SEARCH FOR LENSED GRAVITATIONAL WAVE SIGNALS FROM BINARY BLACK HOLE MERGERS

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with
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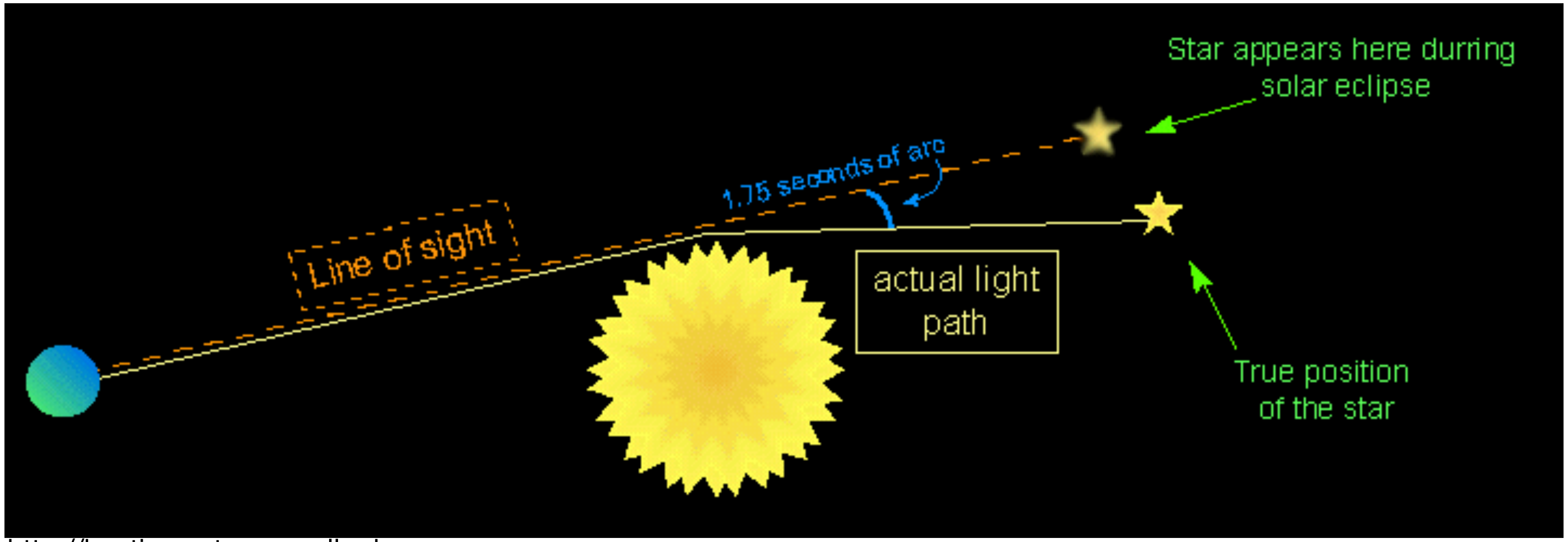
OUTLINE

- Overview of gravitational lensing.
- Strong lensing of gravitational waves.
- Bayesian model selection technique to identify lensed gravitational signals.
- Simulations.
- Future directions.
- Conclusion.

WHAT IS GRAVITATIONAL LENSING ?



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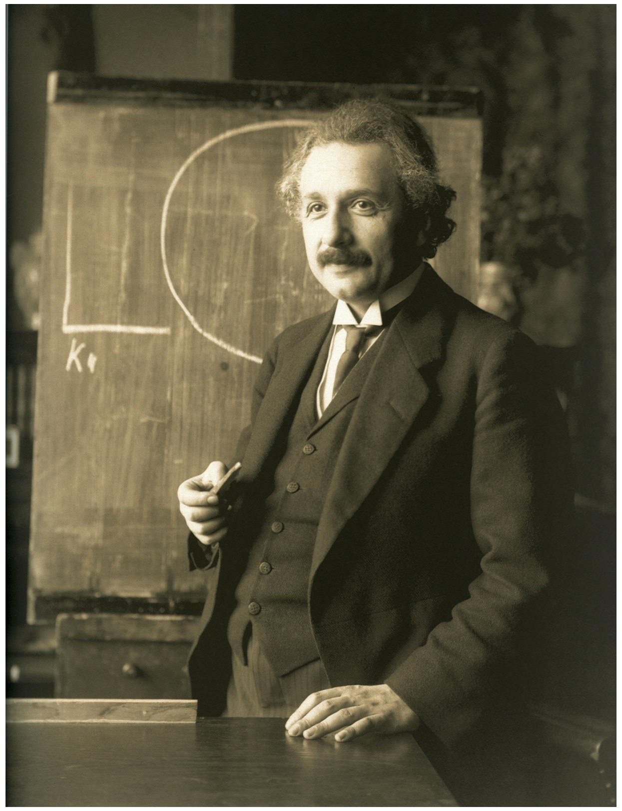

- Radiation is deflected in gravitational field of massive objects.
- A consequence of General Relativity.
- Confirmed by Eddington by his solar eclipse observation in 1919 from Principe, Africa.

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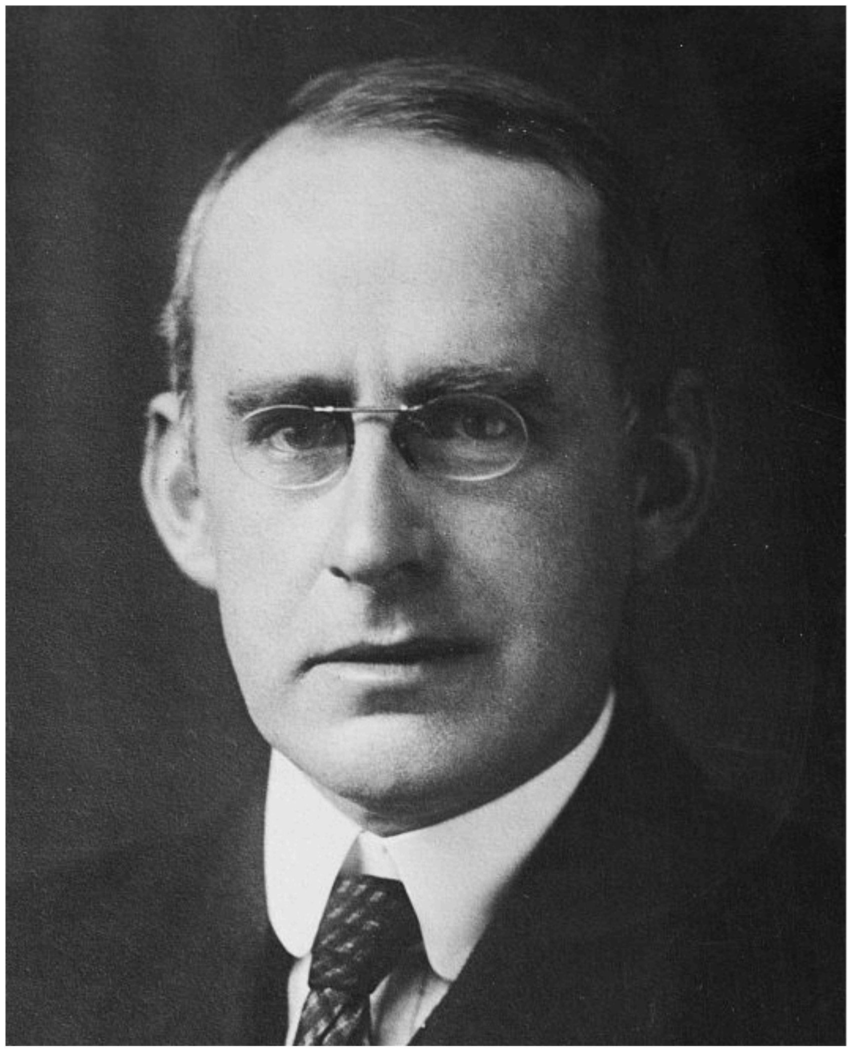

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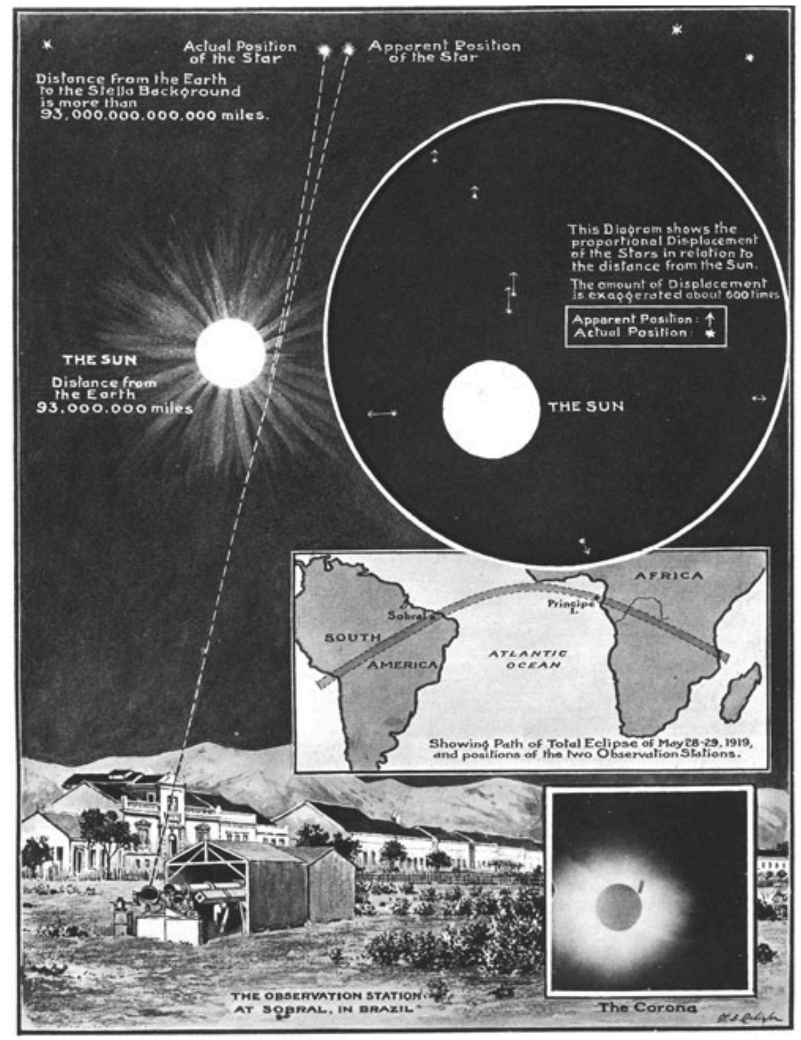

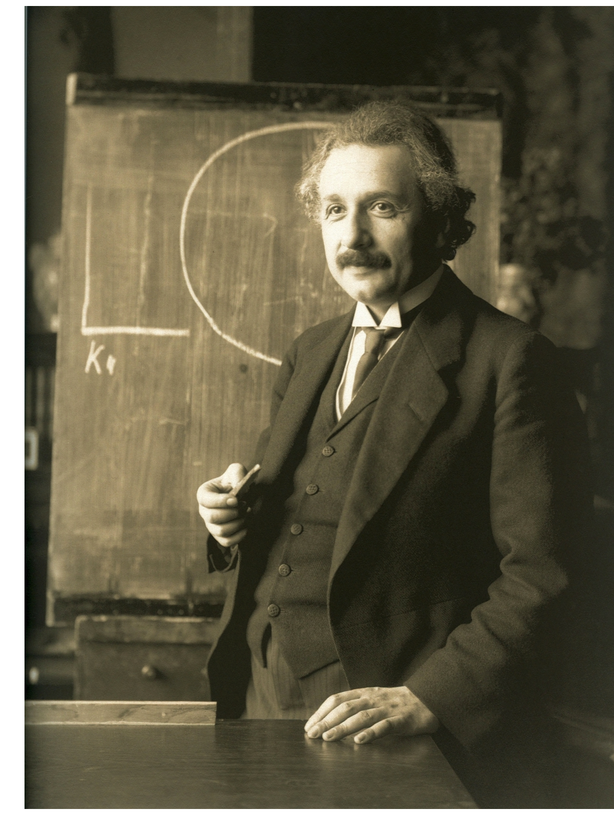

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WHAT IS GRAVITATIONAL LENSING ?

- As for conventional lenses, images will form at extrema in the light travel time surface (Fermat's principle).
- Produce visible distortions such as the Einstein rings, arcs, and multiple images.
- Examples: cluster of galaxies, dark matter.

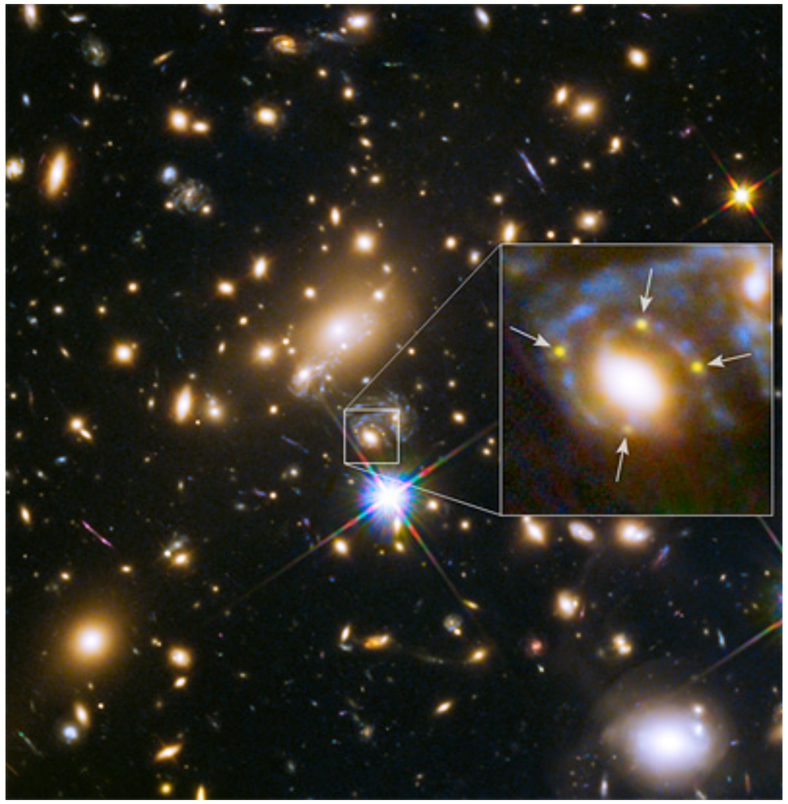
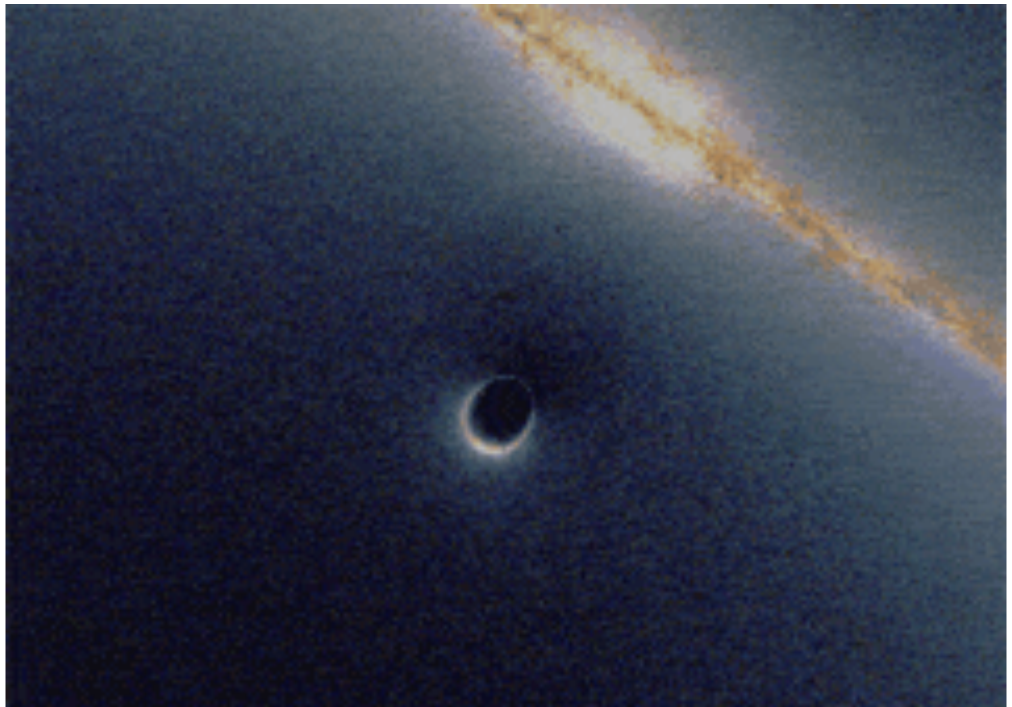
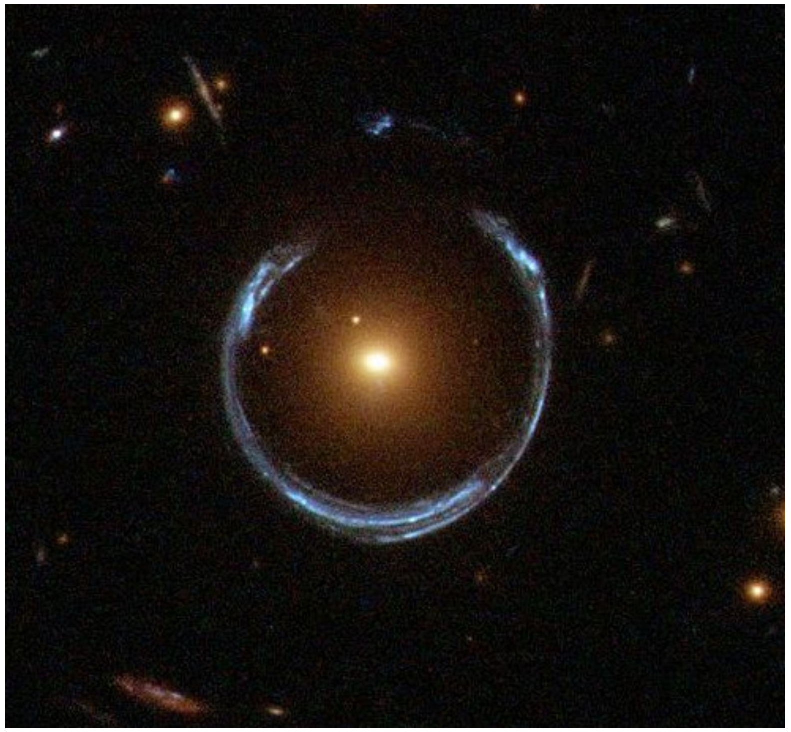


Image credit: NASA, ESA



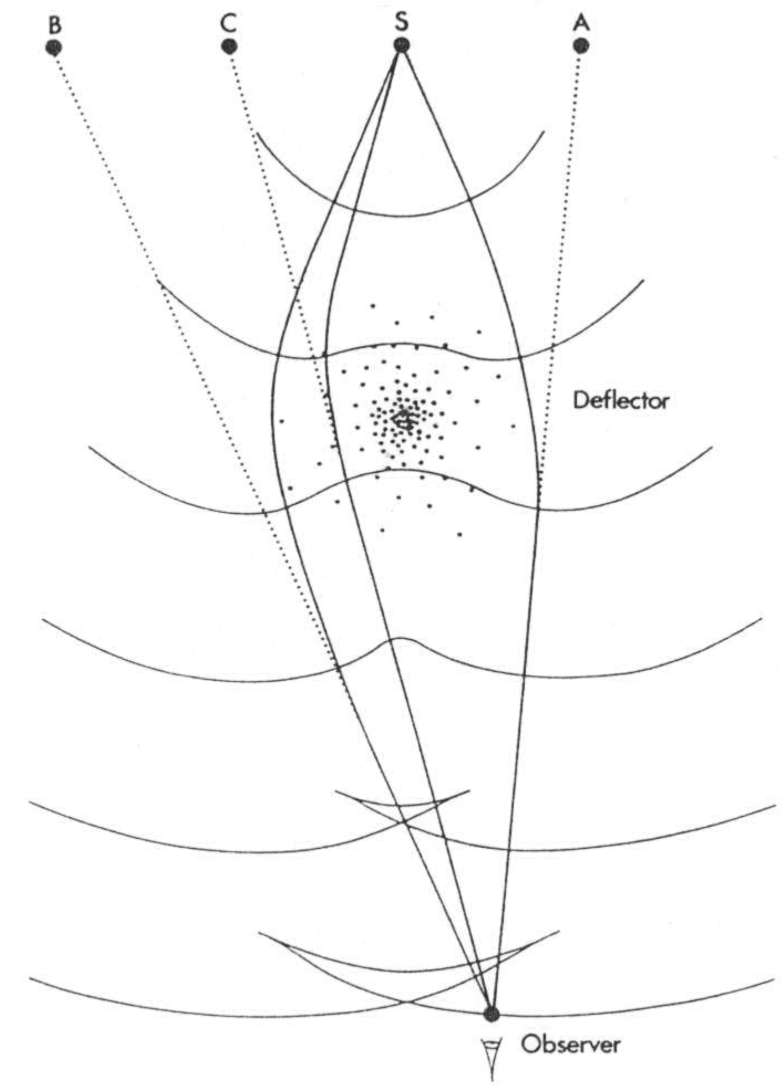
Simulated gravitational lensing (black hole passing in front of a background galaxy).



Einstein Ring. Image credit: NASA,

TIME DELAYS AND MAGNIFICATION

- Multiple light paths connecting source and observer result in multiple images.
- Images seen in directions perpendicular to the wavefronts.
- Wavefronts from the same event in the object arrive at different times (time delay between images).
- The gravitational lensing changes the apparent solid angle of a source results in magnification of the images.



$$magnification = \frac{image\ area}{source\ area}$$

- Strong, Weak and Micro lensing based on the lensing strength.

APPLICATIONS OF LENSING OBSERVATION

- Cosmic telescopes: The magnification effect enables us to observe objects which are too distant or intrinsically too faint to be ob- served without lensing.
- Probing compact dark matter: Gravitational lensing depends solely on the mass distribution of the lens.
- Cosmology: The Hubble constant (from time delays) and the density distribution of the universe can be significantly constrained through lensing.

OUTLINE OF LENSING THEORY

- From Fermat's principle, the deflection angle is given by,

$$\hat{\alpha} = -\int \nabla_{\perp} \ln n \; dl \; = \frac{2}{c^2}\int \nabla_{\perp}\Phi \; dl$$

Gravitational potential

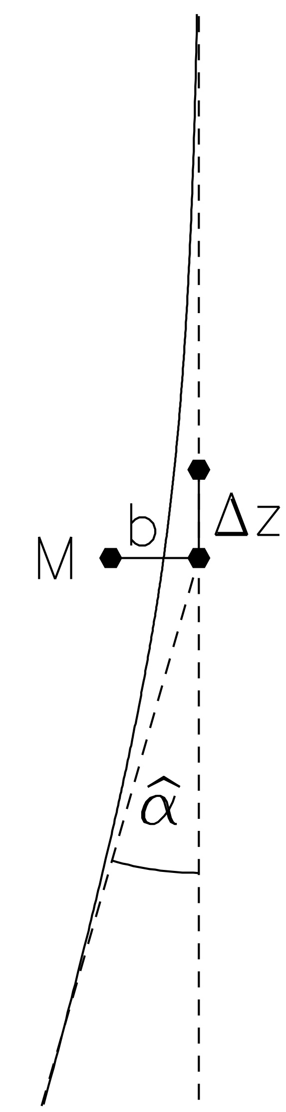
Refractive index

- For a point mass lens,

$$\hat{\alpha} = \frac{4GM}{c^2 b}$$

Lens mass

Impact parameter



Narayanan & Bartelmann, 1995

LENSING EQUATION: THIN LENS APPROXIMATION

- Scaled lens Equation

$$\vec{y} = \vec{x} - \vec{\alpha}(\vec{x})$$

With,

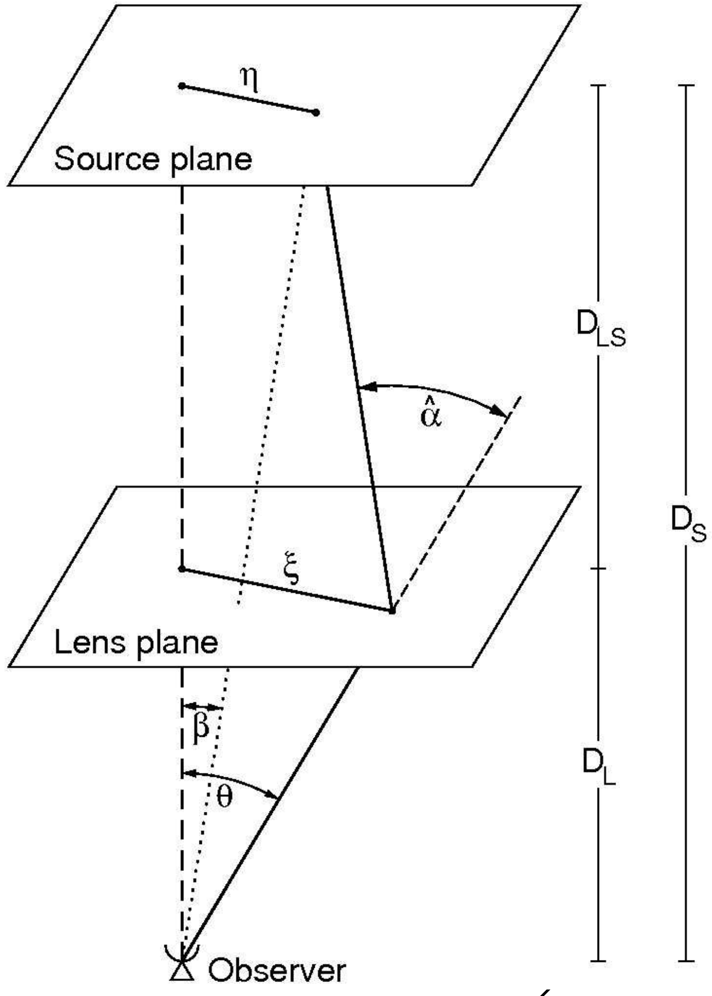
$$\vec{x} = \frac{\xi}{\xi_0}, \quad \vec{y} = \frac{D_L}{D_s}\frac{\eta}{\xi_0}, \quad \vec{\alpha} = \frac{D_L D_{LS}}{\xi_0 D_S}\hat{\alpha}$$

Scaling constant

- Scaled deflection,

$$\vec{\alpha} = \frac{1}{\pi}\int_{lens} d^2x' \kappa(\vec{x}') \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|}$$

Scaled lens mass density



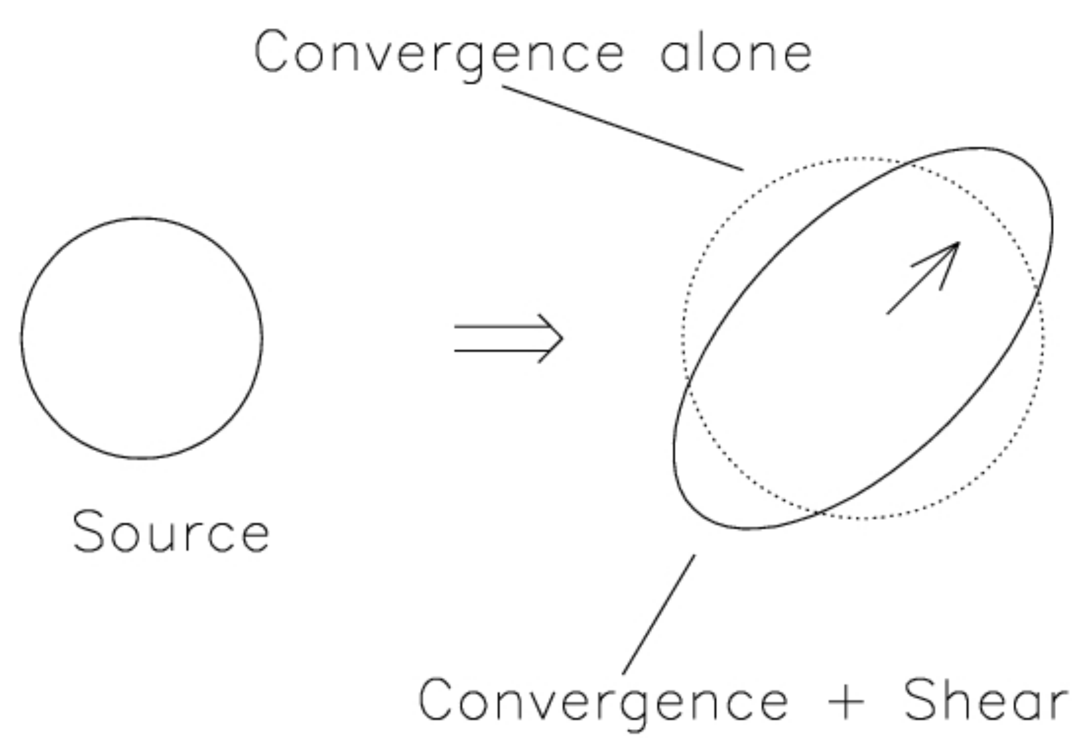
Angular diametric distances

Narayanan & Bartelmann, 1995

MAGNIFICATION

- The distortion arises because light bundles are deflected differentially. The shape of the images can be determined by solving the lens equation for all the points within the extended source.
- Magnification of the image is given by the inverse of Jacobian of transformation from source plane to lens plane.

$$\mu = \left| \frac{\partial \vec{y}}{\partial \vec{x}} \right|^{-1}$$



Narayanan & Bartelmann, 1995

TIME DELAY

- This time delay has two components:

$$t = t_{geometric} + t_{gravitational}$$

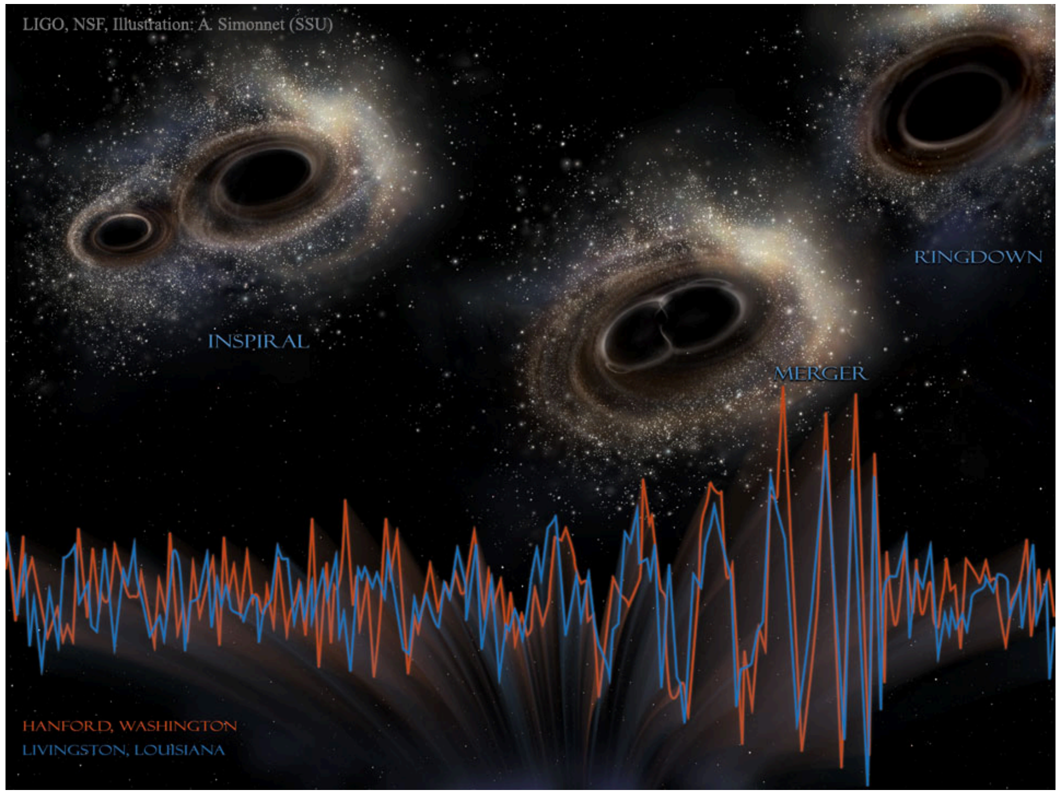
Due to the different path length of the light rays.

Due to the slowing down of photons traveling through the gravitational field of the lens.

$$t(\overrightarrow{x}) = \frac{1+z_L}{c}\left[\frac{D_s\xi_0^2}{2D_LD_s}(\overrightarrow{x}-\overrightarrow{y})^2 - \frac{2}{c^2}\Phi(\overrightarrow{x})\right]$$

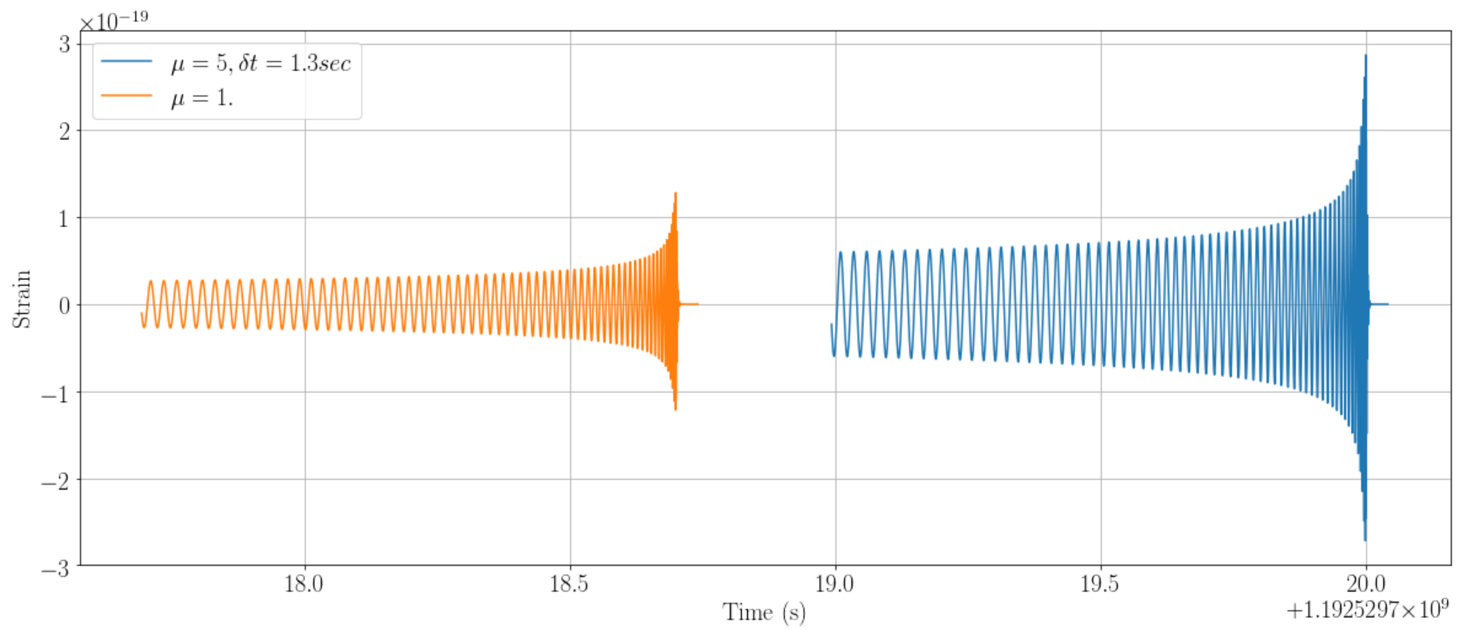
Lens surface potential

STRONG LENSING OF BBH MERGERS



- Strong lensing can produce multiple GW events from the same binary black hole merger (analogous to multiple images in EM observations) typically separated by time delays of weeks to months.
- Multiple images dominantly arise due to lensing by galaxies [Fukugita et al, 1991].
- A small fraction (~ 1%) of BBH mergers detectable by LIGO & Virgo could be strongly lensed by intervening matter distributions [Ken K. Y. Ng et al, 2017].

STRONG LENSING OF BBH MERGERS



- Under geometric optics approximation ($\lambda_{\mathrm{GW}} \ll R^{\mathrm{lens}}_{\mathrm{Schwarzschild}}$) frequency profile of the images remains the same. Only change is in the magnification, and hence the estimated luminosity distance.
- Poor sky localization, good time resolution.

Estimated luminosity distance

Actual luminosity distance

$$\rho' = \sqrt{\mu}\rho \implies d'_L = d_L / \sqrt{\mu}$$

lensed SNR

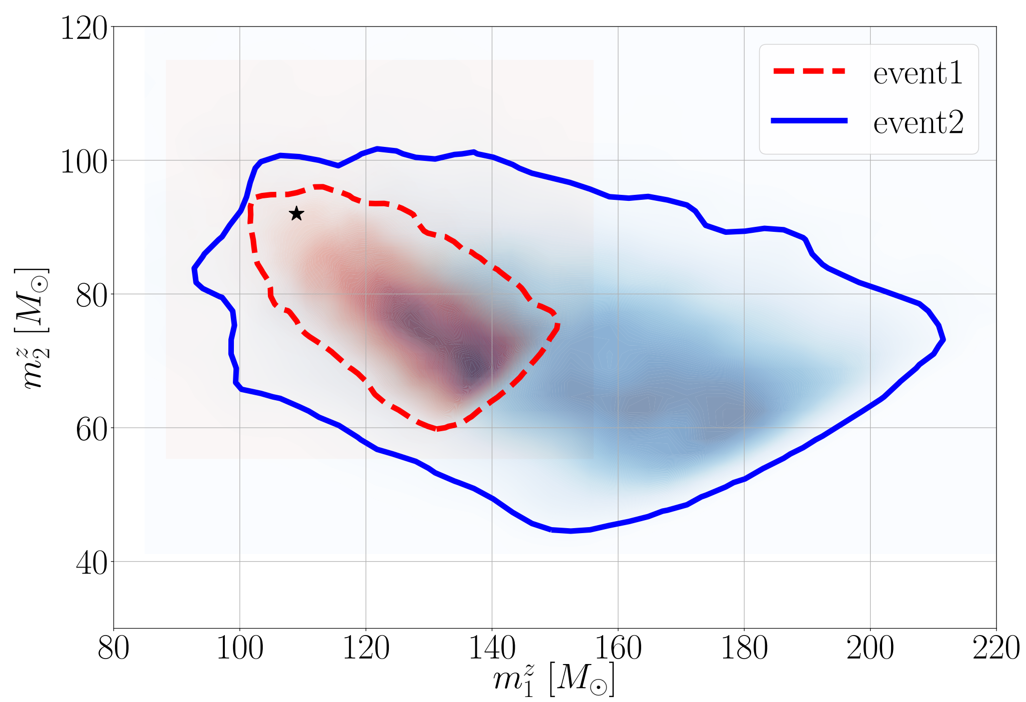
magnification

original SNR

IDENTIFYING LENSED BBH MERGERS:BAYESIAN FORMALISM

- Since the frequency evolution of the signal remain unchanged, the estimated intrinsic parameters of the waveform that determines the evolution of the waveform (eg: redshifted masses, inclination angles etc.) will be consistent between the images.

- From the two events $\{d_1, d_2\}$, compute the Odds ratio between two hypotheses:



Two GW events are produced by the same merger

$$\mathcal{O}^L_U := \frac{P(\mathcal{H}_L \mid \{d_1, d_2\})}{P(\mathcal{H}_U \mid \{d_1, d_2\})}$$

Two GW events are unrelated

IDENTIFYING LENSED BBH MERGERS:BAYESIAN FORMALISM

$$\mathscr{O}^L_U = \frac{P(d_1, d_2 \,|\, \mathscr{H}_{\mathscr{L}})}{P(d_1, d_2 \,|\, \mathscr{H}_{\mathscr{U}})} \, \frac{P(\mathscr{H}_L)}{P(\mathscr{H}_U)}$$

Bayes factor, $\mathscr{B}^L_U$

Prior odds

Posteriors

$$\mathscr{B}^L_U := \int d\vec{\theta} \; \frac{P(\vec{\theta} \,|\, d_1) \; P(\vec{\theta} \,|\, d_1)}{P(\theta)}$$

Prior distribution

FURTHER IMPROVING THE MODEL SELECTION

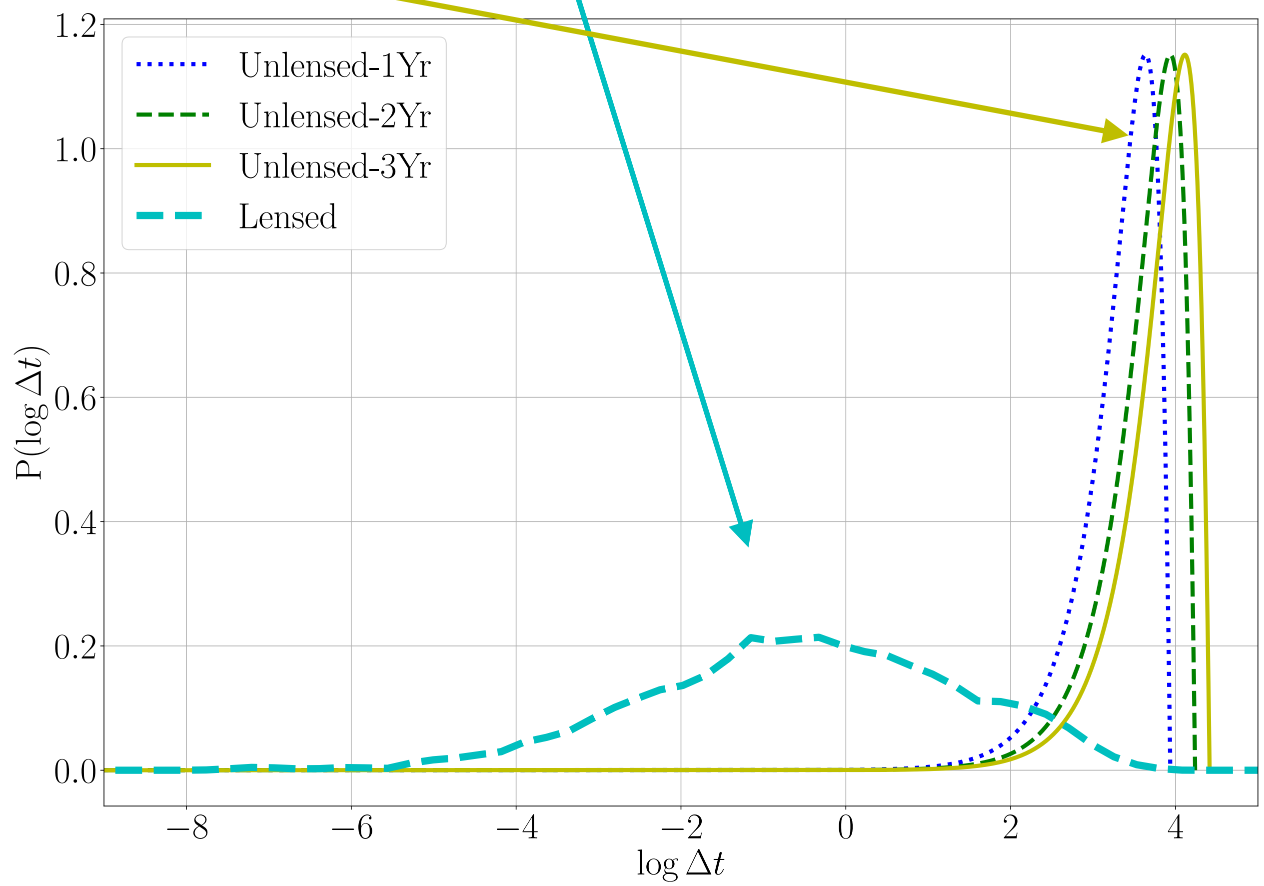
- Distribution of time delay between lensed events (elliptic galaxy lenses) is different from that of unlensed events (assuming poisson process).

Bayes factor computed from the time delay distribution:

lensed

$$\mathcal{R}^{\mathrm{L}}_{\mathrm{U}} = \frac{P(\Delta t_0 | \mathcal{H}_{\mathrm{L}})}{P(\Delta t_0 | \mathcal{H}_{\mathrm{U}})}$$

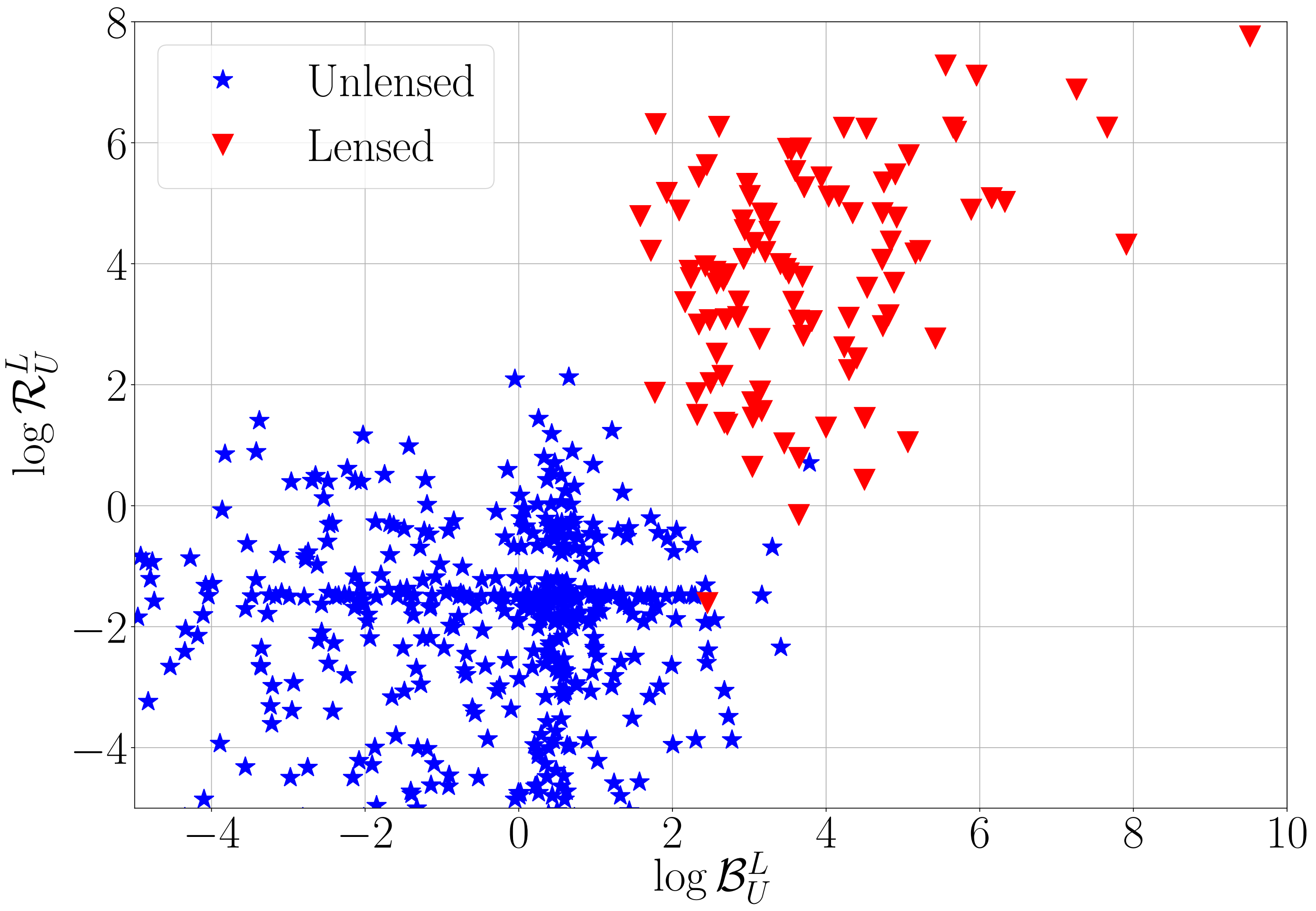
unlensed



FURTHER IMPROVING THE MODEL SELECTION

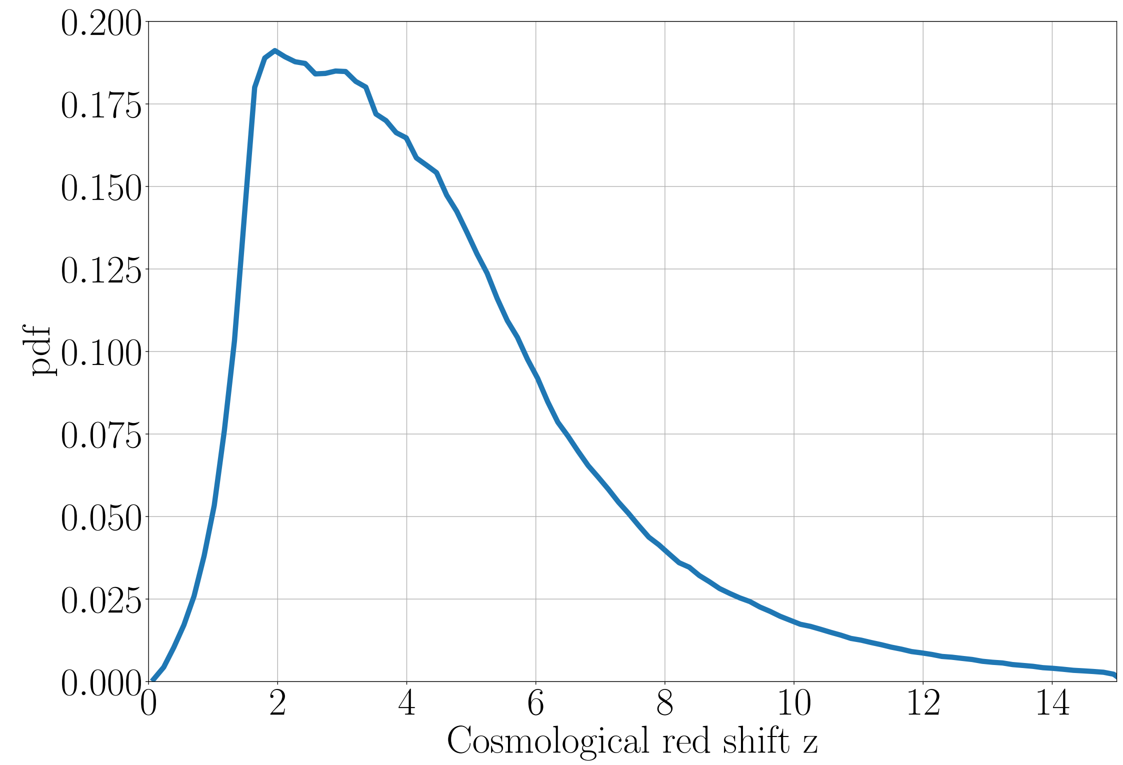
- The two Bayes factors could be combined to improve the discriminatory power.

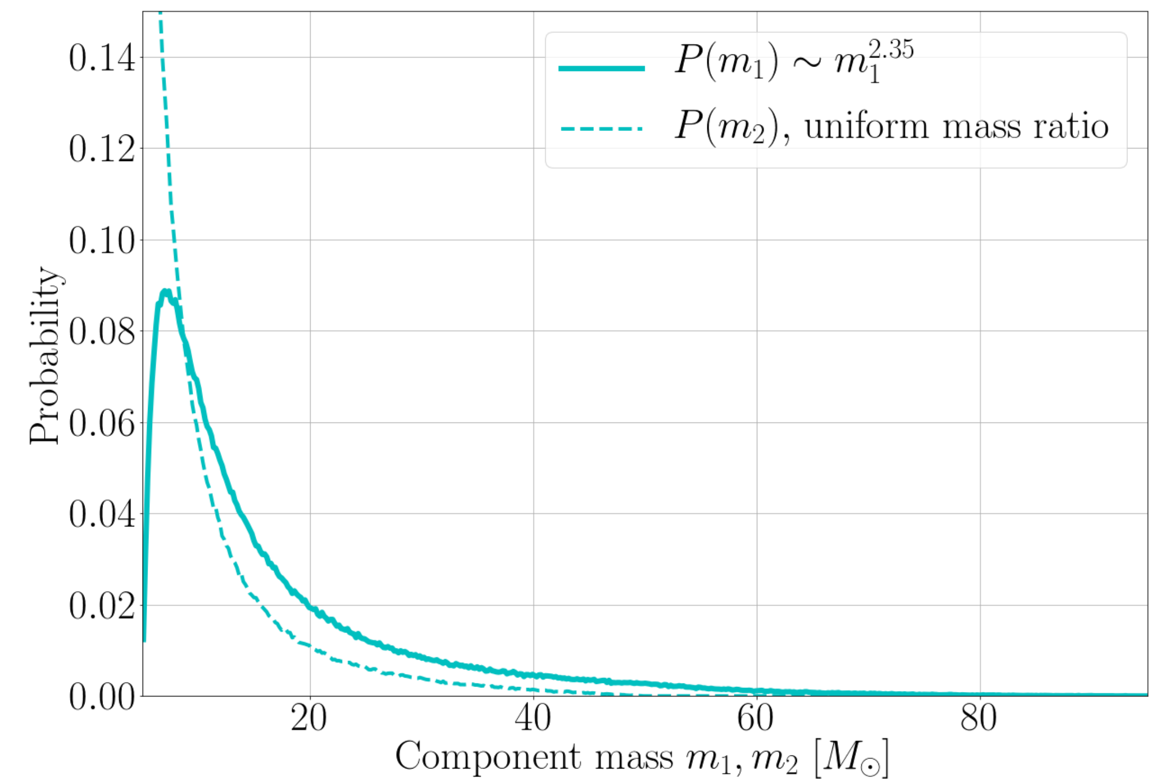
$$\mathcal{B}^{\mathrm{L}}_{\mathrm{U}} \times \mathcal{R}^{\mathrm{L}}_{\mathrm{U}}$$



TESTING THE MODEL SELECTION: DISTRIBUTION OF LENSES AND BBHS

- Astrophysical simulation
  - Singular isothermal elliptic lenses (SIE), whose surface mass density diverges at the center [Collett, T. E. 2015].
  - Redshift distribution of mergers [Dominik et al, 2013].
  - Component masses follows two different power laws [LVC GW150914 Rates paper, 2016].





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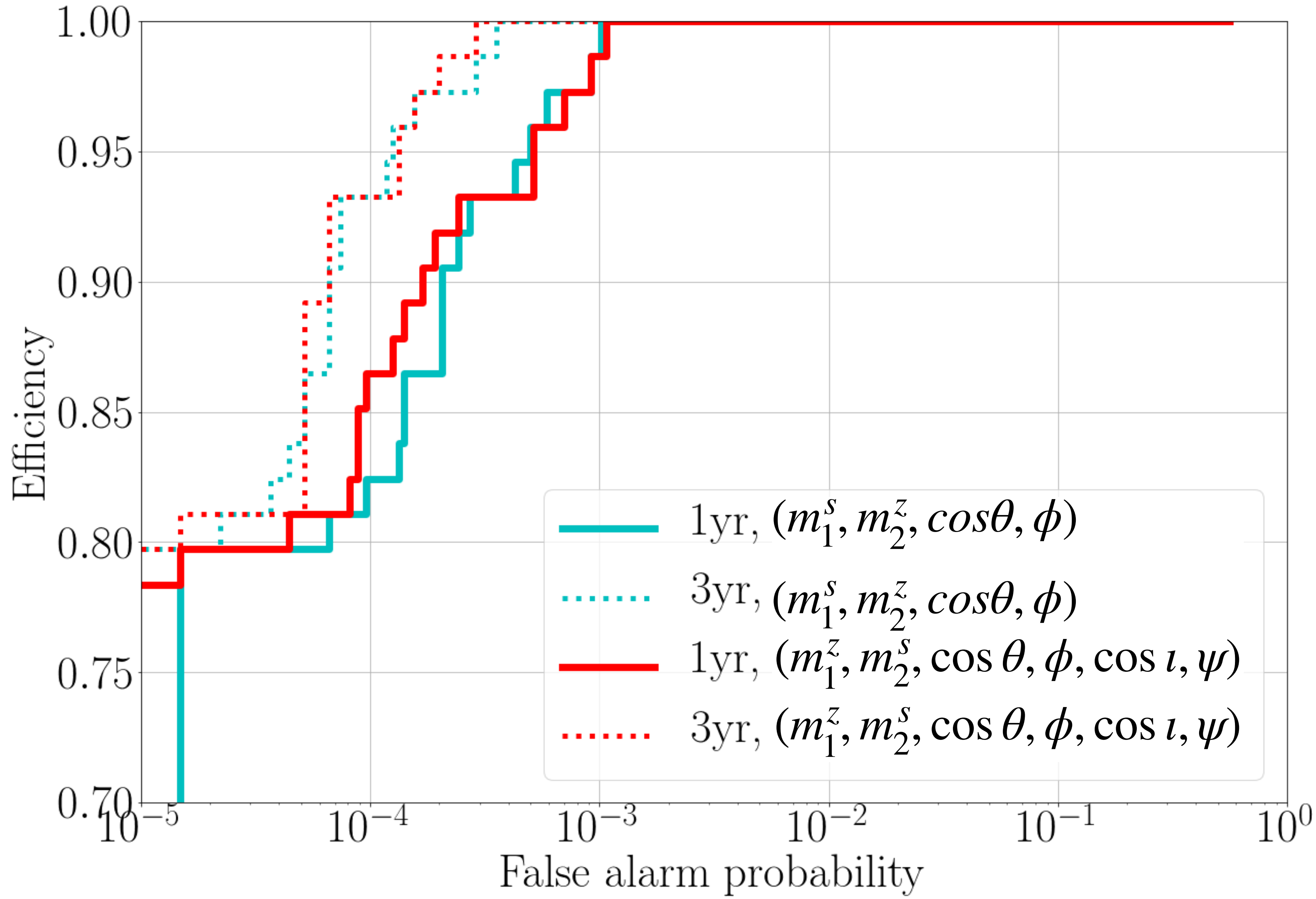
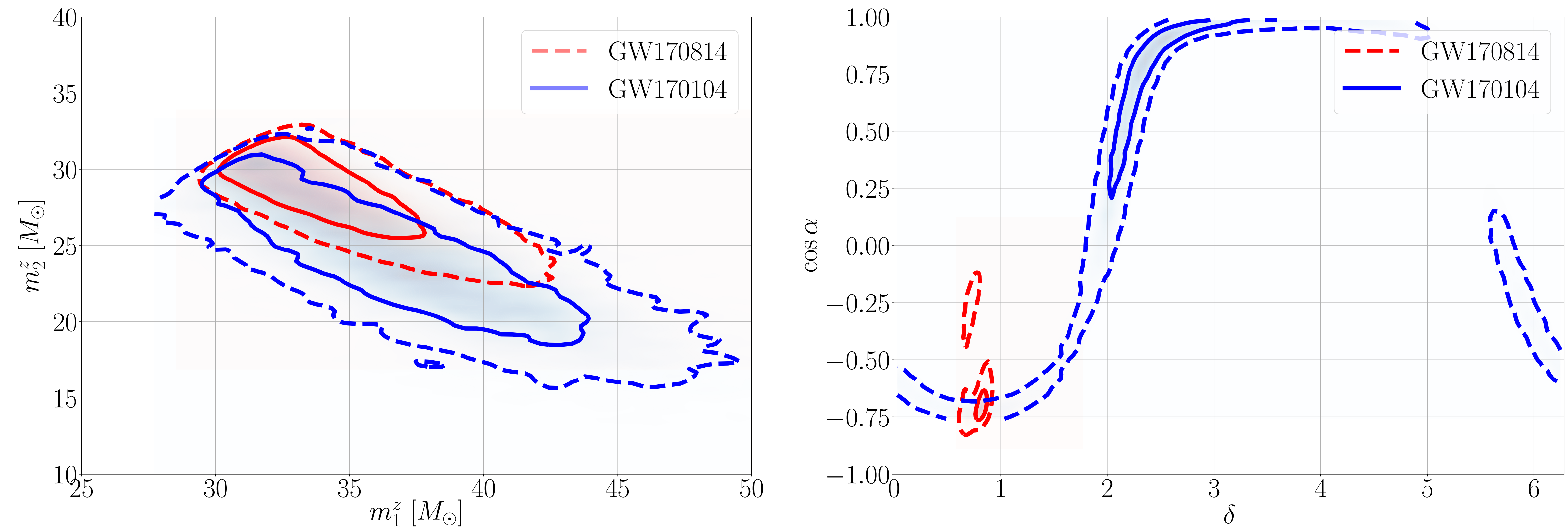


Can detect 80% of lensed events with a false alarm prob of 10^{-5}

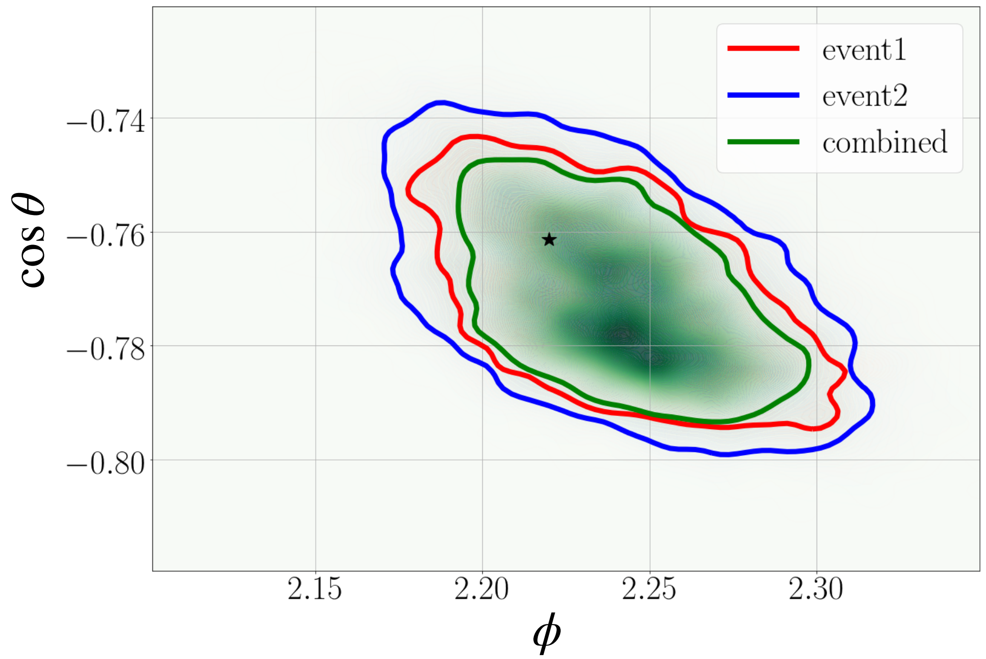
REAL EVENTS: GW170104 AND GW170814



Combined Bayes factor ~ 0.2.

Note that the prior odds is < 0.01. Hence the odds ratio does not support the lensing hypothesis.

APPLICATIONS OF BBH LENSING OBSERVATIONS

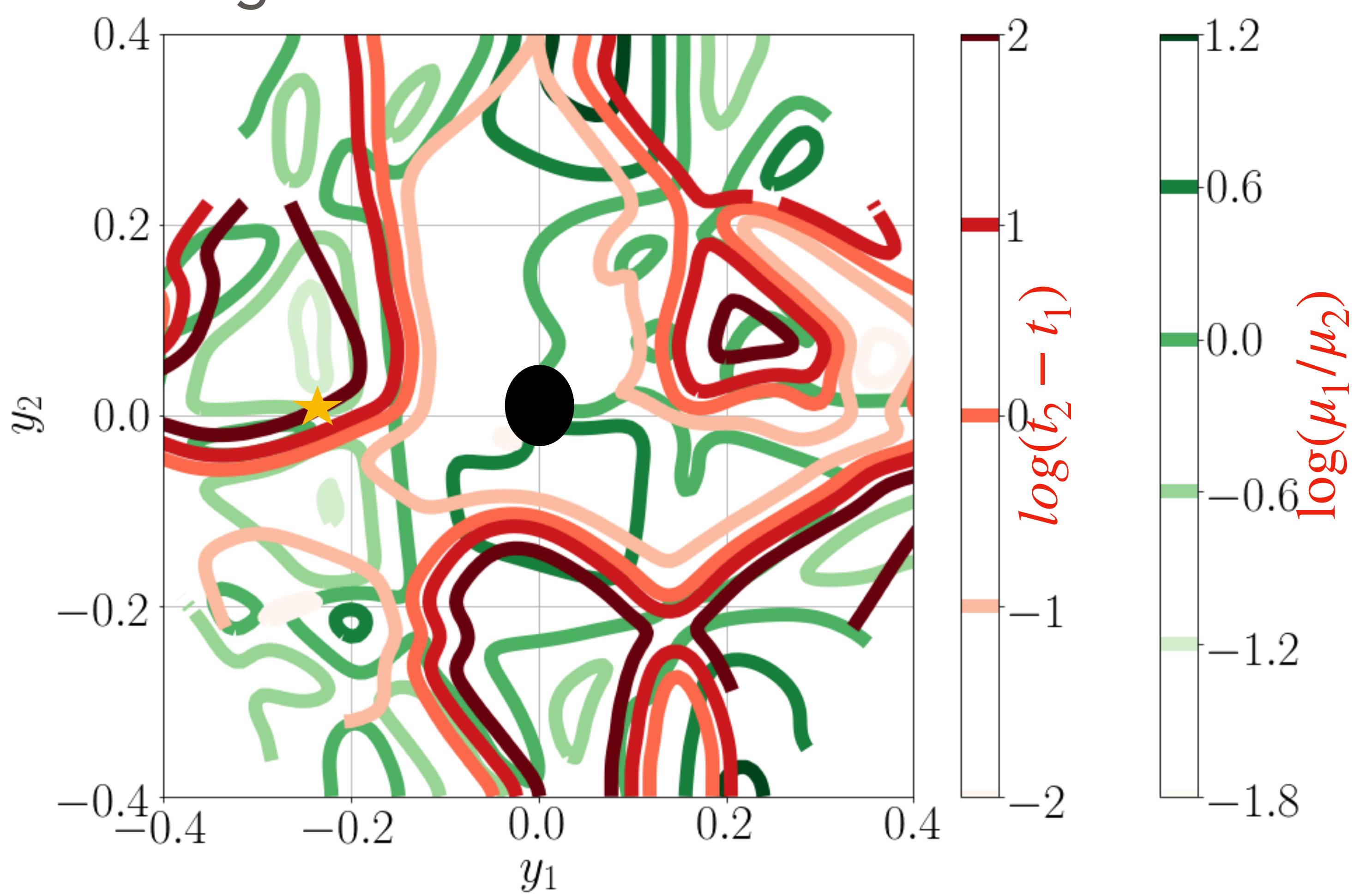


- Improved parameter estimation: As the lensed images share same set of intrinsic parameters, one can combine the posteriors of the images to obtain improved distribution.

$$P(\overrightarrow{\theta}\,|\,d_1, d_2) = P(\overrightarrow{\theta}\,|\,d_1) \times P(\overrightarrow{\theta}\,|\,d_2)$$

APPLICATIONS OF BBH LENSING OBSERVATIONS: LOCALIZE THE SOURCE POSITION WITH LENS MODEL

- The EM lens survey data can be used to model the lens galaxies and obtain the time delay - magnification ratio map.
- If one can identify one or more lensed galaxies with in the LIGO-Virgo posterior sky patch of lensed BBH event, we can compare the time delay and magnification ratio of the images with the model to identify the actual location of the BBH merger.



CONCLUDING REMARKS

- A non-negligible fraction of BBH mergers detectable by LIGO & Virgo could be strongly lensed by intervening matter distributions.
- The proposed Bayesian model selection technique can detect ~80% of lensed events with a false alarm prob of 1e-5.
- Identification of such lensed images can help us to improve the parameter estimation of the mergers.
- Currently we are doing the search on the catalogue of BBH mergers observed by LIGO-Virgo.

Thank you!