Computational Group Cohomology

Bangalore, November 2016

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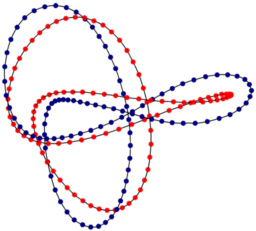
NUI Galway, Ireland

Slides available at **http://hamilton.nuigalway.ie/Bangalore**

Password: **Limerick**

Outline

$L =$  $G = \pi_1(\mathbb{R}^3 \setminus L)$ has presentation

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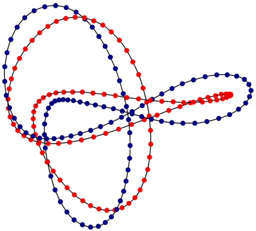
$L =$  $G = \pi_1(\mathbb{R}^3 \setminus L)$ has presentation

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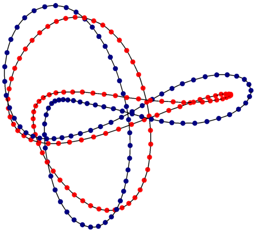
$L =$  $G = \pi_1(\mathbb{R}^3 \setminus L)$ has presentation

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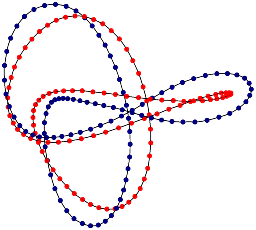
$L =$  $G = \pi_1(\mathbb{R}^3 \setminus L)$ has presentation

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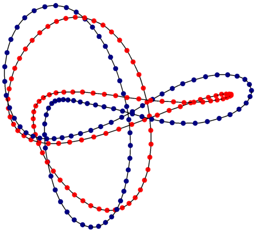
$L =$  $G = \pi_1(\mathbb{R}^3 \setminus L)$ has presentation

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Hurewicz: $X = BG \iff \tilde{X}$ contractible $\iff \pi_2(X) = 0$

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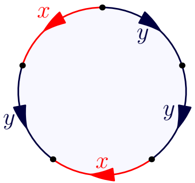

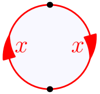

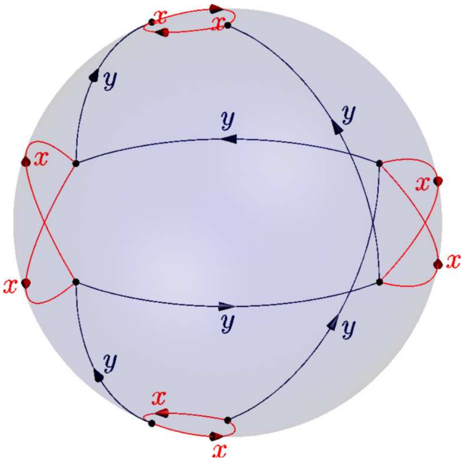

Star graph of $\mathcal{P} = \langle \underline{x} \mid \underline{r} \rangle$

Assume $x \in \underline{x} \Rightarrow x^{-1} \in \underline{x}$

- One vertex for each generator x
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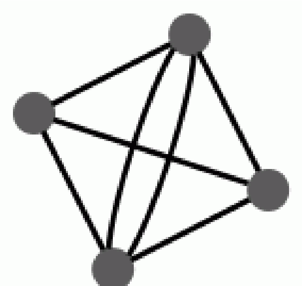

Labelled Star graph of $\mathcal{P} = \langle \underline{x} \mid \underline{r} \rangle$

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- corners assigned numerical angles: for each n-corner relator

$$\textbf{relator angle sum} \ = (n-2)\pi$$

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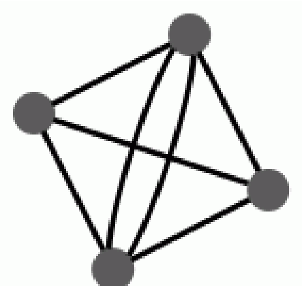

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If each loop in a labelled star graph of $\mathcal{P}$ satisfies

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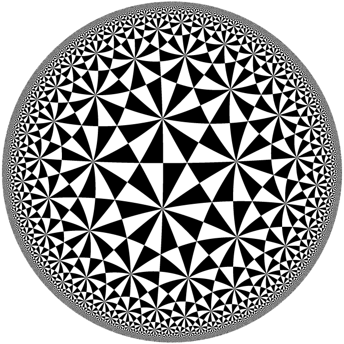
act on

$$X = \begin{cases} \mathbb{H}^2 & \text{(hyperbolic plane)}, \quad \text{if } 1/l + 1/m + 1/n < 1 \\ \mathbb{R}^2 & \text{(real plane)}, \quad \text{if } 1/l + 1/m + 1/n = 1 \\ \mathbb{S}^2 & (2-\text{sphere}), \quad \text{if } 1/l + 1/m + 1/n > 1 \end{cases}$$

with finite cyclic stabilizers.

T(2,3,7) and T(2,3,5)



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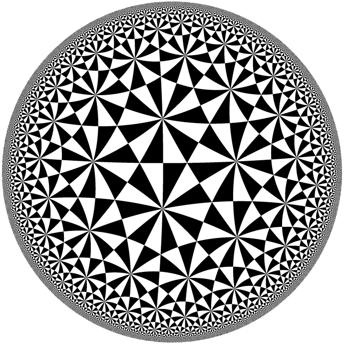

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For $X = \mathbb{R}^2$ and $X = \mathbb{H}^2$

$$R_*^T : \cdots \to (\mathbb{Z}T)^3 \to (\mathbb{Z}T)^3 \to (\mathbb{Z}T)^3 \to (\mathbb{Z}T)^4 \to (\mathbb{Z}T)^5 \to (\mathbb{Z}T)^3$$

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(first studied by Coxeter, Sinkov)

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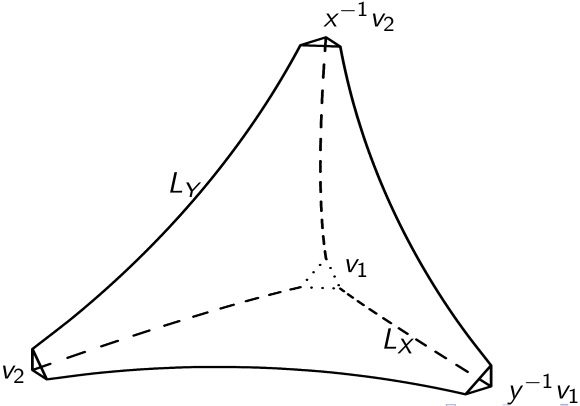

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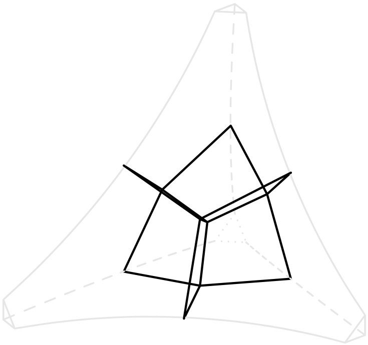

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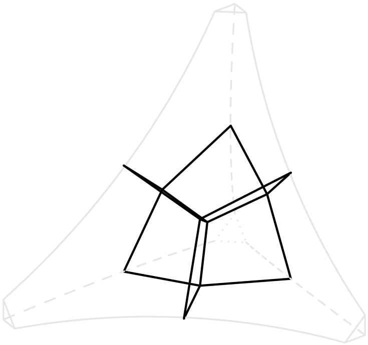

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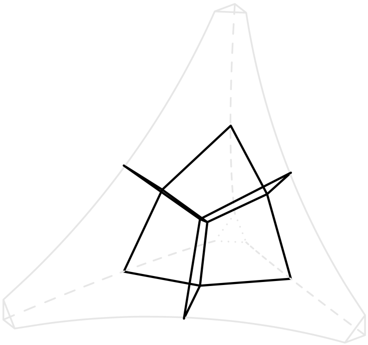

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$$C_*(X) : 0 \to (\mathbb{Z}_G \otimes_{\langle x \rangle} \mathbb{Z}) \oplus (\mathbb{Z}_G \otimes_{\langle y \rangle} \mathbb{Z}) \oplus (\mathbb{Z}_G \otimes_{\langle [x,y] \rangle} \mathbb{Z}) \to (\mathbb{Z}G)^2 \to \mathbb{Z}G$$

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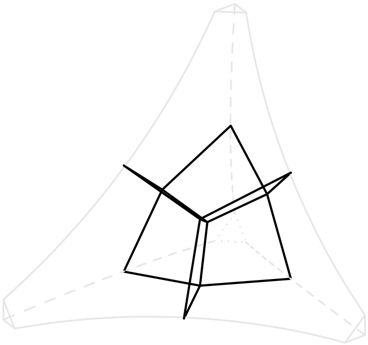

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$$R_*^G : \xrightarrow{\beta} (\mathbb{Z}G)^3 \xrightarrow{\alpha} (\mathbb{Z}G)^3 \xrightarrow{\beta} (\mathbb{Z}G)^3 \xrightarrow{\alpha} (\mathbb{Z}G)^3 \to (\mathbb{Z}G)^3 \to (\mathbb{Z}G)^2 \to \mathbb{Z}G$$

Contraction of X= $\mathbb{R}^n$ or $\mathbb{H}^n$

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Theorem (Gromov et al.)

A **piecewise euclidean/hyperbolic** CW complex X is a **unique geodesic space** if $\pi_1 X = 0$ and the **link** of each cell is a **piecewise spherical** CW complex with no geodesic loop of length less than 2π.

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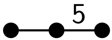
Example

For the cubical tesselation of $\mathbb{R}^3$, the link of a point x in an edge e^1 is a sphere. The link of e^1 is a circle.

Coxeter Matrix, Graph and Group

$$(m_{ij}) = \begin{pmatrix} 1 & 3 & 2 \\ 3 & 1 & 5 \\ 2 & 5 & 1 \end{pmatrix}$$



$$G = \langle\ \sigma_1, \sigma_2, \sigma_3\ \rangle$$

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$$\langle e_i, e_j\rangle = -\cos(\pi/m_{ij})$$

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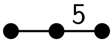

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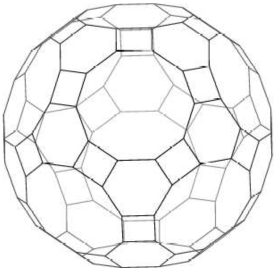
Convex hull of v^G



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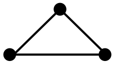

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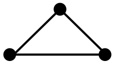

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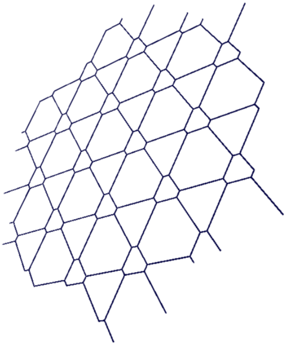

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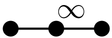

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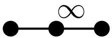

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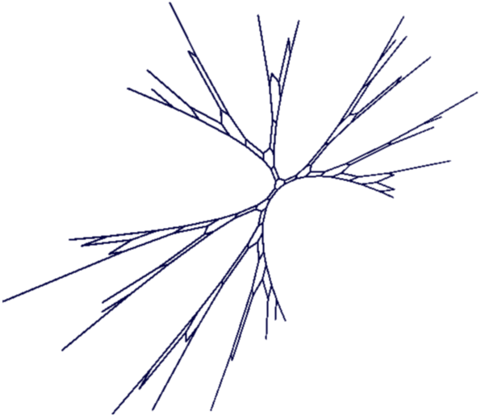

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$$\mathrm{Link}(1, X) \cong \mathrm{Simplicial\ complex}\ \{T \subset S \ : \ 1 \neq |\langle T \rangle| < \infty\}$$

Theorem (Davis '93)

The Coxeter complex X is contractible and admits an action of G with all stabilizers finite Coxeter groups.

Theorem (De Concini & Salvetti, '00)

There is a free $\mathbb{Z}G$-resoluton R_*^G whose generators in degree k correspond to sub k-multisets of S that generate a finite group.

Finite Coxeter Groups

$S = \{\sigma_1, \ldots, \sigma_n\}$, $G = \langle S \rangle$
$v \in \mathbb{R}^n$, $t(v) = \{\sigma \in S \;:\; \sigma(v) \neq v\}$

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Combinatorial description of $P(G, v)$ available:

- faces e^k of
$$P(G, v) = \mathrm{Convex\ Hull}(v^G)$$
indexed be their centres $c(e^k)$;
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$P(M_{24}, v) = P(S_{24}, v)$ for $v = (1, 2, 3, 4, 5, 0, \ldots, 0) \in \mathbb{R}^{24}$

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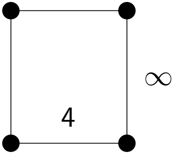
There is a free $\mathbb{Z}M_{24}$-resolution $R_*^{M_{24}}$ with $1, 9, 50, 204, 649, \ldots$ free generators in degrees $0, 1, 2, 3, 4, \ldots$

Artin groups

To any Coxeter diagram

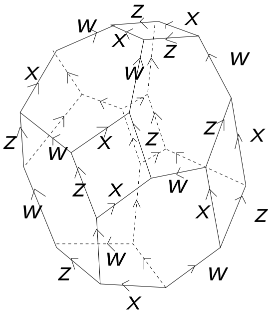
$D =$

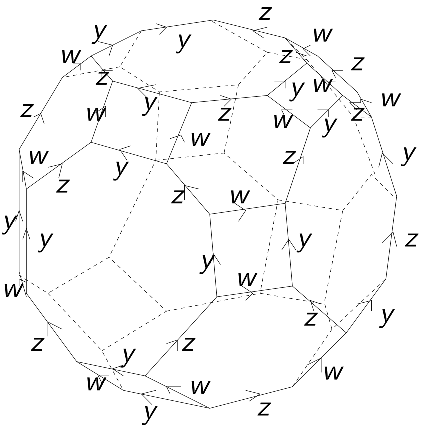


is associated a Coxeter group W_D and Artin group

$$A_D = \langle w, x, y, z : \quad wxw = xwx, wy = yw, wzw = zwz, \\ xz = zx, yzyz = zyzy \rangle$$

Associated to D is a finite CW space B_D.





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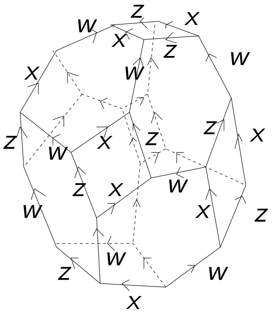

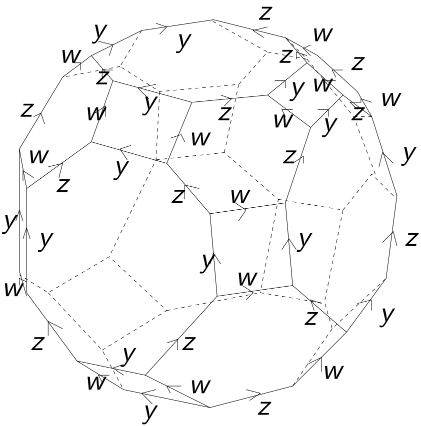

The $\mathbf{K(\pi, 1)}$ Conjecture: $\widetilde{B_D}$ is contractible.

The conjecture is true for many D. [Deligne '72, Appel & Schupp '83, Charney & Davis '95, ...]

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Theorem:(E, Sköldberg '10)
An Artin group A_D satisfies the conjecture if $A_{D'}$ satisfies conjecture for every ∞-free full subgraph D' in D.

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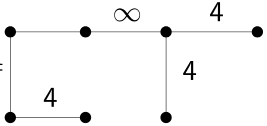
$D =$ 

$$
\begin{array}{ll}
H^0(A_D,\mathbb{Z}) \cong \mathbb{Z}, & H^1(A_D,\mathbb{Z}) \cong \mathbb{Z}^5, \\
H^2(A_D,\mathbb{Z}) \cong \mathbb{Z}^{11}, & H^3(A_D,\mathbb{Z}) \cong \mathbb{Z}_2 \oplus \mathbb{Z}^{14}, \\
H^4(A_D,\mathbb{Z}) \cong \mathbb{Z}_2^2 \oplus \mathbb{Z}^{12}, & H^5(A_D,\mathbb{Z}) \cong \mathbb{Z}_2 \oplus \mathbb{Z}^6, \\
H^6(A_D,\mathbb{Z}) \cong \mathbb{Z}, & H^n(A_D,\mathbb{Z}) = 0 \ (n \geq 7).
\end{array}
$$

bahut dhanyavaad