Covariant Spectator Theory© (CST) of Nuclear Forces

Mumbai Conference, November 22, 2009

Franz Gross

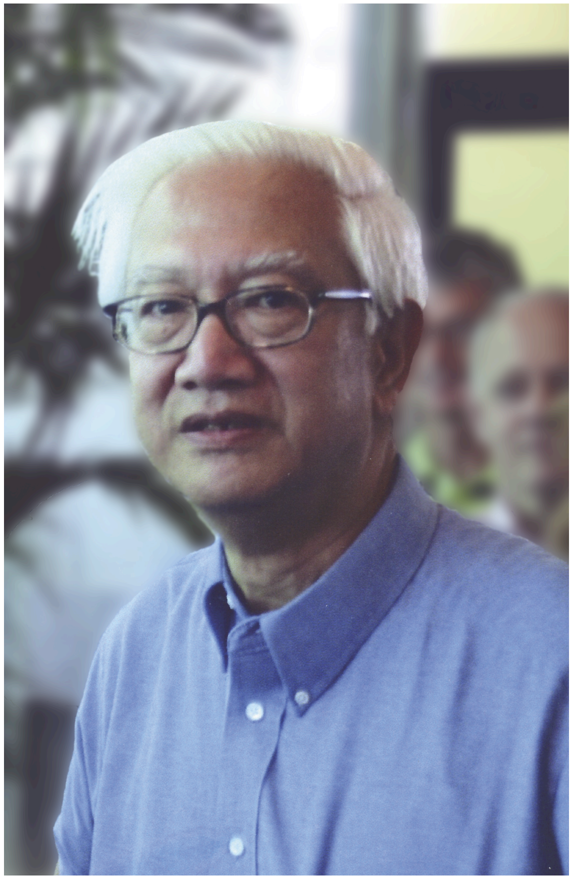
JLab and W&M

- Motivation and description of the CST
- Precision fits to the np data
- Binding energy of the triton and three-body forces
- Currents and form factors
- Outlook and conclusions

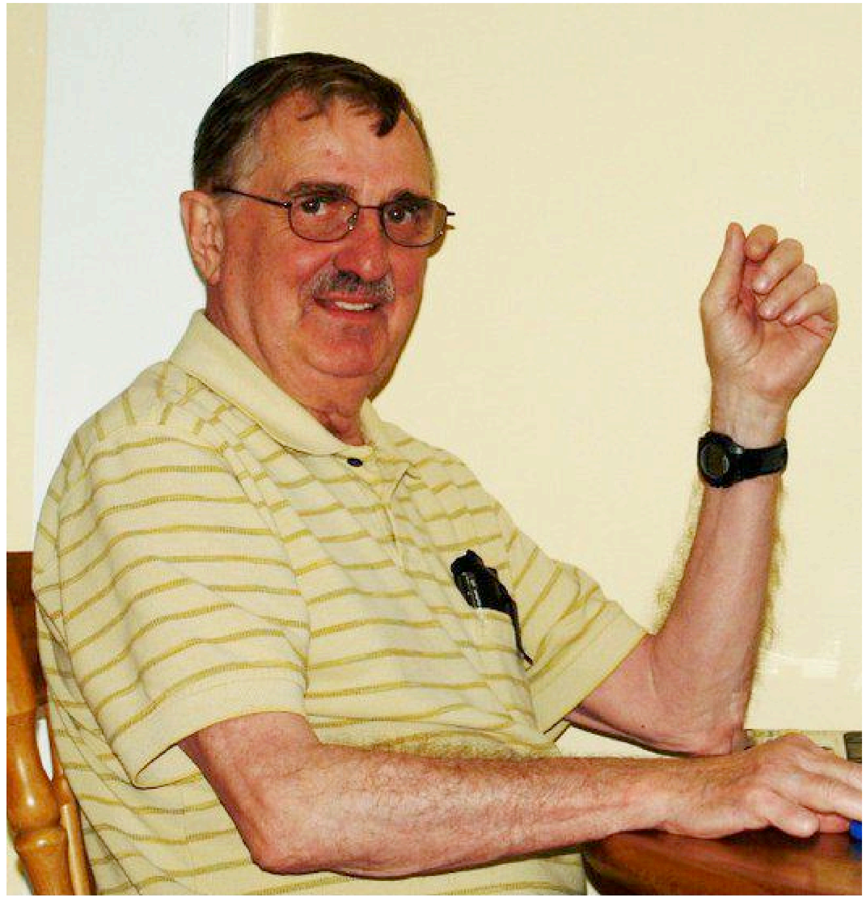
Done in collaboration with
Alfred Stadler

Thanks to:
J. W. Van Orden -- 1991 models and deuteron from factors
Karl Holinde (and R. Machleidt) -- Bonn code
Dick Arndt -- SAID
Johan de Swart (and others in the Nijmegen group --Timmermans and Rentmeester) -- advice

In Memoriam:



John Tjon (1937 -- Sept 20, 2010)



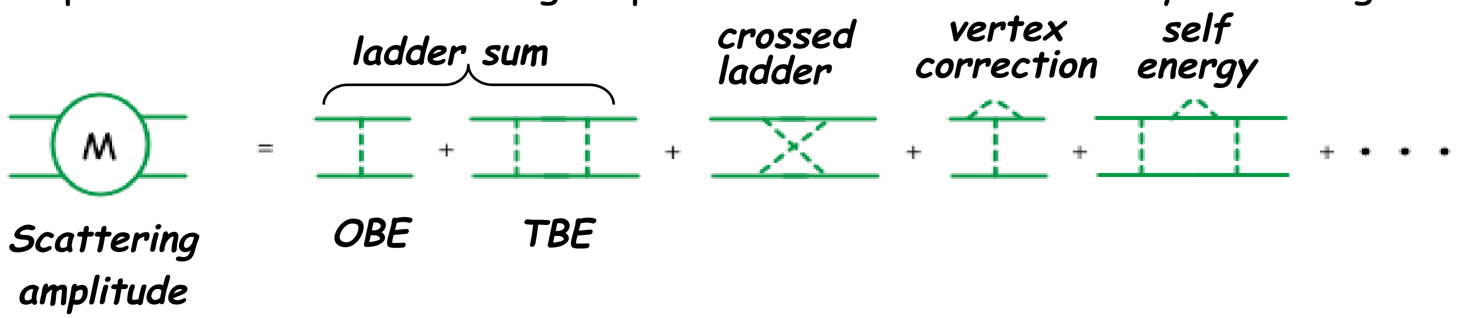
Dick Arndt (1933 -- April 10, 2010)

Motivation and description of the

Covariant Spectator Theory© (CST)

Diagrammatic derivation of field theory scattering equations

- ★ Step 1: As in χEFT, assume nucleons and mesons are point-like particles
- ★ Step 2: the exact scattering amplitude is the sum of all Feynman diagrams



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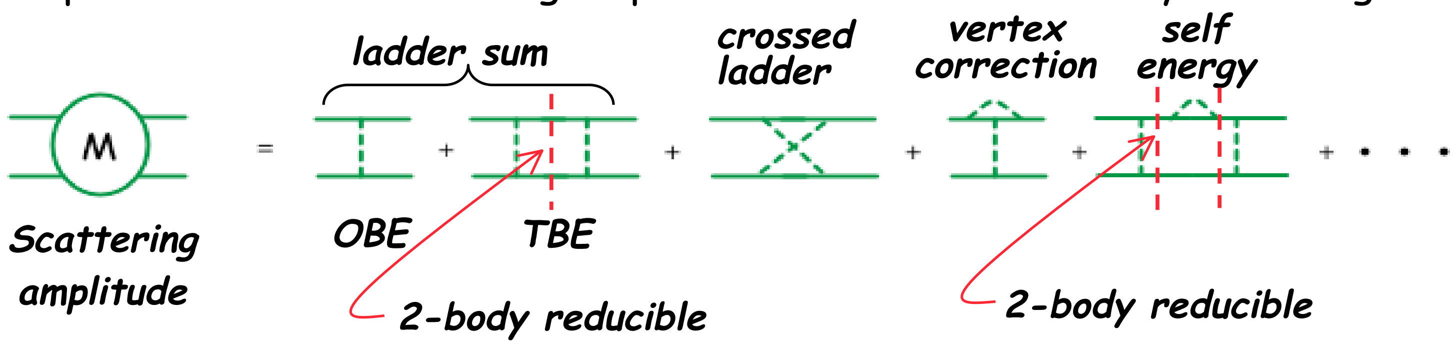

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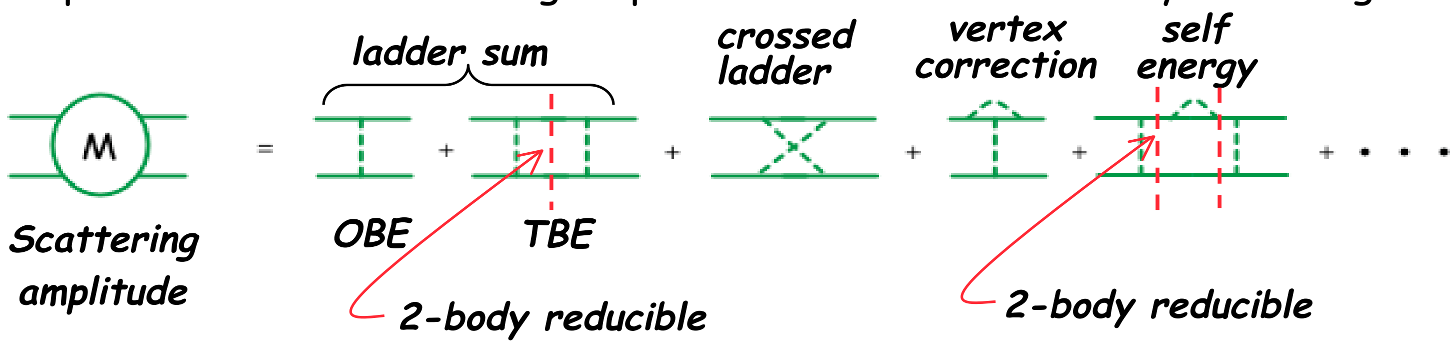

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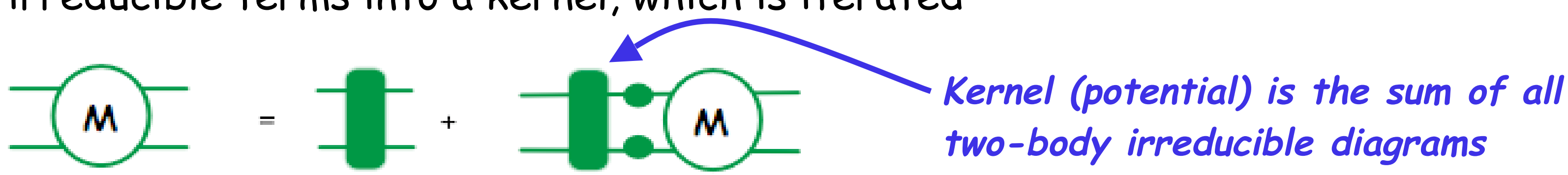

★ Step 4: *Field theory* becomes *field dynamics* when the kernel is phenomenological

$$M(p',p;P) = V(p',p;P) + \int V(p',k;P)G(k;P)M(k,p;P)$$

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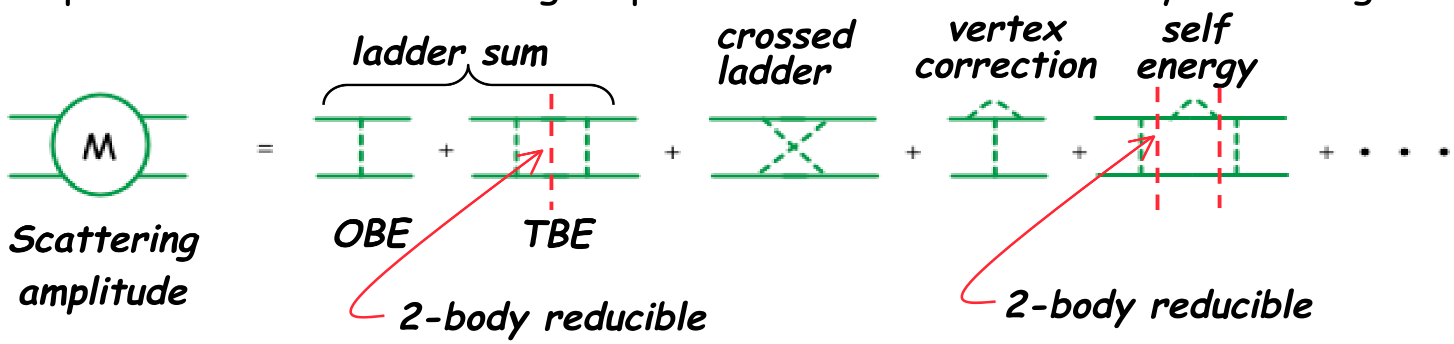

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$$M(p',p;P)=V(p',p;P)+\int V(p',k;P)G(k;P)M(k,p;P)$$

- Spin 1/2 particles have a Dirac structure

When is it necessary to sum diagrams to all orders?

★ In schematic form, the integral equation looks like

$$M = V + VgM = V + VgV + VgVgV + \bullet\bullet\bullet = \frac{V}{1 - gV}$$

★ must sum when $gV \approx 1$ (even if V is small, g might be large)

- example: atomic states

★ Necessary for the description of bound states

- A bound state is a new particle (not in the Lagrangian).
- shows up as a pole in the scattering matrix (i.e. $gV = 1$)
- generated *non-perturbatively* from the sum of an infinite number of diagrams

★ Necessary to describe unitarity ($g \propto \sqrt{4m^2 - W^2} \to i\sqrt{W^2 - 4m^2}$ when $W > 2m$)

$$M = \frac{V}{1 - igV} \quad \Rightarrow \quad \operatorname{Im} M = \frac{gV^2}{1 + (gV)^2} = g|M|^2$$

Why use the Dirac equation for a spin 1/2 particle?

★ The Dirac equation for the coulomb interaction (with $A^\mu = \{\phi, \mathbf{A}\}$) is

$$i\frac{\partial}{\partial t}\Psi = \left(\alpha\cdot(\mathbf{p}-e\mathbf{A}) + \beta m + e\phi\right)\Psi$$

★ Taking the non-relativistic limit gives [to order $(v/c)^2$]

$$i\frac{\partial}{\partial t}\psi = \left\{\boxed{\frac{(\mathbf{p}-e\mathbf{A})^2}{2m}} - \frac{\mathbf{p}^4}{8m^3} - \frac{e}{2m}\sigma\cdot\mathbf{B} + \frac{e\left[\nabla^2\phi\right]}{8m^2} + \boxed{\frac{e}{4m^2 r}\frac{d\phi}{dr}\sigma\cdot\mathbf{L}}\right\}\psi$$

★ Each of these terms has a special history:

$$\boxed{-\frac{\mathbf{p}^4}{8m^3}}$$ **relativistic mass increase**

$$\boxed{\frac{e}{8m^2}\nabla^2\phi = -\frac{e}{8m^2}\nabla\cdot\mathbf{E} = \frac{Ze^2}{8m^2}\delta^3(r)}$$ **Darwin term**

$$\boxed{\frac{e}{4m^2 r}\frac{d\phi}{dr}\sigma\cdot\mathbf{L} = \frac{e}{2m^2 r}\frac{d\phi}{dr}\mathbf{S}\cdot\mathbf{L}}$$ **spin-orbit term**

$$\boxed{-\frac{e}{2m}(\mathbf{p}\cdot\mathbf{A}+\mathbf{A}\cdot\mathbf{p}) - \frac{e}{2m}\sigma\cdot\mathbf{B} = -\frac{e}{2m}\mathbf{B}\cdot(\mathbf{L}+\sigma) = -\frac{e}{2m}\mathbf{B}\cdot(\mathbf{L}+2\mathbf{S})}$$ **Zeeman effect (with the gyromagnetic ratio of 2)**

So, why doesn't everyone use the Dirac equation?

Pros and cons of the Dirac equation

- ★ The Dirac equation has negative energy solutions
  - can be reinterpreted as anti-particle states when we use field theory
- ★ Two choices (or points of view):
  1. Avoid the Dirac equation, because we abhor negative energy states; they are unphysical and definition of a Hilbert space in unclear
  2. Keep the Dirac equation, because
     - We are impressed with the physics it contains
     - We are willing to truncate the field theory (i.e. invent a new "field dynamics" which may require uncontrolled approximations),
     - We are willing to have a formalism with "off-shell" particles and negative energy states
- ★ *Field Dynamics* is a truncated field theory that keeps the full Dirac structure.

Relation between the BS and CST© two-body equations

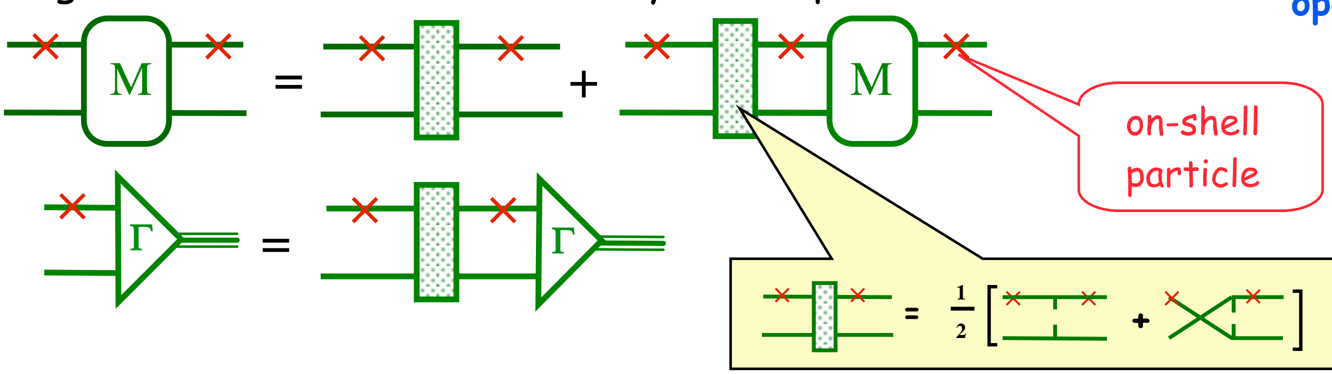
★ The Bethe-Salpeter (BS) propagator depends on all four components of the relative momentum, $\{k_0,\mathbf{k}\}$. For two spinor particles it is

$$G_{BS}(k;P)=\frac{1}{\left(m_1-\not{p}_1+\Sigma\left(\not{p}_1\right)-i\varepsilon\right)\left(m_2-\not{p}_2+\Sigma\left(\not{p}_2\right)-i\varepsilon\right)} \quad \textbf{with} \quad \begin{cases} p_1=\frac{1}{2}P+k \\ p_2=\frac{1}{2}P-k \end{cases}$$

★ The Covariant Spectator Theory© propagator depends on only three components of the relative momentum, $\mathbf{k}$. One particle is on-shell

$$G_{CS}(k;P)=\frac{2\pi i\,\delta_+\left(m_1^2-p_1^2\right)\left[m_1+\hat{\not{p}}_1\right]}{\left(m_2-\not{p}_2+\Sigma\left(\not{p}_2\right)-i\varepsilon\right)}=\frac{2\pi i\,\delta\left(p_{10}-E_1\right)}{\left(m_2-\not{p}_2+\Sigma\left(\not{p}_2\right)-i\varepsilon\right)}\underbrace{\frac{m_1}{E_1}\sum_s u(\mathbf{p}_1,s)\bar{u}(\mathbf{p}_1,s)}_{\text{on-shell projection operator}}$$

★ Diagrammatic notation for 2-body CST equations:

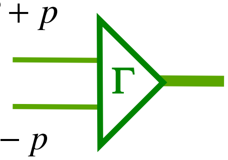


CST: must explicitly (anti)symmetrize the kernel

Bound state equations emerge automatically: NO extra assumptions

- The vertex function Γ describes how the bound state couples to particles in the Lagrangian:

$p_1 = \frac{1}{2}P + p$

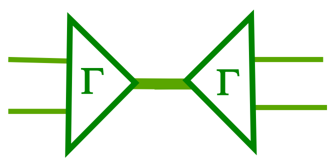


$p_2 = \frac{1}{2}P - p$

$\Gamma(p)$

Notation:
P=total momentum (always conserved)
p relative momentum

- The bound state equation follows from the assumption the M matrix has a pole at the bound state, and substituting



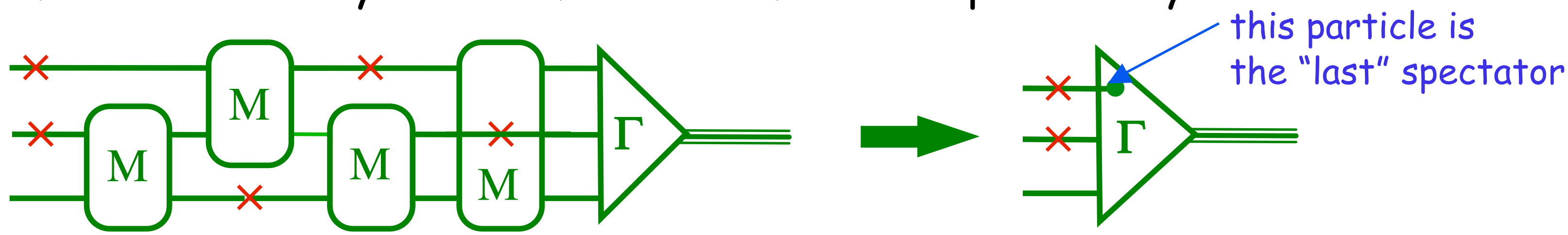
$$M(p',p;P) = \frac{\Gamma(p')\overline{\Gamma}(p)}{M_B{}^2 - P^2} + R(p',p;P)$$

$$M(p',p;P) = \frac{\Gamma(p')\overline{\Gamma}(p)}{M_B^2 - P^2} + R(p',p;P) = V(p',p;P) + \int V(p',k;P)G(k;P)\left\{\frac{\Gamma(k)\overline{\Gamma}(p)}{M_B^2 - P^2} + R(k,p;P)\right\}$$

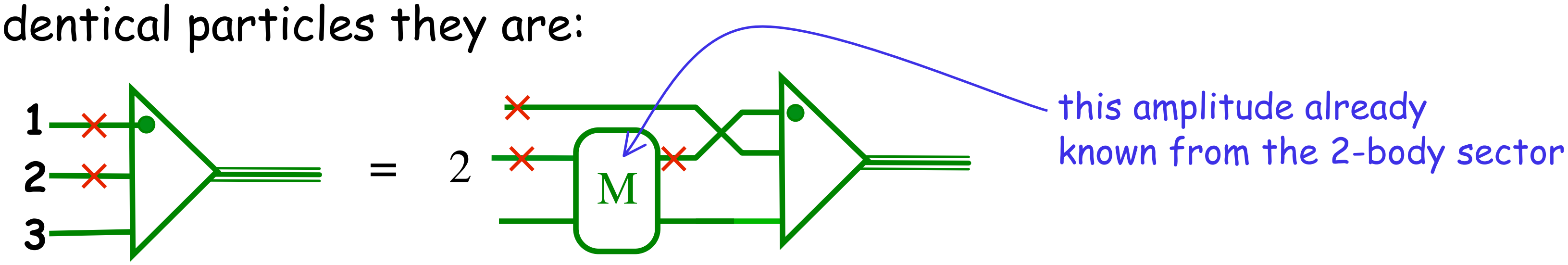
at the pole: $\Gamma(p') = \int V(p',k;P)G(k;P)\Gamma(k)$

CST equations for three-body bound state*

★ Define three-body vertex functions for each possibility



★ Then three body Faddeev-like equations emerge automatically. For identical particles they are:



$$\left|\Gamma_2^1\right\rangle = 2M_{22}^1 G_2^1 P_{12} \left|\Gamma_2^1\right\rangle$$

*Alfred Stadler, FG, and Michael Frank, Phys. Rev. C **56**, 2396 (1997)

Generalizing the CST to n-body systems

- In the *n* nucleon problem, make the following substitution for *n* – 1 nucleon propagators (with $\bar{u}_\alpha(\mathbf{p}_i, s) u_\alpha(\mathbf{p}_i, s') = \delta_{ss'}$)

$$S_{\alpha\beta}(p_i) = \frac{(m + \not{p}_i)_{\alpha\beta}}{m^2 - p_i^2 - i\varepsilon} \Rightarrow 2\pi i \delta_+(m^2 - p_i^2) 2m \sum_s u_\alpha(\mathbf{p}_i, s)\bar{u}_\beta(\mathbf{p}_i, s)$$

- The off-shell propagator is (in the CM with $k = P - \Sigma_i p_i$)

$$S_{\alpha\beta}(k) = \frac{(m + \not{k})_{\alpha\beta}}{m^2 - k^2 - i\varepsilon} \Rightarrow \left(\frac{m}{E_k}\right) \sum_s \left\{ \frac{u_\alpha(\mathbf{k}, s)\bar{u}_\beta(\mathbf{k}, s)}{(2E_k - W)} - \frac{v_\alpha(-\mathbf{k}, s)\bar{v}_\beta(-\mathbf{k}, s)}{W} \right\}$$

- Integration over all internal $p_{i\,0}$'s places *n* – 1 particles on their *positive energy* mass-shell. All 4-d integrations reduce to 3-d integrations.
- If n ≥ 4, additional equations emerge (as in the AGS equations)
- Deriving the equations for n ≥ 4 is a work in progress; extensions to many body systems will be difficult, and might best be done through reduction to the nonrelativistic limit

Normalization conditions obtained directly from the CST equations

- Covariant bound state normalization conditions follow from examination of the residue of the bound state pole
  - 2-body case
    $$1=\langle\Gamma|\frac{dG}{dM_d^2}|\Gamma\rangle-\langle\Gamma|G\frac{dV}{dM_d^2}G|\Gamma\rangle$$
  - 3-body case
    $$1=3\left\langle\Gamma_2^1\right|\left(1+2P_{12}\right)\frac{dG_2^1}{dM_d^2}\left|\Gamma_2^1\right\rangle-3\left\langle\Gamma_2^1\right|\left(1+2P_{12}\right)G_2^1\frac{dV_{22}^1}{dM_d^2}G_2^1\left(1+2P_{12}\right)\left|\Gamma_2^1\right\rangle$$
- Define the 2-body relativistic wave function: $|\Psi\rangle=G|\Gamma\rangle$. Then, if $\dfrac{dV}{dM_d^2}=0,$

  $$\boxed{2M_d=\left\langle\Psi\right|\gamma^0\left|\Psi\right\rangle}\qquad\left(\text{because }\frac{dG}{dM_d^2}=\frac{1}{2M_d}\frac{dG}{dM_d}=\frac{1}{2M_d}G\gamma^0G\right).$$
  - Identical to the normalization condition for the Dirac equation
- Similar interpretation for the 3-body normalization condition
- Similar derivation for the Bethe-Salpeter (and Schrödinger) equations

Why are particles put on shell?

- ★ Original motivation: 1965 study of the deuteron form factors in dispersion theory (too complicated!)
- ★ Preserves cluster separability: when one particle is separated to infinity, the equation for the remaining system is not affected by its presence
- ★ Maintains manifest covariance without adding more momentum variables
  - Gives a smooth nonrelativistic limit
  - Equations are "easy" to solve; like NR equations but with more channels
- ★ CST in OBE gives the *exact result* for the sum of all ladders and crossed ladders in special cases
  - Gives the correct "one body limit"

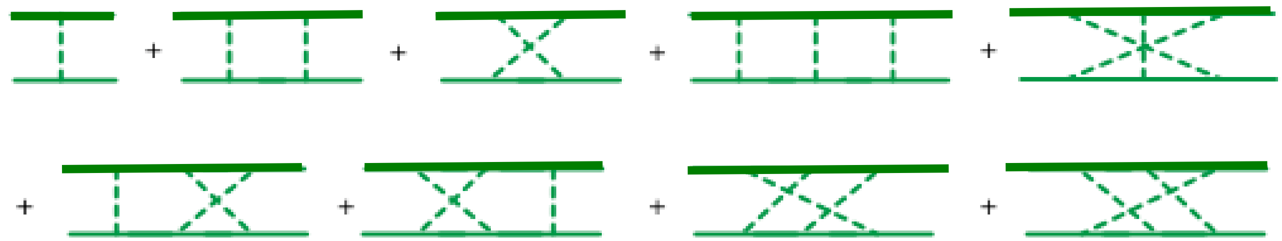
Illustrate this point on the following slides

- ★ In general, we believe CST converges more rapidly that the BS equation

BS and CST are equivalent when both are solved exactly

★ To 6th order, the generalized ladder sum is M > m

M
m



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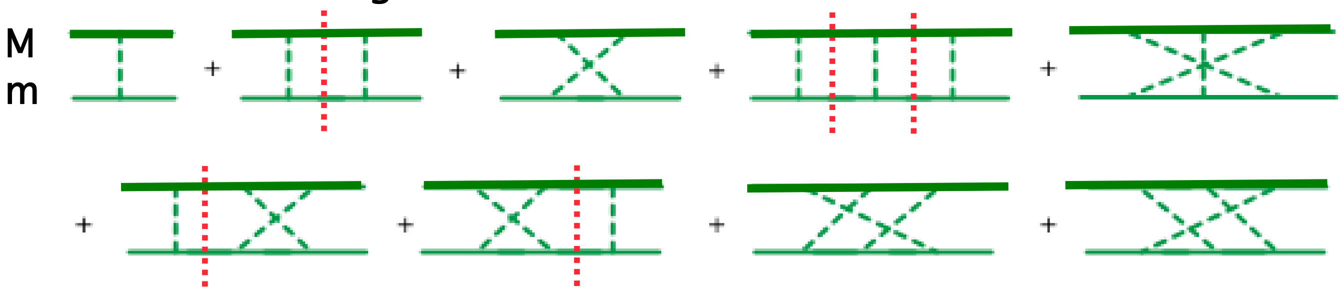
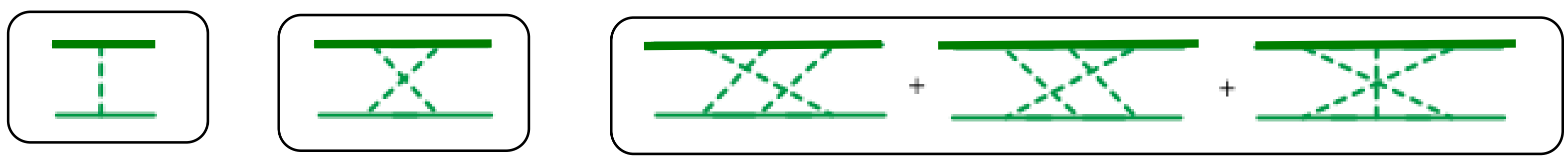


- In the BS theory, these terms require the following *irreducible* kernel:



2nd order *4th order* *6th order*

BS and CST are equivalent when both are solved exactly

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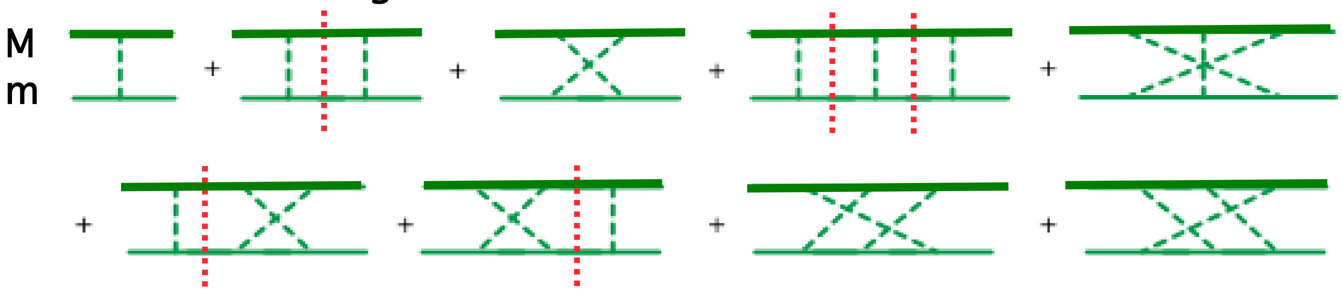

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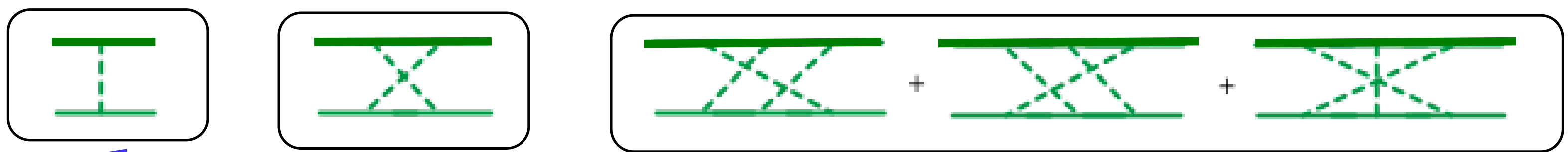
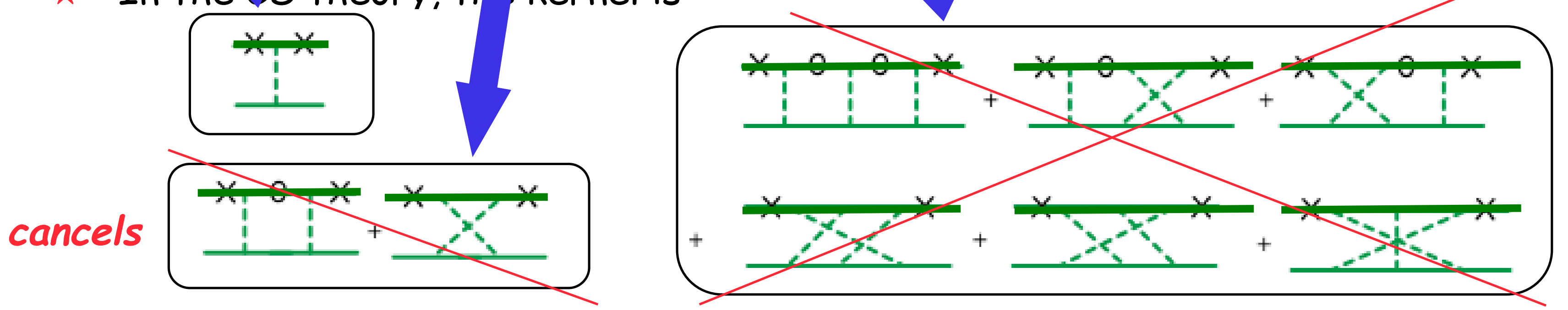


2nd order   *4th order*   *6th order*

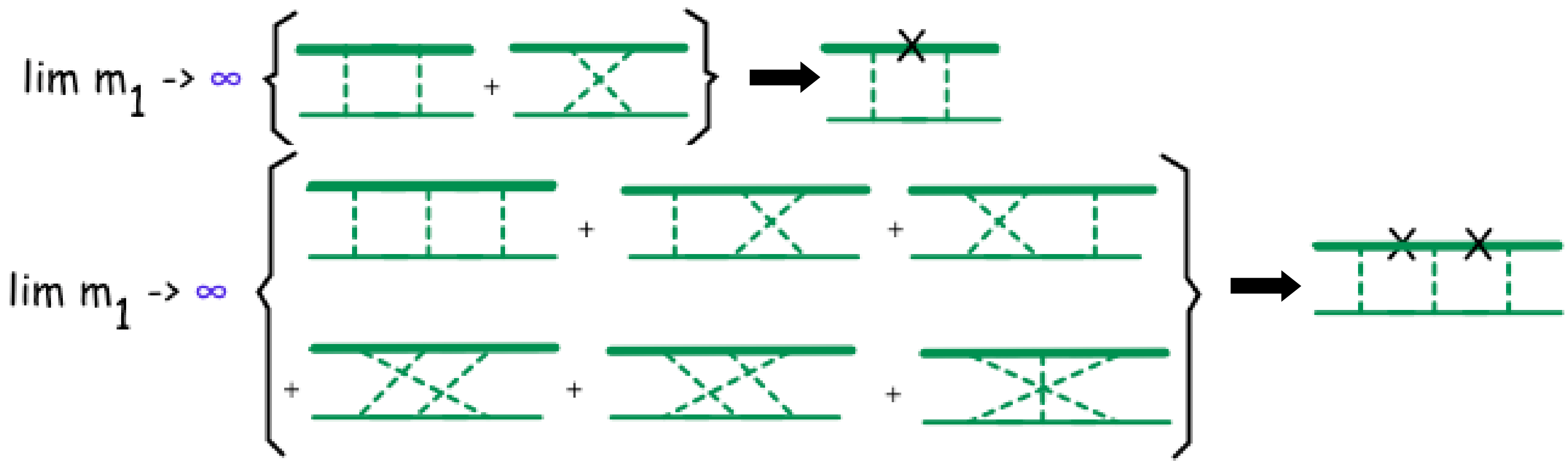
★ In the CS theory, the kernel is

If $M \to \infty$
these cancel



The one-body limit and the cancellation theorem

- If particle 1 is a neutral scalar particle and $m_1 \Rightarrow \infty$, the equation should reduce to a one-body equation for m_2. This is the *one-body limit*.
- The *generalized* ladder sum has this property to *each* order. Diagrammatically, for the 2nd and 4th orders



- For scalar theories in the $m_1 \Rightarrow \infty$ limit, the OBE approximation in CS theory gives the *exact* result for the generalized ladder sum.

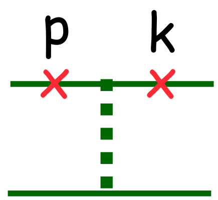
Show stopper: Singularities ..

- ★ In Minkowsky space
  - BS equation is full of singularities;
  - Path integrals (Lattice Gauge Theory) also do not converge
- ★ The “solution” is to work in Euclidian space.
  - Wick rotation for the BS equation
  - By fiat for Lattice Gauge Theory and Schwinger-Dyson equations
  - BUT amplitudes are not physical in Euclidian space !
- ★ The CST equations live in Minkowsky space
  - therefore have some singularities.
- ★ Major new development
  - beautiful (?) new philosophy -- called option C
  - redefine propagators so that all singularities are removed!

modified propagator -- Option C

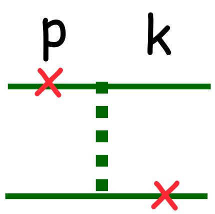
- The option C propagator is $$\frac{1}{\mu^2 - q^2} \Rightarrow \frac{1}{\mu^2 + \left|q^2\right|}$$

- the "direct" term is unaffected by the cutoff because $-q^2$ is always positive



$$-q^2 = (\mathbf{p} - \mathbf{k})^2 - \left(E_p - E_k\right)^2 \geq (p-k)^2 - \left(E_p - E_k\right)^2$$
$$\geq 2E_pE_k - 2pk - 2m^2 \geq 0$$

- the "exchange" term (required to maintain particle symmetry) can have negative $-q^2$ and singularities. This term is affected by the cutoff



$$-q^2 = (\mathbf{p} - \mathbf{k})^2 - \left(W - E_p - E_k\right)^2 \geq (p-k)^2 - \left(W - E_p - E_k\right)^2$$
$$= 2E_pE_k - 2pk - 2m^2 - \left(2E_p - W\right)\left(2E_k - W\right)$$

Illustration of the effect of option C

- Regions for total energy W > 2m

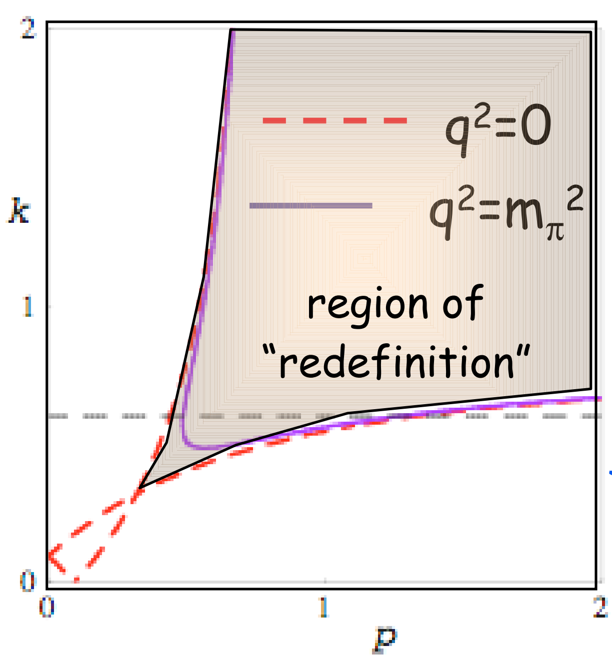
Option C

$$F(p) = \int_{-1}^{+1} \frac{dz}{m_\pi^2 + |q^2|}$$

Option A

$$F(p) = \mathcal{P}\int_{-1}^{+1} \frac{dz}{m_\pi^2 - q^2}$$

k=0.6



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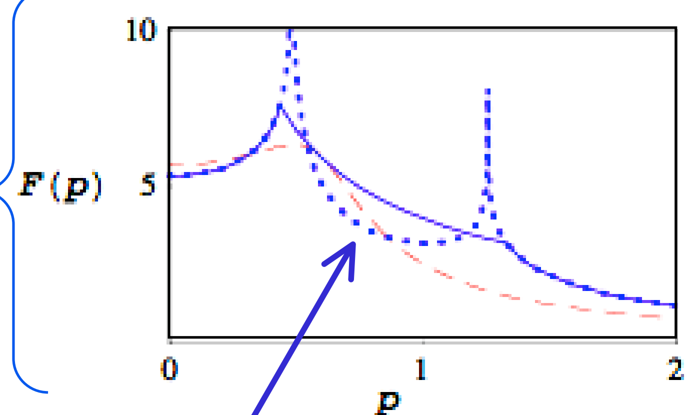


nice model independent smoothing permitted by the cancellation

Conclusions to Part I (1):
Why develop a relativistic theory (in particular, *field dynamics*)?

- ★ Intellectual: preserve an exact symmetry (Poncare' invariance)
- ★ Practical: calculate boosts and Lorentz kinematics to all orders
  - essential when energies are of the order of 1 GeV
- ★ Consistent: use *field dynamics* for guidance in the construction of
  - forces (2⇔3 body consistency)
  - currents consistent with forces
- ★ Efficient: "phenomenological economy"
  - spin 1/2 particles (Dirac equation)
  - interpretation of L•S forces (covariant scalar-vector theory of N matter)
  - One CST one boson exchange (OBE) model of NN forces (WJC-2) has only 15 parameters (will discuss soon)!
- ★ Provide an alternative picture which may give insight

Conclusions to Part I (2):
Principals of the Covariant Spectator Theory (CST)

- ★ Start from relativistic quantum field theory: manifest covariance built in
- ★ Maintain manifest covariance in all modifications; keep Dirac structure of off-shell particles
- ★ Bound states and unitarity require an an infinite sum of diagrams; perturbation theory is NOT enough.
- ★ Formulate integral equations to sum infinite series of diagrams
- ★ Base the dynamics on generalized ladder sum (i.e. ladders and crossed ladders). Treat vertex corrections and self energies phenomenologically $\Rightarrow$ "Field dynamics"
- ★ To sum ladders and crossed ladders efficiently leads to the placement of particles on shell

Precision fits to the np data

Model the kernel using one boson exchange (OBE) diagrams

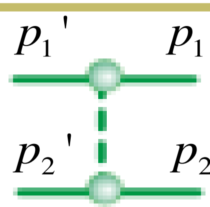
Justification for OBE:

- ★ simplest possible model
- ★ directly related to hadronic phenomenology
- ★ can be extended *consistently* to N-body systems
- ★ can be extended *consistently* to electromagnetic currents
- ★ implements the cancellation theorem
- ★ connection to QCD through large N_c(?)

CS Dynamics: OBE with off-shell couplings

★ Kernel is a sum of One Boson Exchange diagrams

$$\delta = \begin{cases} 1 & \text{isoscalar} \\ \tau_1 \cdot \tau_2 & \text{isovector} \end{cases}$$

V $= \sum$ 

$= \sum \varepsilon_b \delta \dfrac{\Lambda_{\alpha'\alpha}(k_1,p_1) \otimes \Lambda_{\beta'\beta}(k_2,p_2)}{m_m^2 + |q^2|} f^4(\Delta_b, q)$

$$f^4(\Lambda_b, q) = \left[\frac{\Lambda_b^2}{\Lambda_b^2 + |q^2|}\right]^4$$

boson	$J^P(b)$	ε_b	$\Lambda_1 \otimes \Lambda_2$	$\Lambda(k,p)$ or $\Lambda^\mu(k,p)$
σ_0 σ_1	$0^+(s)$	−	$\Lambda_1\,\Lambda_2$	$\Lambda(k,p) = g_s - \nu_s[\theta(k) + \theta(p)]$
π η	$0^-(p)$	+	$\Lambda_1\,\Lambda_2$	$\Lambda(k,p) = g_p\{\gamma^5 - (1-\lambda_p)[\theta(k)\gamma^5 + \gamma^5\theta(p)]\}$
ρ ω	$1^-(v)$	+	$\Lambda_1^\mu \Lambda_2^\nu \Delta_{\mu\nu}$	$\Lambda^\mu(k,p) = g_v\left\{\gamma^\mu + \dfrac{\kappa_v}{2m} i\sigma^{\mu\nu}(k-p)_\nu + \nu_v[\theta(k)\gamma^\mu + \gamma^\mu\theta(p)]\right\}$
a_1 h_1	$1^+(a)$	+	$\Lambda_1^\mu \Lambda_2^\nu g_{\mu\nu}$	$\Lambda^\mu(k,p) = g_a\gamma^5\gamma^\mu$

$\boxed{\theta(p) = \dfrac{m - \not{p}}{2m}}$ ***vanishes on-shell***

$$\Delta_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / m_v^2$$

OBE parameters obtained from the recent precision fits

b	I	$G_b = \frac{g_b^2}{4\pi}$		m_b		λ_b or ν_b		κ_v		Λ_b	
π^0	1	14.608	14.038	134.9766		0.153	0.0	---		4400	3661
$\pi^\pm$	1	13.703	14.038*	139.5702		-0.312	0.0	---		4400*	3661*
η	0	10.684	4.386	604	547.51	0.622	0.0	---		4400*	3661*
σ_0	0	2.307	4.486	429	478	-15.169	-2.594	---		1435	3661*
σ_1	1	0.539	0.477	515	454	4.763	9.875	---		1435*	3661*
ω	0	3.456	8.711	657	782.65	0.843	0.0	0.048	0.0	1376	1591
ρ	1	0.327	0.626	787	775.50	-1.263	-2.787	6.536	5.099	1376*	1591*
h_1	0	0.0026	0.0							1376*	
a_1	1	-0.436	0.0							1376*	
									Λ_N	1656	1739

left column: WJC-1 27 parameters

right column: WJC-2 15 parameters

Comments on the parameters

- The $G_{\pi^\pm}$ that emerges from the WJC-1 fit is close to Nijmegen (13.567), but the others are larger
- The off-shell pion couplings (λ_π) are either 0 (WJC-2) or small (WJC-1) in agreement with chiral symmetry
- The meson masses that were adjusted are all near 500 MeV as expected if a dispersion integral is saturated by a mass near the 2π =280 MeV threshold. Only the ρ is larger.
- The axial vector contributions are 0 (WJC-2) or very small (WJC-1) implying convergence in the exchange quantum numbers/masses. G_{a_1} (WJC-1) is negative; must be a contact interaction and not a boson exchange.

WJC-1 and -2 are precision fits to the 2007 data base

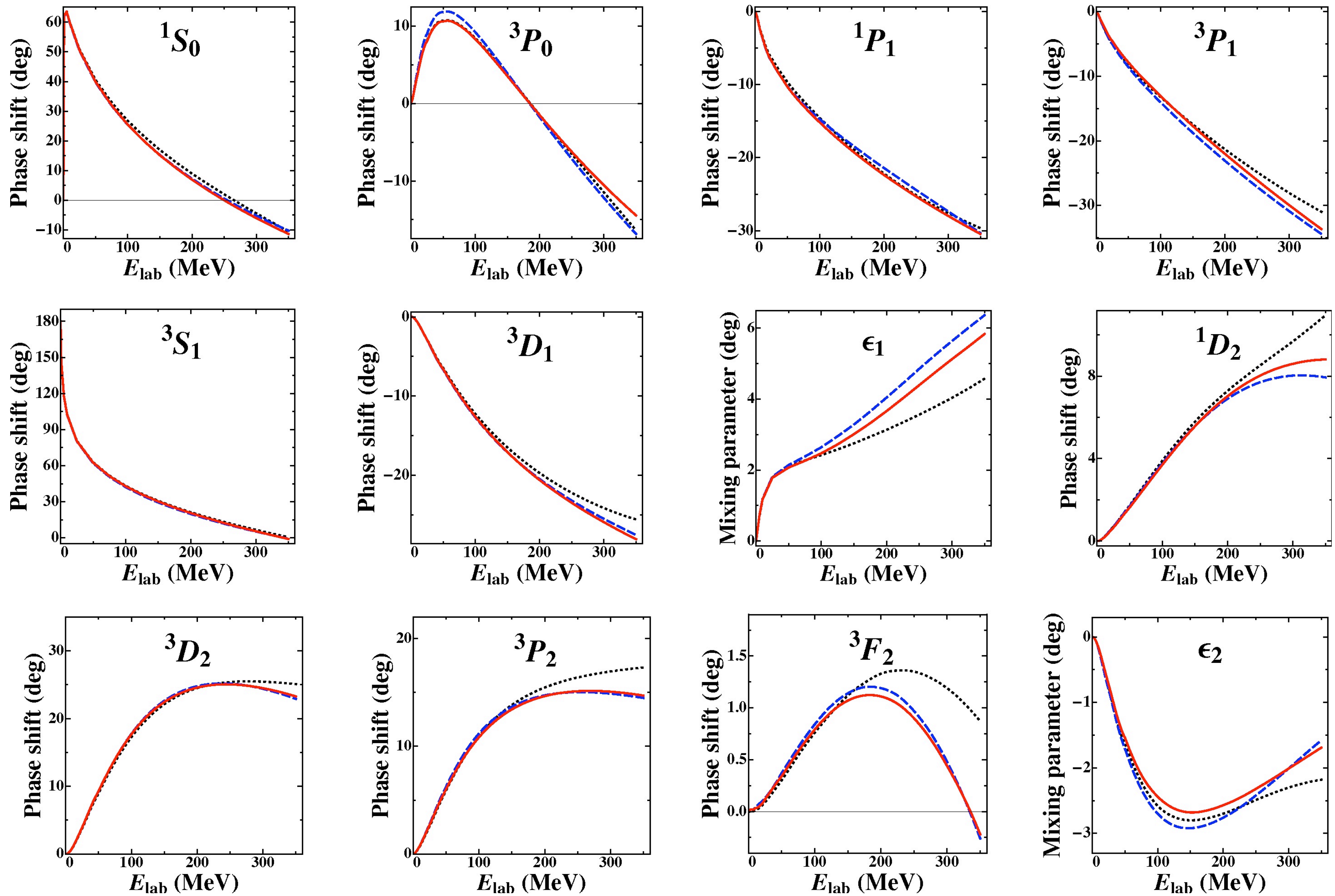
★ Comparison with other precision fits (all to 350 MeV)

Models			$\chi^2/N_{\text{data}}\ (N_{\text{data}})$		
Ref	year	# para	1993	2000	2007
PWA93	1993	39	0.99 (2514)		
			1.09 (3011)	1.12 (3336)	1.13 (3788)
Nijm I	1993	41	1.03 (2514)		
AV18	1995	40	1.06 (2526)		
CD-Bonn	2000	43		1.02 (3058)	
WJC-1	2007	27	**1.03** (3011)	**1.05** (3336)	**1.06** (3788)
WJC-2	2007	15	**1.09** (3011)	**1.11** (3336)	**1.12** (3788)

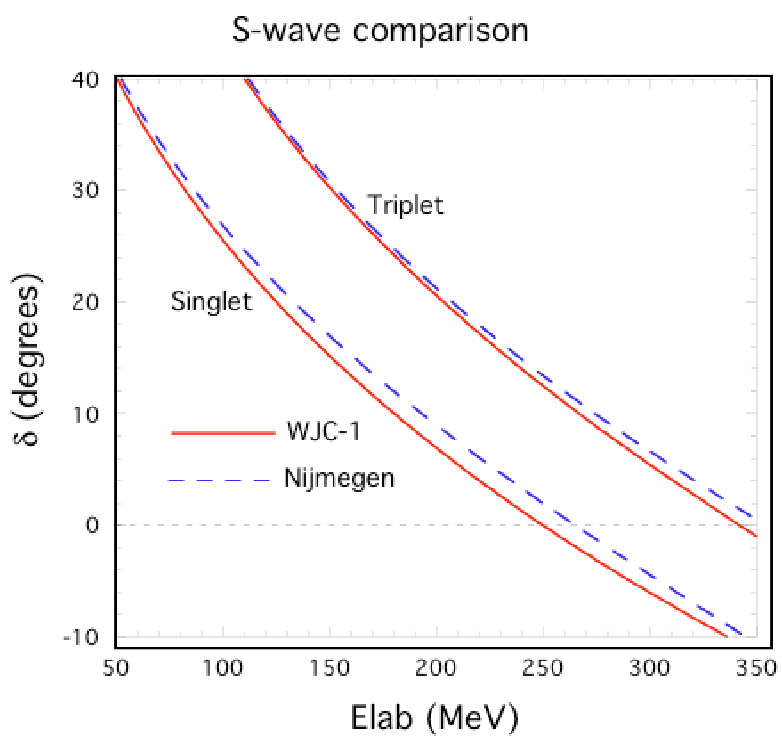
#'s in green are for fits to BOTH np and pp data

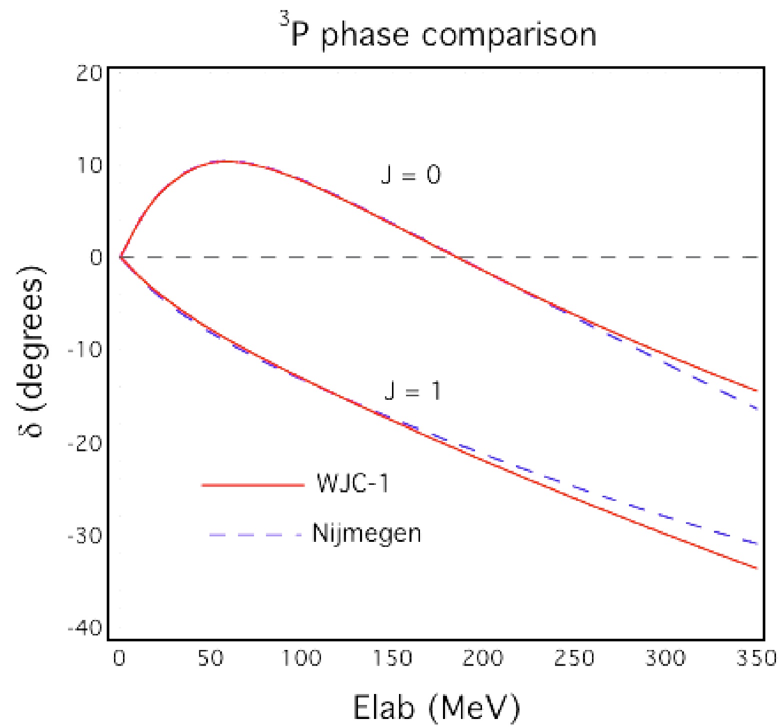
J=0, 1, 2 Phases

WJC-1 WJC-2 Nijmegen PSA93



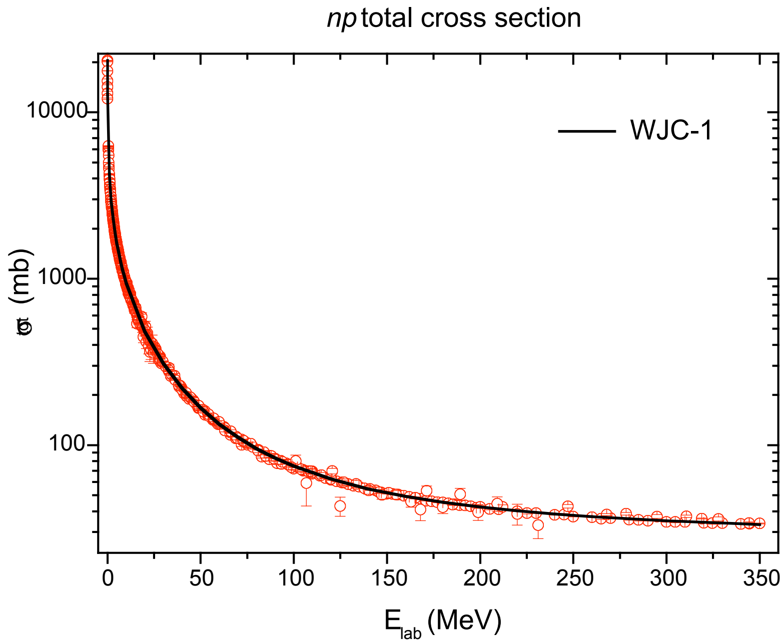
Changes in the phase shifts





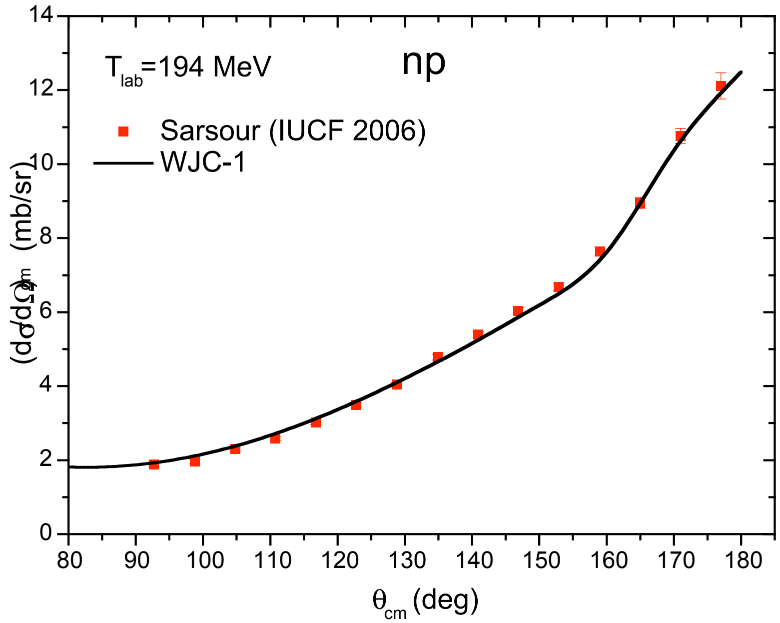
- ★ Nijmegen phases differ by several degrees from the WJC-1 phases. (Explains earlier problem fitting the data.)
- ★ This is a NEW PHASE SHIFT ANALYSIS!

Low χ^2 implies excellent fits to data (of course)



Total cross sections fit over the entire energy range

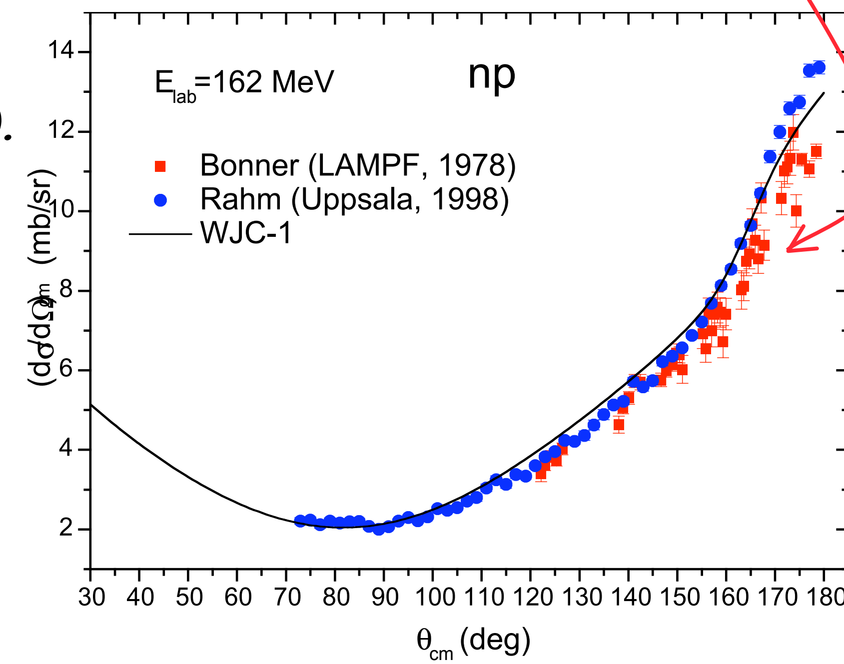
New accurate differential cross sections

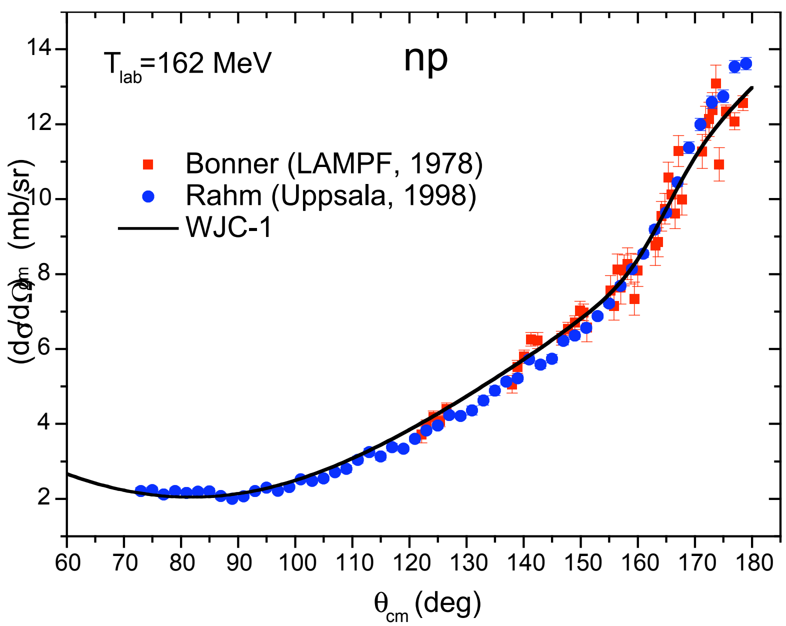


Scaling and rejection of data sets (1)

Experimentalists may specify that data has a systematic error; it may be scaled (within the error) to agree with theory. The red Bonner data has been scaled (below).

This data initially unscaled

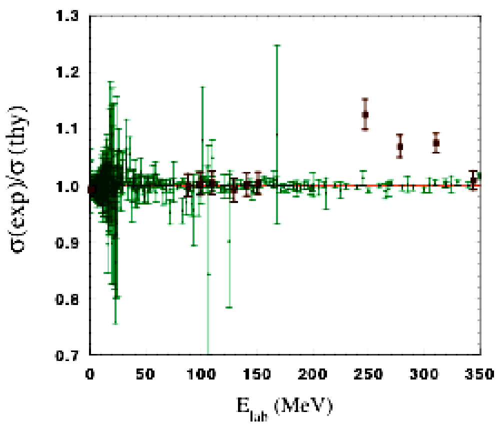




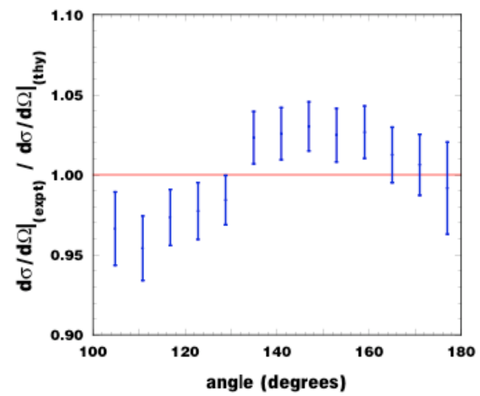
However, the Uppsala data (blue) is rejected; no scaling can change its incorrect shape. RA(98)

Scaling and rejection of data sets (2)

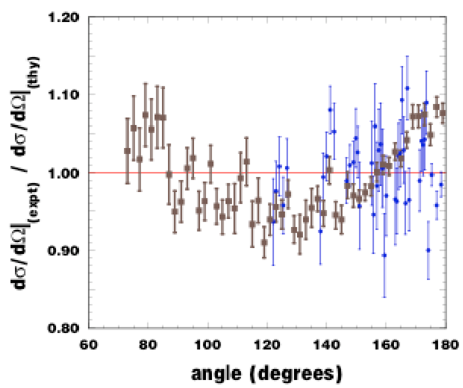
- Fits to the data are excellent; all data shown are scaled by the fit; some data with large systematic errors is excluded



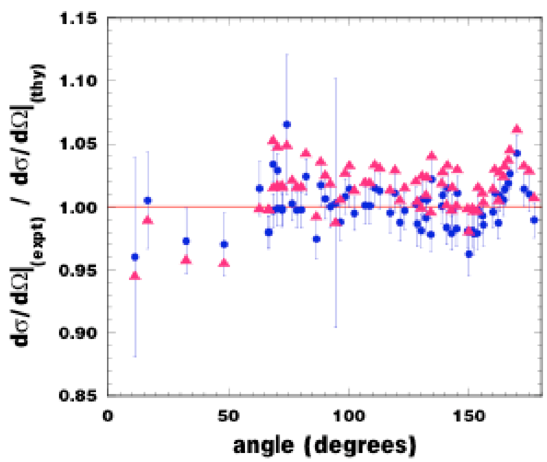
total cross section; entire energy range



194 MeV differential cross section



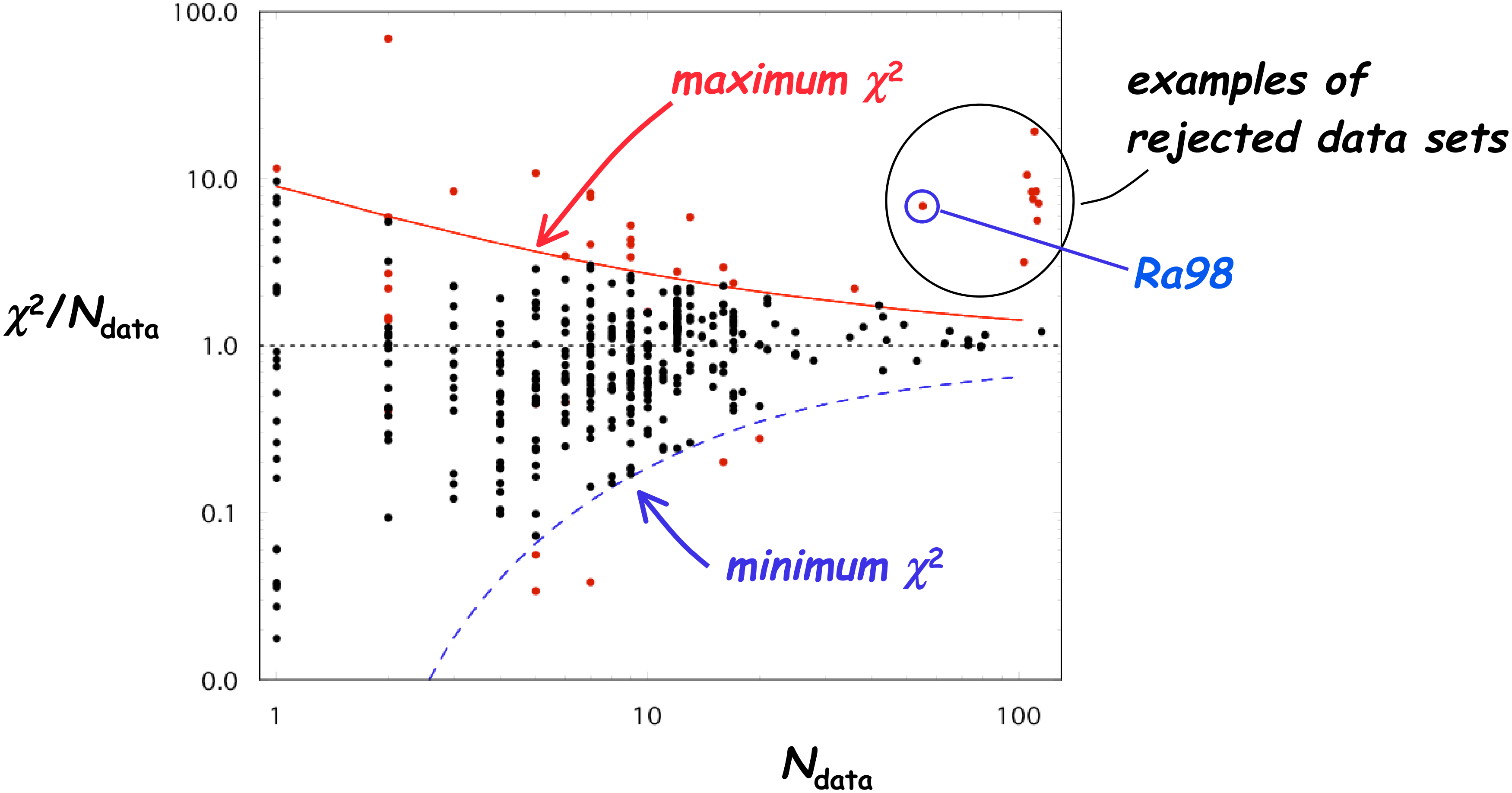
162 MeV differential cross section; brown data excluded



319 MeV differential cross section; shows scaling permitted by systematic errors

Rejected data sets can be identified

★ Nijmegen identifies a 3σ criterion. Data sets with χ^2 too large or too small are rejected.

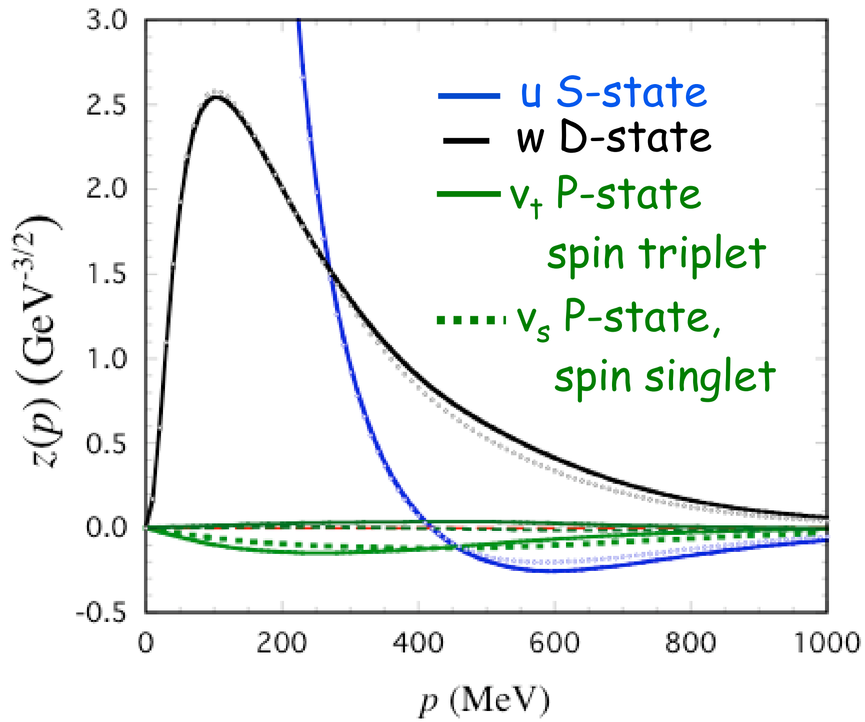


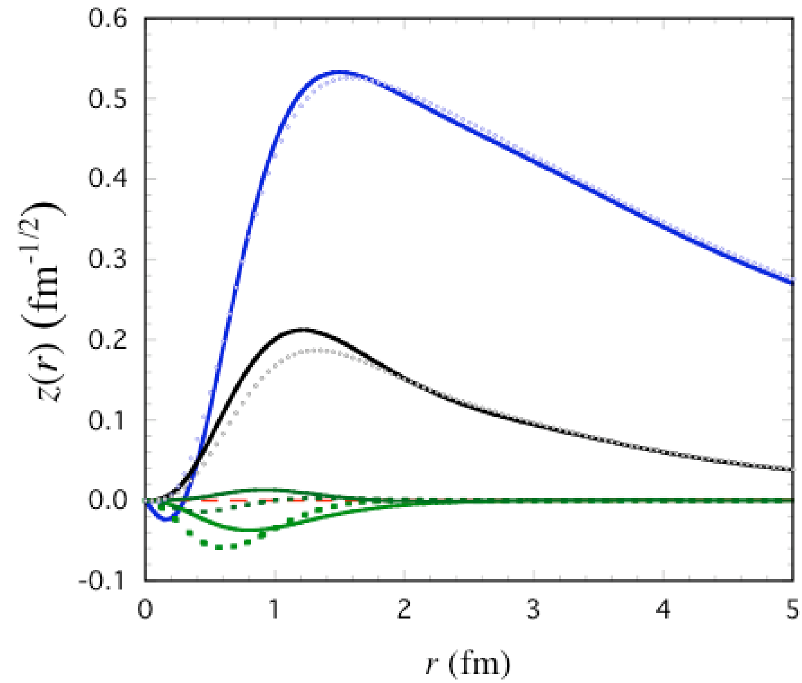
Deuteron wave functions

Normalization condition

$$\int_0^\infty dr\left\{u^2+w^2+v_t^2+v_s^2\right\}+\langle V'\rangle=1$$

Probability	WJC-1 exact	WJC-1 scaled	WJC-2 exact	WJC-2 scaled
P_S	97.3876	92.3330	95.7607	93.5985
P_D	7.7452	7.3432	6.5301	6.3827
P_{Vt}	0.1180	0.1119	0.0103	0.0101
P_{Vs}	0.2234	0.2118	0.0090	0.0088
ΣP	105.4743	100.0000	102.3101	100.0000
$\langle V'\rangle$	-5.4743	-----	-2.3101	------
total	100.0000		100.0000	

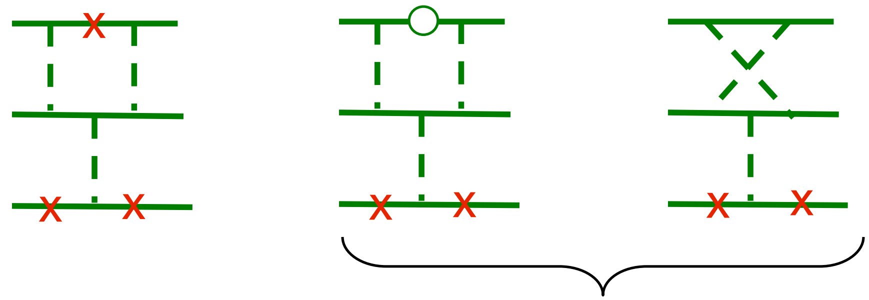




Binding energy of the triton and three-body forces

In a pure OBE theory there are NO three-body forces

★ Possible 3-body interactions can be reducible or irreducible. Irreducible ones are 3-body forces)

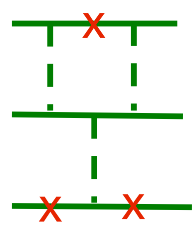


reducible
iteration of OBE

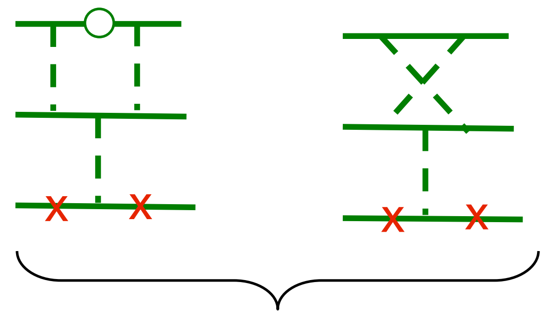
irreducible
3-body force diagrams

In a pure OBE theory there are NO three-body forces

- ★ Possible 3-body interactions can be reducible or irreducible. Irreducible ones are 3-body forces)

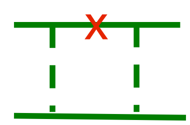


reducible
iteration of OBE

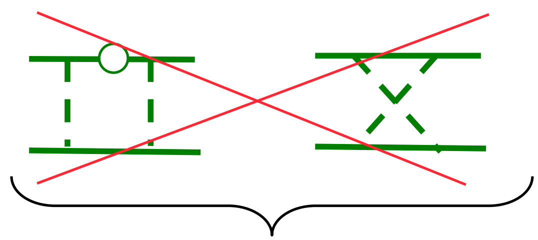


irreducible
3-body force diagrams

- ★ But irreducible non-OBE diagrams have been excluded



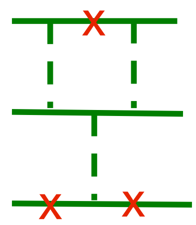
OBE iteration



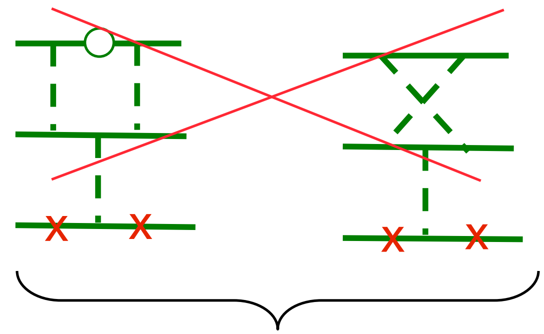
excluded irreducible
2-body force diagrams

In a pure OBE theory there are NO three-body forces

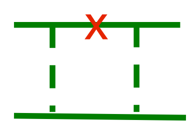
- ★ Possible 3-body interactions can be reducible or irreducible. Irreducible ones are 3-body forces)



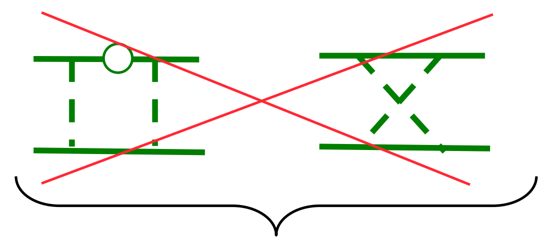
reducible
iteration of OBE



irreducible
3-body force diagrams



OBE iteration

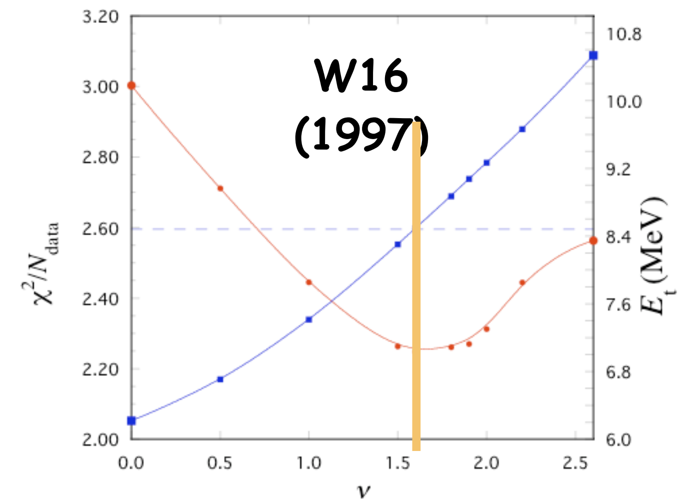


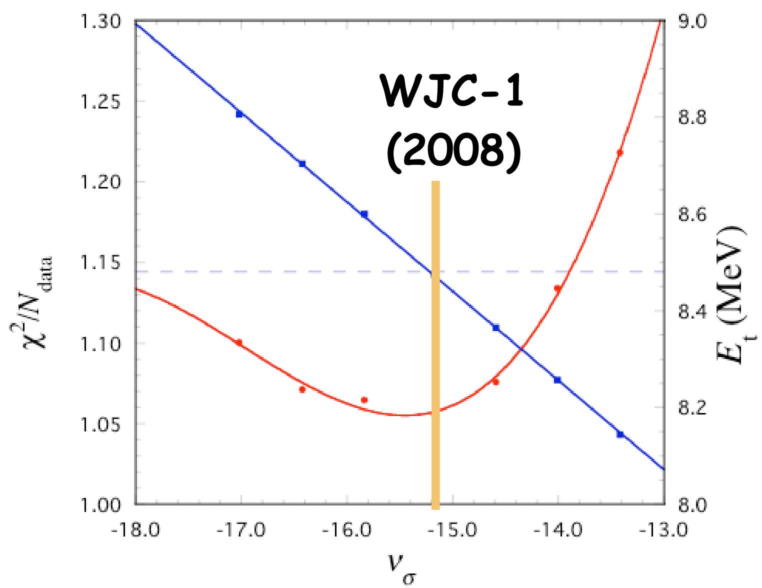
excluded irreducible
2-body force diagrams

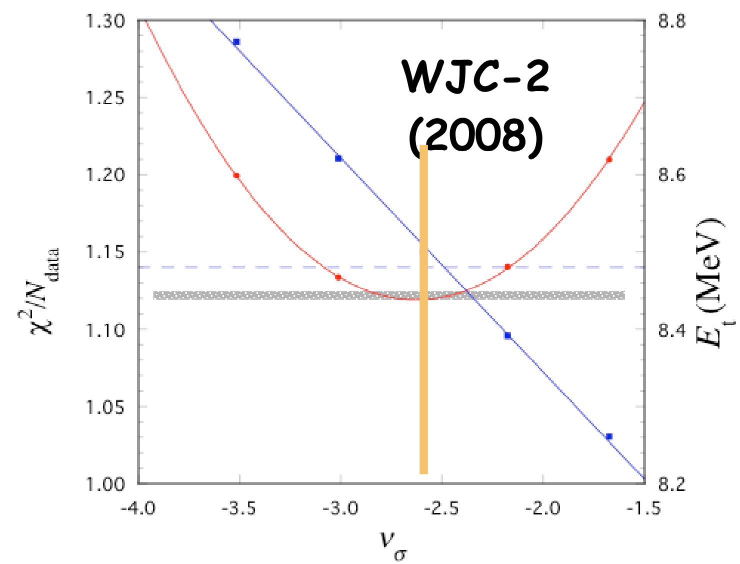
- ★ But irreducible non-OBE diagrams have been excluded
- ★ Therefore, consistency requires that they also be excluded from the 3-body sector

How well can the CST OBE model predict the ^{3}H binding energy?

- Recall the off-shell scalar coupling
  $\Lambda(k,p) = g_s - \nu_s\left[\theta(k)+\theta(p)\right]$ WJC
  $\Lambda(k,p) = g_s\left(1+0.75\nu\left[\theta(k)+\theta(p)\right]\right)$ WXX (1997)
- In ALL three cases we found that the best value of ν_σ also gave the correct binding energy for ^{3}H !



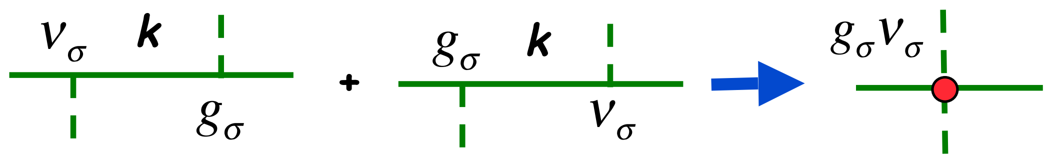




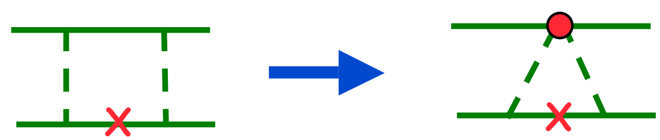
How can we get the right binding with NO 3-body force?

- Off-shell couplings remove the off-shell propagator, contracting the interaction to a point

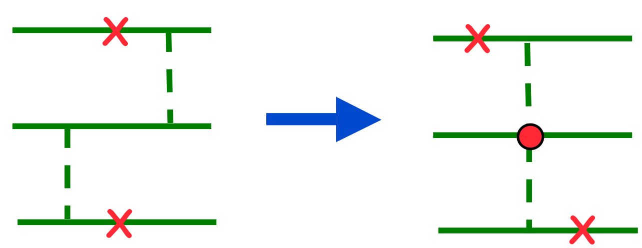
$$v_\sigma \frac{m-\not{k}}{2m}\left(\frac{1}{m-\not{k}}\right) g_\sigma + g_\sigma\left(\frac{1}{m-\not{k}}\right)\frac{m-\not{k}}{2m} v_\sigma = \frac{g_\sigma v_\sigma}{m}$$



- In the 2-body space, off-shell couplings are equivalent to effective non-OBE type interactions with loops



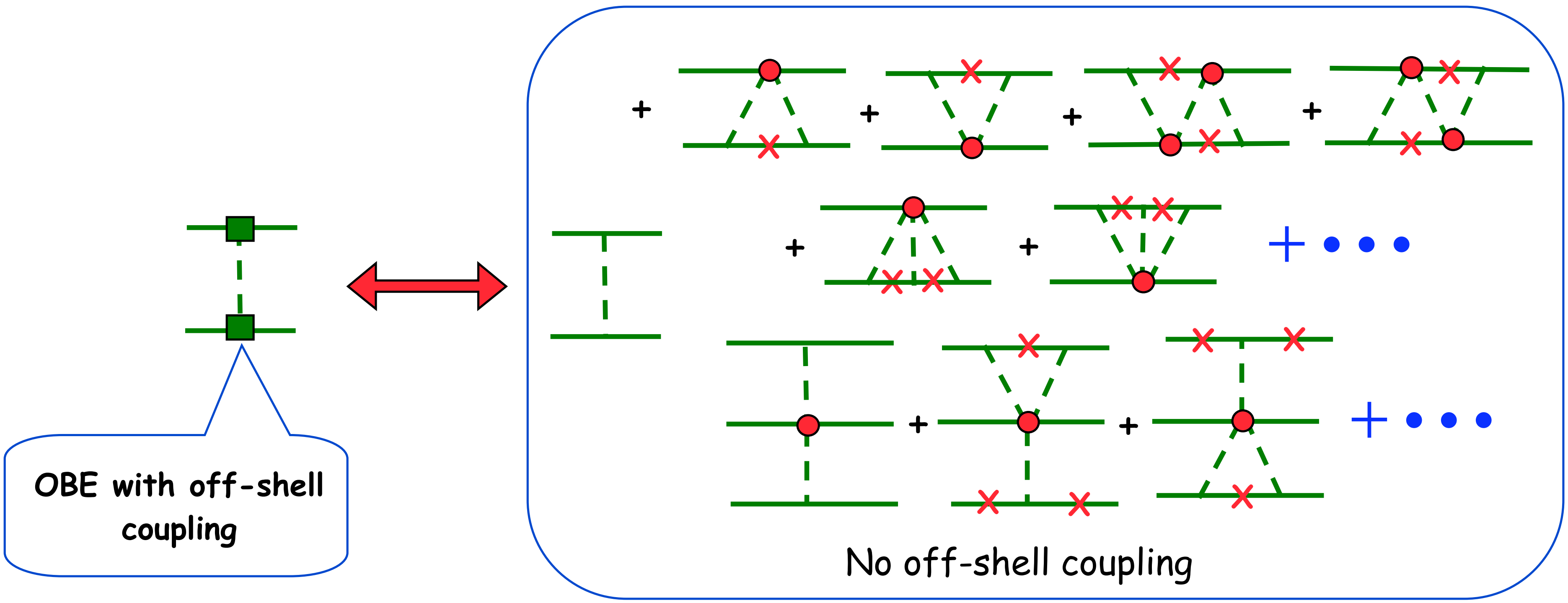
- In the 3-body space, off-shell couplings are equivalent to 3-body forces



Equivalence theorem

OBE with off-shell couplings ⟷ *OBE without off-shell couplings PLUS a specific set of N-meson exchange and N-body forces*

★ The equivalence is very complicated !!



Can we make a direct connection with χEFT?

Currents and form factors

Construction of the current operator in CST©
-- Gauge invariant* one-body current operator

- Exact gauge invariance currents can be constructed following the method of FG and Riska.* These have been used for both relativistic and non-relativistic calculations

- Proceed in two steps:
  - Step 1: construct one body currents that satisfy the Ward-Takahashi identity
  - Step 2: couple these currents to all charges (or momentum dependent couplings) in ALL of the infinite number of diagrams under consideration.

- Step 1: Construct a one-body current that satisfies the WT identity:

$$q_\mu j_N^\mu(p',p) = S^{-1}(p) - S^{-1}(p') \quad \text{where} \quad S(p) = \frac{h^2(p)}{m - p}$$

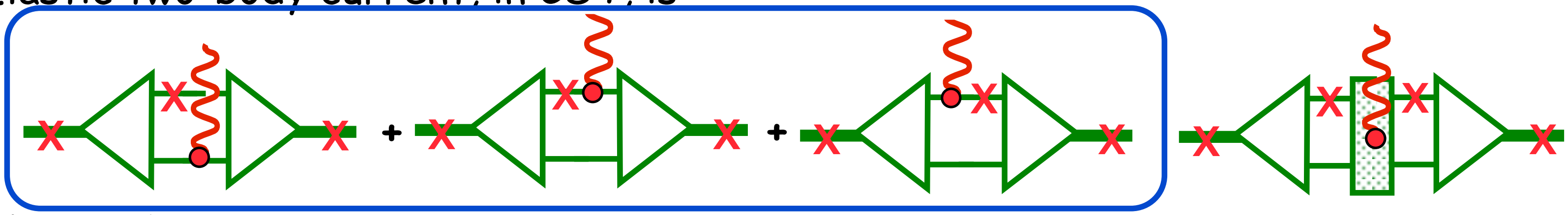
with $h(p)$ a nucleon form factor that dresses the propagator.

---

*FG, and D. O. Riska, PRC 36, 1928 (1987)

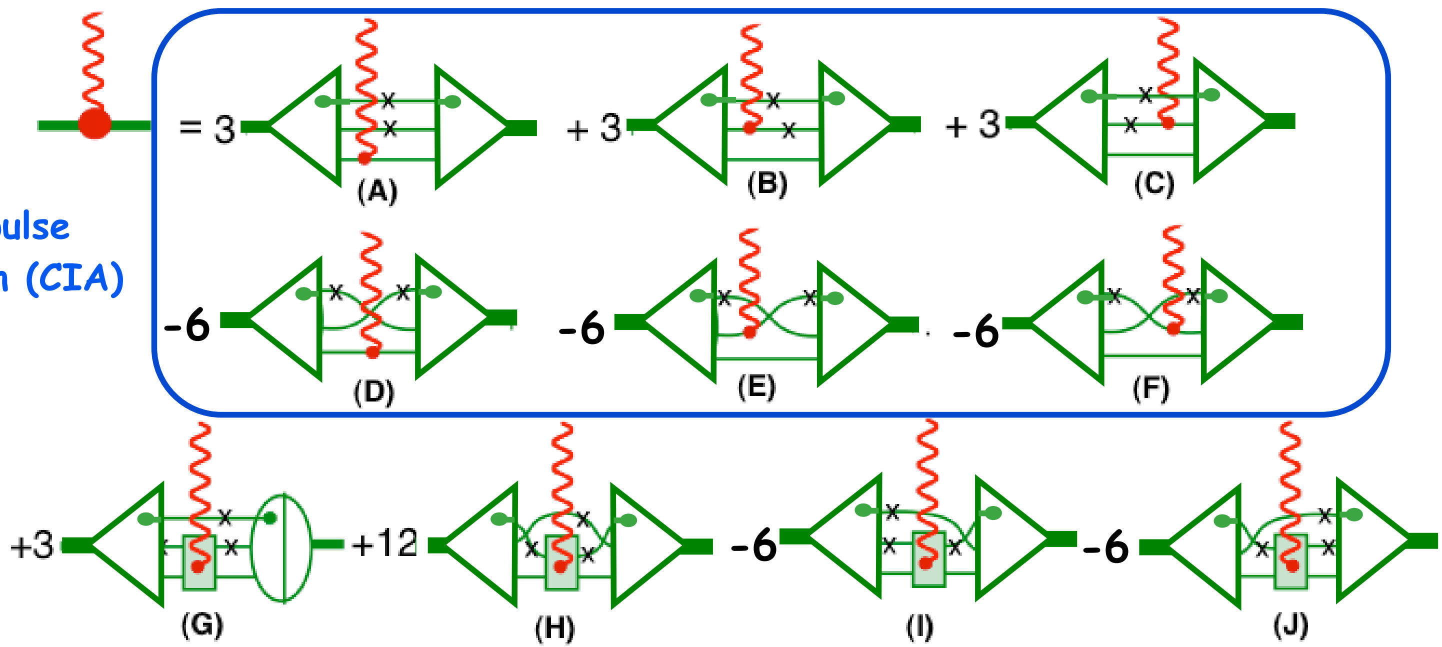
Construction of the current operator in CST©
-- Gauge invariant* two- and three-body current operators

- ★ Step 2: coupling to ALL charges not so difficult -- if the equations are used.
- ★ Elastic two-body current, in CST, is



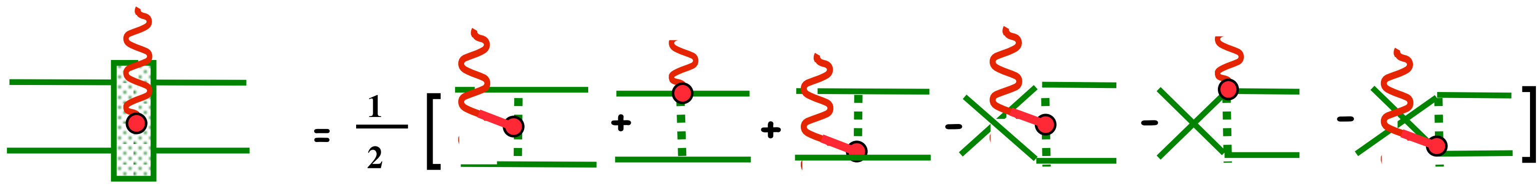
- ★ Elastic three-body current, in CST, is

Complete Impulse Approximation (CIA)



Construction of the current operator in CST©
-- Gauge invariant* interaction current operator

★ Interaction current for the OBE model:



★ Simplest one-body current operator that satisfied the WT identity

$$j^\mu(p',p) = F_0\left\{\gamma^\mu + (F_1-1)\left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right) + F_2\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right\} + G_0\,\Lambda_-(p')\left\{\gamma^\mu + (F_3-1)\left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right)\right\}\Lambda_-(p)$$

purely transverse

$$j^\mu(p',p) = F_0\left\{F_1\gamma^\mu + F_2\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right\} + G_0F_3\,\Lambda_-(p')\gamma^\mu\Lambda_-(p)$$

off-shell effects

$$F_0 = \frac{h(p)}{h(p')}\left(\frac{m^2-p'^2}{p^2-p'^2}\right) - \frac{h(p')}{h(p)}\left(\frac{m^2-p^2}{p^2-p'^2}\right) \qquad G_0 = \left(\frac{h(p')}{h(p)} - \frac{h(p)}{h(p')}\right)\frac{4m^2}{p^2-p'^2}$$

★ $F_3(Q^2)$ is unknown, except $F_3(0)=1$. This freedom can be exploited.

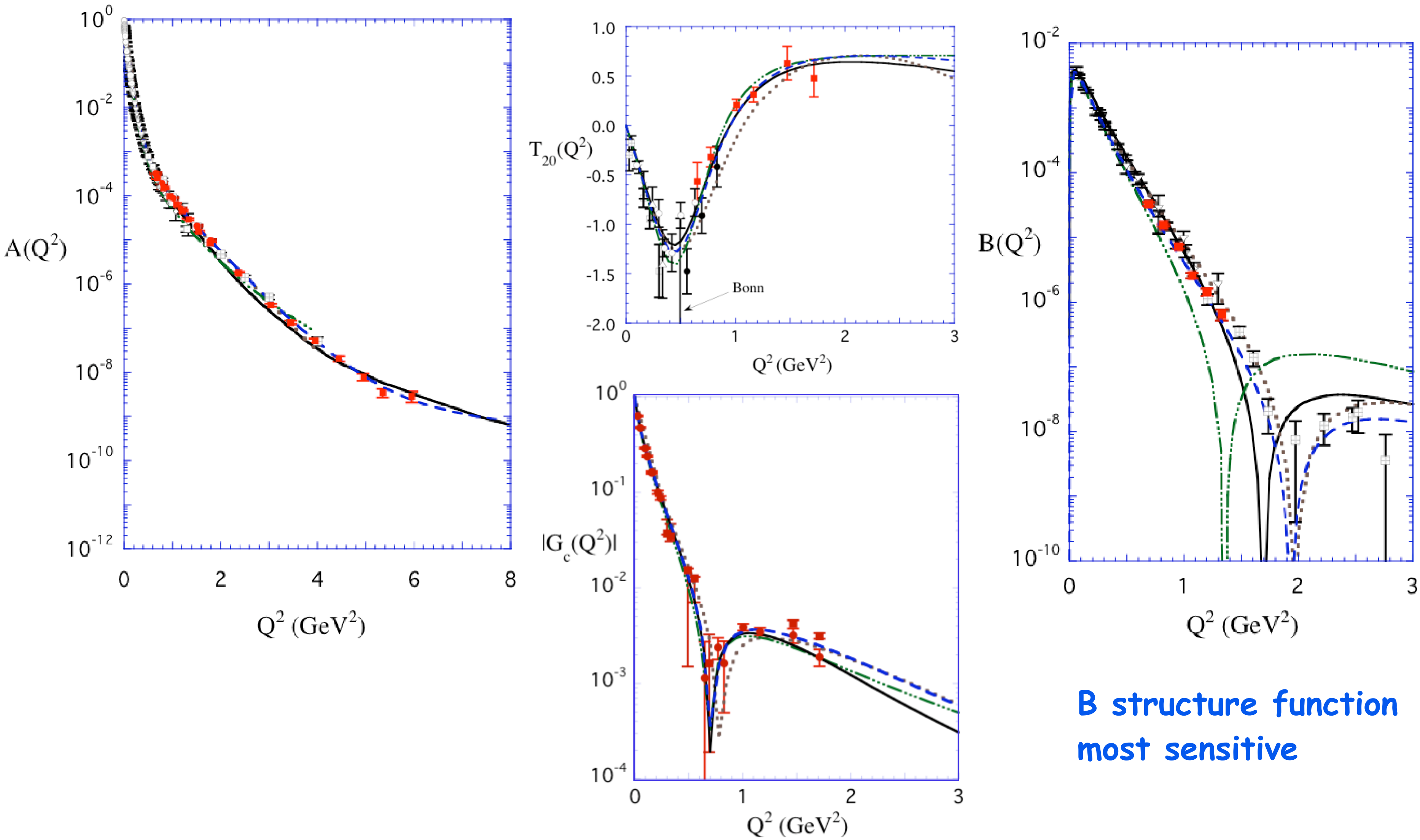
Form Factor calculations

- Current is conserved exactly, even in the presence of hadronic form factors
- CIA approximation: all calculations (so far) ignore interaction currents
- Deuteron[1]: Model IIB (1992); no off-shell couplings
  - Needs to be updated
  - ν-dependent off-shell couplings generate interaction currents expected to be important
- Three-body[2]: two cases:
  - WXX models of 1997 - ν-dependent couplings but fits have $\chi^2/N_{\mathtt{data}} \approx 2$
  - WJC models of 2008 -- but in CIA-0 approximation (full off-shell wave functions were not available then)

[1] J. W. Van Orden, N. Devine, & FG, Phys. Rev. Lett. **75**, 4369 (1995)
[2] Sergio A. Pinto, A. Stadler, & FG, Phys. Rev. C **79**: 054006 (2009); **81**: 014007 (2010)

Deuteron Form Factors (comparisons)

CST-IIB (van Orden, Devine, FG)
LF-HD (Huang and Polyzou)
MW (Phillips, Mandelsweig and Wallace)
QCB (Dijk and Bakker)



B structure function most sensitive

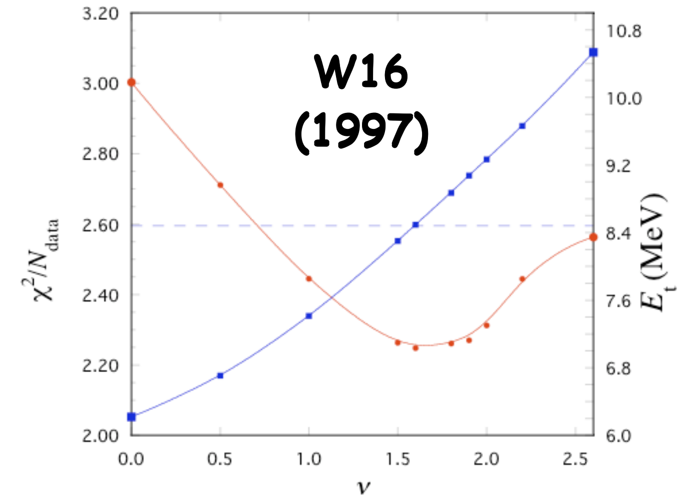
Recall the CST ν-dependent families

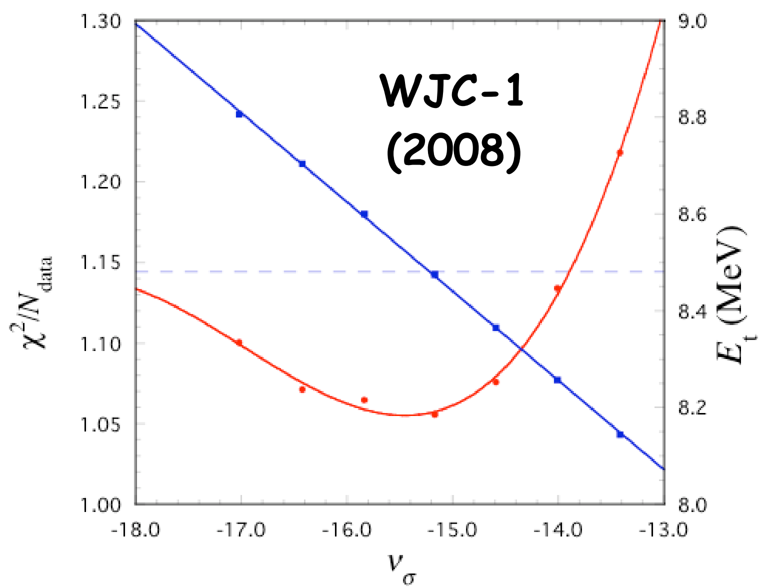
- Families depend on the off-shell scalar coupling

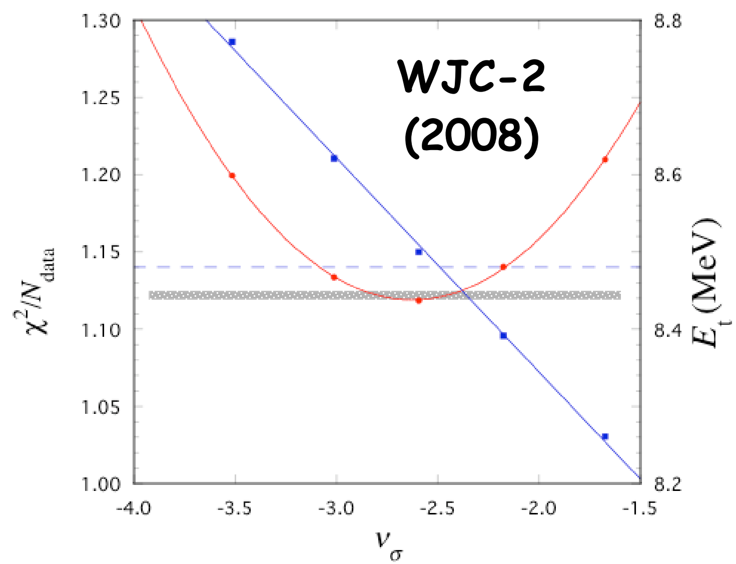
$$\Lambda(k,p) = g_s - \nu_s\left[\theta(k) + \theta(p)\right]$$

$\Lambda(k,p) = g_s\left(1 + 0.75\nu\left[\theta(k) + \theta(p)\right]\right)$ WXX (1997)

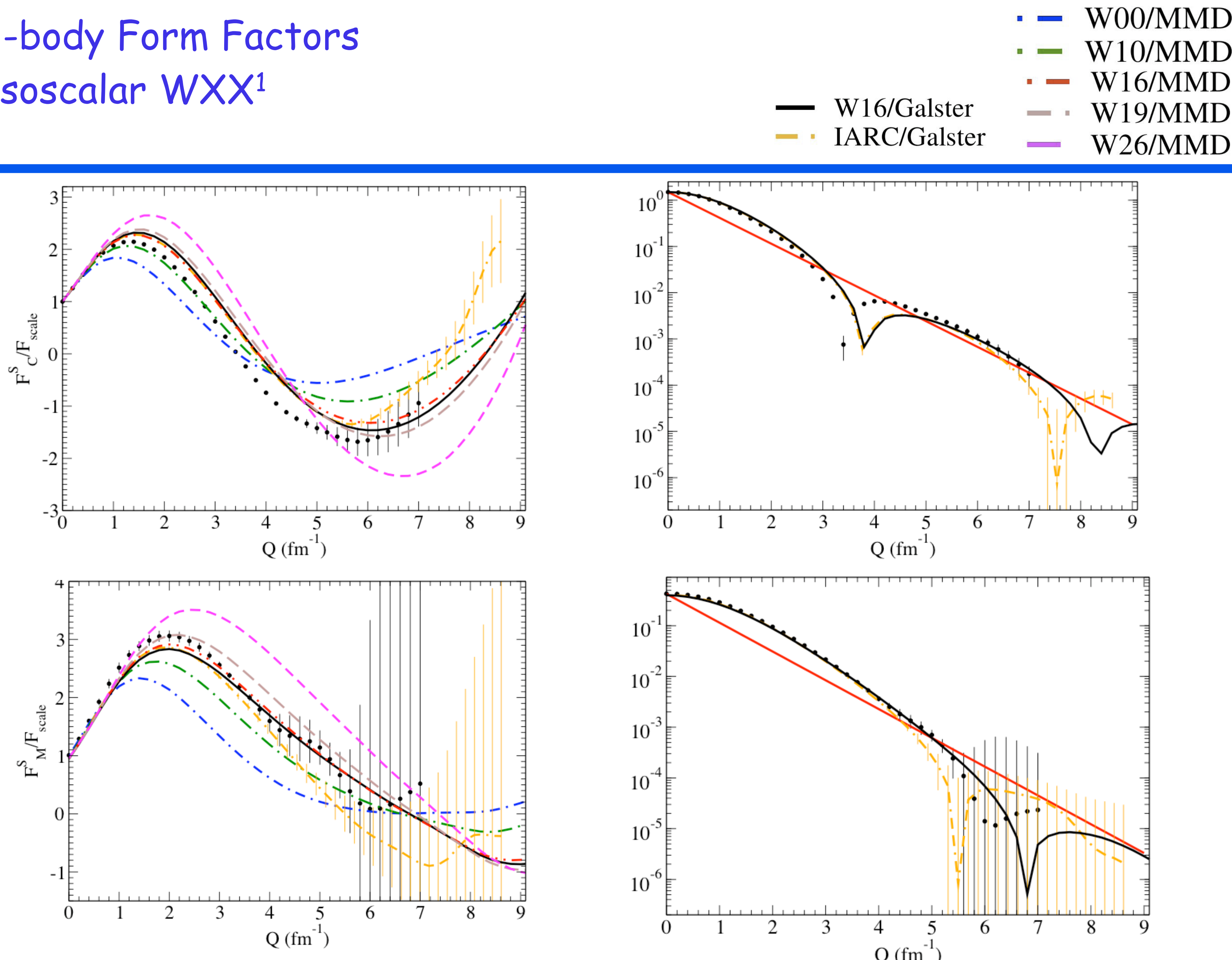
- The binding energy of ^{3}H depends almost linearly on ν





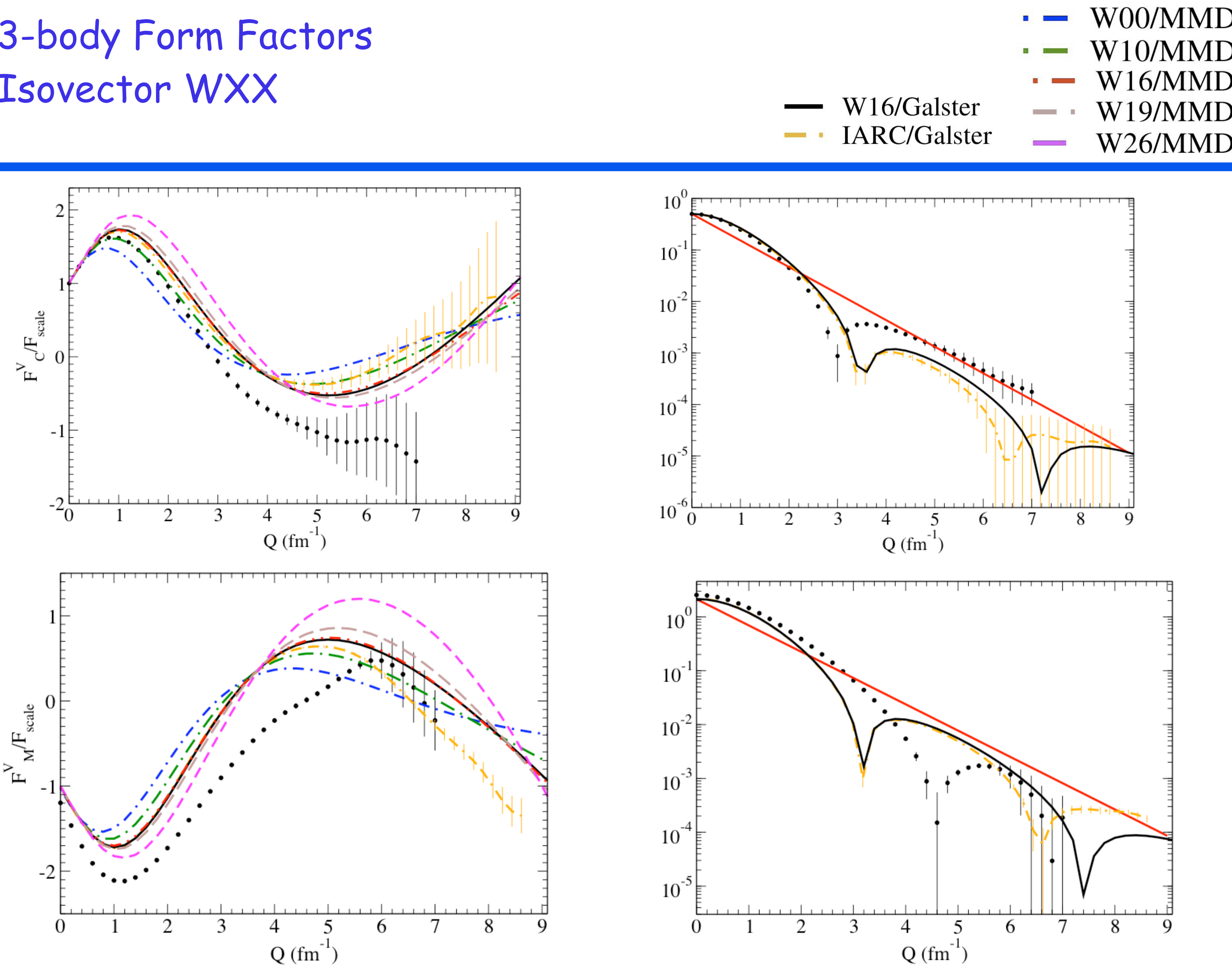


3-body Form Factors
Isoscalar WXX[1]

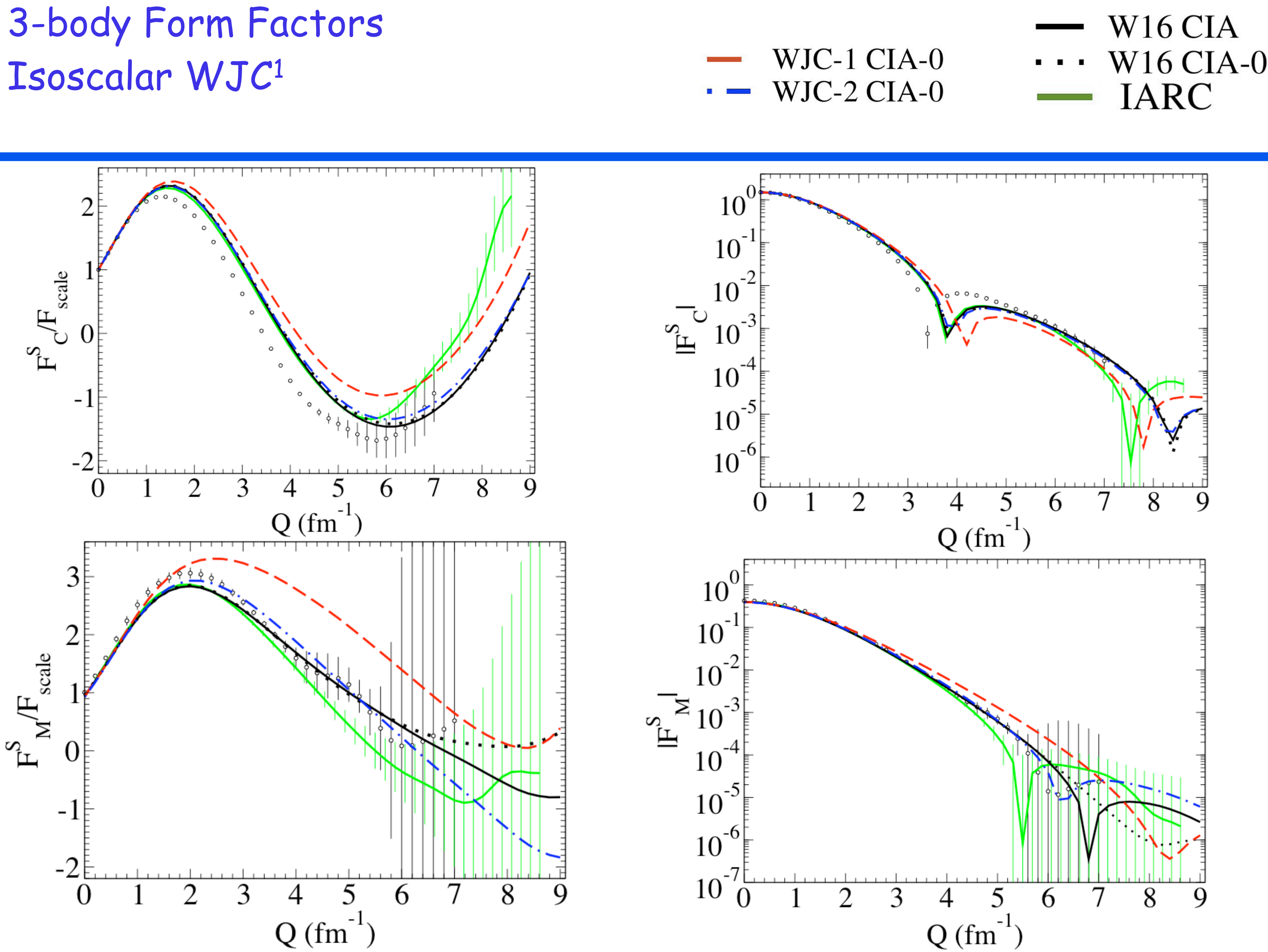


[1]Sergio A. Pinto, A. Stadler, & FG, Phys. Rev. C **79**: 054006 (2009)

3-body Form Factors
Isovector WXX

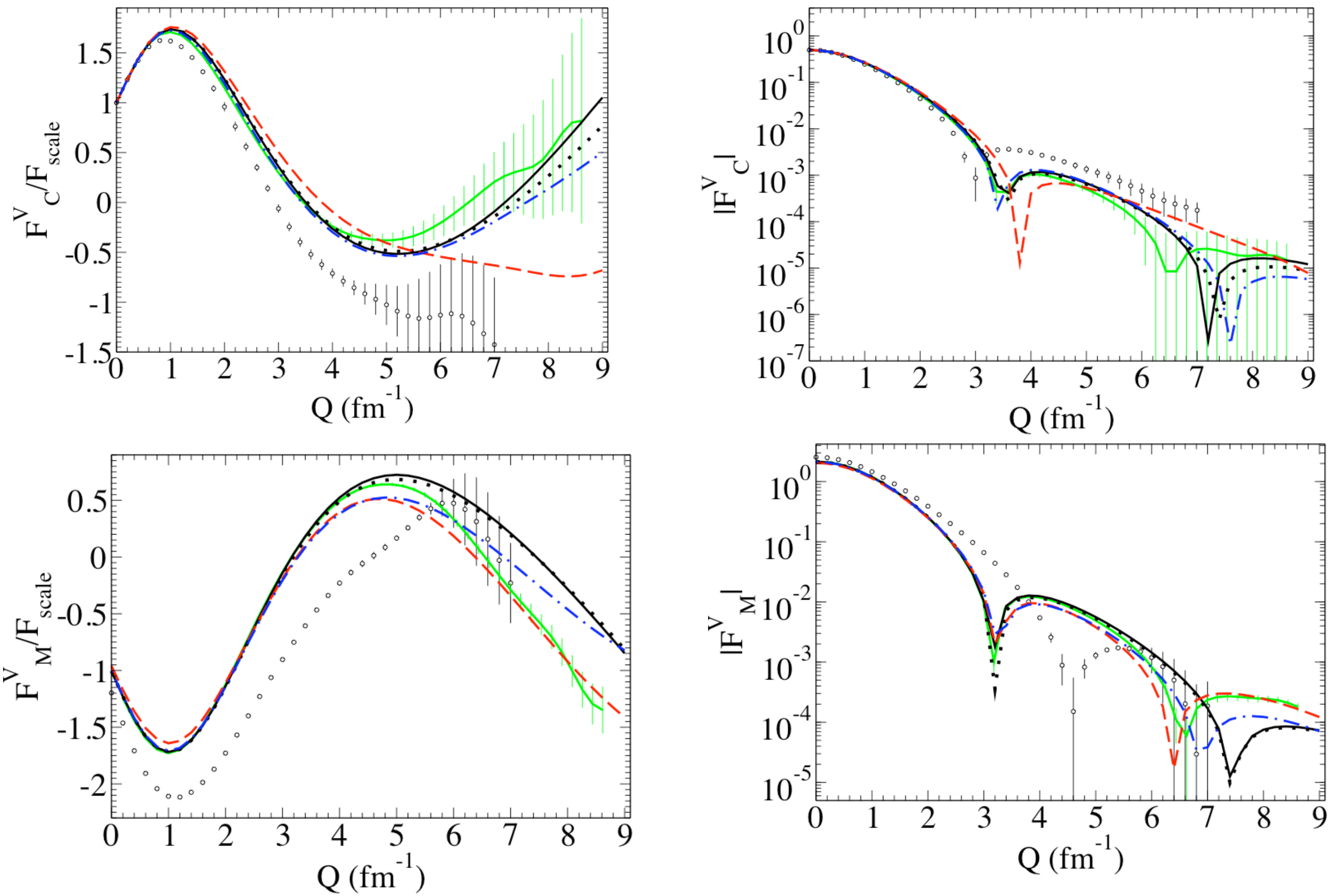


3-body Form Factors
Isoscalar WJC[1]



[2]Sergio A. Pinto, A. Stadler, & FG, Phys. Rev. C **81**: 014007 (2010)

3-body Form Factors
Isovector WJC



Conclusions from the three-body form factors

- Results for W16 and both WJC models are very reasonable
  - In remarkable agreement with IARC despite very different framework and dynamical input
  - IARC: Impulse Approximation with Relativisitc Corrections using AV18 and UIX 3-N force (Marcucci, et. al.)
- We do not expect to fit data
  - No interaction currents -- known to be large for the isovector terms
    - Isoscalar terms in fair agreement
  - Same wave functions for ^3H and ^3He (no Coulomb)
    - but figures compare to a special IARC with no Coulomb (prepared by Laura Marcucci
- Binding energy $E_t \sim 1/\langle r^2 \rangle$ determines the form factors to relatively large Q
- WJC-1 shows the largest differences with the others
  - probably due to admixture of pion γ^5 coupling with large pair terms
- REMEMBER: These currents are gauge invariant, EVEN THOUGH STRONG FORM FACTORS ARE PRESENT!

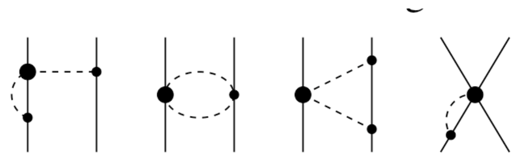
Outlook and conclusions

Future

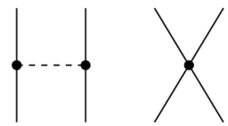
-- What it the connection to χEFT?

Potentials to N^{4}LO

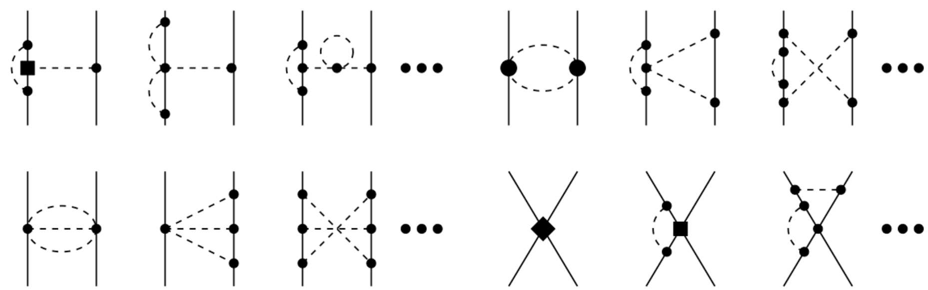
N^{3}LO



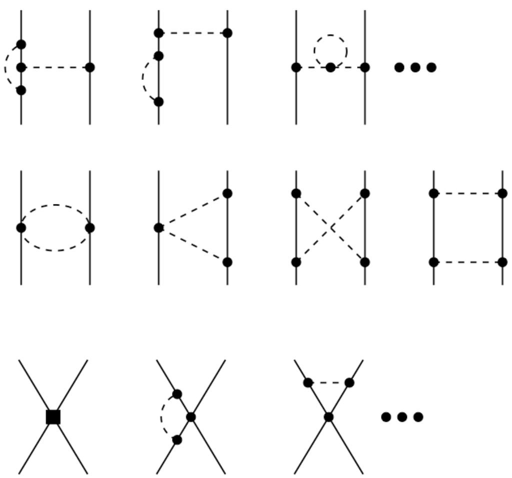
LO

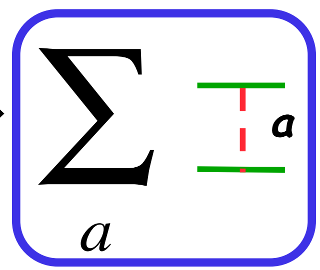


N^{4}LO



N^{2}LO



$\Rightarrow$ 

this whole sum is approximated By the OBE model

***E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Reviews of Modern Physics, arXiv:0811.1338 [nucl-th]**

Future

-- Extracting a potential from the CST equations

★ CST equations have additional channels for the negative energy components of the wave functions:

$$(W-2E_p)\psi^+(p)=\int\frac{d^3k}{(2\pi)^3 2E_k}\left\{V_{++}(p,k;W)\psi^+(k)+V_{+-}(p,k;W)\psi^-(k)\right\}$$

$$W\psi^-(p)=\int\frac{d^3k}{(2\pi)^3 2E_k}\left\{V_{-+}(p,k;W)\psi^+(k)+\boxed{V_{--}(p,k;W)}\psi^-(k)\right\}$$

★ Replacing $V_{--}(p,k;W)\to(2\pi)^3 2E_k\lambda\delta^3(p-k)$ gives an exact one-channel equation

$$\boxed{(W-2E_p)\psi^+(p)=\int\frac{d^3k}{(2\pi)^3 2E_k}V_{eff}(p,k;W)\psi^+(k)}$$

with an effective non-local, energy dependent potential

$$V_{eff}(p,k;P)=\int\frac{d^3k'}{(2\pi)^3 2E_{k'}}\frac{V_{+-}(p,k';W)V_{-+}(k',k;W)}{W-\lambda}$$

★ We are currently searching for the 16 possible parameters λ which best fit the data, and a mean energy W* to remove the energy dependence.

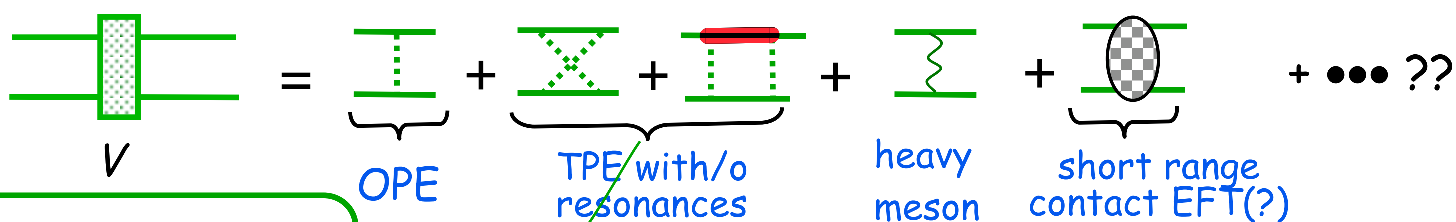
★ The resulting equation with a non-local potential can be used in Schrödinger theory.

Future

-- Where are the quarks?

- ★ In this picture the quarks are "frozen" out, and do not need to be included explicitly until momenta >> X GeV (where X=??)
- ★ Still, quarks might "explain," through duality, the relative strength of meson exchange models:

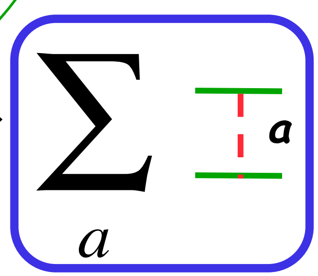
The "exact kernel is a complicated sum of many contributions



these can be replaced by I = 0 and I = 1 scalar exchanges (σ and δ)*

*Riska and Brown, NP **A153**, 8 (1970)
*Peña, Gross, and Surya, PRC, **54**, 2235 (1996)

⇒



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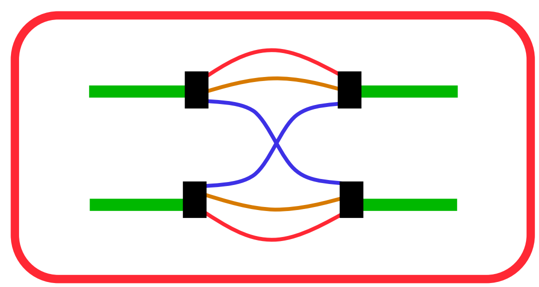


this whole sum is approximated By the OBE model

OBE model might approximate quark exchange

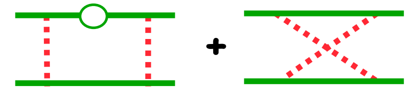
Conclusions

- We have found two precision covariant models of the *NN* kernel based on OBE (WJC-1 and WJC-2)
  - Comparatively simple with only a few parameters
    - WJC-2 has 15 compared to the 24 of χEFT
  - EXCELLENT description of the low energy *NN* data
    - $\chi^2/N \sim 1$ for the 2007 data base (most complete data base)
  - New phase shift analysis
- OBE mechanism works VERY WELL, and (NEW) *gives* precision fits to data
  - probably because the relativistic (Dirac) structure of the equation is efficient
- ALL Poincaré transformations are kinematic -- i.e. exact
  - Form factor calculations include recoil corrections to ALL orders
  - Can be expected to work at high Q^2
- Three body forces are incorporated as off-shell effects arising from two body interactions
  - NO three body forces are needed!
  - Robust role of off-shell couplings to be tested and understood
- These models can be used for precision calculations of few body interactions
- The kernel provides a “bridge” between hadronic physics and QCD

END

Future
-- Extensions of the cancellation theorem

- Has been proved only for scalar theories and QED

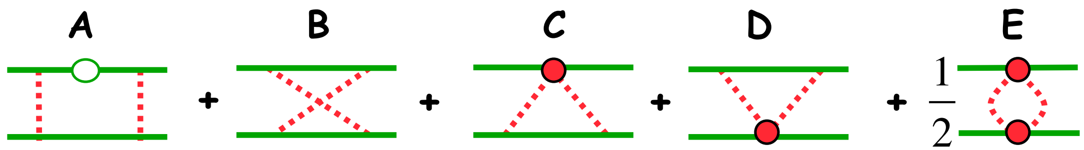


$$-\frac{N}{m_X^2-t}+\frac{N}{m_X^2-t}\cong 0$$

- For pion exchanges with chiral symmetry treated as in the sigma model (i.e. γ^5 coupling with sigma type contact term δ^{ij}/m required by chiral symmetry, and use

$$\bar u\left[\gamma^5\frac{1}{(m-\not p)}\gamma^5\right]u=\bar u\gamma^5\left(\frac{m}{E_p}\right)\left\{\frac{u\bar u}{E_p-p_0}-\frac{v\bar v}{E_p+p_0}\right\}\gamma^5 u\Rightarrow-\frac{1}{2m}=-\eta$$

The 4th order kernel becomes



$$-\frac{\eta^2 N}{m_X^2-t}\left\{\overset{A}{\tau_1^k\tau_1^m}\left(\tau_2^k\tau_2^m+\overset{B}{\tau_2^m\tau_2^k}\right)\overset{C}{-6}\overset{D}{-6}+\overset{E}{\tfrac{1}{2}(12)}\right\}\Rightarrow 0$$

- This can be generalized: see study of the large N_c limits by T. Cohen. et.al.*

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*see, for example, Phys.Rev.C65:064008,2002.