velocity (SO+MTV): add velocity at 8km.
- Complete Observations (CO).

Observation networks: Surface Observations



Unrealistic estimates for precipitation with MSM-filter (squares), the complete 3D-VAR (circles), "dry and cold" 3D-VAR (diamonds)!

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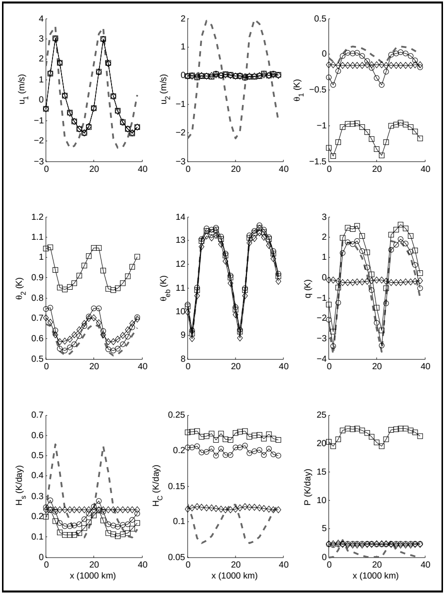

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Precipitation budget: $P_0 = \frac{1}{\tau_{conv}}\left[a_1\theta_{eb} + a_2(q - \hat{q}) - a_0(\theta_1 + \gamma_2\theta_2)\right]^+$

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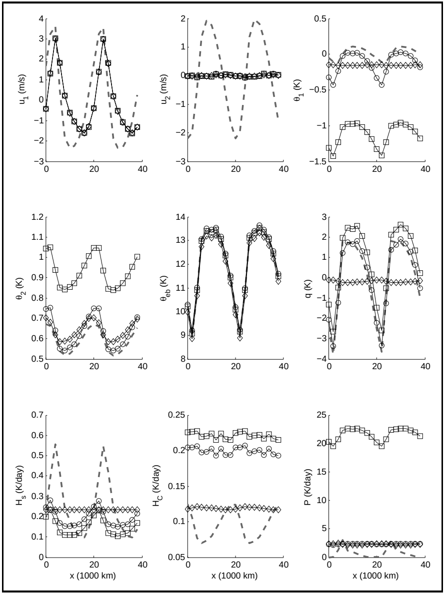

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$a_1 = 0.1, a_2 = .5, a_0, \gamma_2 = 0.1,$ but $a_0 = 12$!

Observation networks: Surface Observations + Midtroposphere Temperature & Velocity

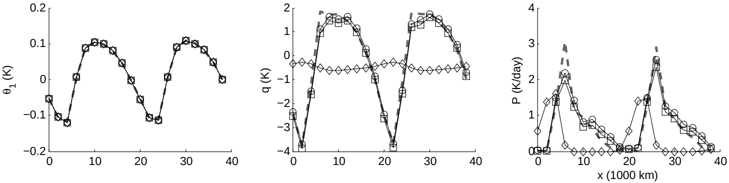
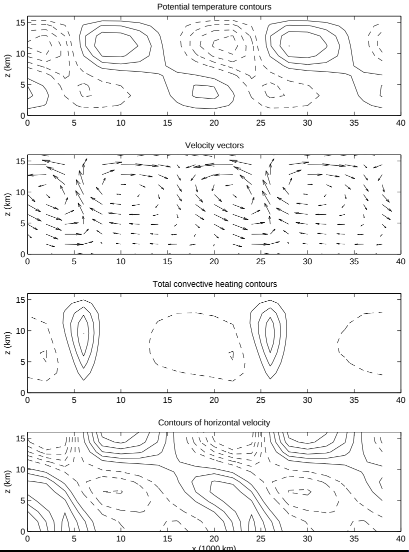
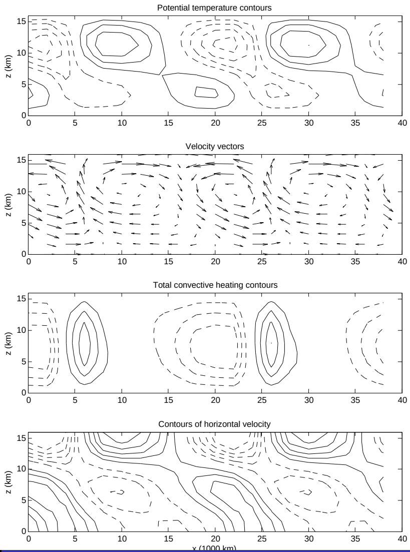


Figure: Moving average of the multi-cloud model variables with their vertical structure reconstructed from observing only wind and temperature spatially at every 2000km and 24h. True (grey dashes), posterior mean estimates from the 3D-VAR with moist background covariance matrix (circles), the MSM-Filter (squares), and the 3D-VAR with dry background covariance matrix (diamonds).

True (left) vs Observation networks: Surface Observations + Midtroposphere Temperature & Velocity (right)





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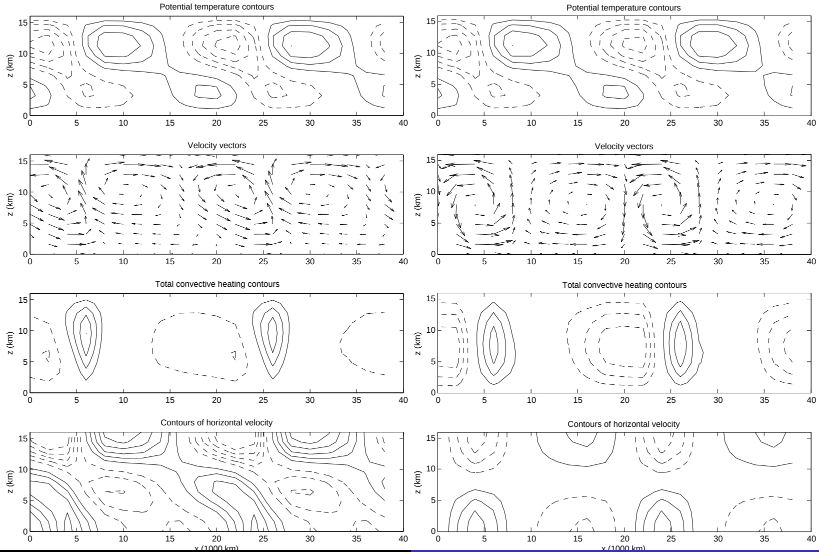

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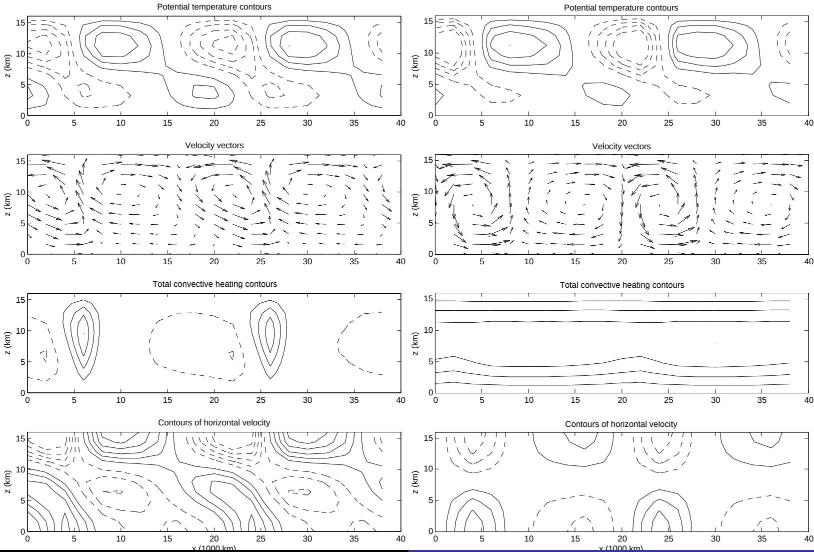
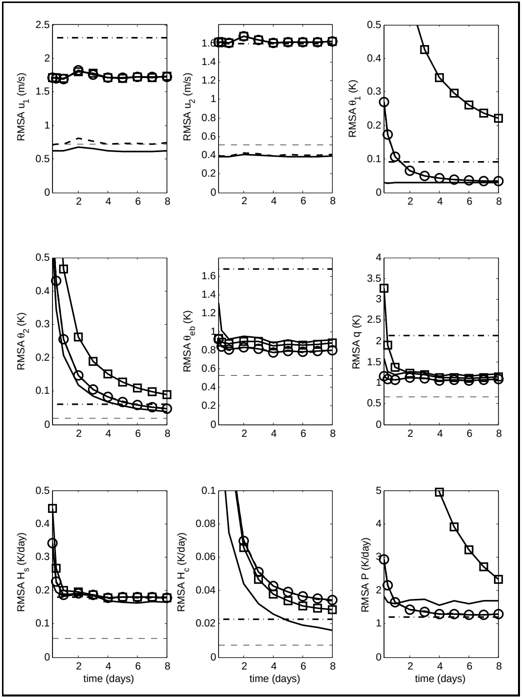
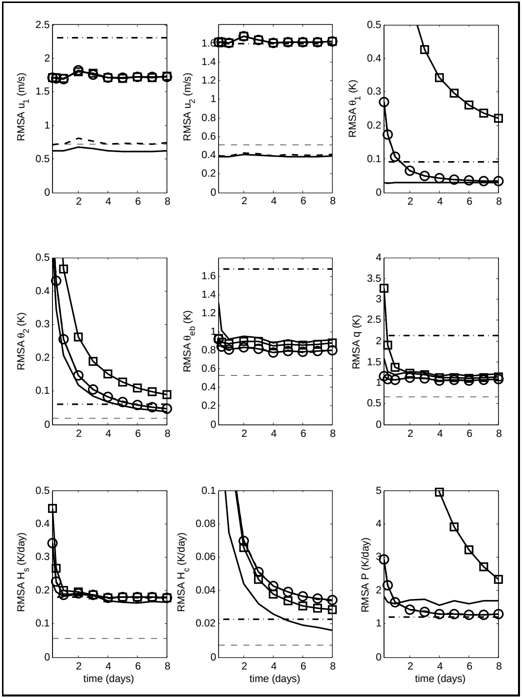




Observation error variance (thin dashes) is about 10% of the climatological error variance (dash-dotted line): CO (thick solid line), SO+MTV (thick dashes), SO+MT (circles) and SO(squares).



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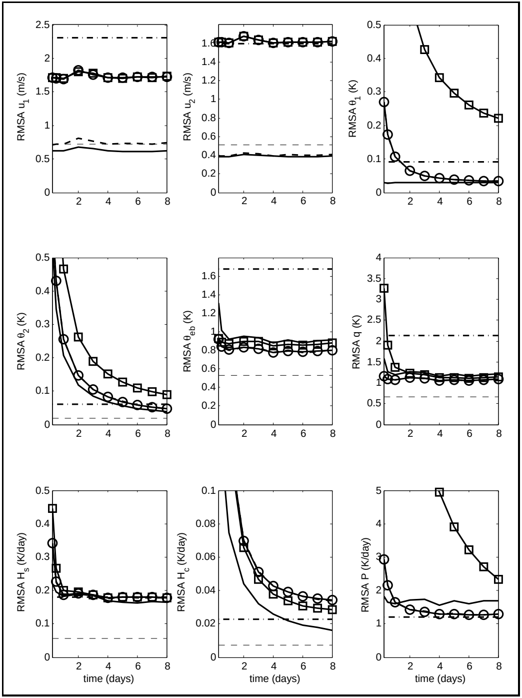
The filter except for SO is skillful (better than climatological errors) for $u_1, \theta_1, \theta_{eb}, q$



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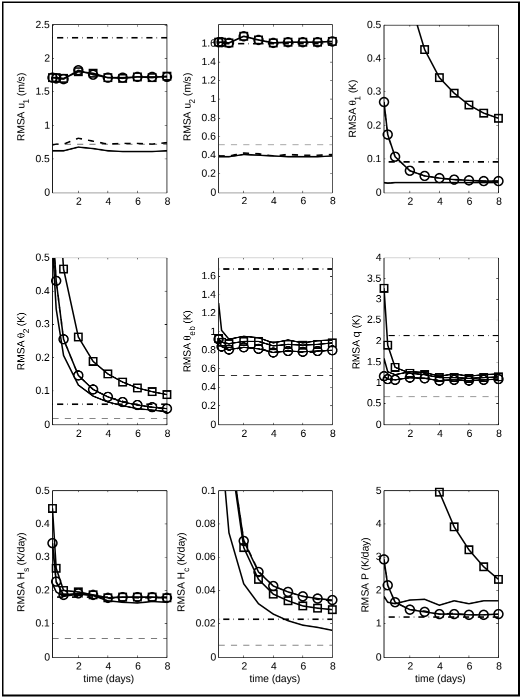

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For SO+MT: When $\Delta t = 6$h, $\lambda_1(\mathcal{F}_k) = 0.9899$ and when $\Delta t = 72$h, $\lambda_1(\mathcal{F}_k) = 0.8836$. The shorter time is marginally stable! In this case, the observability condition that is necessary for filter stability [Anderson and Moore 1979, MH 2012] is practically violated here! In this case, the observability matrix is ill-conditioned, $\det\left([\mathbf{G}^T\,(\mathbf{G}\mathcal{F}_k)^T]\right) \approx 10^{-20}$.

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- Our simple reduced stochastic filters are able to recover moisture and precipitation field profile (even when online observations of these variables are not available) provided that the filter forward prior model is designed with a moisture coupled eigenmode basis.
- A better estimate for the tropical convection wave patterns requires more than surface wind and potential temperature observations.
- The skill of the reduced filtering methods with horizontally and vertically sparse observations suggests that more accurate filtered solutions are achieved with less frequent observation times. Such a counterintuitive finding is justified through an analysis of the classical observability and controllability conditions which are necessary for optimal filtering especially when the observation timescale is too short relative to the timescale of the true signal.

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