

VINCENT VAN DER PAS

Collective Modes in Nuclei

Collective Modes in Nuclei



Hans Emling, ANUP, Goa, Nov. 2011

Collective motion

– a universal phenomenon
in nature

BI-SW: <http://www.youtube.com/watch?v=V71hz9wNsgs>

SHIBUYA:

http://www.youtube.com/watch?v=QXtOdSgf6lc&feature=BFa&list=PL881859D3DB1AFA9F&lf=results_video

Ped: <http://arxiv.org/abs/cond-mat/9805074>

Fl: <http://www.youtube.com/watch?v=clgHEhziUxU>

Computer Simulations of **Pedestrian Dynamics** and Trail Formation

D. Helbing, P. Molnar, F. Schweitzer

(Submitted on 6 May 1998 (v1), last revised 7 May 1998 (this version, v2))

Abstract: A simulation model for the **dynamic behaviour of pedestrian crowds** is mathematically formulated in terms of a **social force model**, that means, pedestrians behave in a way as if they would be subject to an **acceleration force** and **to repulsive forces** describing the reaction to borders and other pedestrians. The computational simulations presented yield many realistic results that can be compared with video films of pedestrian crowds. Especially, they show the self-organization of **collective behavioural patterns**. By assuming that pedestrians tend to choose routes that are frequently taken the above model can be extended to an active walker model of trail formation. The topological structure of the evolving trail network will depend on the disadvantage of building new trails and the durability of existing trails. Computer simulations of trail formation indicate to be a valuable tool for designing systems of ways which satisfy the needs of pedestrians best. An example is given for a non-directed trail network.

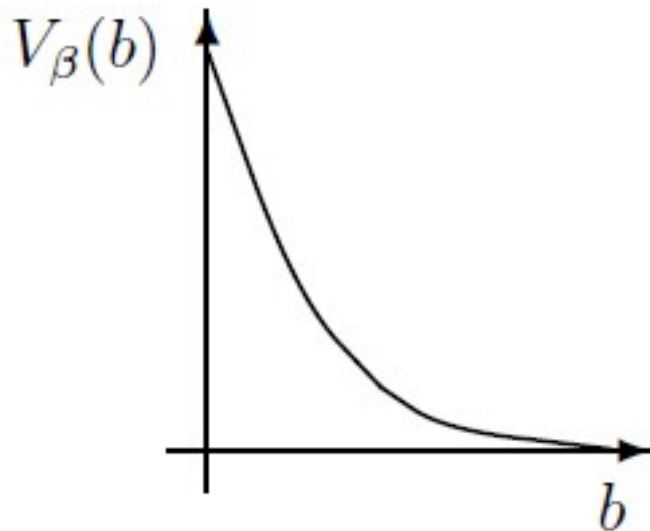
*For related work see this [http URL](http://URL) Statistical Mechanics (cond-mat.stat-mech); Pattern Formation and Solitons (nlin.PS) Pages 229-234 in: Evolution of Natural Structures (Sonderforschungsbereich Journal reference: 230, Stuttgart, 1994)
Cite as:arXiv:cond-mat/9805074v2 [cond-mat.stat-mech]*

'SOCIAL' repulsive force:

$$\vec{f}_{\alpha\beta}(\vec{r}_\alpha - \vec{r}_\beta) = -\nabla_{\vec{r}_\alpha} V_\beta[b(\vec{r}_\alpha - \vec{r}_\beta)]. \quad (5)$$

The sum over the repulsive potentials V_β defines the *interaction potential* which influences the behaviour of each pedestrian:

$$V_{\text{int}}(\vec{r}, t) := \sum_{\beta} V_\beta\{b[\vec{r} - \vec{r}_\beta(t)]\}. \quad (6)$$



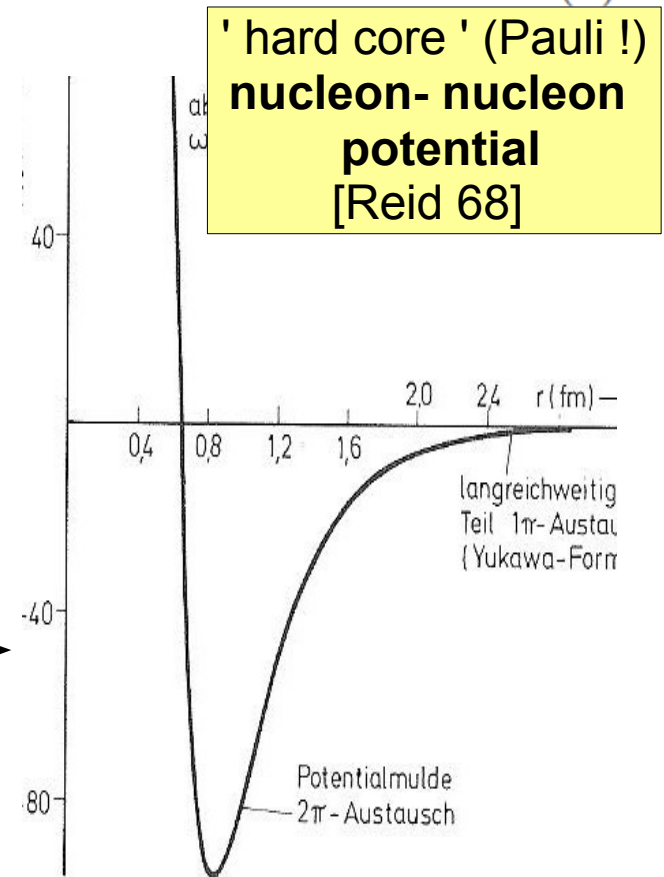
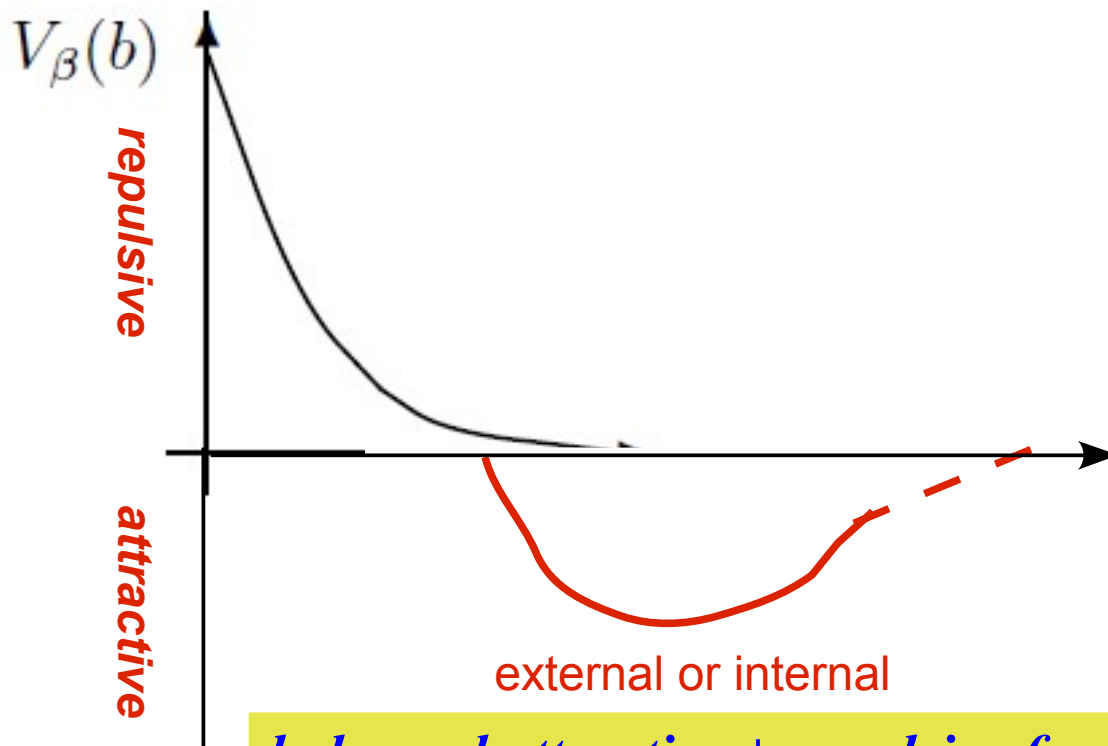
$b = \text{distance to neighbor}$

Repulsive 'social' force:

$$\vec{f}_{\alpha\beta}(\vec{r}_\alpha - \vec{r}_\beta) = -\nabla_{\vec{r}_\alpha} V_\beta[b(\vec{r}_\alpha - \vec{r}_\beta)]. \quad (5)$$

The sum over the repulsive potentials V_β defines the *interaction potential* which influences the behaviour of each pedestrian:

$$V_{\text{int}}(\vec{r}, t) := \sum_{\beta} V_\beta\{b[\vec{r} - \vec{r}_\beta(t)]\}. \quad (6)$$



balanced attractive + repulsive forces ==> collective motion

How do we recognize COLLECTIVE MODES in Nuclei ??

(Here: Nuclear Eigenstates,
not, e.g., collective flow in collisions)

Recipe: Quantization of a classical collective system and search for
the respective characteristic patterns in the nuclear spectral response

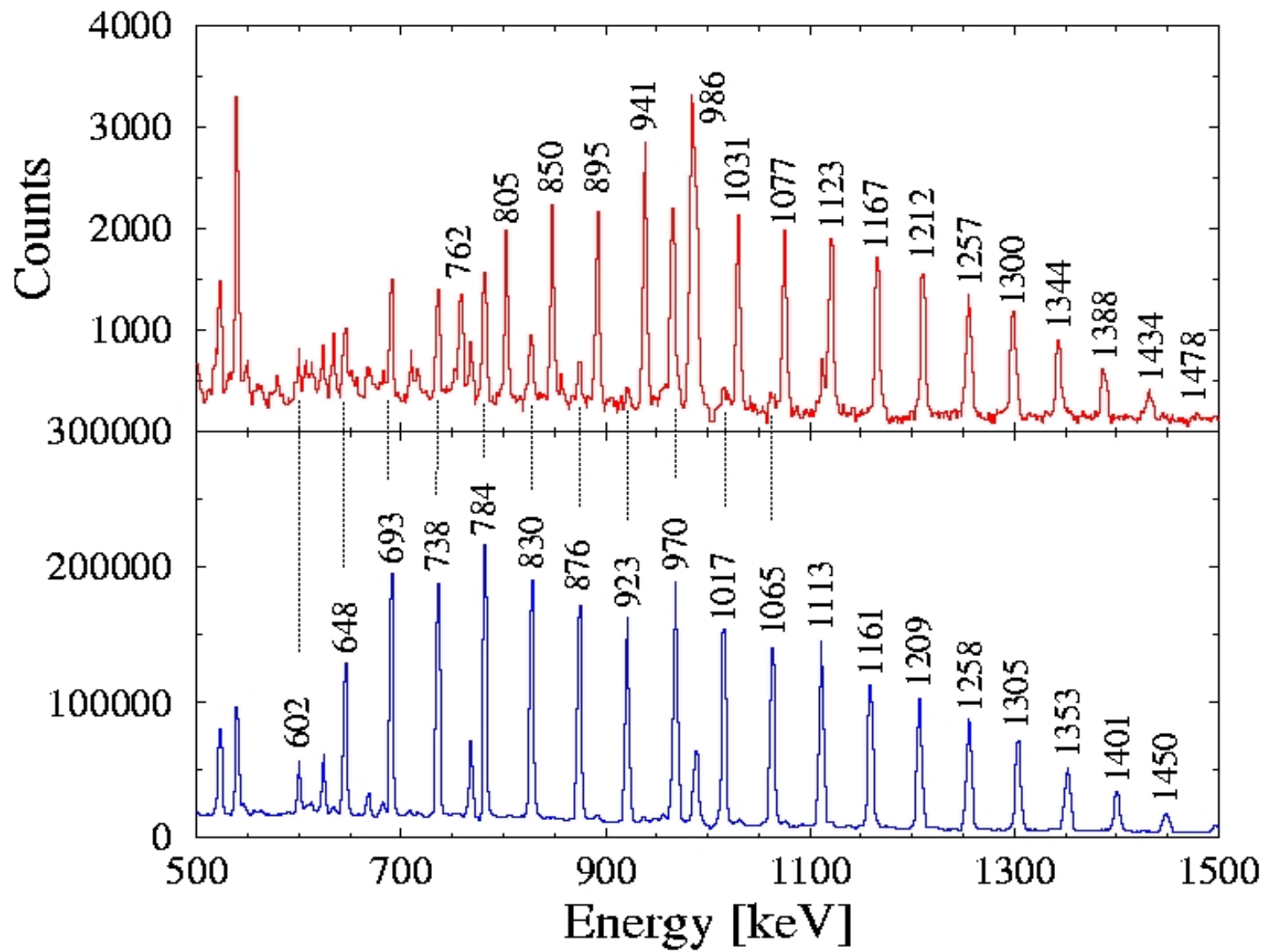
for example: rotational spectra

Classical rigid rotation: $E_L = L^2 / 2\Theta$ (continuous)

Quantized system: $E_I = \hbar^2 I (I + 1) / 2\Theta$ (discrete
eigenstates)



E2- γ -transitions: $E_\gamma = \hbar^2 2 (I - 1) / \Theta$ or $\Delta E_\gamma = \text{const.}$



- T. Lauritsen *et al.*, Phys. Rev. Lett. **88**, (2002). 042501
- T. Lauritsen *et al.*, Phys. Rev. Lett. **89** (2002) 282501

Vibrational nuclear states:

- Surface (β -, γ -, octupole) vibrations
- Density oscillations (' giant resonances ')

However:

**Not much exp.
information
on two- (higher-)
phonon states
available**

Quantized harmonic oscillator: $E_n = n E_0$

or $\Delta E = E_0$ ('phonon' !)

Rotational nuclear spectra and surface vibrations discussed already,
see M. Carpenter, P. Regan, P. Reiter ...

this lecture restricted to (specific aspects of) giant resonances !

Lecture I: Giant Resonances (GR) – Introduction, Basics

Phenomenology, properties, main facts

Classification schemes

Theoretical description – concepts

What we (might) learn from GR's

Tools (reactions) for GR excitation

History – milestones

Lecture II-IV: Giant Resonances – selected topics

Spin-flip resonances : Gamow-Teller puzzle

Isoscalar GMR and GDR : nuclear matter (in)compressibility

Multi-phonon Giant Resonances

Multipole Strength in (neutron-rich) 'exotic' nuclei

Pygmy resonances, soft modes

Symmetry energy

Neutronstar – neutronskin and pygmy

Outlook

OXFORD STUDIES IN NUCLEAR PHYSICS • 24

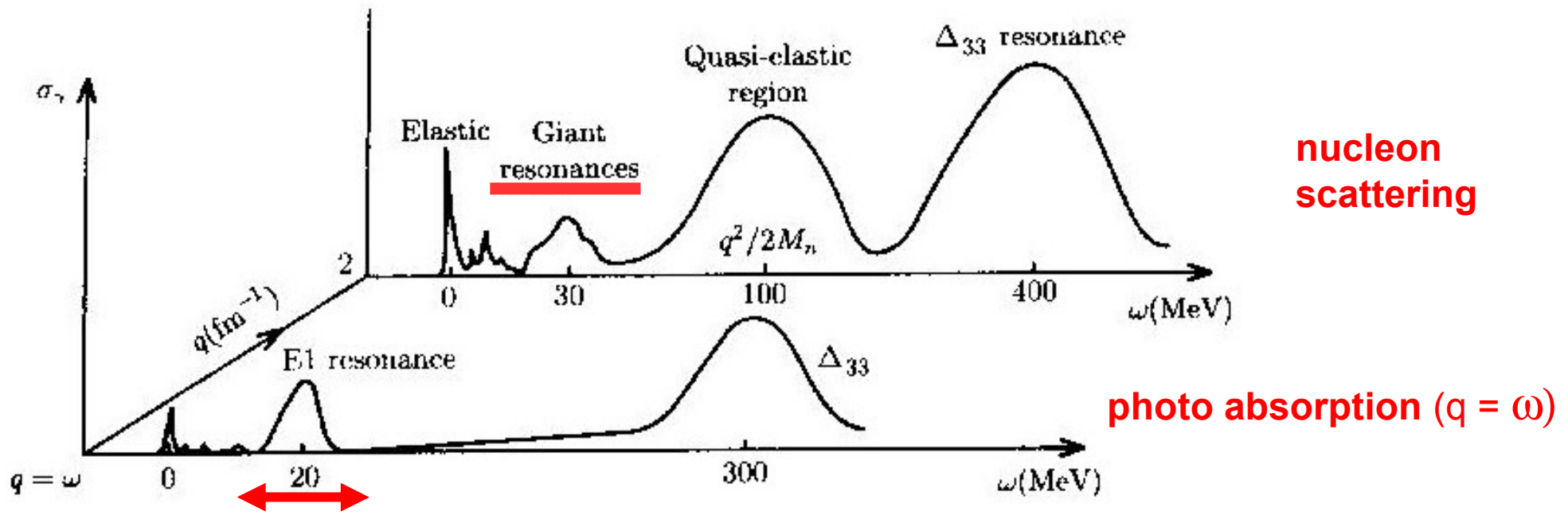
Giant Resonances
Fundamental
High-Frequency Modes of
Nuclear Excitation

M. N. HARAKEH
and
A. van der WOUDE



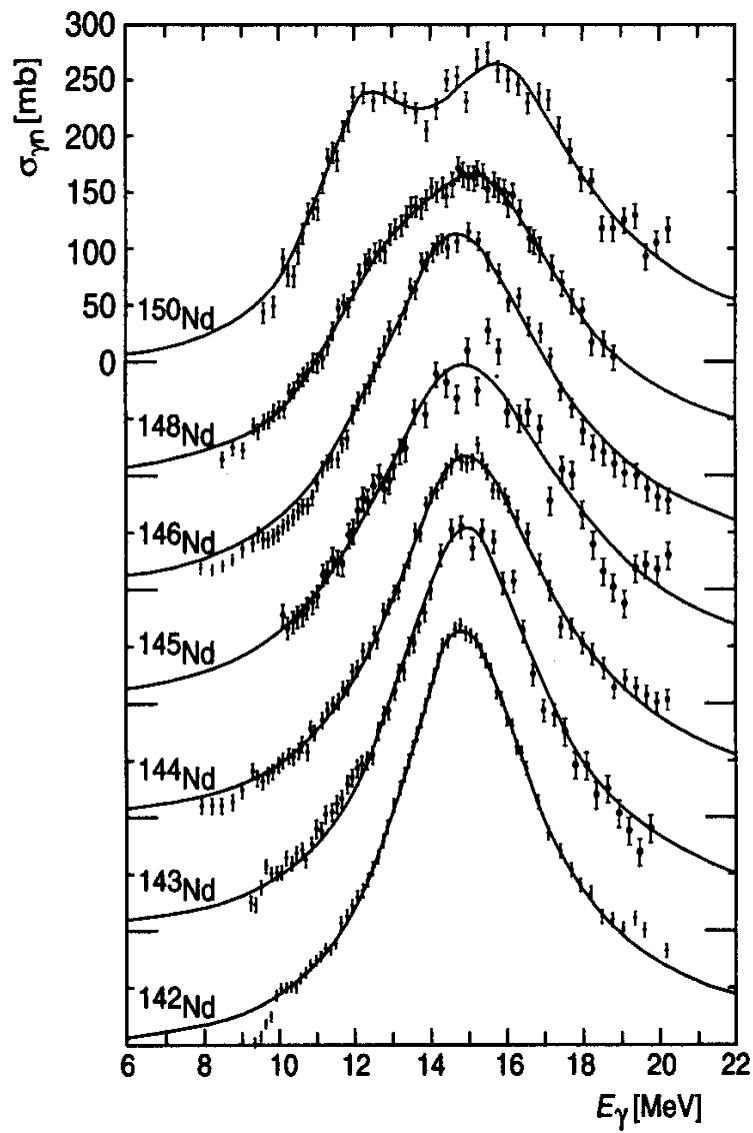
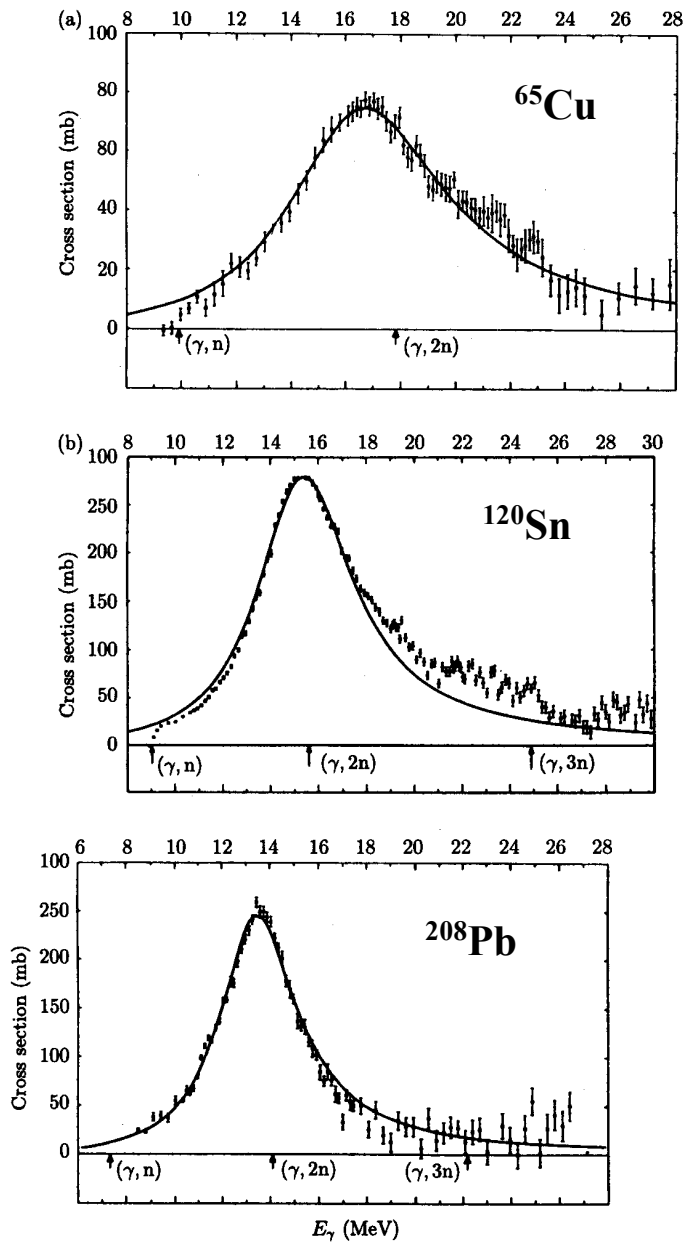
OXFORD SCIENCE PUBLICATIONS

Nuclear response to external field , at moderate energy (ω) and momentum (q) transfer



Photoabsorption (GDR)

Berman and Fulz, Rev. Mod. Phys. 47 (1975) 47



Inelastic α scattering (GMR + GQR)

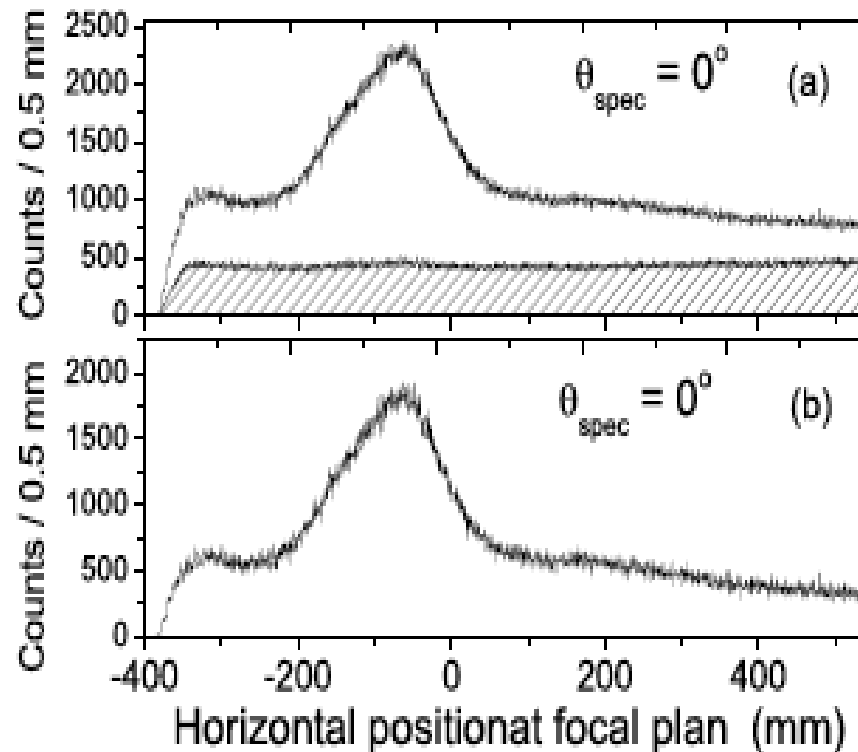
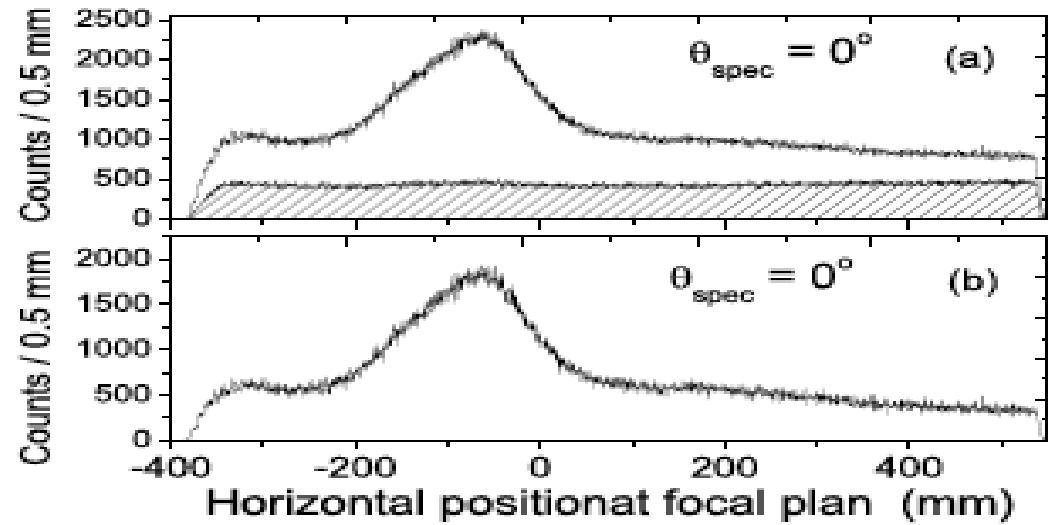
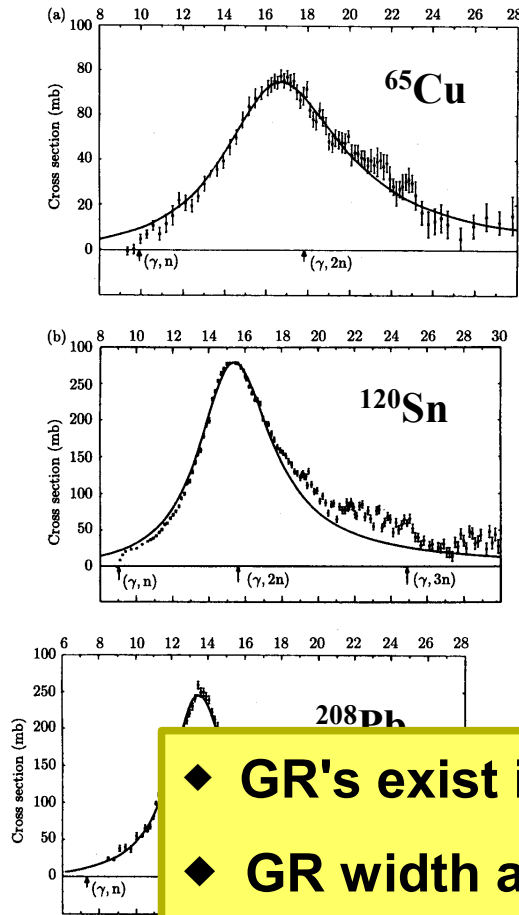


FIG. 2: (a) Horizontal-position spectrum of the $^{112}\text{Sn}(\alpha, \alpha')$ reaction at 0° . The hatched region is background events. (b) Background-free spectrum



- ◆ GR's exist in all nuclei – typically around $E^* \approx 10 \dots 30$ MeV
- ◆ GR width a few MeV
- ◆ GR shape of Lorentzian type (e.g., GDR); frequently fragmented
- ◆ GR parameter vary smoothly with Z, N, A
- ◆ GR strength exhausts large fraction of sum rules
- ◆ appear as prominent structures in the spectra obtained from various scattering experiments

==> reflecting bulk properties of nucl. matter

Classification of giant resonances (macroscopic view)

Four fluids in the nucleus, characterized by spin and isospin

neutron ($T_z = 1/2$)

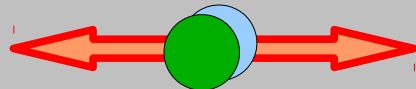
proton ($T_z = -1/2$)



Spin up ($\sigma_z = 1/2$) – *down* ($\sigma_z = -1/2$)

Accordingly, 4 basic modes :

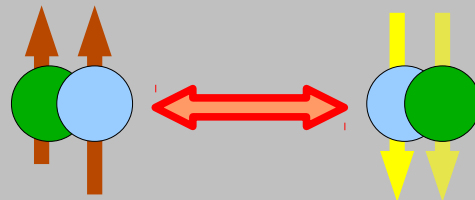
Electric isoscalar



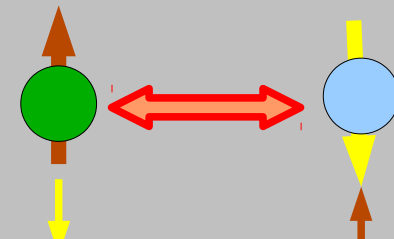
Electric isovector



Magnetic isoscalar



Magnetic isovector

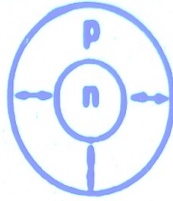


rich pattern of GR's :

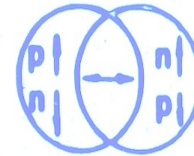
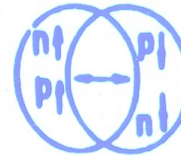
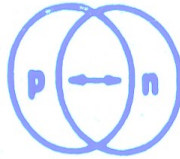
----- electric -----

----- magnetic -----

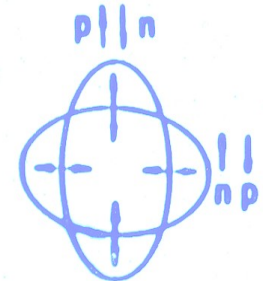
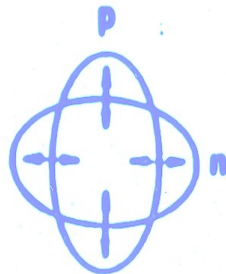
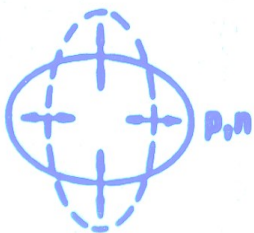
monopole



dipole



quadrupole



isoscalar

isovector

isoscalar

isovector

Nomenclature:

L (λ) – multipolarity

S (σ) – spin

T (τ) – isospin

$\Delta S, \Delta T$ – spin-, isospin transfer

$E\lambda$ – electric transition

$M\lambda$ – magnetic transition

IS (is) – isoscalar (*p – n in phase*)

IV (iv) – isovector (*p – n out of phase*)

GMR, GDR, GQR...

– **giant monopole resonance**

e.g. ivSGDR = isovector spinflip giant dipole resonance

Microscopic understanding of the GR

GR created by the action of a
one-body transition operator
onto the ground state of a nucleus:

$$| \Psi_{GR}^{\lambda, \sigma, \tau} \rangle = O^{\lambda, \sigma, \tau} | \Psi_{g.s.} \rangle$$

The transition operator contains
a multipole-type operator

$$(e.g. \sum_i r_i^\lambda Y_m^\lambda)$$

and/or may involve appropriate
spin and/or isospin operator

Microscopic classification of giant resonances

-- electric ($\Delta S = 0$) ----

-- magnetic ($\Delta S = 1$) --

	$\Delta S = 0$ $\Delta T = 0$	$\Delta S = 0$ $\Delta T = 1$	$\Delta S = 1$ $\Delta T = 0$	$\Delta S = 1$ $\Delta T = 1$
--	----------------------------------	----------------------------------	----------------------------------	----------------------------------

isoscalar ($\Delta T = 0$)

isovector

$L = 0$	X	$\sum \tau_i$ IAS	X	$\sum \vec{\sigma}_i \tau_i$ GTR
Second order	$\sum r_i^2$ ISGMR	$\sum r_i^2 \tau_i$ IVGMR	$\sum r_i^2 \vec{\sigma}_i$ ISSMR	$\sum r_i^2 \vec{\sigma}_i \tau_i$ IVSMR
$L = 1$	X	$\sum r_i Y_m^1 \tau_i$ IVGDR	$\sum r_i Y_m^1 \vec{\sigma}_i$ ISSDR	$\sum r_i Y_m^1 \vec{\sigma}_i \tau_i$ IVSDR
Second order	$\sum r_i^3 Y_m^1$ ISGDR			

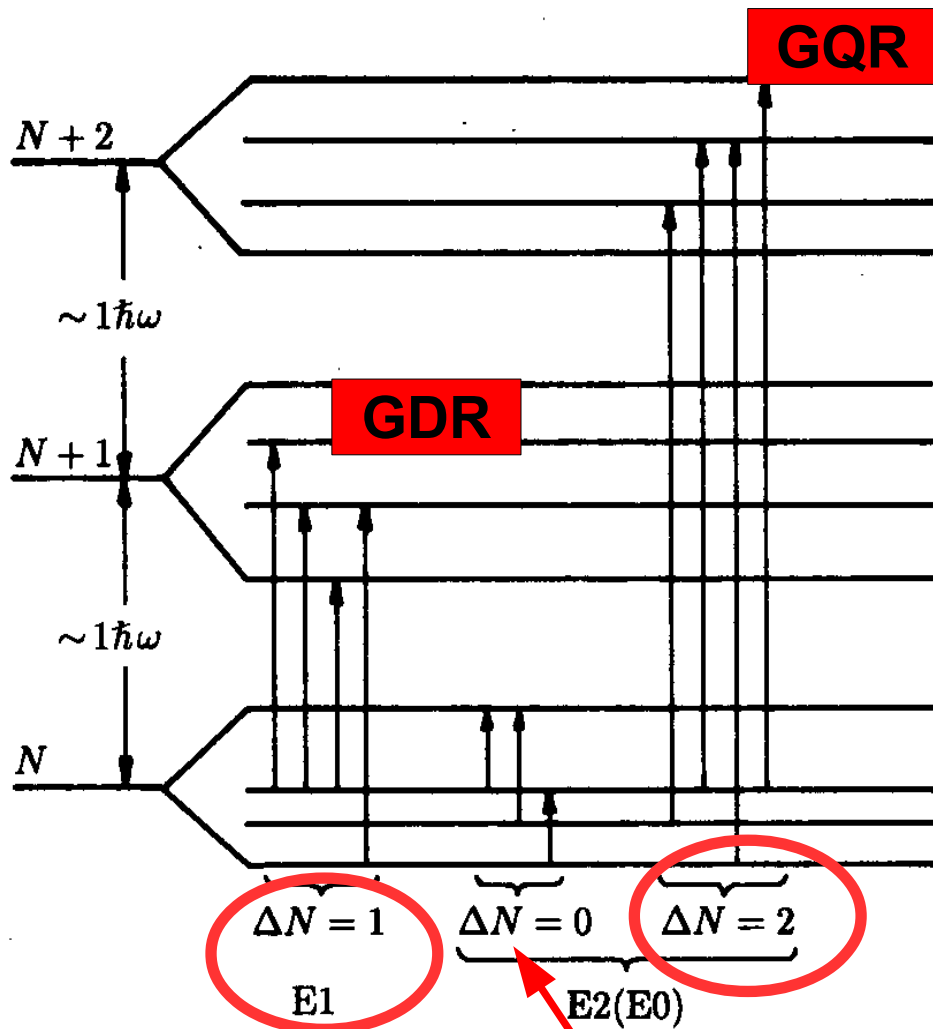
not allowed: center of gravity to be conserved

How are such operators realized in nuclear reaction experiments ?

i.e., how do we relate

nuclear reaction cross section \leftrightarrow strength distribution ??

consider a (naive) shell model (harmonic osc.):



Excited states from p-h excitations within / across major shells N

remember:

Parity alternates with N, thus, for multipolarity, e.g.,

$$\Delta L = 1 \implies \Delta N = 1$$

$$\Delta L = 2 \implies \Delta N = 0 \text{ or } 2 \text{ etc...}$$

e.g. for ^{16}O nucleus. $\sim 7 \text{ MeV}$

however, exp. value: 13 MeV !!

GR's are understood as a

Coherent superposition of **many** 1p-1h excitations across major shells.

Strongly shifted in energy by **residual** interactions.

Low-lying E2: rotation, surface vibration

Theoretical description of giant resonances – microscopic approach

Keywords:

Level of sophistication

mean field

(relativistic and non-relativistic approaches)

TDA Tamm-Danco

RPA random-phase

QRPA quasi-particle RPA

S-,E-,C-RPA

second-, extended, continuum RPA

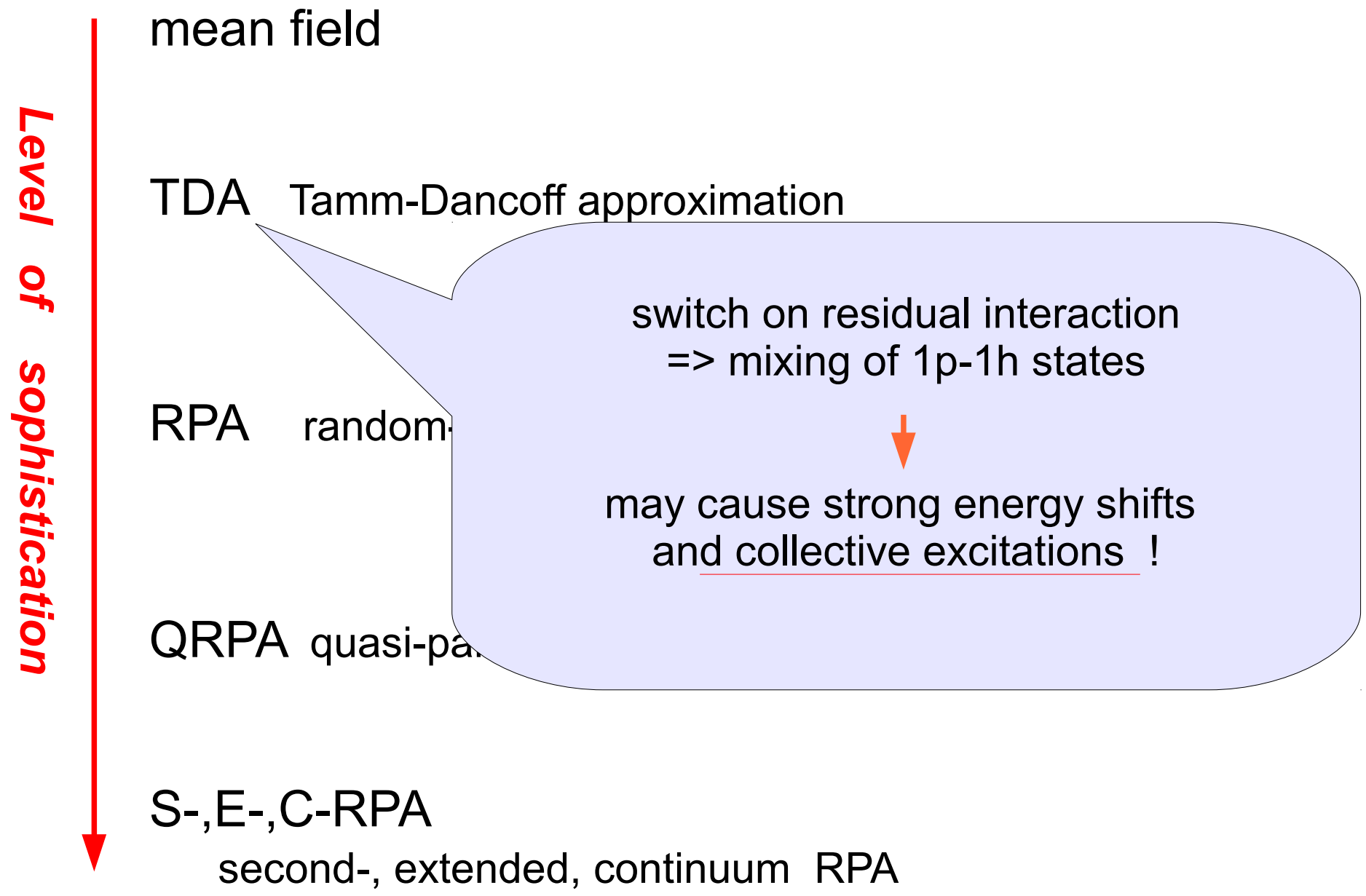
average one-body potential
obtained empirically (e.g. Woods-Saxon)
or from effective nn interactions
via self-consistent Hartree-Fock method

↓

Excitations: 1p-1h np-nh,
no two-body interactions
=> no coherent excitations !

Theoretical description of giant resonances – microscopic approach

Keywords:



Eigenstates of single-particle Hamiltonian ($\mathbf{H} = \sum_i \mathbf{h}_i$) with energy : ϵ_i

thus, the 1p-1h states $|\mathbf{n}\mathbf{i}^{-1}\rangle$ with excitation energies : $\epsilon_n - \epsilon_i$

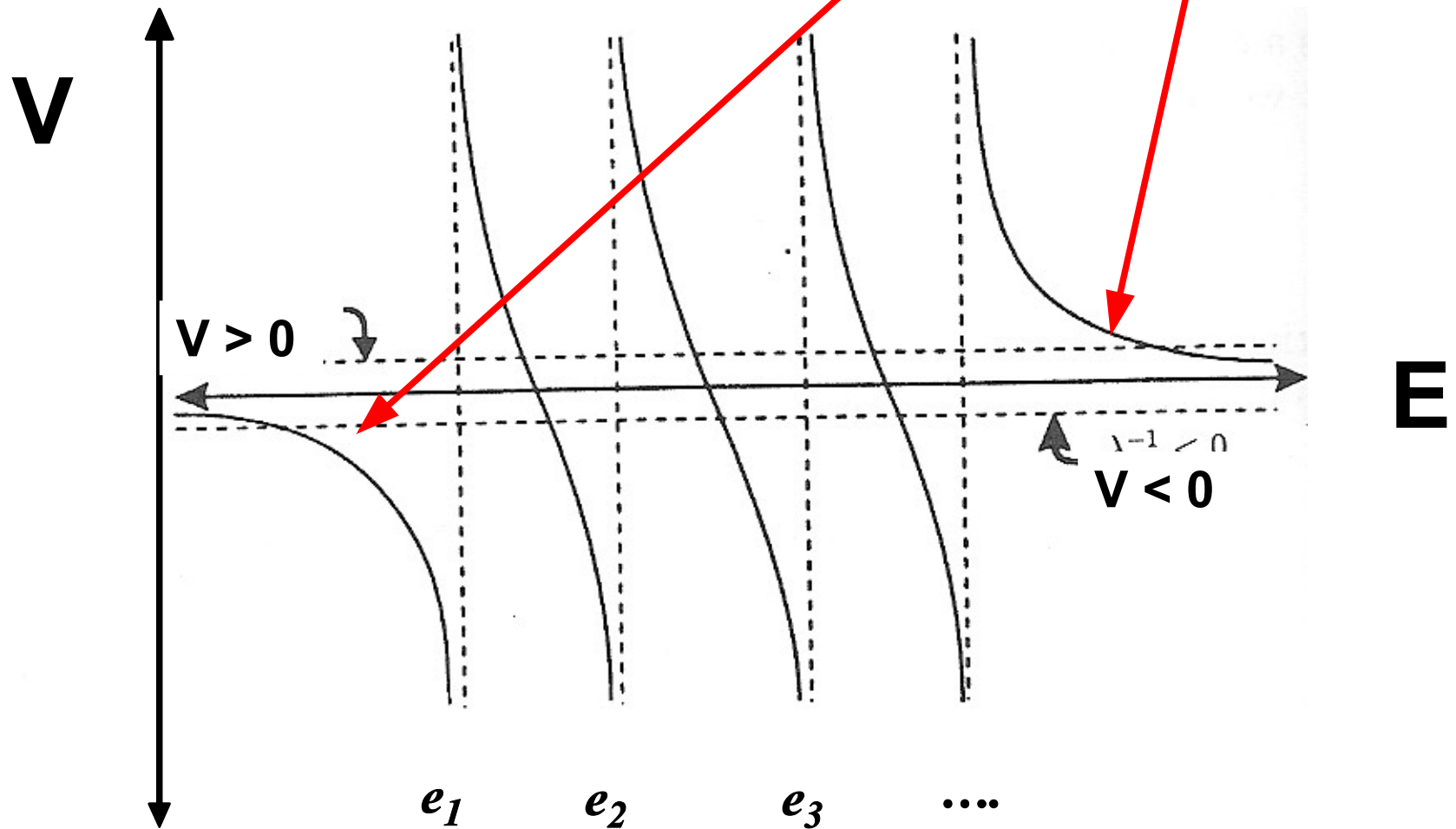
The 1p-1h states $|\mathbf{n}\mathbf{i}^{-1}\rangle$ form a complete orthonormal set, thus

with a (simplified) residual interaction \mathbf{V} ($\mathbf{H} = \sum_i \mathbf{h}_i + \mathbf{V}$), obtain
the 'perturbed' particle-hole energies:

$$\mathbf{E} = \epsilon_n - \epsilon_i + \sum_{\mathbf{m}\mathbf{j}} \langle \mathbf{n}\mathbf{i}^{-1} | \mathbf{V} | \mathbf{m}\mathbf{j}^{-1} \rangle c_{\mathbf{m}\mathbf{j}} / c_{\mathbf{n}\mathbf{i}}$$

(notice: ground state remains untouched)

*Residual interaction is positive for isovector transitions \Rightarrow upshift
negative for isoscalar transitions \Rightarrow downshift*



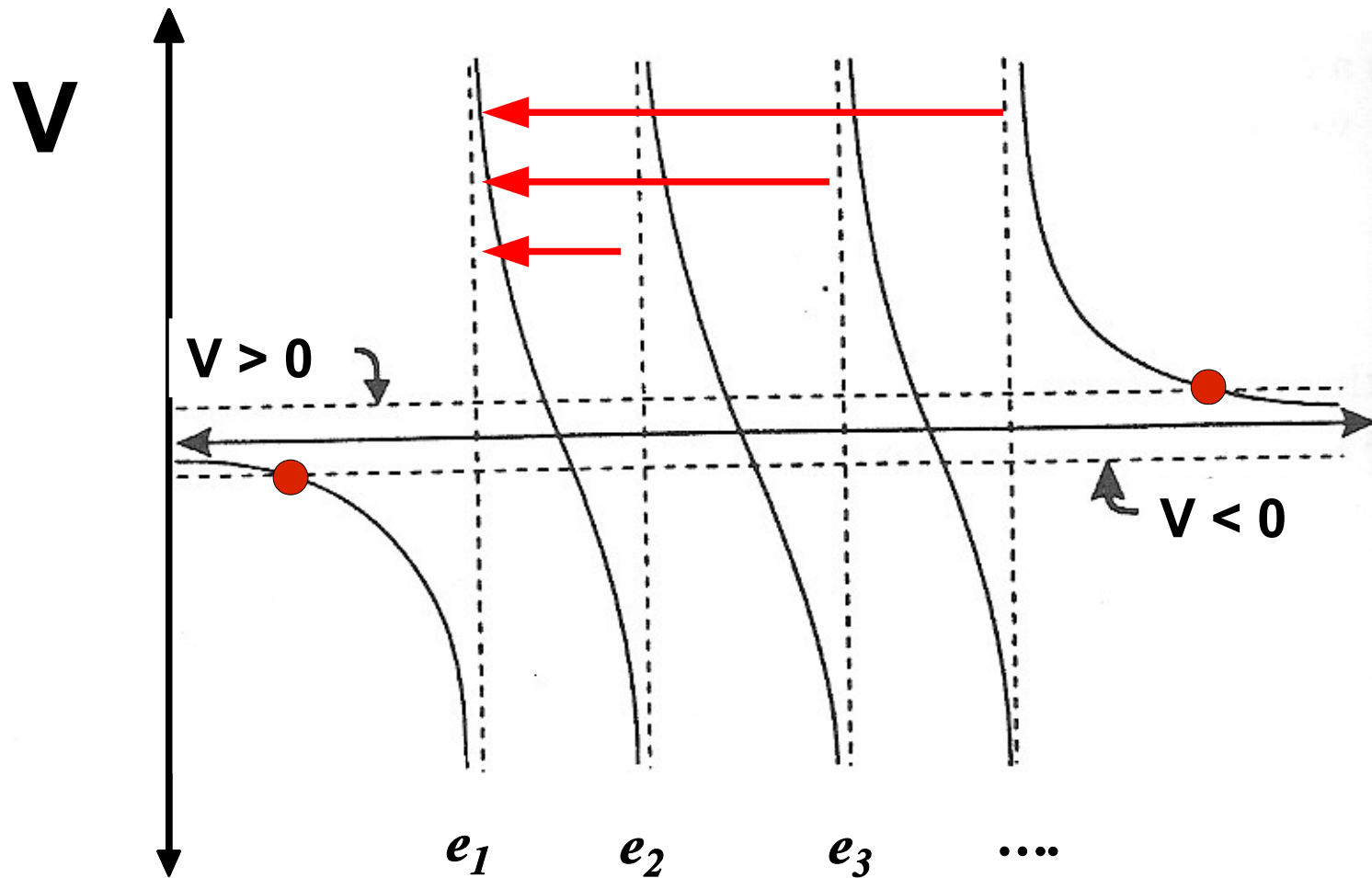
Further (over-) simplifying by adopting all e_k to be equal, apparently only one state is shifted up (down) maximally and

most importantly:

it receives 100% of the transition probability ('the winner takes it all' -ABBA)

(i.e. the sum of the transition probability between the unperturbed states)

==> the collective state !



Theoretical description of giant resonances – microscopic approach

Keywords:

Level of sophistication



mean field

TDA Tamm-Dancoff approximation

STANDARD method:
allow for correlated ground state

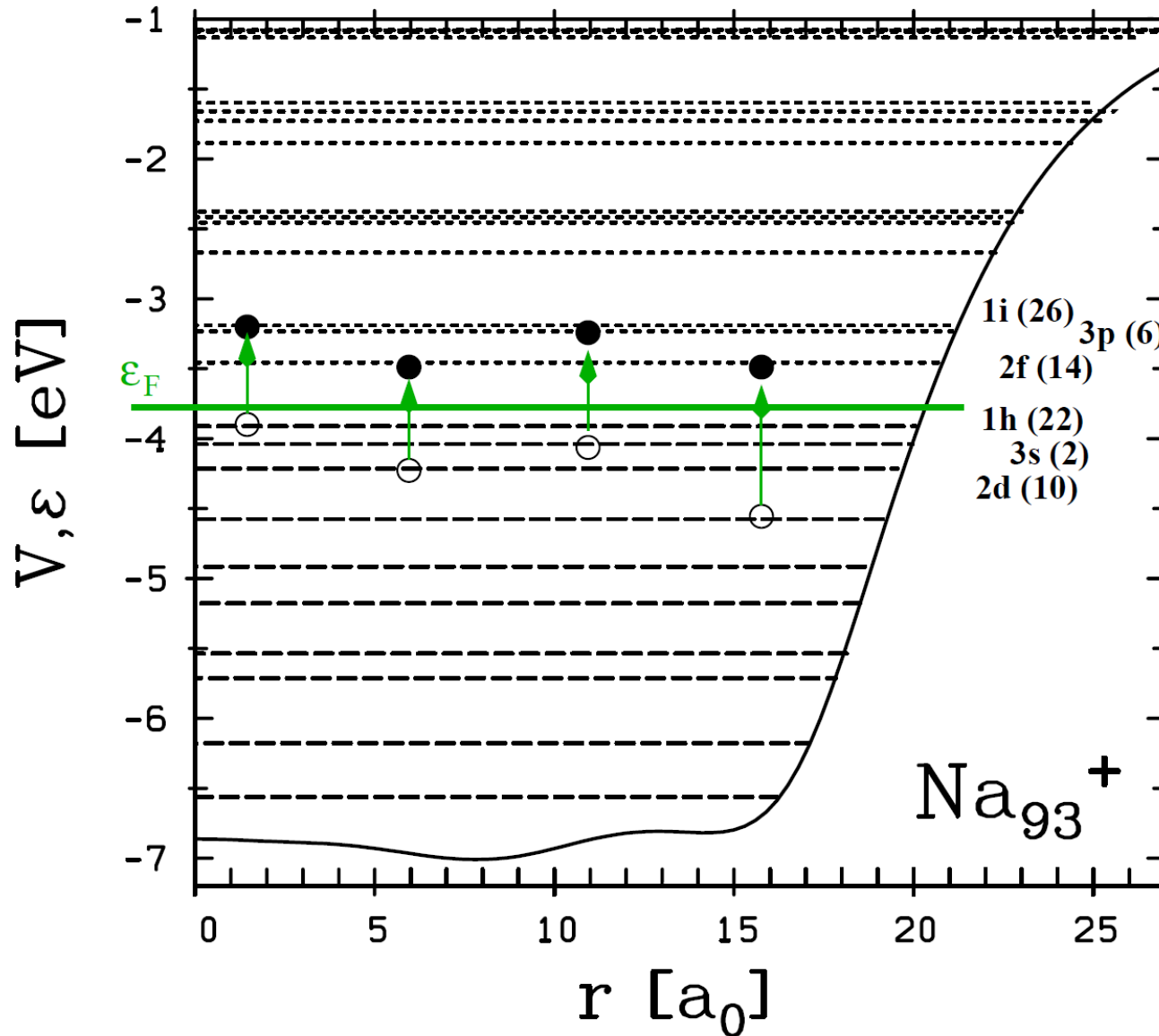
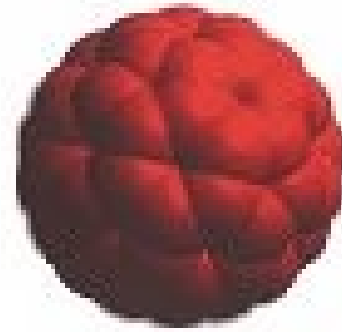
RPA random-phase approximation

QRPA quasi-particle RPA

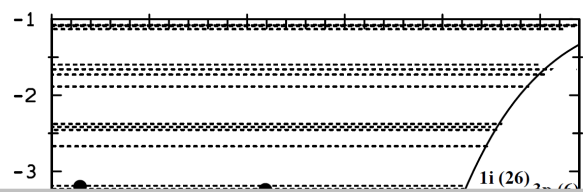
S-,E-,C-RPA

second-, extended, continuum RPA

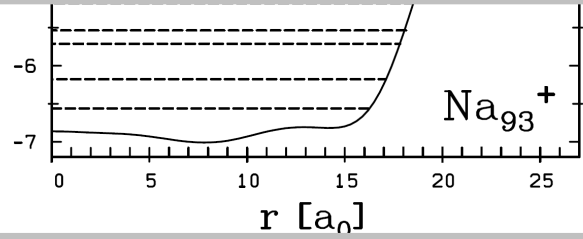
atomic Na_{93}^+ Cluster



single-electron levels
and
some ph - excitations



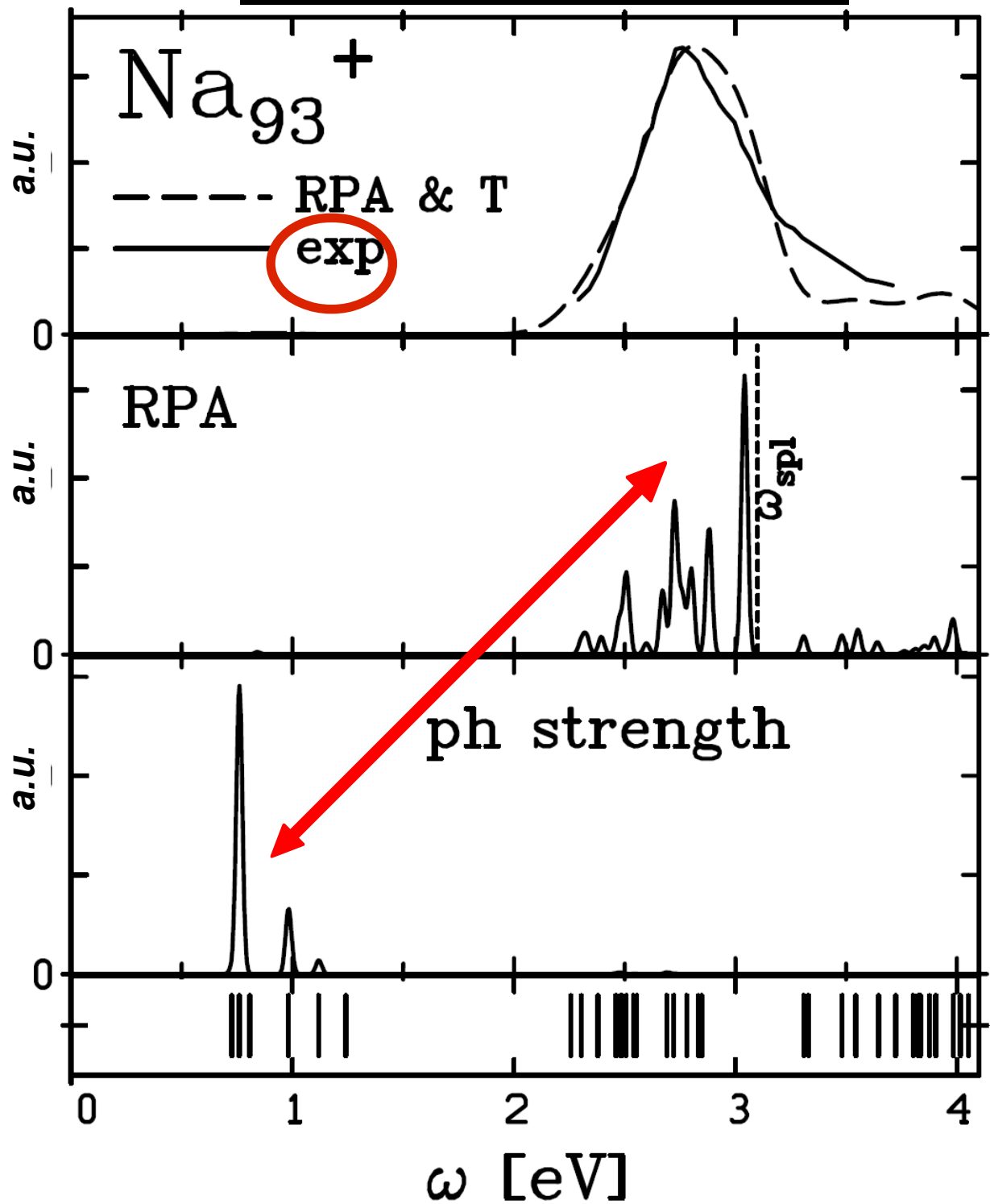
RPA dipole strength + thermal shape fluctuations

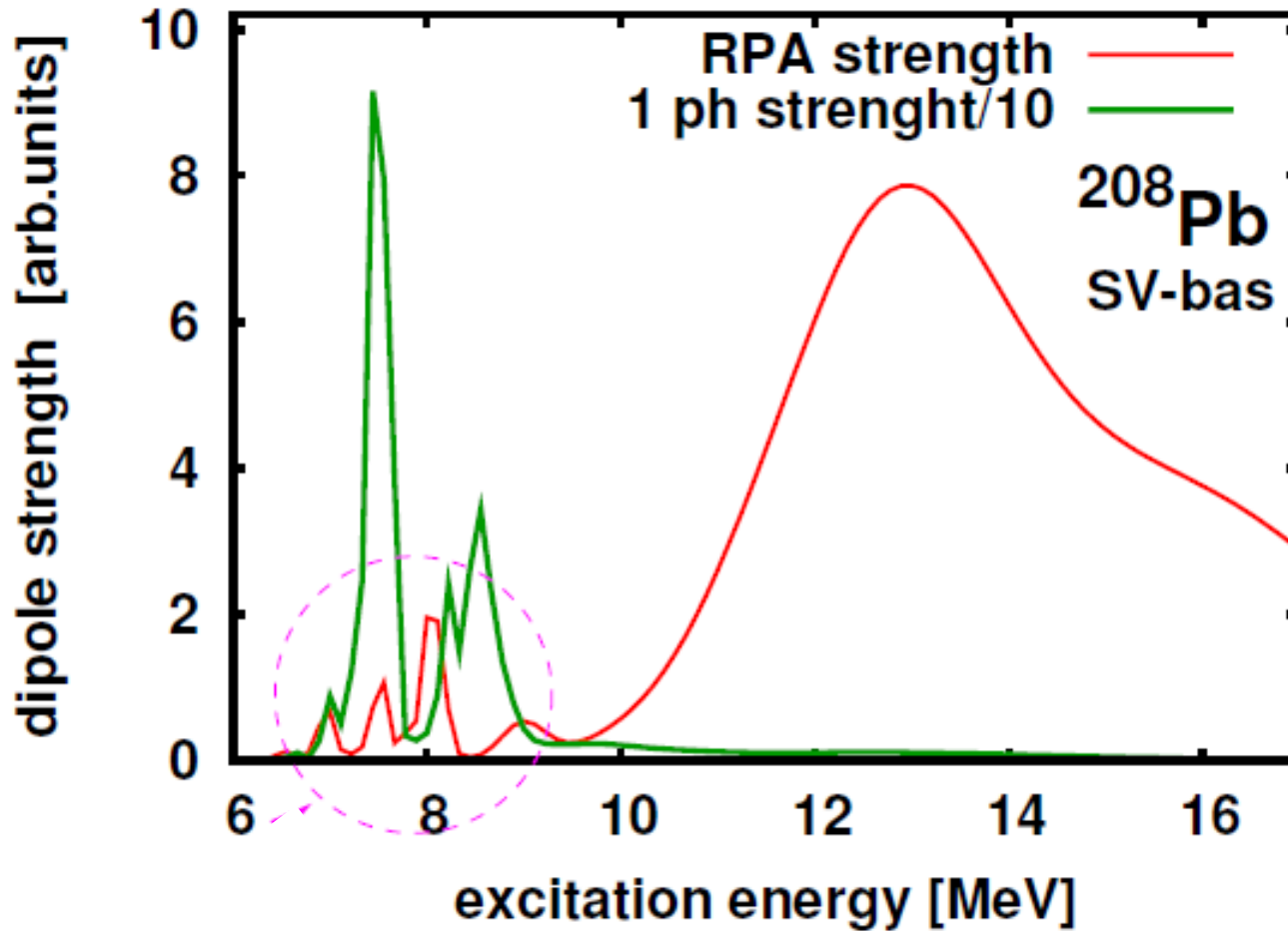


RPA dipole strength (notice up-shift !)

Dipole strength pure 1p-1h transitions

Dipole ($L = 1$) pure 1p-1h transitions





*1ph dipole strength
gathers in narrow
energy band*

*looks "resonance like"
but is composed of
several different
1ph states*

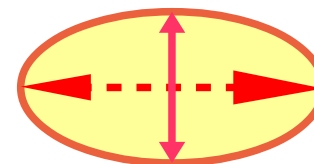
*RPA dipole strength
collected in
resonance peak
(and somewhat
fragmented to 1ph ...)*

Watch green curve only !

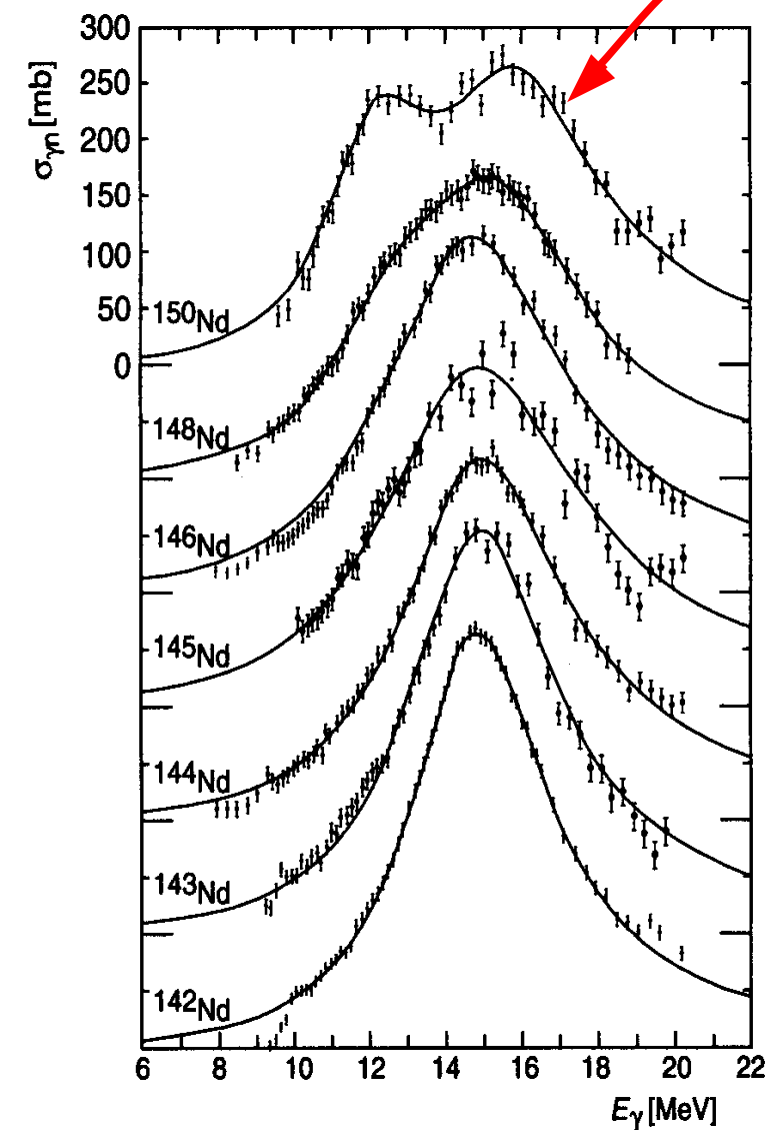
Theoretical description of giant resonances – microscopic approach

Keywords:

mean field



deformation splitting



anccoff approximation

-phase approximation

Include pairing:
Open-shell nuclei, deformation

article RPA

ended, continuum RPA

Theoretical description of giant resonances – microscopic approach

Keywords:

Level of sophistication

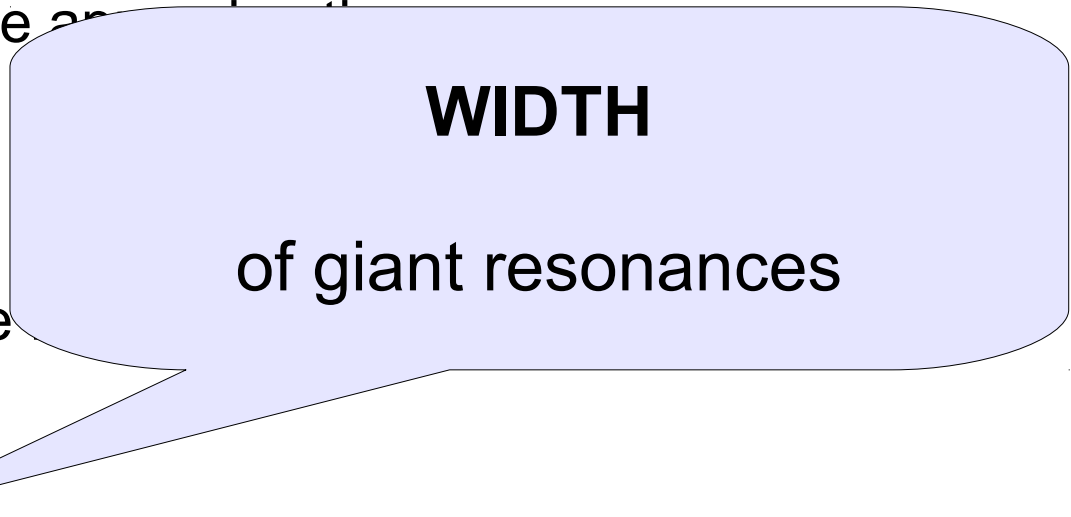
mean field

TDA Tamm-Dancoff approximation

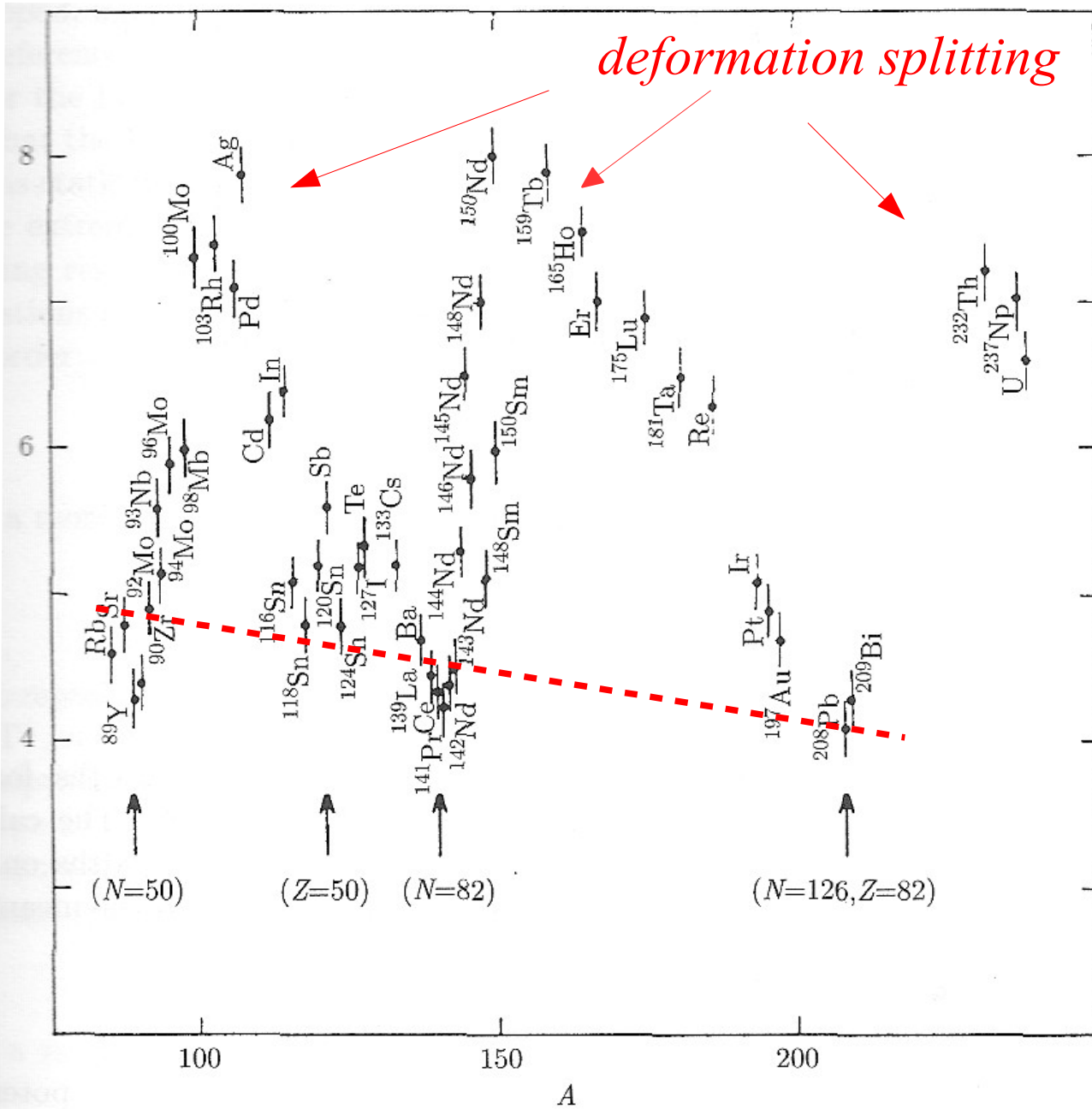
RPA random-phase approximation

QRPA quasi-particle

S-,E-,C-RPA
second-, extended, continuum RPA

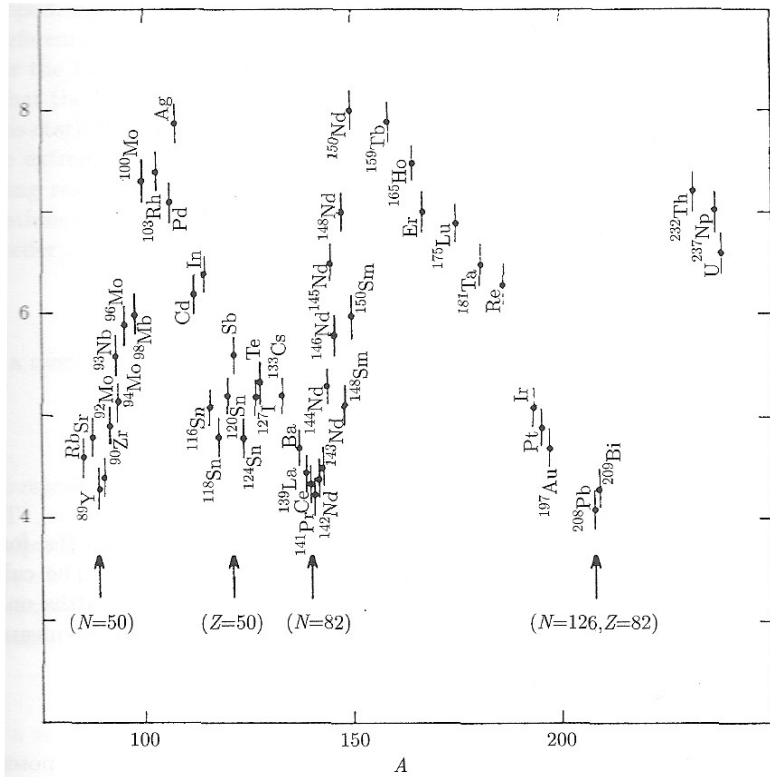


WIDTH of GDR in heavy nuclei



closed shell nuclei:
 $\Gamma \sim 4 - 5 \text{ MeV}$

giant resonances – WIDTH and Damping



^{208}Pb :
 $E_{\text{GDR}} = 13 \text{ MeV}$
 $\Gamma_{\text{GDR}} = 4 \text{ MeV}$

GDR vibration period T_s : $E_{\text{GDR}} = h\nu = h / T_s$
 Heisenberg - lifetime τ : $\Gamma_{\text{GDR}} = h / 2\pi\tau$

$^{208}\text{Pb} \implies \tau = 0.5 T_s$

strongly damped motion !

giant resonances – WIDTH - damping

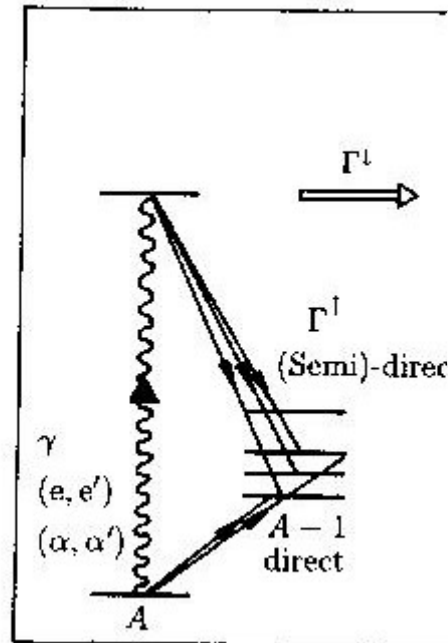
Apparent GR width due to -

- fragmentation of strength ('Landau damping')
- particle decay

GR as a 1p-1h state should decay predominantly into particular hole states in the (A-1) nucleus

However:
in heavier nuclei
mainly statistical decay
observed !

==> coupling to
2p-2h.....



'spreading width'

Damping / Width of giant resonances :

Strength fragmentation (Landau damping)

Deformation splitting (open-shell nuclei)

Direct particle decay (lighter nuclei): Γ^{\uparrow} *escape width*

Statistical (compound nucleus) decay: Γ^{\downarrow} *spreading width*

What do we (hope to) learn from GR's ?

unperturbed particle-hole states \leftrightarrow mean field

collective state \leftrightarrow interaction between ph states

→ learn about effective in-medium interactions
(can be differentiated .. ΔL , ΔT , ΔS)

GR's are density vibrations → learn about nuclear *bulk* properties :

Finite matter (i.e. nuclei): radii, deformations, skins, clusters ...(GEOMETRY)

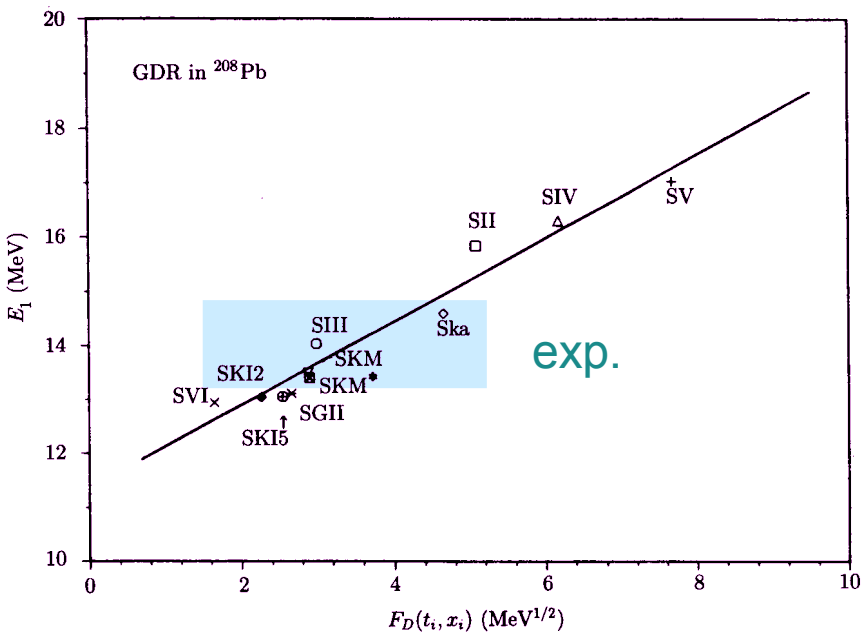
Infinite matter: symmetry energy, compressibility etc.

restricted, however, to around saturation density and (near) zero temperature

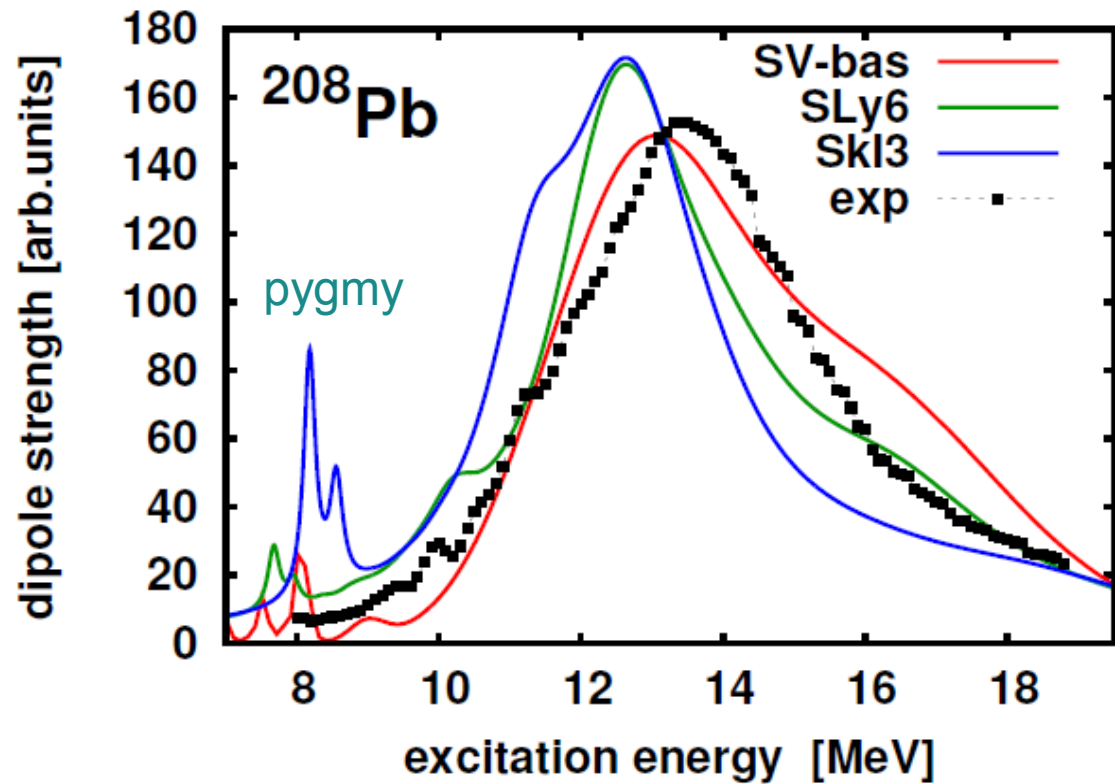
reaction and decay rates (astrophysical interests)

test of effective forces

GDR centroid energy

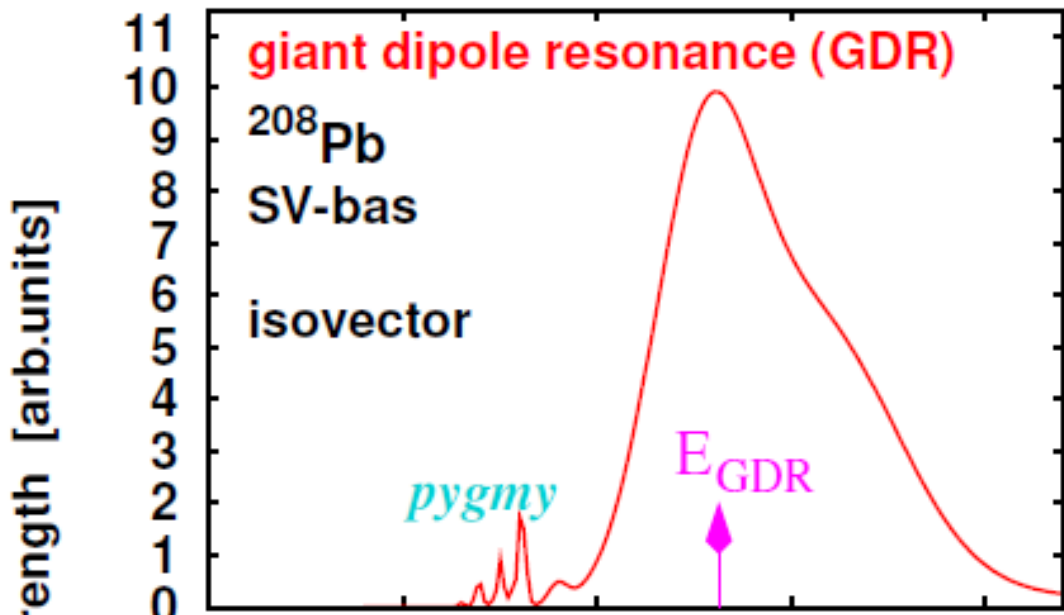


GDR strength distribution for several Skyrme forces



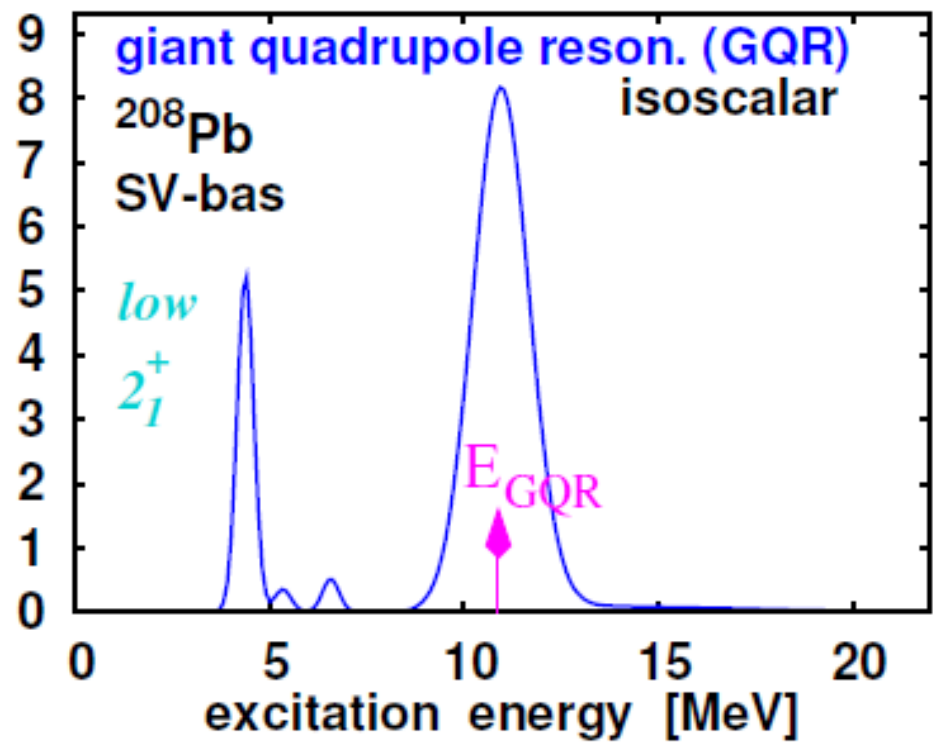
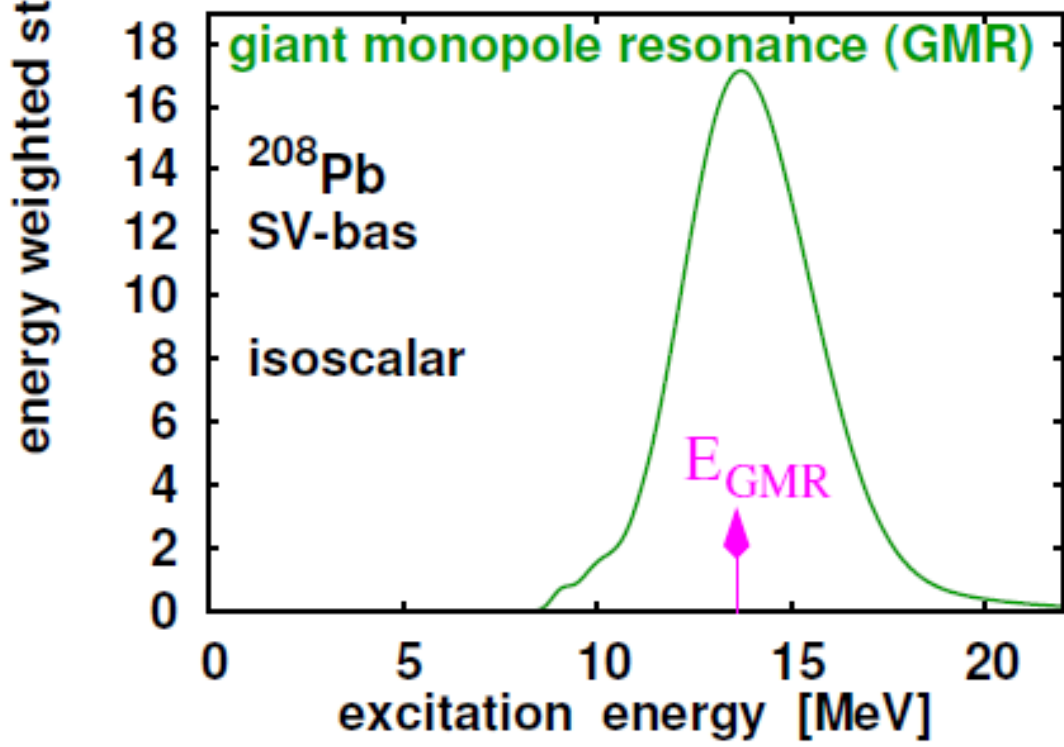
P.-G. Reinhard (Inst.Theor.Physik, Erlangen)

Collective resonance excitations (example ^{208}Pb)

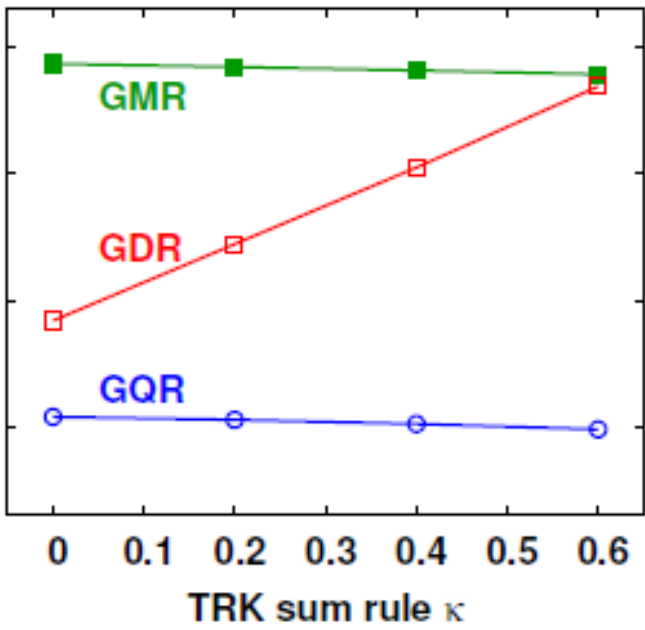
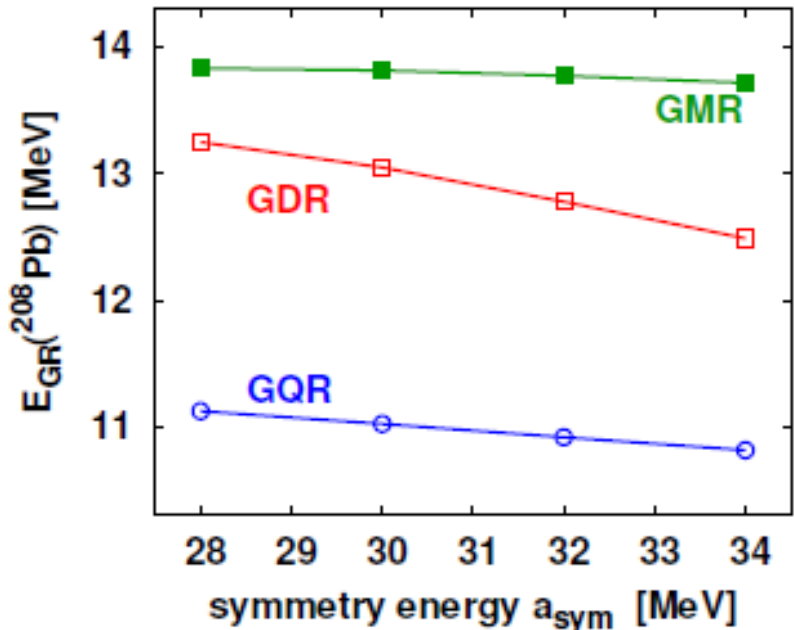


*marked resonance peaks
in the region $E \sim 12$ MeV
well characterizeable by
one resonance energy*

*width: Landau fragmentation
escape, 2ph collisions*

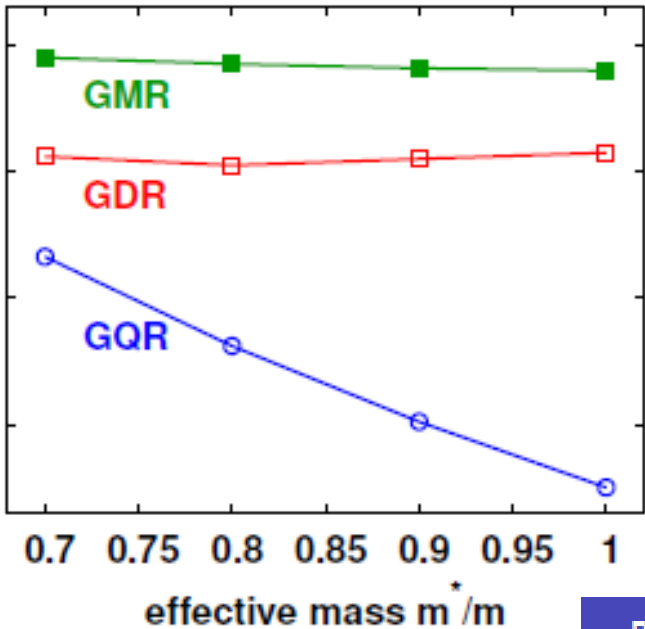
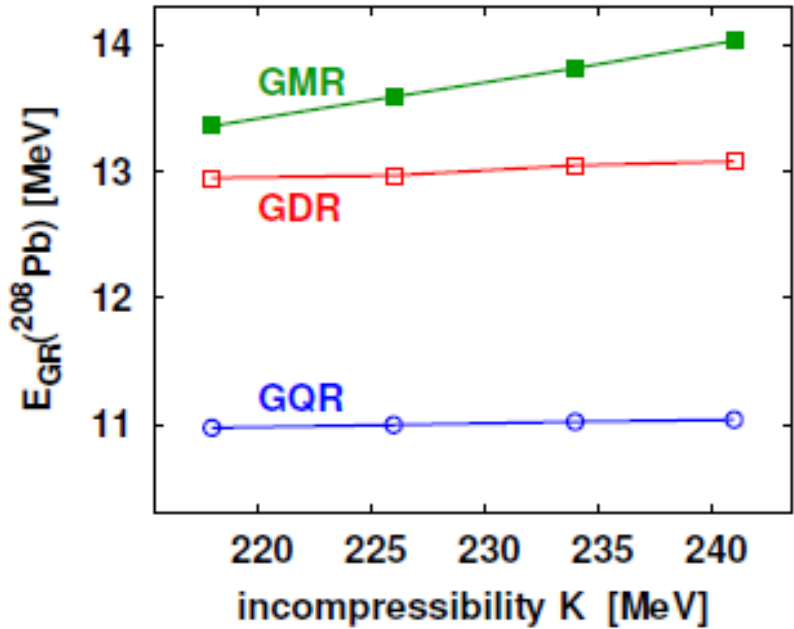


Systematic variation of Skyrme forces



each bulk property is sensitive preferably to one of the 3 modes:

- $K \leftrightarrow GMR$
- $m^*/m \leftrightarrow GQR$
- $a_{sym}, \kappa \leftrightarrow GDR$



Sum Rules

Sum rules may be derived from algebraic relations between transition operators and (powers of) the Hamiltonian

Energy-weighted sum rules are model independent to the extent that

- nn interactions do not depend on the particle momenta (electric isoscalar transitions)
- and charge-exchange components are absent (electric isovector transitions)

some Sum rules

GDR:

$$\mathbf{TRK}: \quad \int \sigma(E) dE \sim N Z / A$$
$$\int \sigma(E)/E dE \sim N Z / A \langle r^2 \rangle_p$$

$$\sigma(E) \sim E \cdot dB(E1, 0 \rightarrow E)/dE$$

$$\mathbf{Migdal}: \quad \int \sigma(E)/E^2 dE \sim R^2 A / E_{\text{sym}}$$

$$\mathbf{Cluster}: \quad \text{TRK} = \text{TRK}_1 + \text{TRK}_2 + \text{TRK}_{12}$$

other sum rules:

$$\mathbf{Ikeda}: \quad S_-(\text{GT}) - S_+(\text{GT}) = 3(N-Z)$$

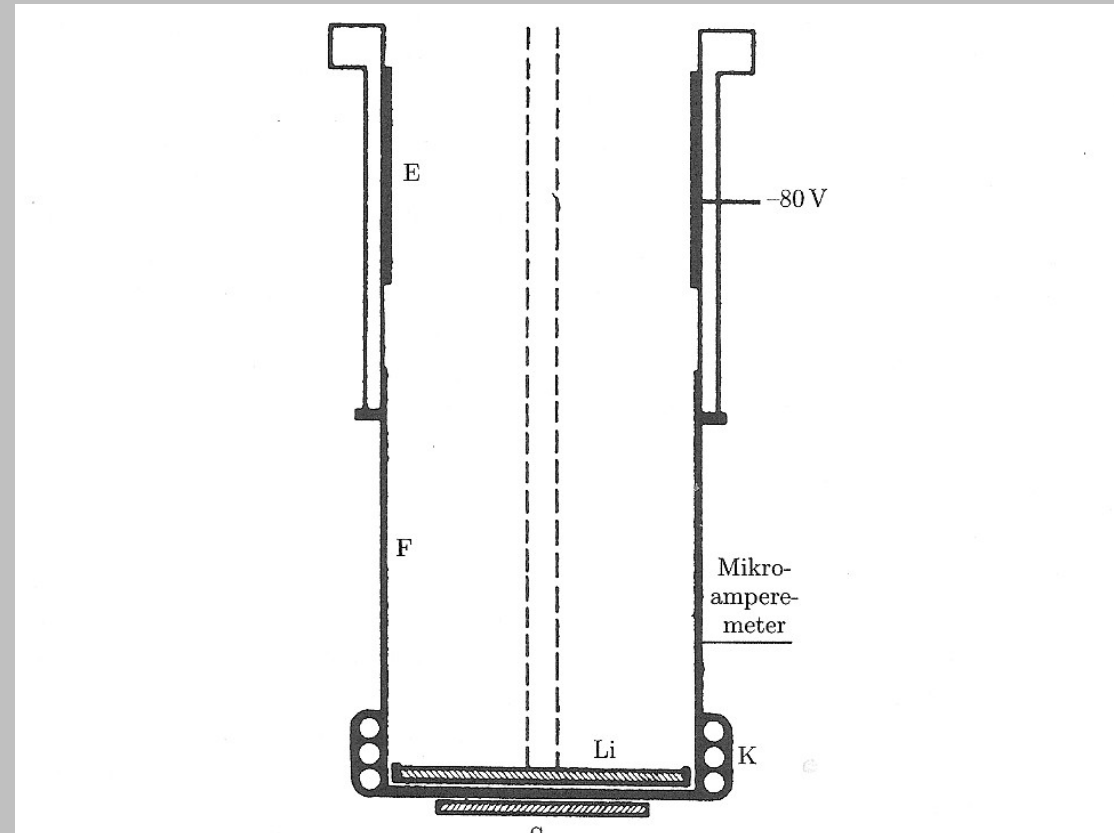
$$S_-(\text{IVSDR}) - S_+(\text{IVSDR}) \sim N \langle r^2 \rangle_n - Z \langle r^2 \rangle_p$$

etc..

History

- Milestones

1937 - Bothe and Gentner



300 keV proton 'beam'

→ $\text{Li}(p, \gamma)$

→ γ -radiation onto sample S, e.g. ^{63}Cu

→ radioactivity from (γ, n) measured

„ vielleicht spielen hier Resonanz -
verhältnisse eine entscheidene Rolle „

History

- Milestones

1937 - Bothe and Gentner

1947 – Balwin and Klaiber
GDR in γ -absorption spectra

1971 – isGQR Pitthan and Walcher

1977 – isoscalar monopole

·
·
·
·

1980... GR in hot nuclei

1985... M1 scissor mode

1985... multiphonon GDR,GQR

~ 2000 dipole strength in unstable nuclei