COMPUTATIONAL REPRESENTATION THEORY – LECTURE I

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Group Theory and Computational Methods
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CONTENTS

Notation

Throughout this lecture, G denotes a finite group and F a field.

Representations: Definitions

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$$\mathfrak{X} : G \to \mathrm{GL}(V),$$

where V is a d-dimensional F-vector space. (This is also called a representation of G on V.)

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To accord with GAP, we let $\mathrm{GL}(V)$ act from the right on V.

For computations one chooses a basis of V and obtains a matrix representation $G \to \mathrm{GL}_d(F)$.

Irreducible Representations

$\mathfrak{X} : G \to \mathrm{GL}(V)$ is reducible, if either $V = \{0\}$, or if there exists a subspace $W < V$, $0 \neq W \neq V$, s.t. $w\,\mathfrak{X}(g) \in W$ for all $w \in W$ and $g \in G$. (W is G-invariant.)

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(W.r.t. suitable bases of V and W, the matrices for $\mathfrak{X}(g)$ and $\mathfrak{Y}(g)$ are simultaneously similar.)

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4. Use a computer for sporadic simple groups.

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An F-character of G is the character of some F-representation.

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4. *Every character is a sum of irreducible characters.*
5. *Two **irreducible** representations of G are equivalent, if and only if their characters are equal.*
6. *Suppose that $\mathrm{char}(F) = 0$. Then **any** two representations of G are equivalent, if and only if their characters are equal.*

The Ordinary Character Table

From now on let $F = \mathbb{C}$.

Put $\mathrm{Irr}(G) :=$ set of irreducible $\mathbb{C}$-characters of G,
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Let $g_1, \ldots, g_k$ be representatives of the conjugacy classes of G (same k as above!).

The square matrix

$$\left[\chi_i(g_j)\right]_{1 \leq i,j \leq k}$$

is called the ordinary character table of G.

Example: Alternating Group A_5

Example (Character table of A_5)

	$1a$	$2a$	$3a$	$5a$	$5b$
χ_1	1	1	1	1	1
χ_2	3	-1	0	A	$*A$
χ_3	3	-1	0	$*A$	A
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$$1 \in 1a, \quad (1,2)(3,4) \in 2a, \quad (1,2,3) \in 3a,$$

$$(1,2,3,4,5) \in 5a, \quad (1,3,5,2,4) \in 5b$$

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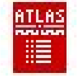

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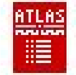

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The character tables of the ATLAS are also contained in GAP.

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3. computed from a generic character table.

The Orthogonality Relations

Let $\mathcal{C}(G)$ denote the set of $\mathbb{C}$-valued class functions on G, and $\mathbb{Z}[\mathrm{Irr}(G)] := \{\sum_{i=1}^{k} z_i\chi_i \mid z_i \in \mathbb{Z} \text{ for all } i\} \subseteq \mathcal{C}(G)$.

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4. *Suppose* $\alpha \in \mathbb{Z}[\mathrm{Irr}(G)]$.
   Then $\alpha \in \mathrm{Irr}(G)$ *if and only if* $\langle \alpha, \alpha \rangle = 1$ *and* $\alpha(1) > 0$.

Class Multiplication Coefficients

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Define the class multiplication coefficients c_{ijl} ($1 \leq i, j, l \leq k$) by

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Theorem (Burnside)

The ordinary character table of G can be computed from

1. *The common column eigenvectors of $M_1, \ldots, M_k$, or*
2. *the common row eigenvectors of $M_1, \ldots, M_k$, or*
3. *the corresponding eigenvalues.*

The Burnside-Dixon-Schneider Algorithm

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Let $\chi \in \mathrm{Irr}(G)$. Then for all $1 \leq i \leq k$, there are $\omega_{\chi,i} \in \mathbb{C}$ such that

$$\omega_{\chi,i}[\chi(g_1), \ldots, \chi(g_k)] = [\chi(g_1), \ldots, \chi(g_k)]M_i.$$

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Computations are done in a finite field and lifted back to $\mathbb{C}$.

The Burnside-Dixon-Schneider Algorithm

Let $C_1, \ldots, C_k$ be the conjugacy classes of G, $g_i \in C_i$, $i = 1, \ldots, k$, $g_1 = 1$.

Let $\chi \in \mathrm{Irr}(G)$. Then for all $1 \leq i \leq k$, there are $\omega_{\chi,i} \in \mathbb{C}$ such that

$$\omega_{\chi,i}[\chi(g_1), \ldots, \chi(g_k)] = [\chi(g_1), \ldots, \chi(g_k)]M_i.$$

Algorithm (Burnside-Dixon-Schneider)

1. *Compute the matrices M_i, $1 \leq i \leq k$.*
2. *Find the common row eigenvectors $\chi'_1, \ldots, \chi'_k$ of these.*
3. $\chi_i = c_i\chi'_i$ *and* $\langle \chi'_i, \chi'_i \rangle = 1/c_i^2 \rightsquigarrow \chi_i$.

Computations are done in a finite field and lifted back to $\mathbb{C}$.

Usually, not all of the matrices M_i have to be computed.

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Product. Let χ, ψ be characters of G. Then the product $\chi \cdot \psi$, defined by

$$[\chi \cdot \psi](g) := \chi(g)\,\psi(g), \quad g \in G$$

is a character as well (proof later).

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Symmetrisation. Let χ be a character of G. Then $S^2(\chi)$ and $\Lambda^2(\chi)$ defined by

$$S^2(\chi)(g) = \frac{1}{2}\left(\chi(g)^2 + \chi(g^2)\right), \qquad \Lambda^2(\chi)(g) = \frac{1}{2}\left(\chi(g)^2 - \chi(g^2)\right)$$

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Restriction. Let $H \leq G$ and χ a character of G. Then the restriction χ_H of χ to H is a character of H.

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ψ^G is called an induced character.

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Theorem (Brauer's Induction Theorem)

$\mathbb{Z}[\mathrm{Irr}(G)] = \sum_{E\in\mathcal{E}} \mathrm{Ind}_E^G(\mathbb{Z}[\mathrm{Irr}(E)])$.

Generate and Split: Overview

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6. Recently, Breuer, Malle and O'Brien have recomputed the character tables of the sporadic groups (except for B and M) using Unger's algorithm.

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The Generic Character Table for $SL_2(q)$, q Even

	C_1	C_2	$C_3(a)$	$C_4(b)$
χ_1	1	1	1	1
χ_2	q	0	1	-1
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$\begin{bmatrix} \mu^a & 0 \\ 0 & \mu^{-a} \end{bmatrix} \in C_3(a)$ ($\mu \in \mathbb{F}_q$ a primitive $(q-1)$th root of 1)

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Specialising q to 4, gives the character table of $\mathrm{SL}_2(4) \cong A_5$.

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Thank you for your attention!