Planetary dynamos below stably stratified layers

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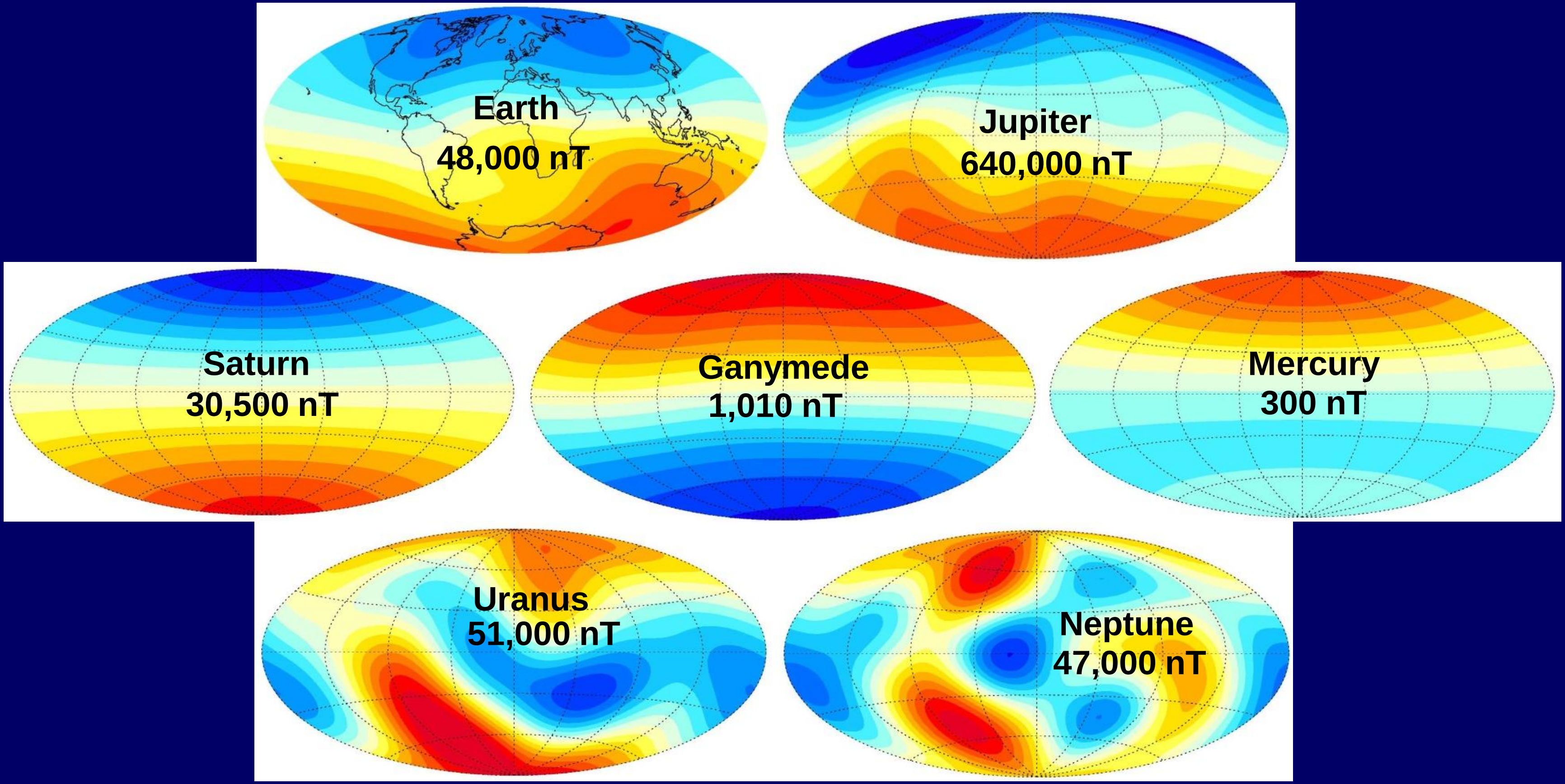
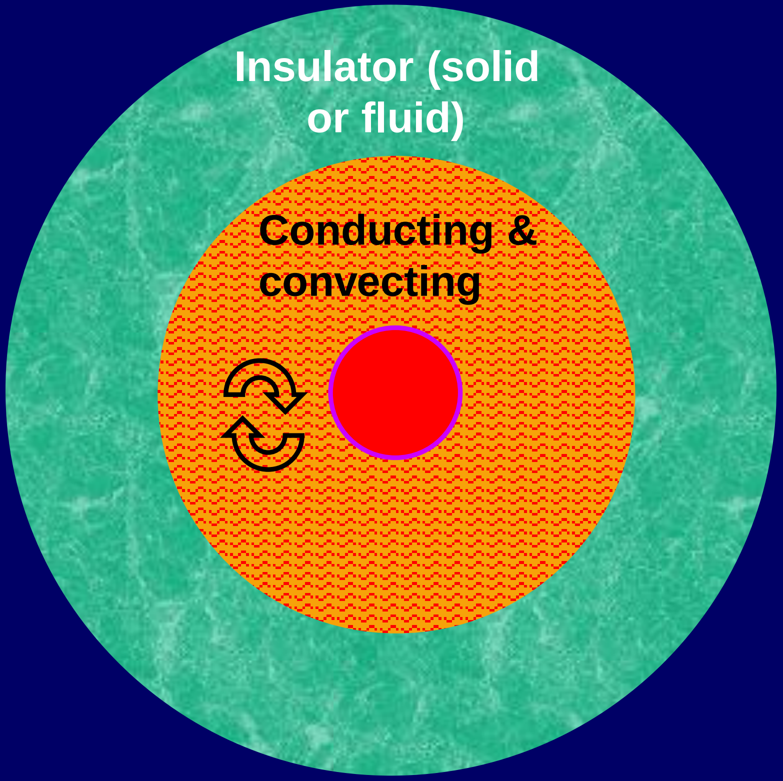
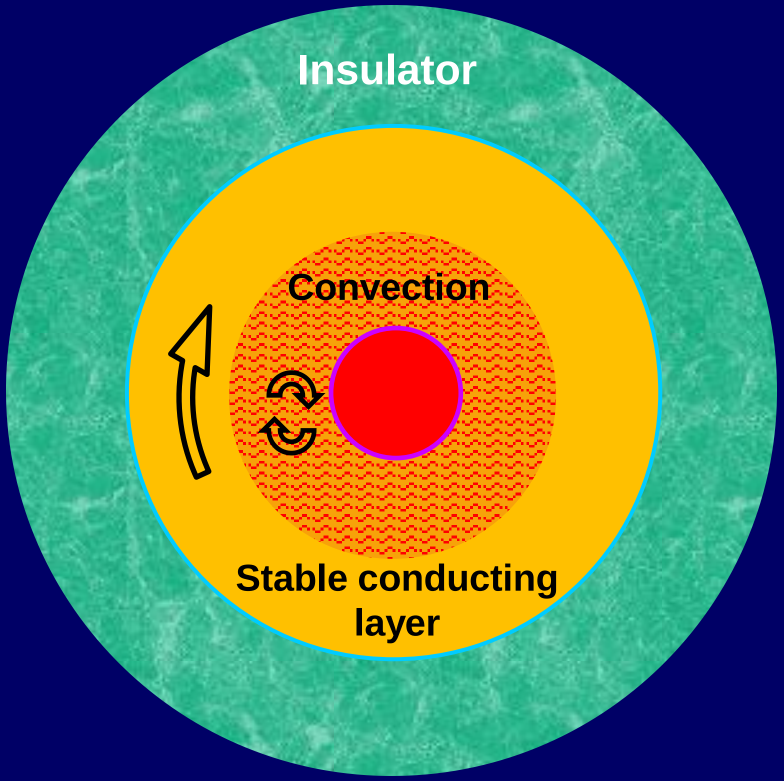


Standard model

Stable layer





Causes for stable layer

Sub-adiabatic T-gradient in outer parts of core

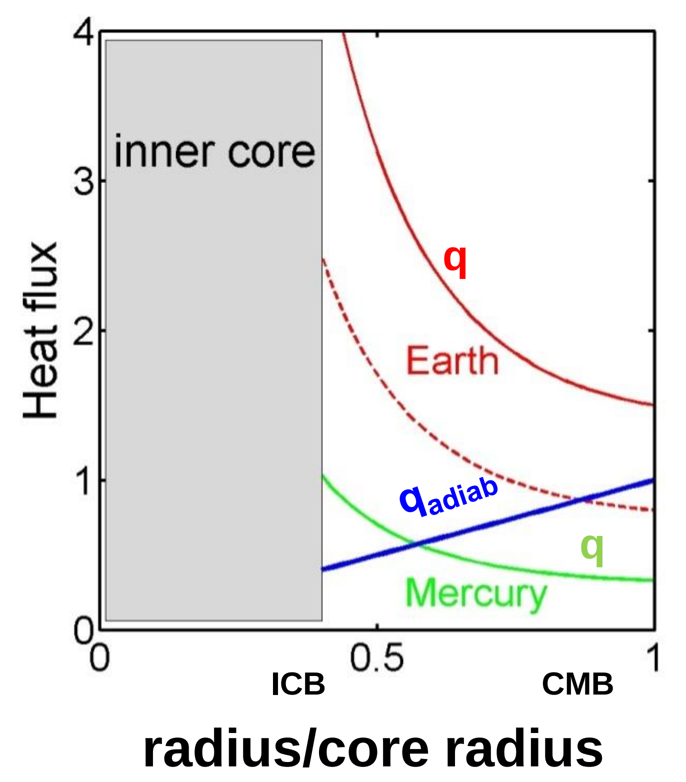
Compositional stratification (e.g. associated with phase separation)

Thermal convection in cores of terrestrial planets

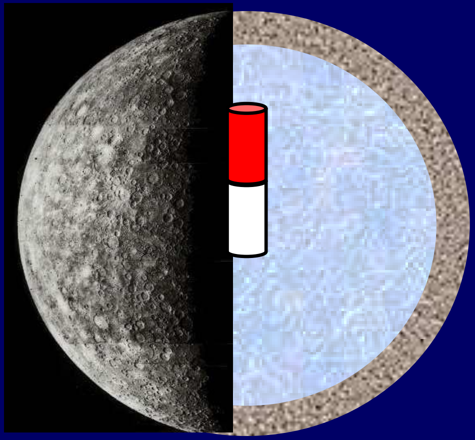
- **Large heat flux qadiab can be conducted along adiabatic T-gradient**
- **Main heat source latent heat of freezing of solid inner core:**

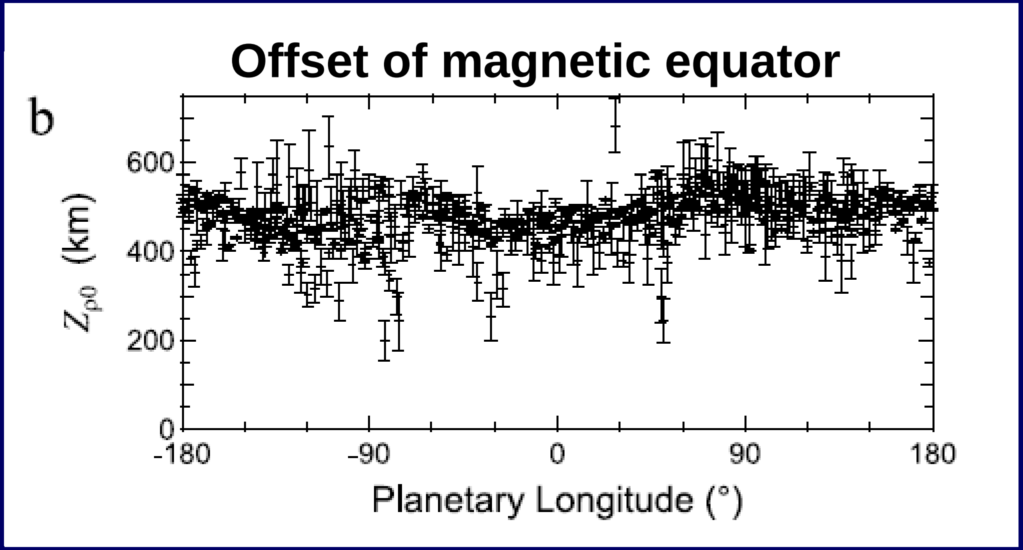
  $q \propto 1/r^2$

  $q_{adiab} \propto r$
- **Possible scenario: thermally unstable at depth, stable near core-mantle boundary**



Mercury‘s magnetic field

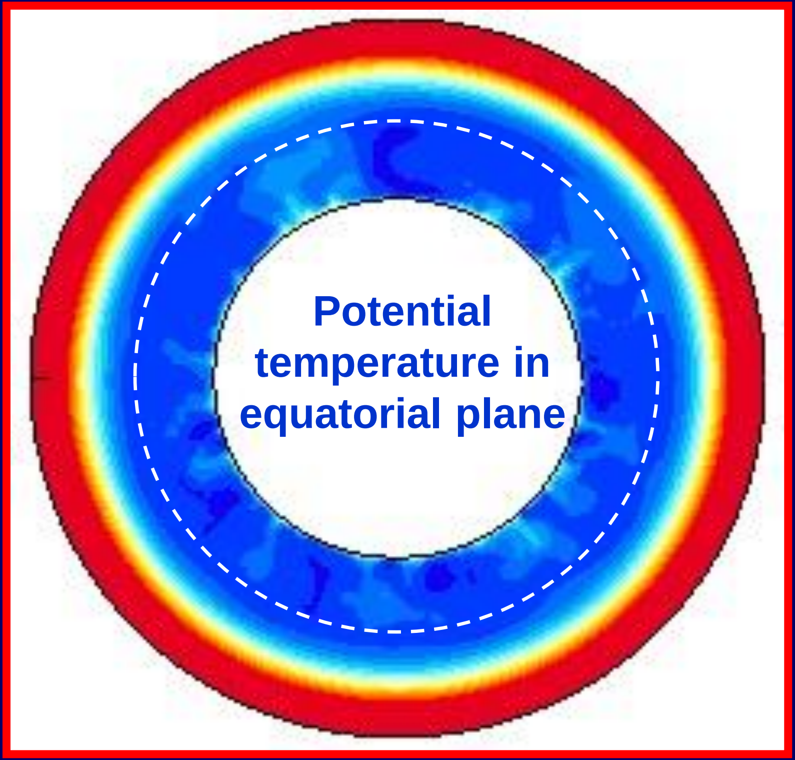


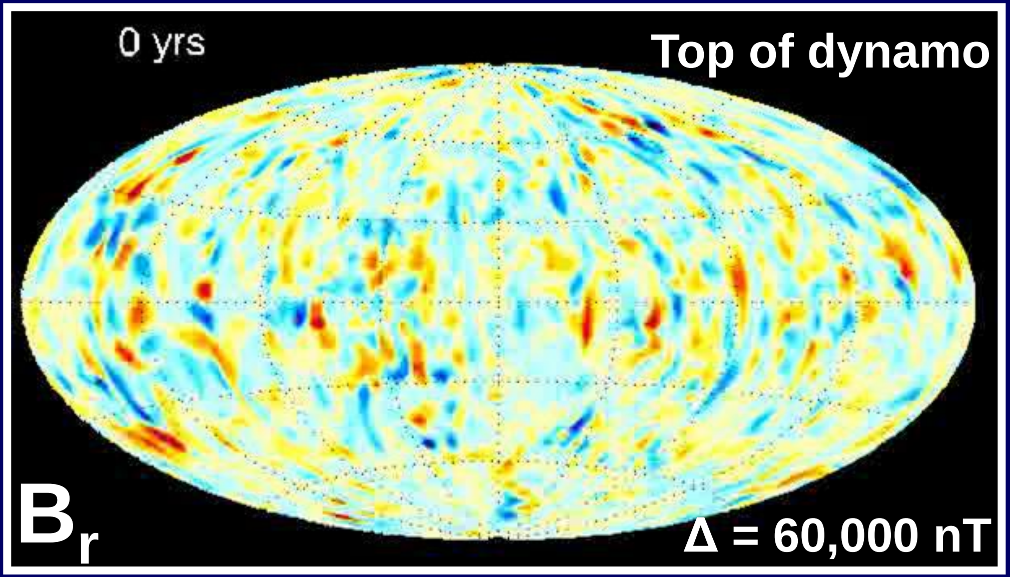

Offset of magnetic equator

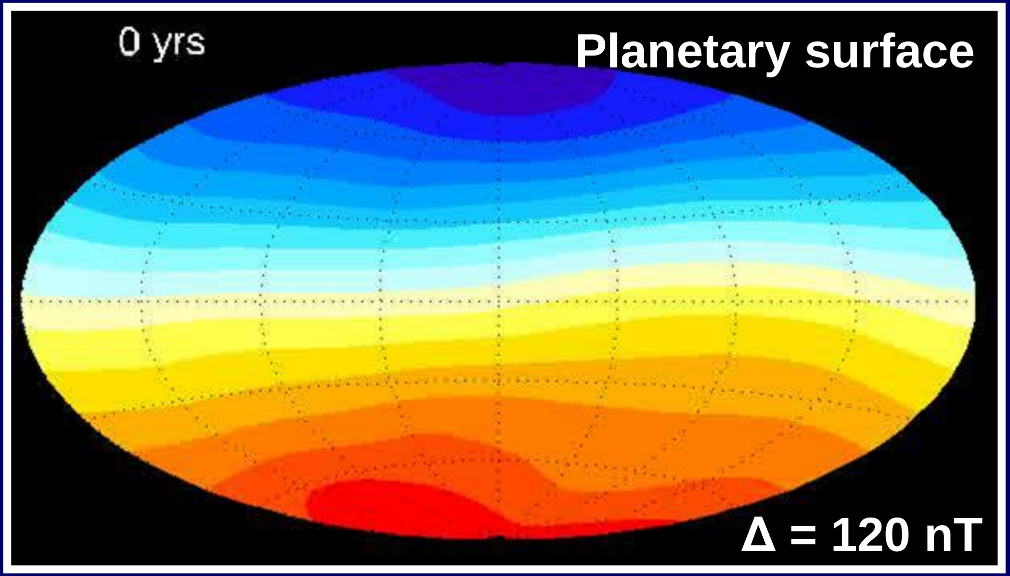
- **Mercury is slow rotator (59 days)**
- **Field dipole-dominated but weak (g_{10}=190 nT)**
- **Dipole tilt wrt rotation axis small (< 0.8°)**
- **Relatively large axial quadrupole (g_{20}/g_{10} = 0.39)**

(Anderson et al., 2011, 2012)

Mercury dynamo below stable layer



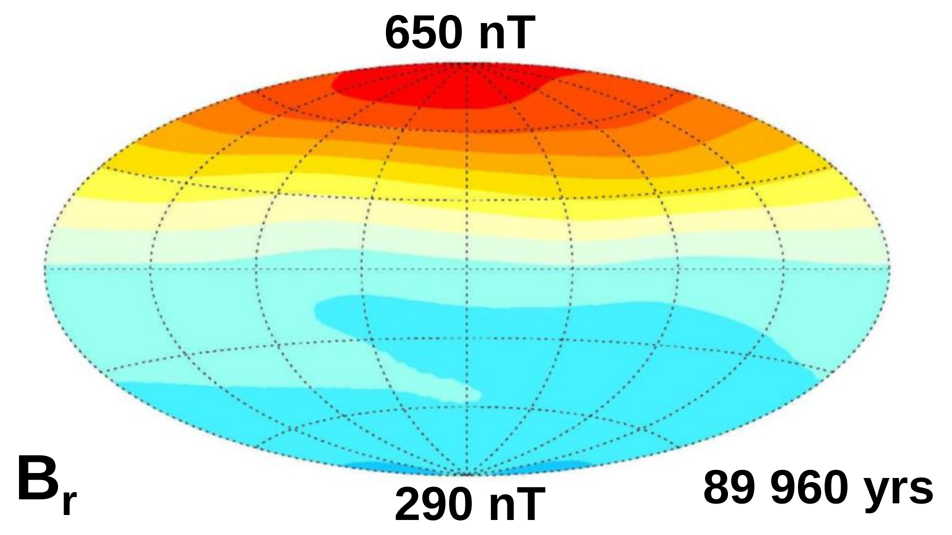


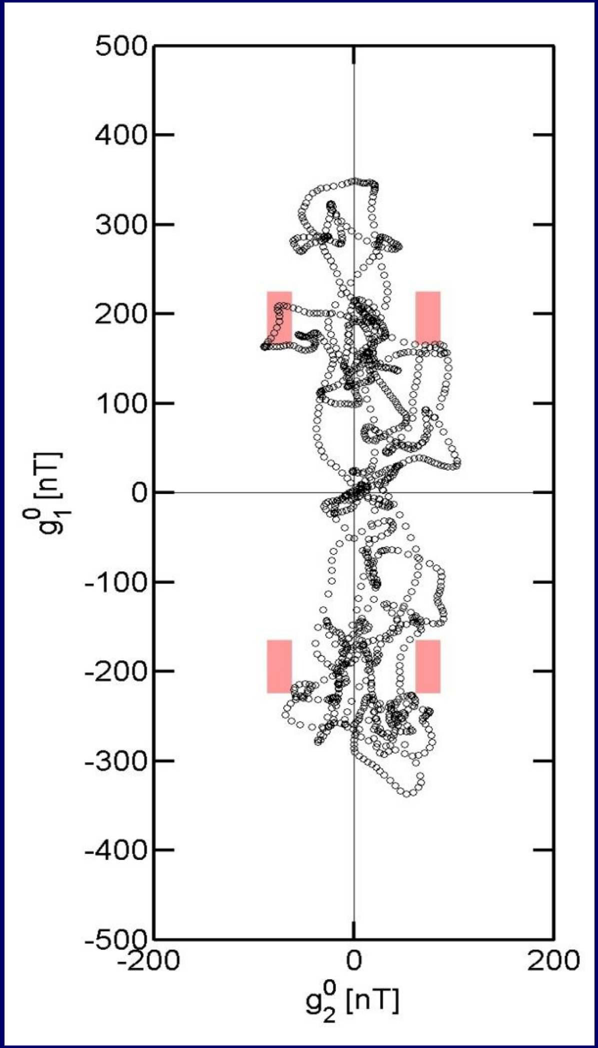


- **Internal field strong, small-scale**
- **Surface field weak, large-scale**

Christensen, 2006
Christensen & Wicht, 2008

Quadrupole / dipole ratio

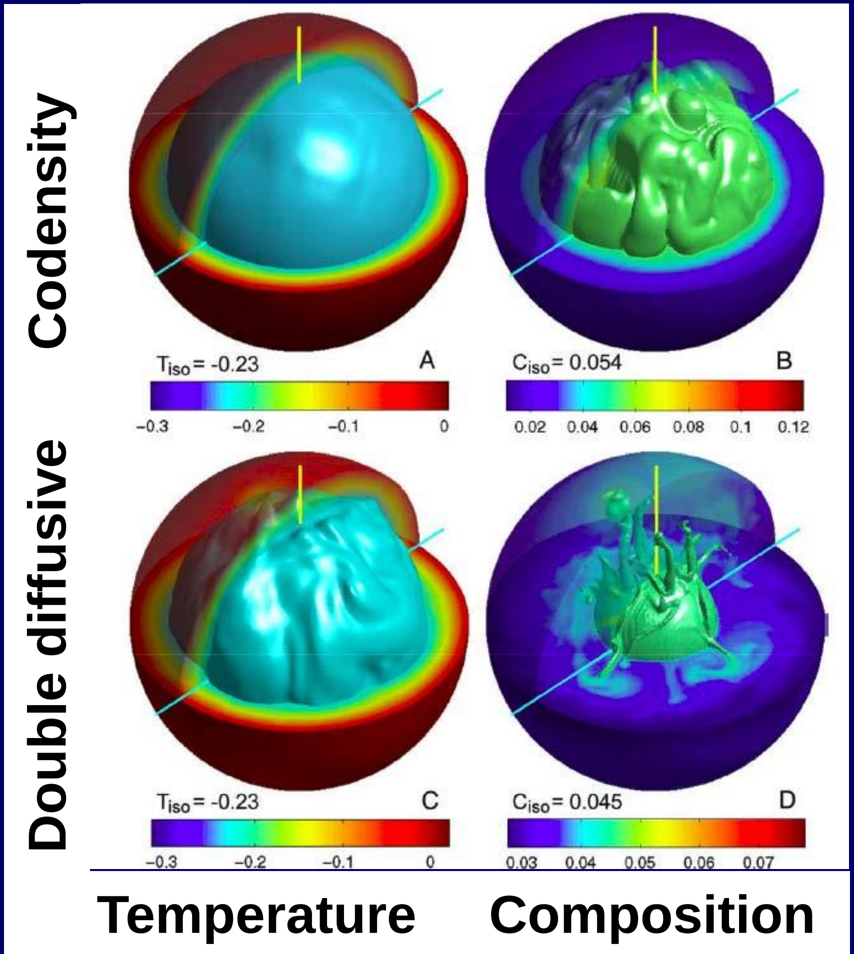




Snapshot fitting the present-day field of Mercury in terms of field strength and g_{20}/g_{10} ratio

One case re-analysed from Christensen & Wicht, 2008

Mercury: double diffusive model



IC growth ⇒ light element flux
Simple model: same diffusivity for heat and concentration: codensity

but

Compos diffusivity << Thermal diff
⇒ Double diffusive convection

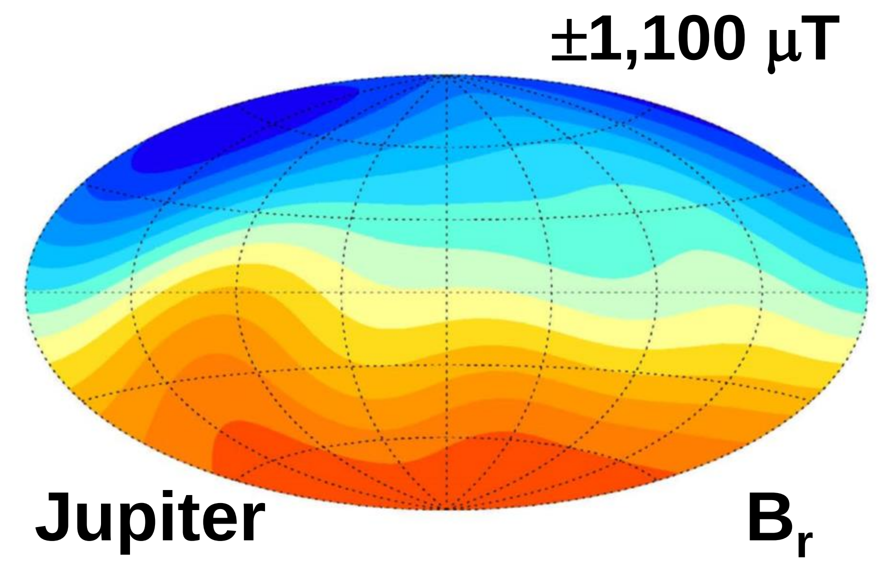
With DDC, convective fingers penetrate into stable layer and cause magnetic induction.

Outside magnetic field too strong compared to observation.

Compatible with observation when < 0.3 % sulphur in core

Manglik, Wicht & Christensen, 2010

Saturn and Jupiter: Unlike siblings



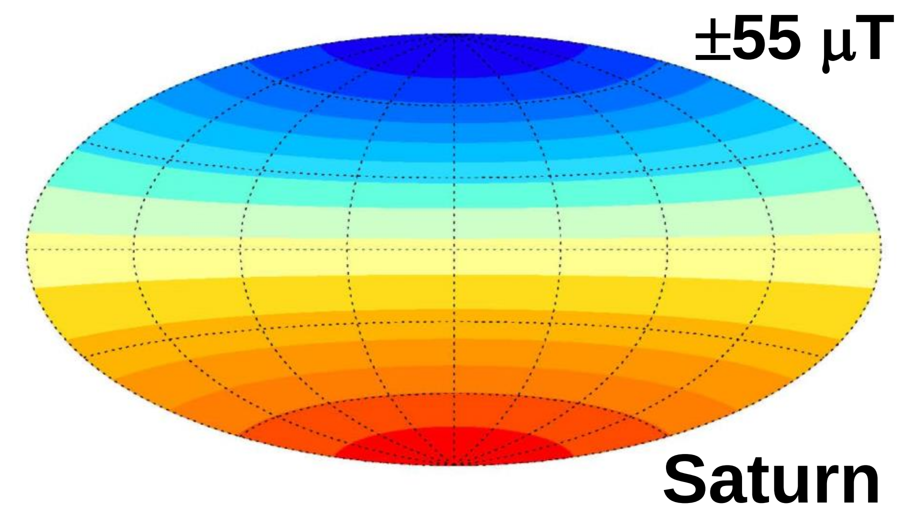
Jupiter's field 15xEarth's

Similar proportion of non-zonal to zonal field as Earth. Dipole tilt 10°

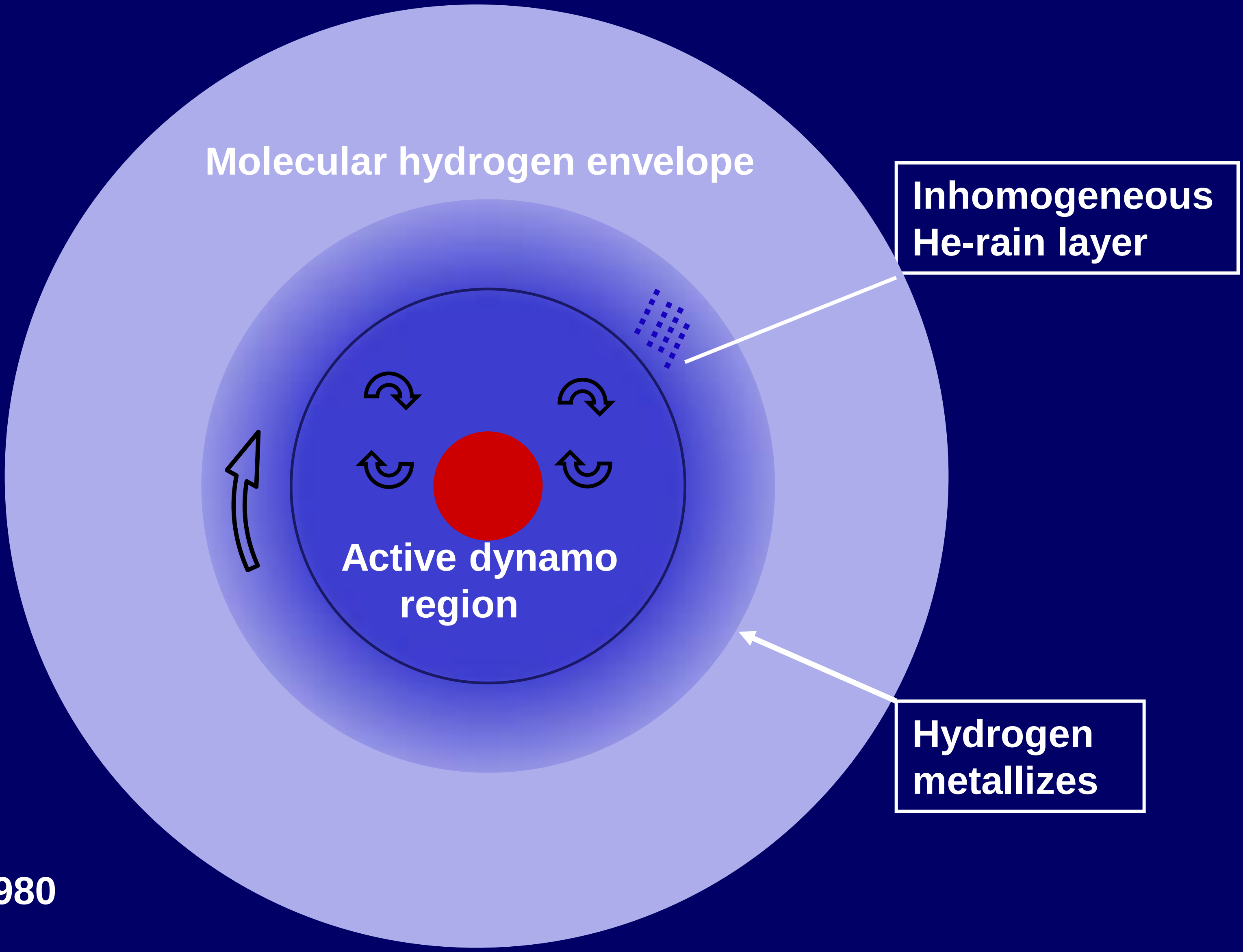
Saturn's field 0.7xEarth's

No non-zonal field detected so far.
Dipole tilt < 0.06°

Cao et al., 2011

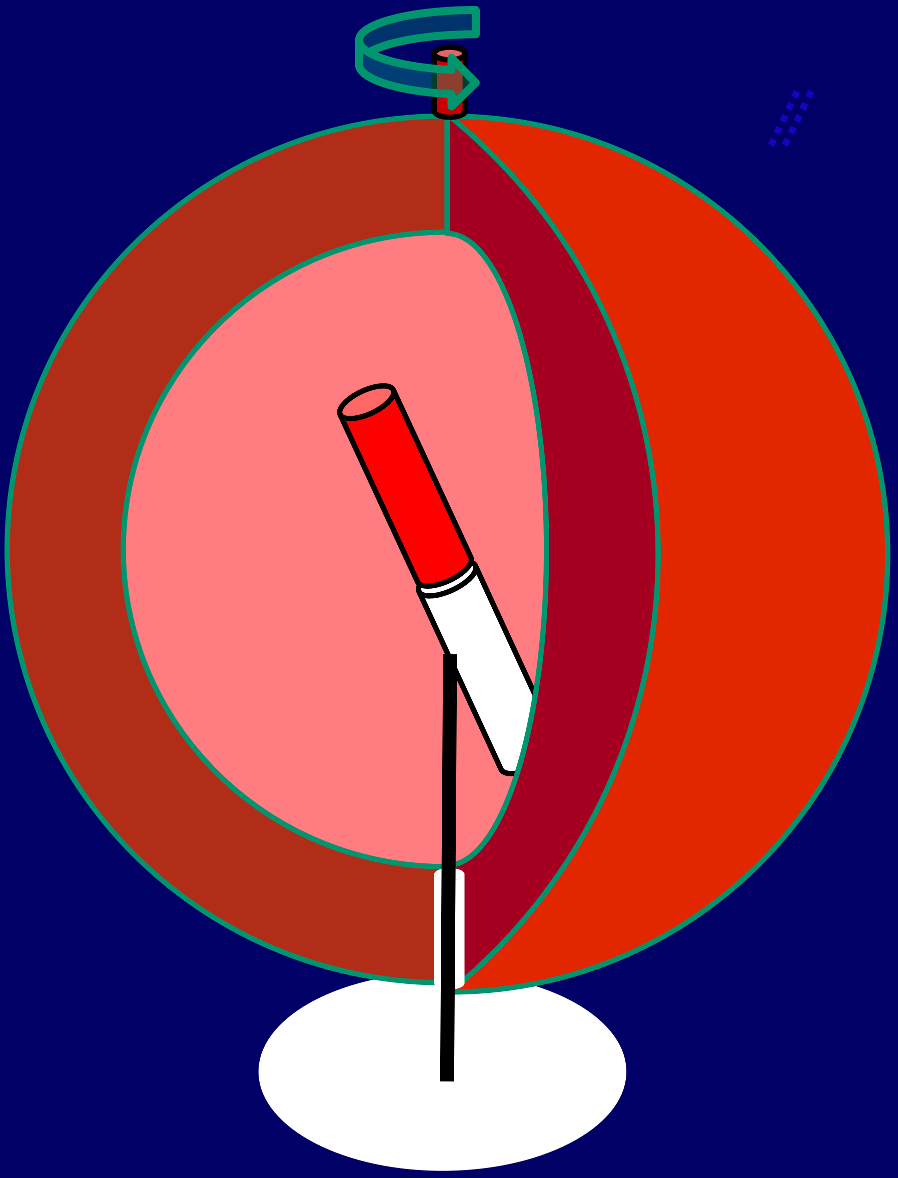


Saturn: Dynamo below He-rain layer



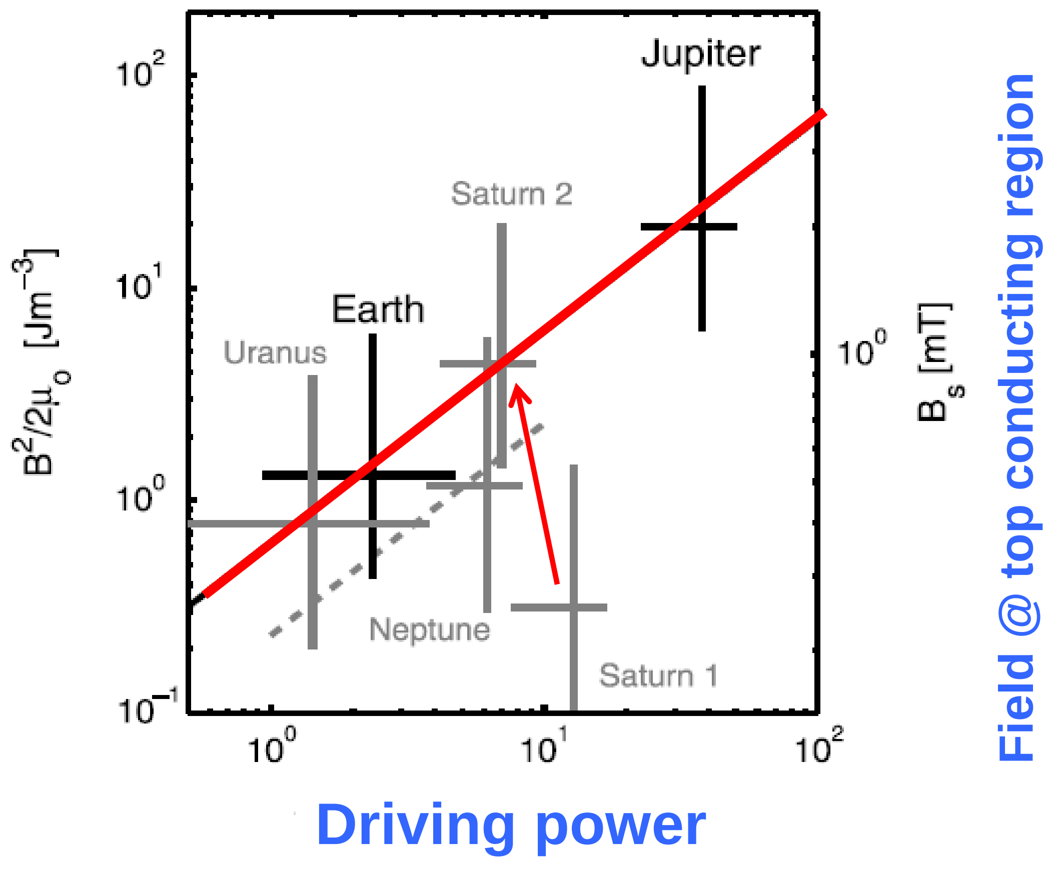
Stevenson, 1980

A Gedankenexperiment



Skin effect eliminates time-variable component (equatorial dipole) and lets static component (axial dipole) pass

Field strength for deep dynamo



With top of Saturn's dynamo at 0.4 R_s rather than at 0.62 R_s observed field complies with scaling relation Christensen, 2010

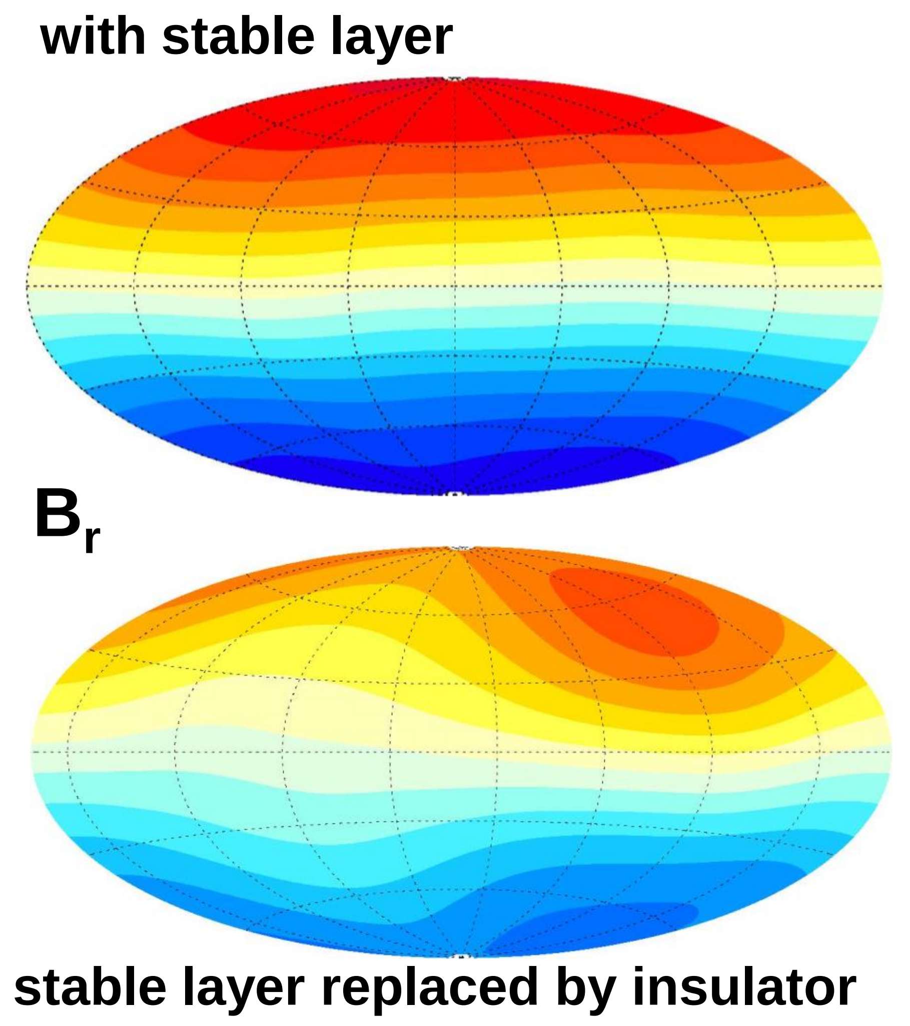
Dipolar dynamo with stable layer

Strong non-zonal field component largely filtered out by stable layer.

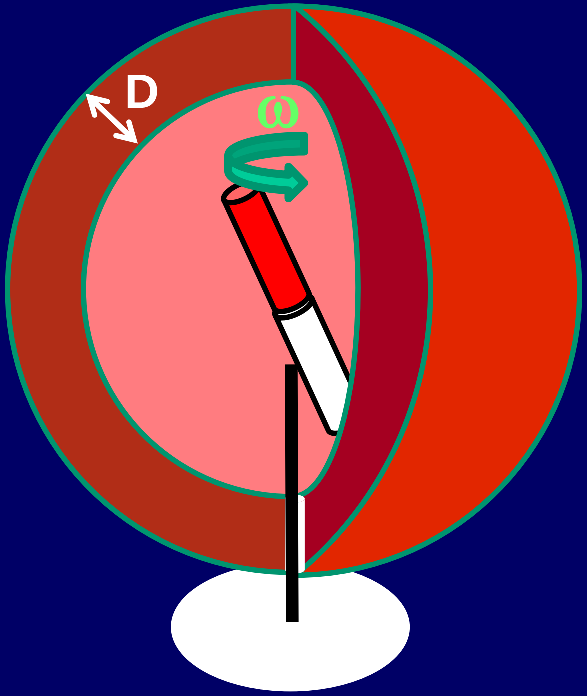
Residual dipole tilt too large compared to observation (1.5° vs. <0.06°)

Model unable to match ratio of dynamo advection time to diffusion time @ stable layer

Christensen & Wicht, 2008



Skin effect



Stagnant stable layer (thickness D) with magn. diffusivity λ

Damping factor $f_\omega = \exp(-[\omega D^2 / 2\lambda]^{1/2})$

With plausible values for λ and ω, a thin layer (D=1000 km, 1.5% of R_S) enough to damp by factor 10^4.

Picture of equatorial dipole oscillating with single frequency too simple.

Consider power spectrum of dipole fluctuations.

Skin effect

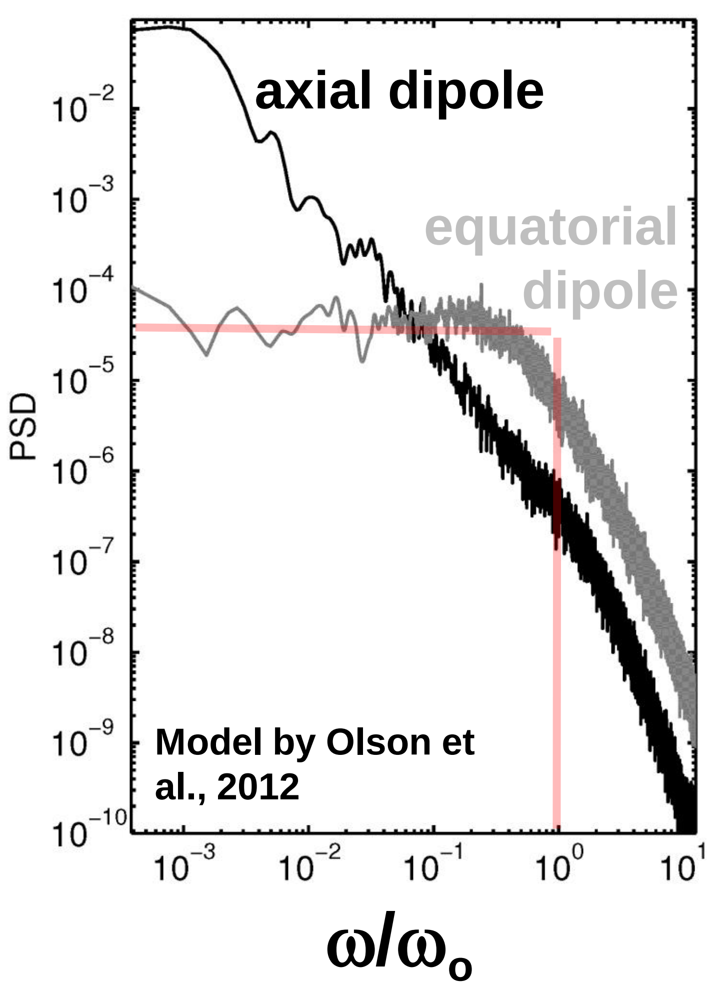
For equatorial dipole, temporal power spectrum flat up to $\omega_o = U/R$ (inverse advection time in dynamo)

Folding white spectrum up to ω_o with f_ω and ignoring power at $\omega > \omega_o$ results in net attenuation factor

$$f = (\lambda R / [2D^2 U])^{1/2}$$

Saturn: $U = 1$ cm/s $R = 30{,}000$ km
$D = 10{,}000$ km $\lambda = 2\ m^2/s \Rightarrow f = 1/170$

10^o dipole tilt at top of dynamo reduced to 0.06^o above stable layer

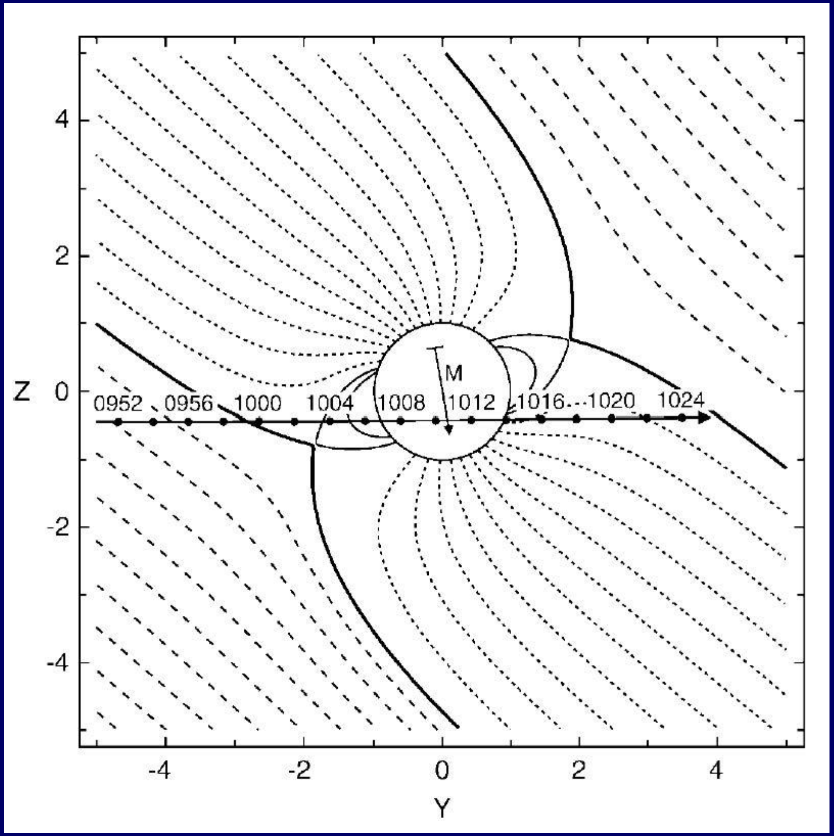


Model by Olson et al., 2012

Christensen et al., 2015 (submitted)

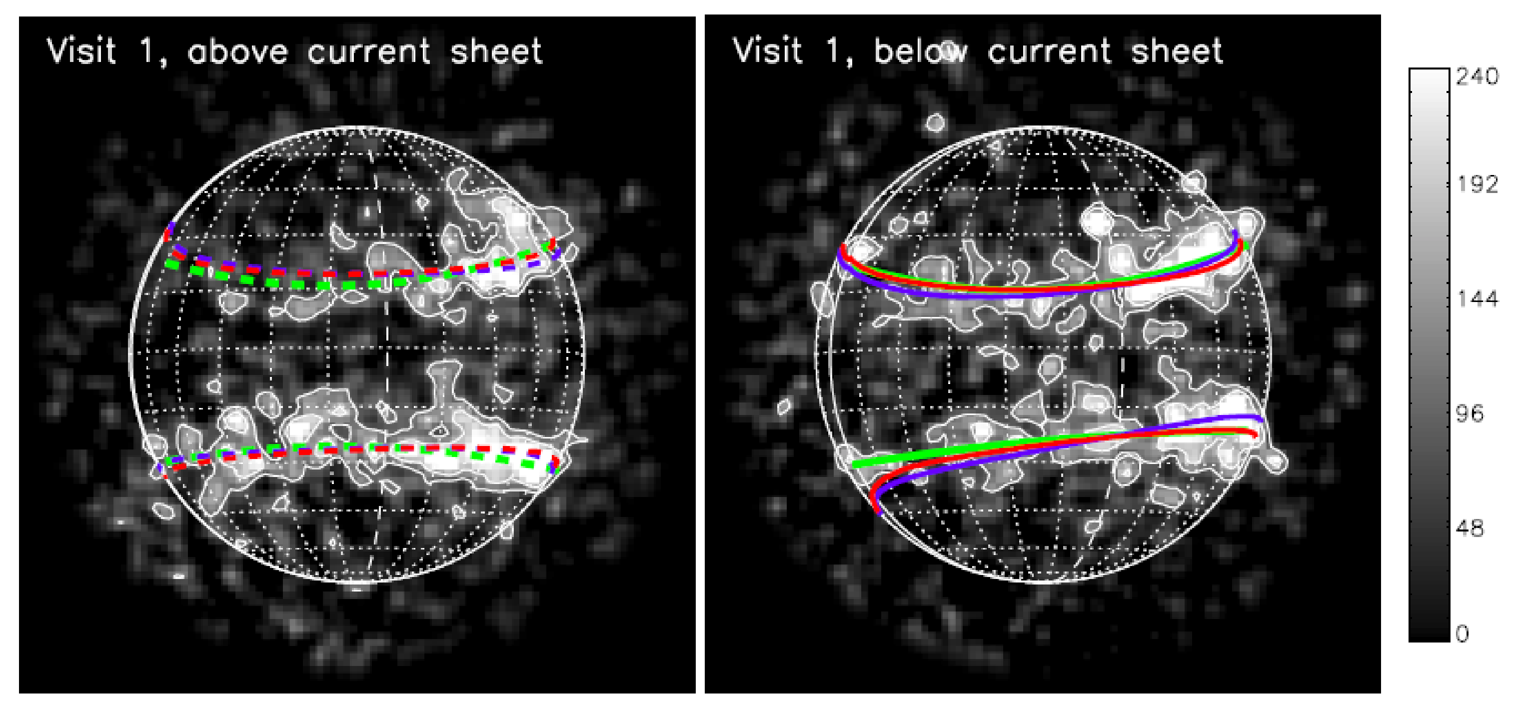
Ganymede‘s magnetic field

- Galileo mission detected intrinsic magnetic field
- g_{10}=711 nT   Dipole tilt 4^o
- Two field models equally consistent with data:
  (1) Dipole + quadrupole
  (2) Dipole + field induced in ocean, no quadrupole



Kivelson et al., 2002

New evidence for induced field



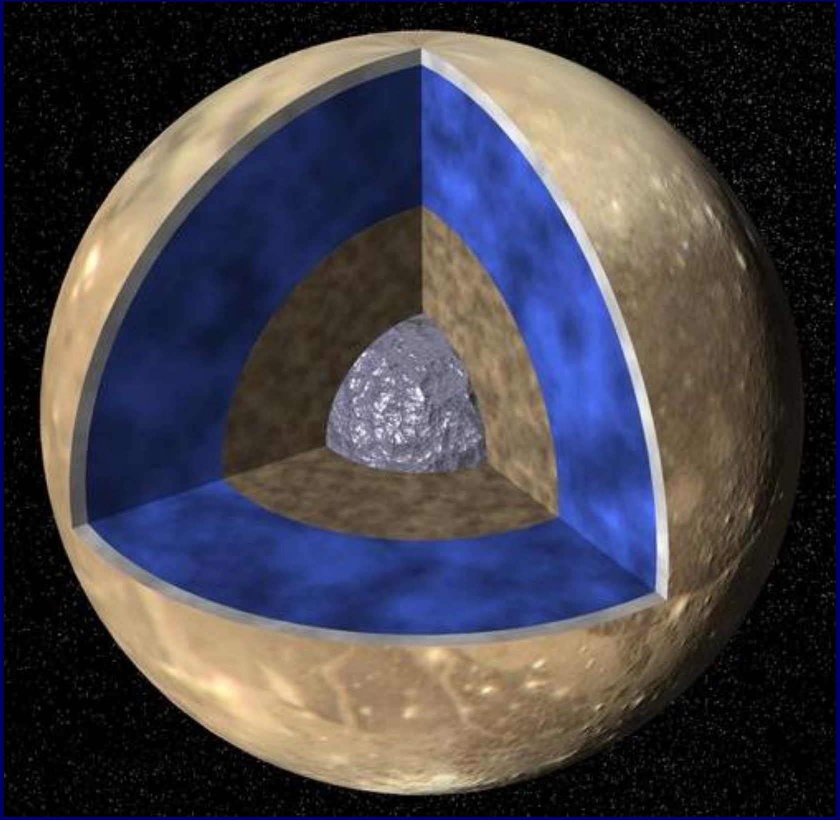
Aurora on Ganymede at open-closed fieldline boundary

Rocking of aurora location with Jupiter‘s rotation in HST images less than expected for non-conducting planet

Saur et al., 2015

Ganymede‘s dynamo field

- Internal field generated by dynamo in metallic core
- Take R_2/R_1 of oceanless field model as upper limit
- Core radius r_c is 1/4 – 1/3 of planetary radius r_G
- Even when calculated at $r_c = 0.25\ r_G$, quadrupole is untypically small



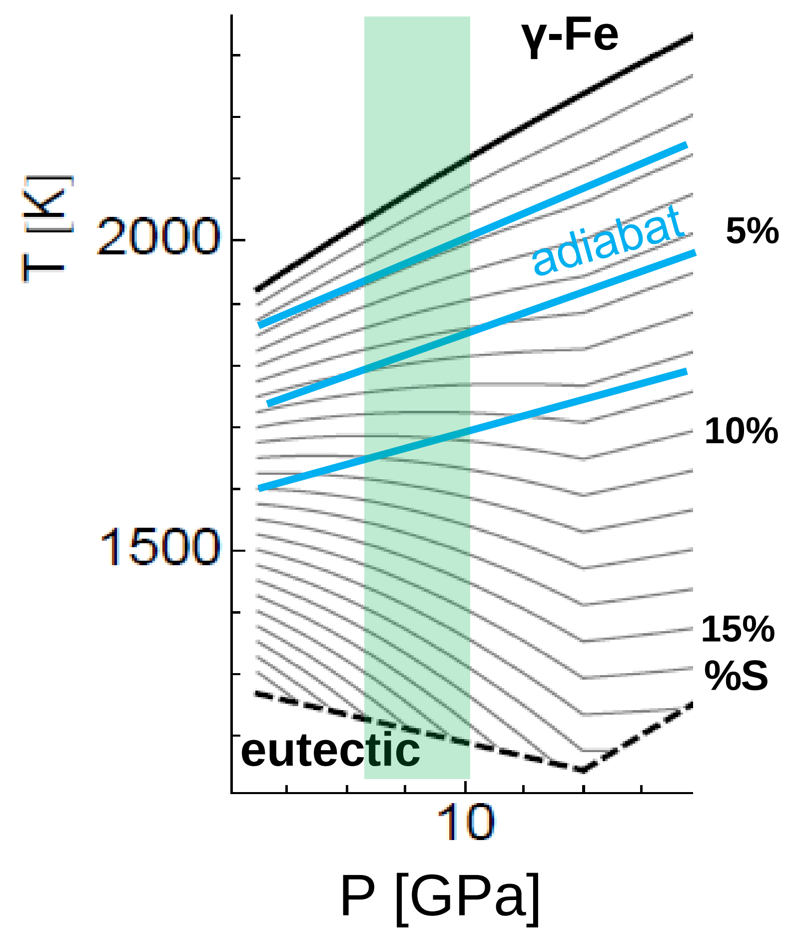
$$R_2 / R_1 < 0.04$$

(Earth CMB 2010: $R_2/R_1 = 0.14$)

Fe – FeS melting curve

- **Sulphur likely abundant in outer solar system**
- **S reduces melting temperature T_M and gradient dT_M/dP**
- **In 5-10 GPa pressure range $dT_M/dP < 0$ for >10% S**
- **For more than few % S, dT_M/dP less steep than adiabatic T-gradient**

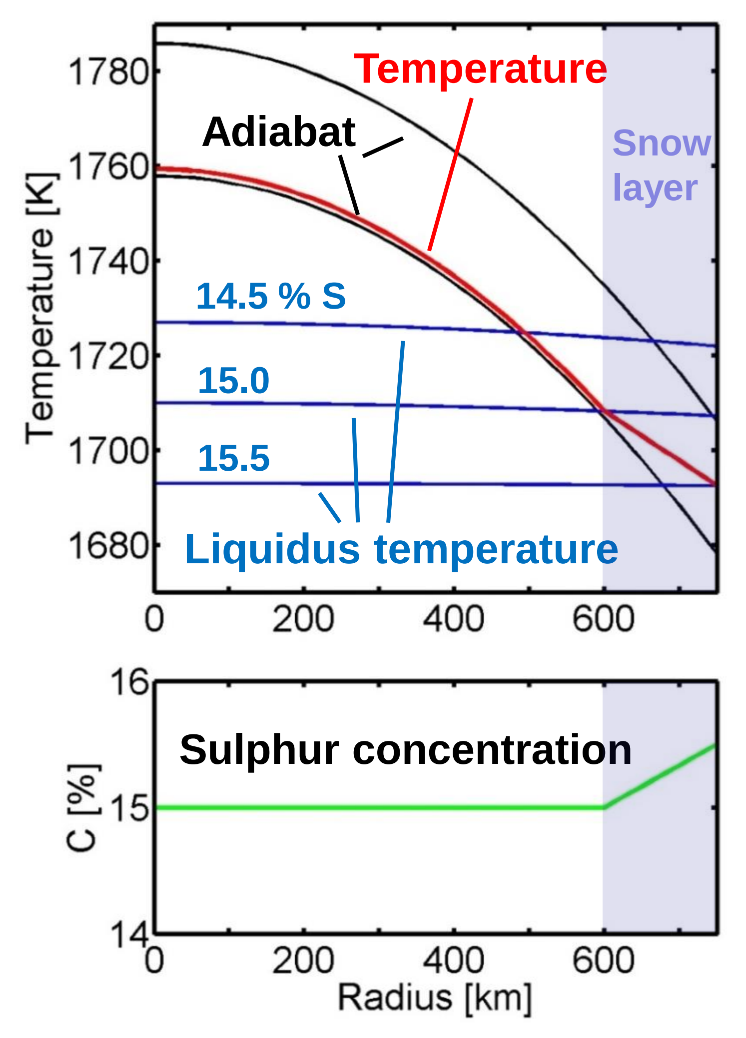
➢ **Crystallization of iron proceeds top-down**



Hauck et al., 2006; Chen et al., 2008; Williams, 2009; Buono & Walker, 2010

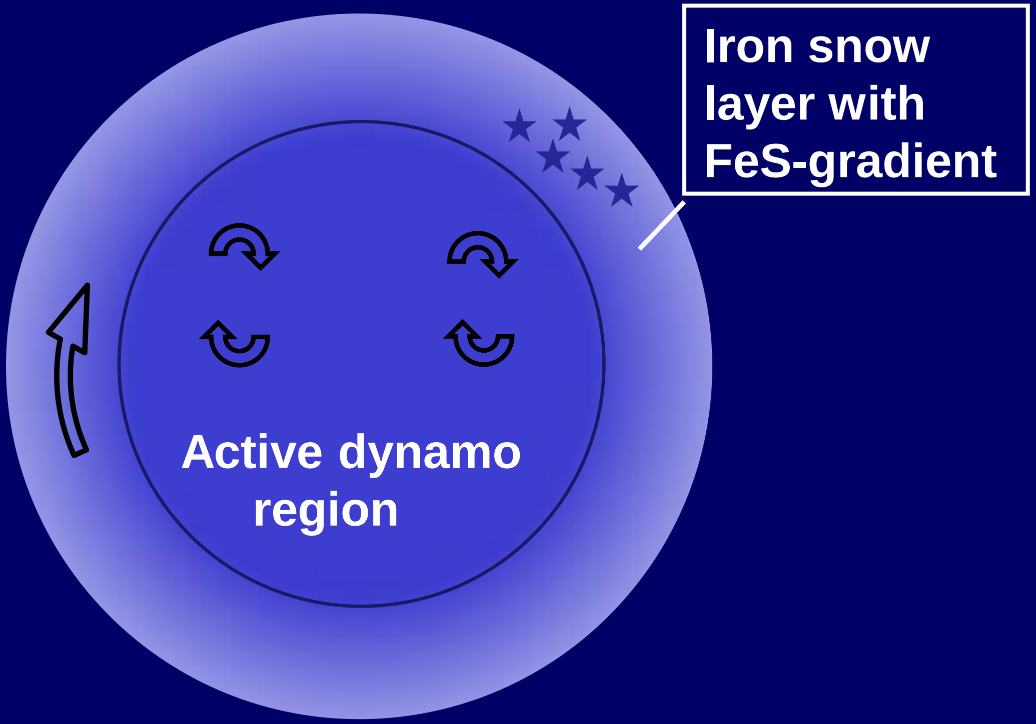
Top-down crystallization

- **Adiabat steeper than melting point gradient**
- **Iron snow forms in top layer. Sinks and dissolves at bottom of this layer**
- **Sulphur-enrichment with stable compositional gradient in snow-forming layer. Here temperature everywhere at liquidus**



Hauck et al., 2006

Dynamo below a snow layer



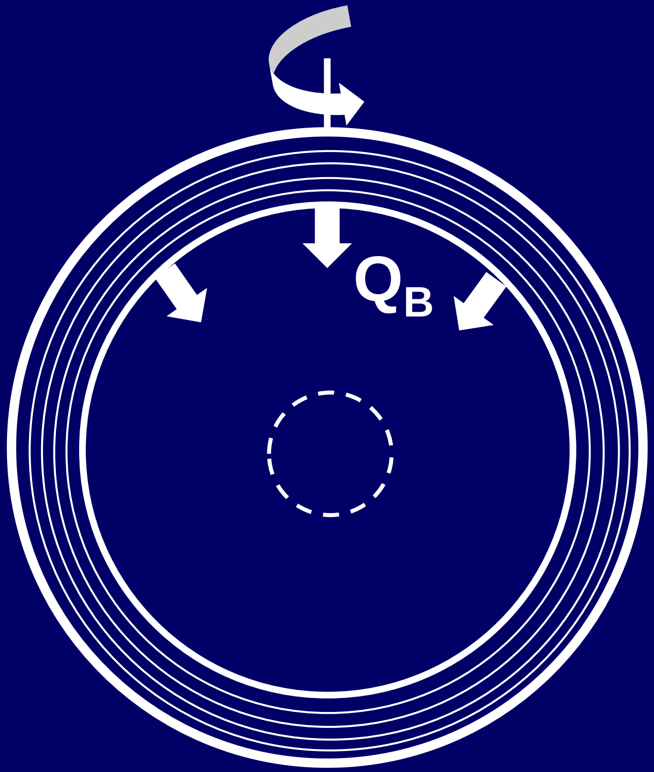
Iron snow layer with FeS-gradient

Active dynamo region

- Fe-enrichment by melting of snow at top of interior region drives compositional convection
- Stable $\Delta\rho$ in snow layer > 10^5 times larger than typical $\Delta\rho$ of convection ⇒ horizontal flow but no radial overturn

Ganymede dynamo models

- MHD dynamo model in rotating sphere, codensity, fixed buoyancy flux Q_B at top of dynamo region
- **Reference model:** No extra outer layer
- **Snow layer model:** Strongly stable fluid layer above dynamo region. Only u_h, subject to viscous, inertial, Lorentz and Coriolis forces. Magnetic induction with $u_r = 0$, $u_h \neq 0$.

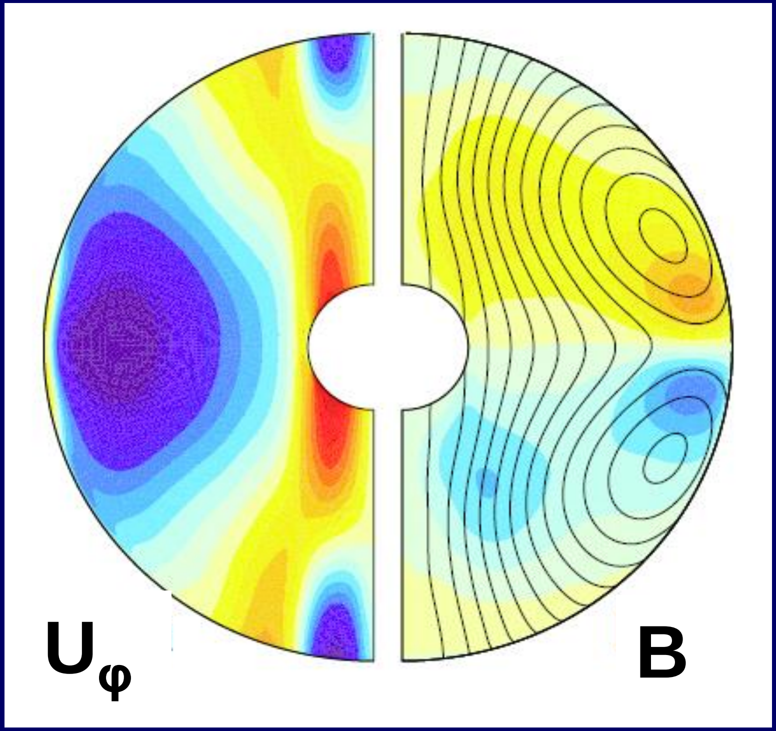


Christensen, 2015a

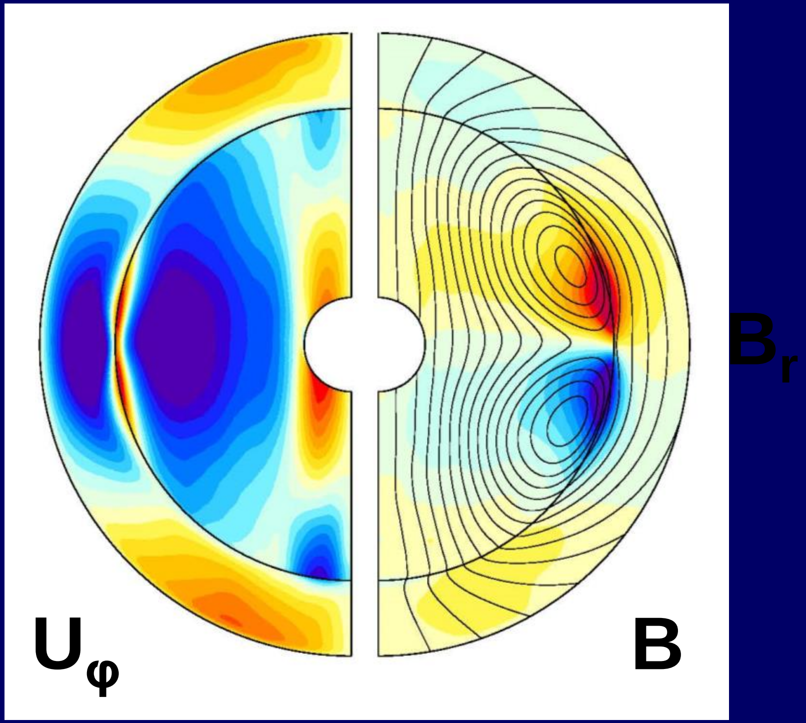
Christensen, 2015b (corrigendum)

Azimuthal and time average

$E = 3 \times 10^{-5}$ $Pm = 3$ $Ra=28xRa_{crit}$



Reference model

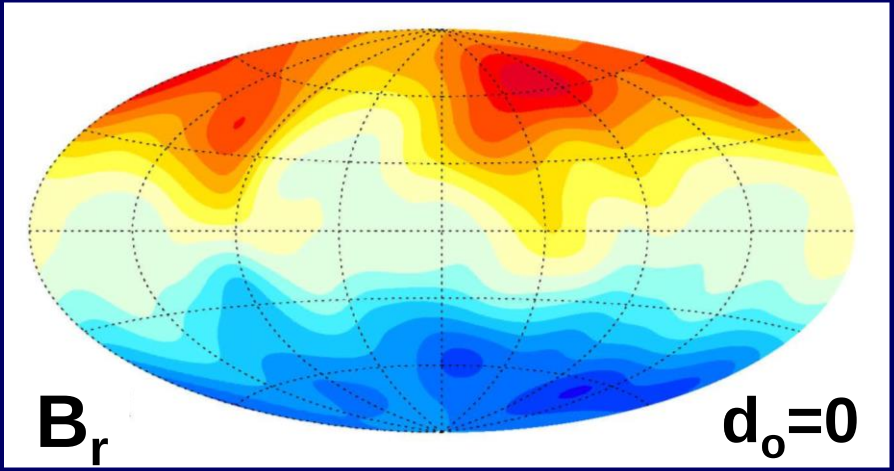


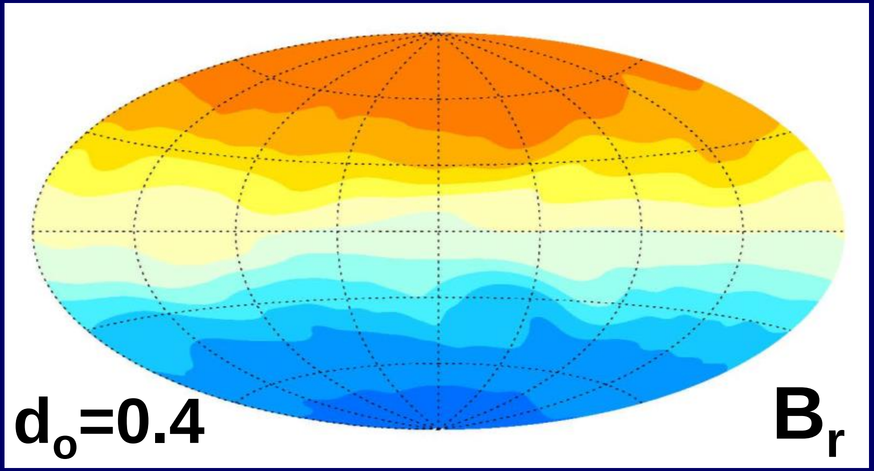
Stable fluid layer

$d_o = d/(R_c-d) = 0.4$

Field morphology

$E = 3 \times 10^{-5}$ Pm = 3 $Ra = 28 \times Ra_{crit}$





Reference model

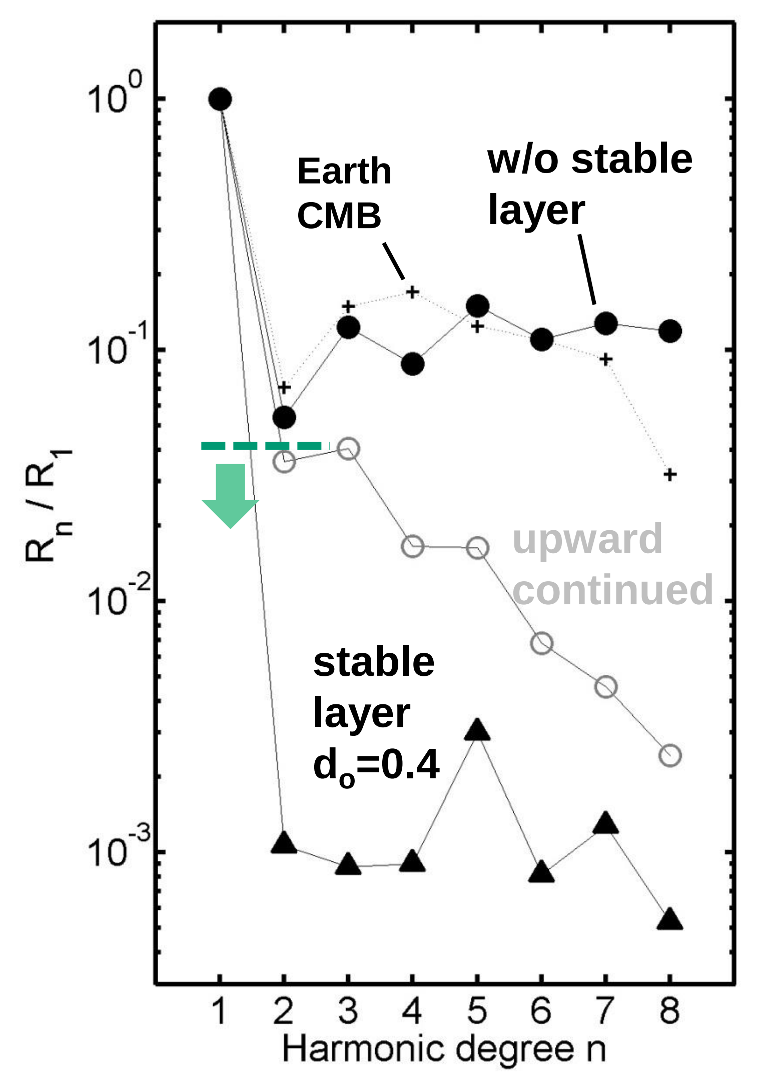
at $r = r_c + 0.4$

Stable fluid layer

at $r = r_c$

Magnetic power spectra

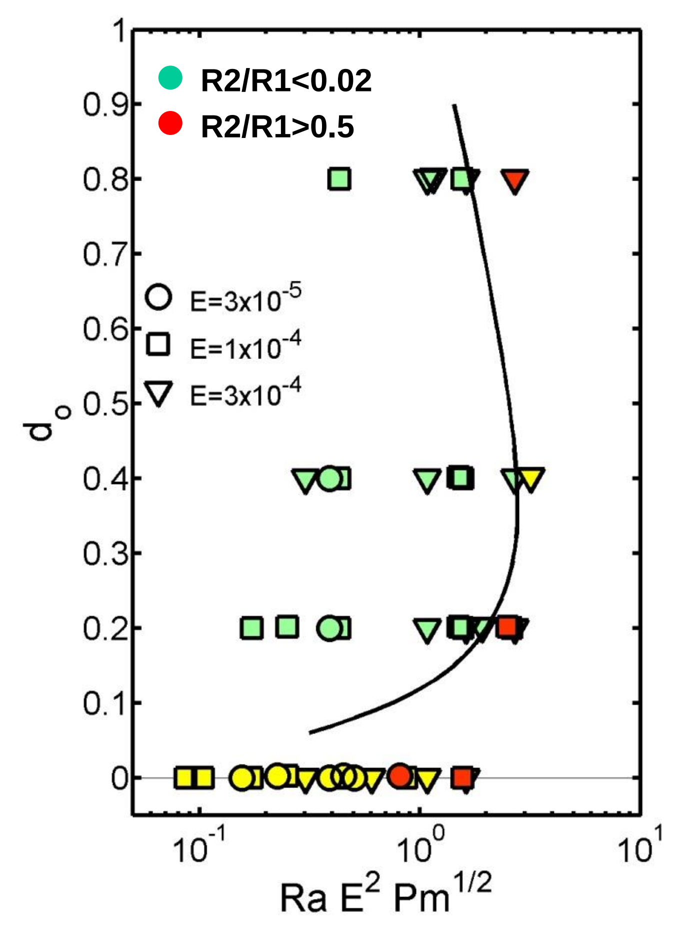
- At top of the core, time-averaged, normalized with dipole
- $E=3\times10^{-5}$ $Pm=3$ $28Ra_c$
- Reference model w/o stable layer similar to geodynamo spectrum
- With snow layer drop in multipoles
- $R_2 / R_1 \sim 0.001$



Dynamo regime

- Empirical ordering parameter $Ra\ E^2\ Pm^{1/2}$
- Dipolar dynamos with low R_2/R_1 for stable layer thickness $d_o \geq 0.2$ (≈ 100 km) and $RaE^2Pm^{1/2} < 2$
- Ganymede value:

$$Ra\ E^2\ Pm^{1/2} \sim O(1)$$



Field strength: How to rescale?

$$B = B^* \sqrt{\varrho\mu\lambda\Omega} \qquad t = t^* D^2/\nu$$

$$\boxed{Ra = \frac{gQ_B D^4}{\rho\nu\kappa^2} \quad E = \frac{\nu}{\Omega D^2} \quad Pm = \frac{\nu}{\lambda} \quad \mathrm{Pr} = \frac{\nu}{\kappa}}$$

- **Take true values of D, ρ, g**
- **To match non-dimensional model parameters, set diffusivites ν, κ, λ to whatever is required**
- **We must also compromise either on the buoyancy flux Q_B (overdrive) or on Ω (underrotate)**
- **Scaling studies suggest that B depends on Q_B, but not strongly on diffusivities or Ω** **(Christensen & Aubert, 2006)**

➢ **Conclusion: take correct Q_B, compromise on Ω**

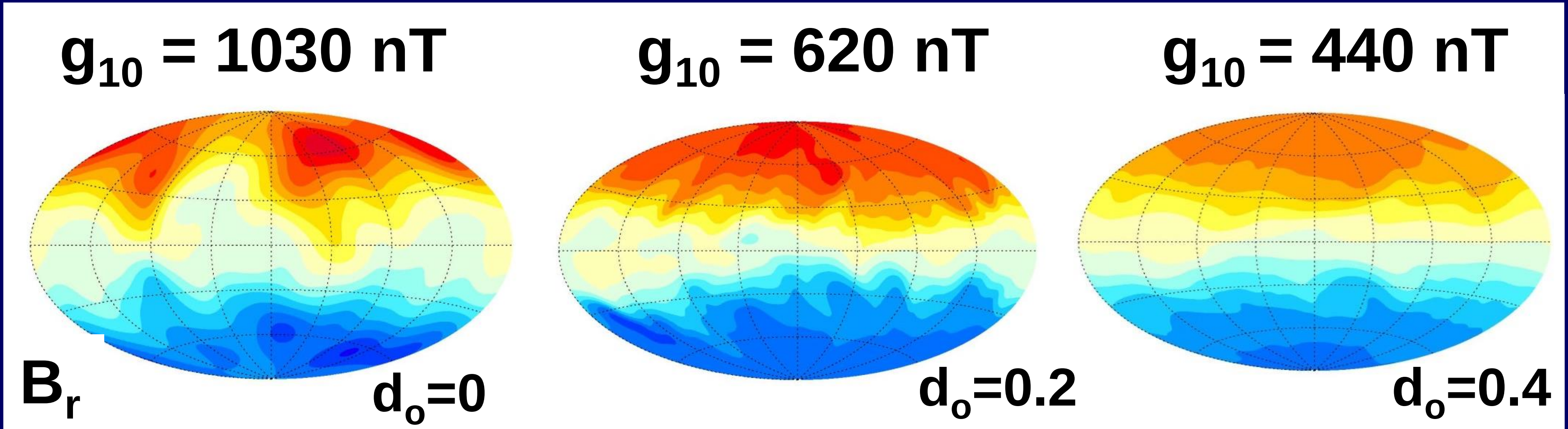
Field strength

$E = 3 \times 10^{-5}$ $Pm = 3$ $Pr=1$ $Ra=2.5 \times 10^8$

Fix D, ρ, g to Ganymede, Q_B = 5000 kg/s

Rückriemen et al., 2015

- $\nu = \kappa = 3.8\ m^2/s$ $\lambda = 1.3\ m^2/s$ $2\pi/\Omega = 160$ d



(Observed g_{10} = 711 nT)

Conclusions

- Stably stratified layer above dynamo region in planets: Mercury, Saturn, Ganymede (Earth?)
- Stable layer tends to make field more axisymmetric (small dipole tilt), lowers dipole moment
- Reduced quadrupole if dynamo field dipole-dominated. Multipolar dynamo field permits large axial quadrupole
- Test of concept by future observations: Skin effect prolongs time-scale of secular variation