Time series analysis of flow reversals

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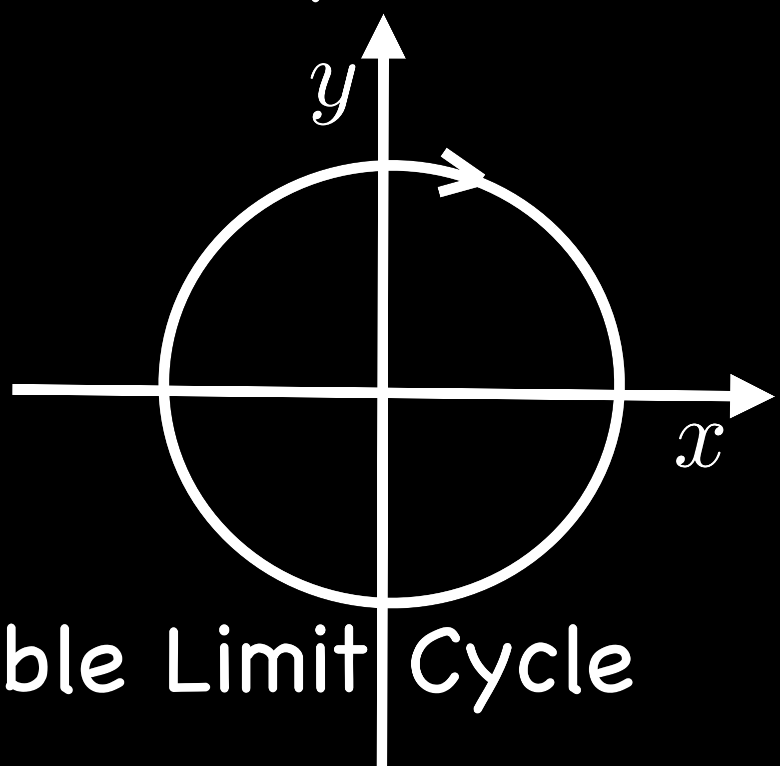
Reconstruction: The Idea

$$\dot{r} = r(1 - r^2);\ \dot{\theta} = -1$$

$$\Rightarrow (x(t), y(t)) = (\sin t, \cos t)$$

(Asymptotically)

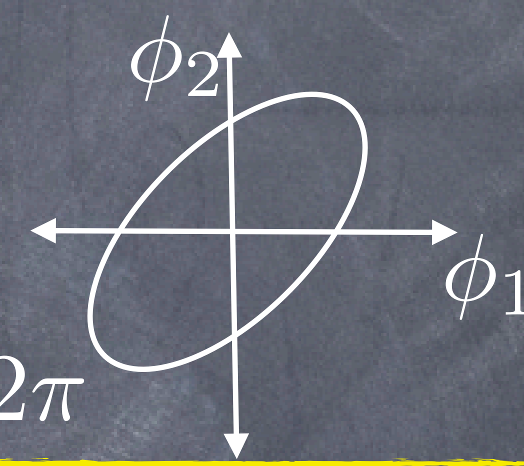


Stable Limit Cycle

$$\phi(x, y) = x(t) = \sin t = \phi_1$$

Embedding dimension (m)=1

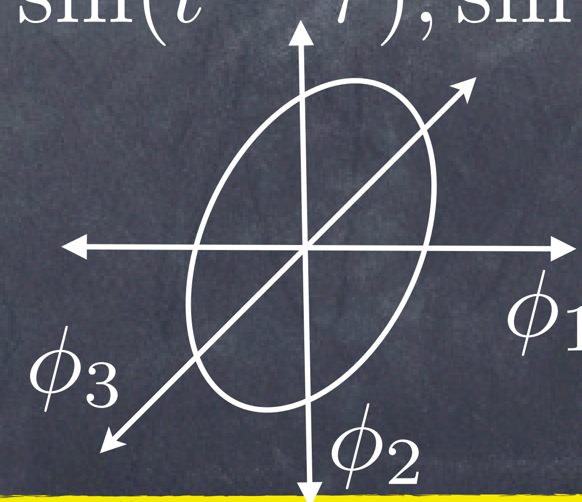
Form: $(\phi_1, \phi_2) \equiv (\sin t, \sin(t - \tau))$



Delay $\tau \neq \pi, 2\pi$

m=2

Construct: $(\phi_1, \phi_2, \phi_3) \equiv$
$(\sin t, \sin(t - \tau), \sin(t - 2\tau))$



m=3

$\tau = ?$

$m = ?$

Reconstruction: Embedding Dimension

Theorem 1. Let M be a compact manifold of dimension m. For pairs (φ, y), $\varphi: M \to M$ a smooth diffeomorphism and $y: M \to \mathbb{R}$ a smooth function, it is a generic property that the map $\Phi_{(\varphi,y)}: M \to \mathbb{R}^{2m+1}$, defined by

$$\Phi_{(\varphi,y)}(x) = (y(x), y(\varphi(x)), \ldots, y(\varphi^{2m}(x))$$

is an embedding; by "smooth" we mean at least C^2.

(Takens, 1981)

Theorem 2.5 (Fractal Delay Embedding Prevalence Theorem). Let Φ be a flow on an open subset U of R^k, and let A be a compact subset of U of box-counting dimension d. Let $n > 2d$ be an integer, and let $T > 0$. Assume that A contains at most a finite number of equilibria, no periodic orbits of Φ of period T or $2T$, at most finitely many periodic orbits of period $3T$, $4T$,..., nT, and that the linearizations of those periodic orbits have distinct eigenvalues. Then for almost every smooth function h on U, the delay coordinate map $F(h, \Phi, T)$: $U \to R^n$ is:

1. One-to-one on A.
2. An immersion on each compact subset C of a smooth manifold contained in A.

(Sauer, Yorke & Casdagli, 1991)

$m > 2D_b$

Sufficient Condition

Reconstruction: Time Delay

Prescription for choosing delay:
look for either first zero of linear auto-correlation,

$$C_\tau \equiv \langle (\phi_1(t) - \langle \phi_1 \rangle)(\phi_1(t-\tau) - \langle \phi_1 \rangle) \rangle$$

or,

first minimum of the average mutual information,

$$I_\tau \equiv \sum_{\phi_1(t),\phi_1(t-\tau)} P(\phi_1(t), \phi_1(t-\tau)) \log_2 [\frac{P(\phi_1(t), \phi_1(t-\tau))}{P(\phi_1(t)) P(\phi_1(t-\tau))}]$$

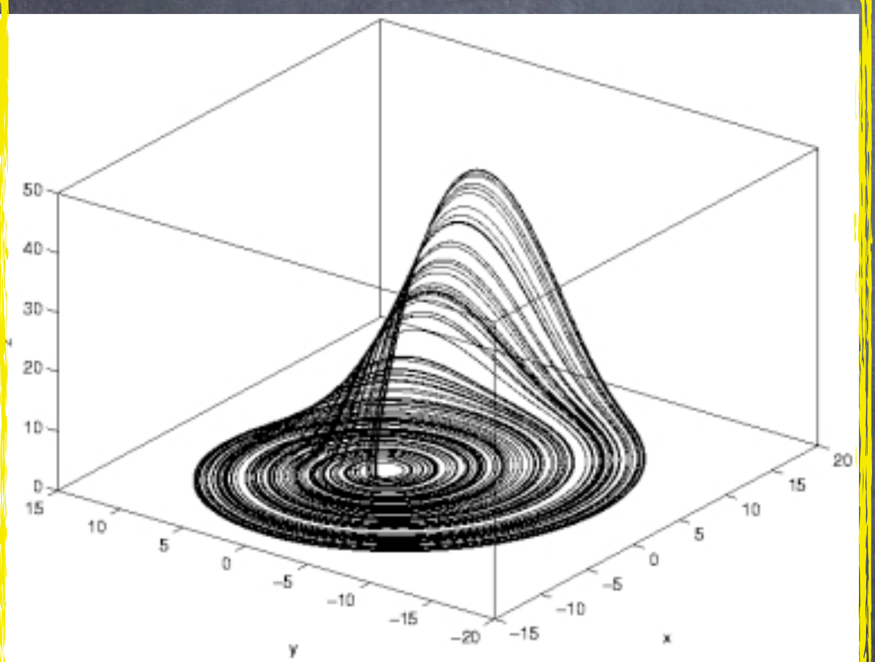
(Fraser & Swinney, 1986)

Reconstruction: Rössler Attractor

$$\dot{x} = -y - z,$$

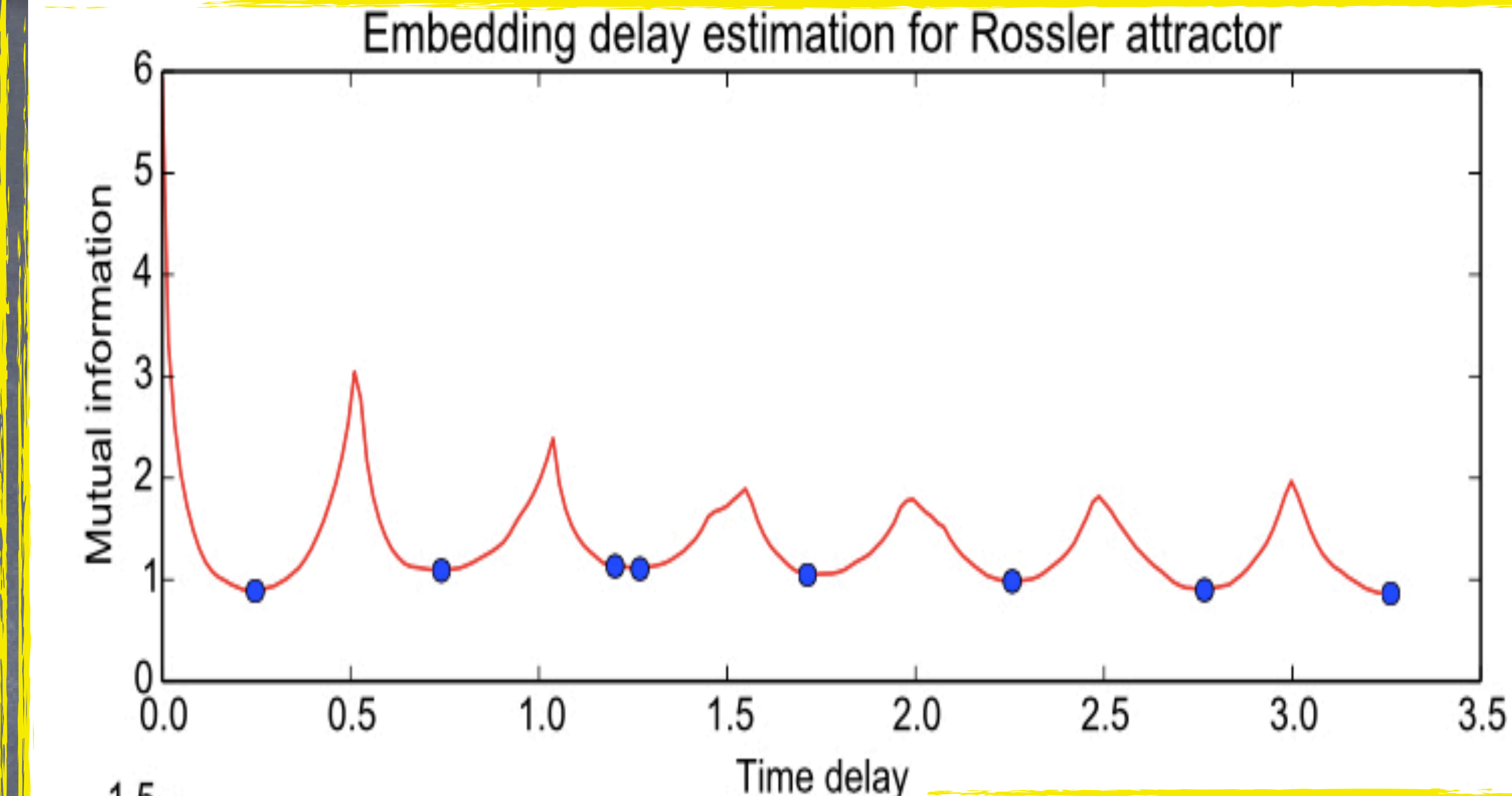
$$\dot{y} = x + ay,$$

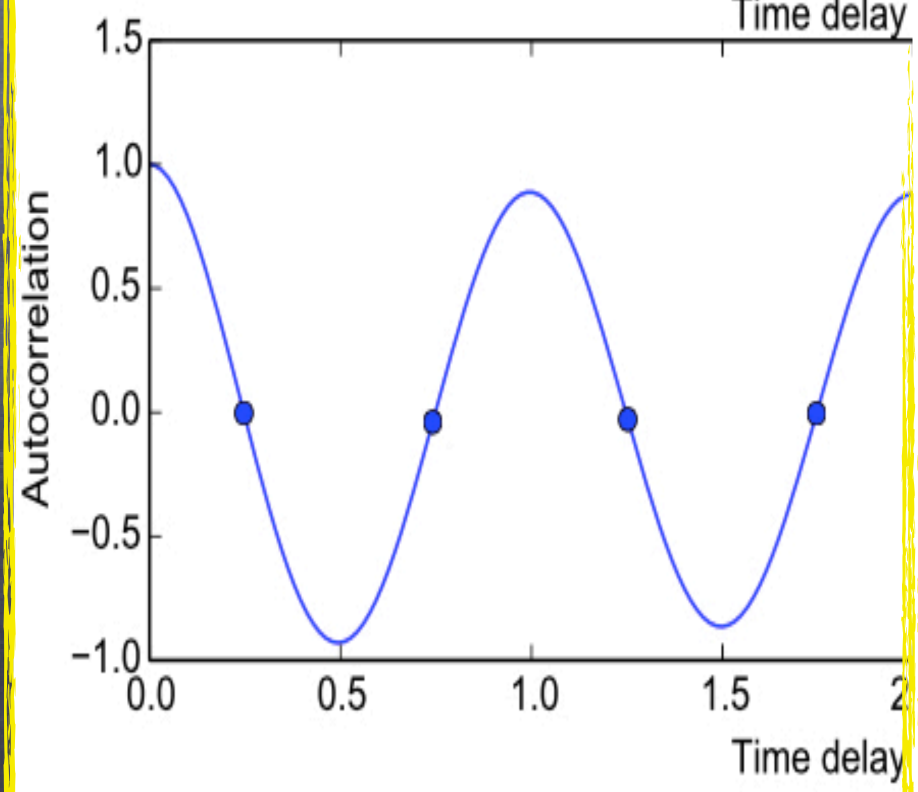
$$\dot{z} = b + z(x - c)$$

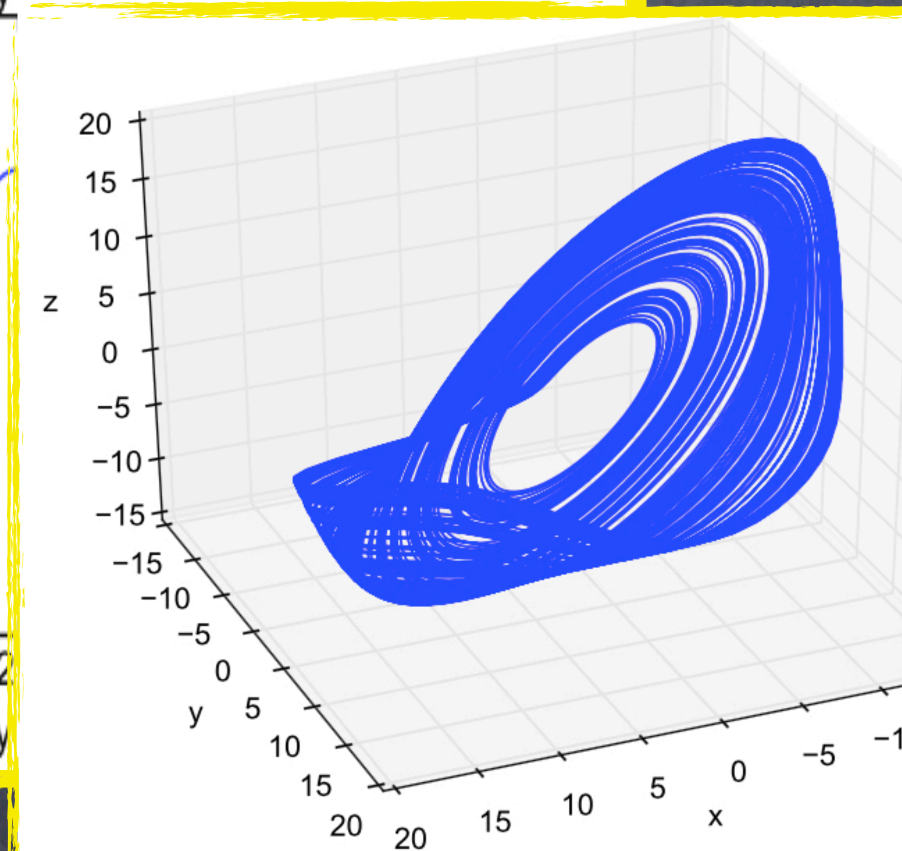


$$D_{KY} \approx 2.01$$

$$D_c \approx 1.99$$







Reconstruction: AFNN (Cao, 1997)

Scalar time-series and delay-reconstructed vector:

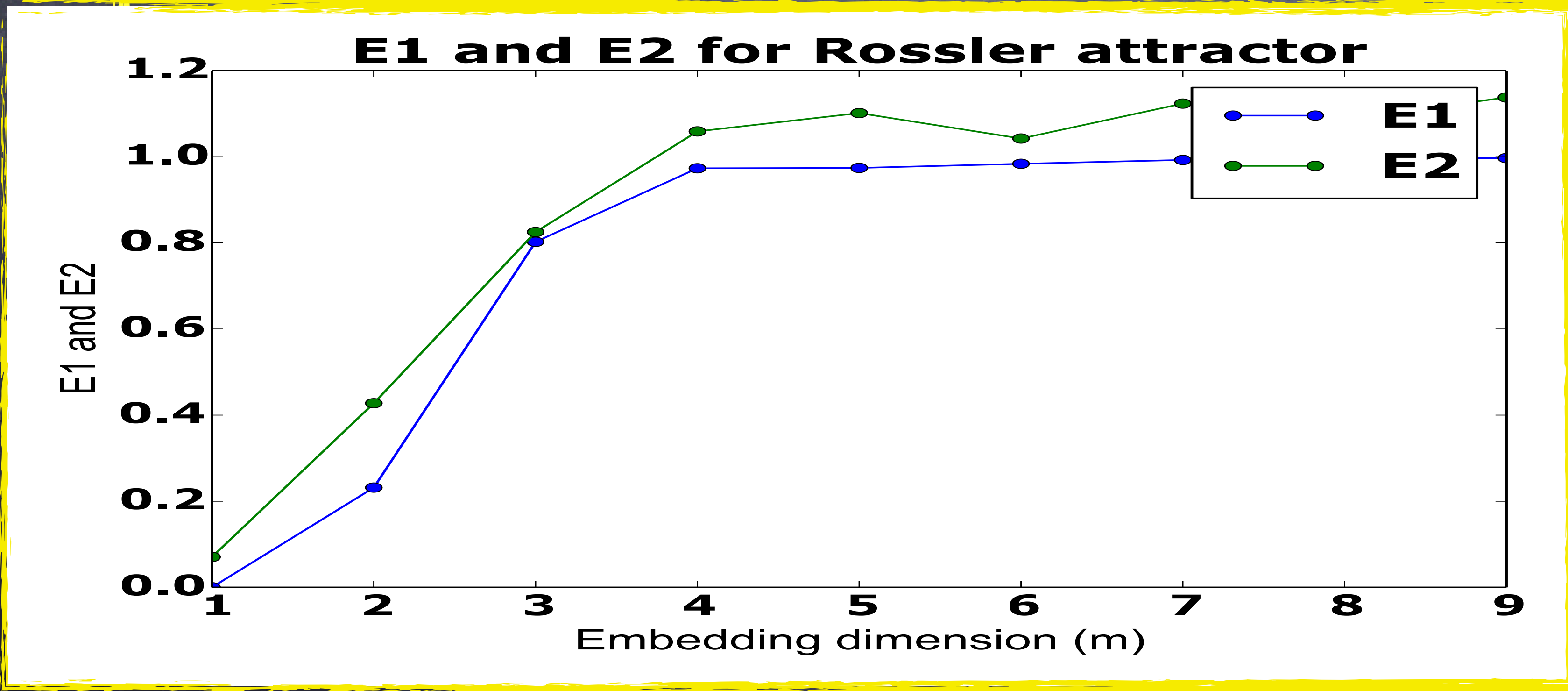
$$\mathbf{x} = (x_1, x_2, \cdots, x_N); \; \mathbf{y}_i = (x_i, x_{i+\tau}, \cdots, x_{i+(d-1)\tau})$$

Average magnifications in NN-distance and their ratios:

$$E(d) \equiv \frac{\sum_{i=1}^{N-d\tau} \frac{\|\mathbf{y}_i(d+1) - \mathbf{y}_{n(i,d)}(d+1)\|}{\|\mathbf{y}_i(d) - \mathbf{y}_{n(i,d)}(d)\|}}{N - d\tau}; \; E1(d) \equiv \frac{E(d+1)}{E(d)}$$

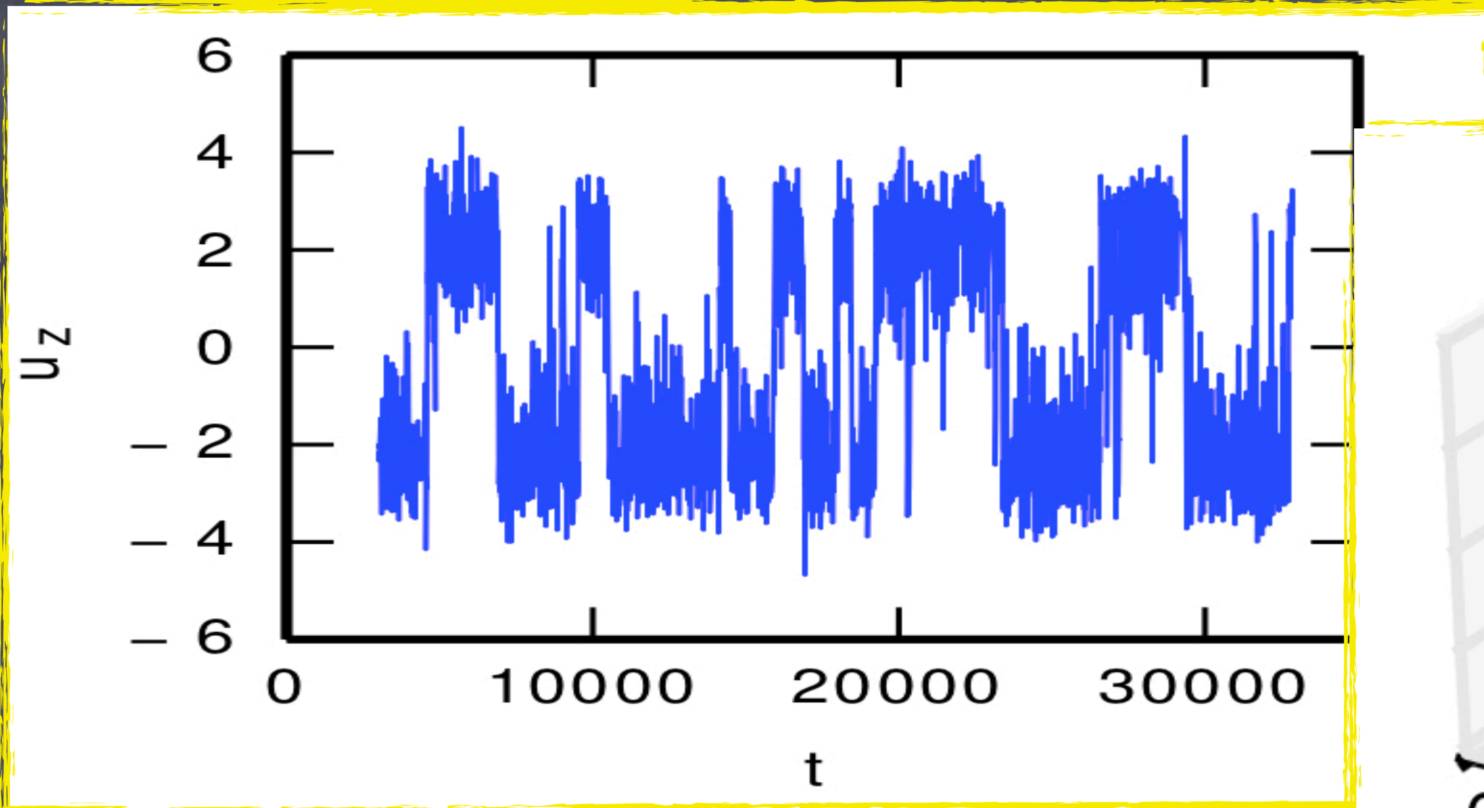
Average increases in NN-distance and their ratios:

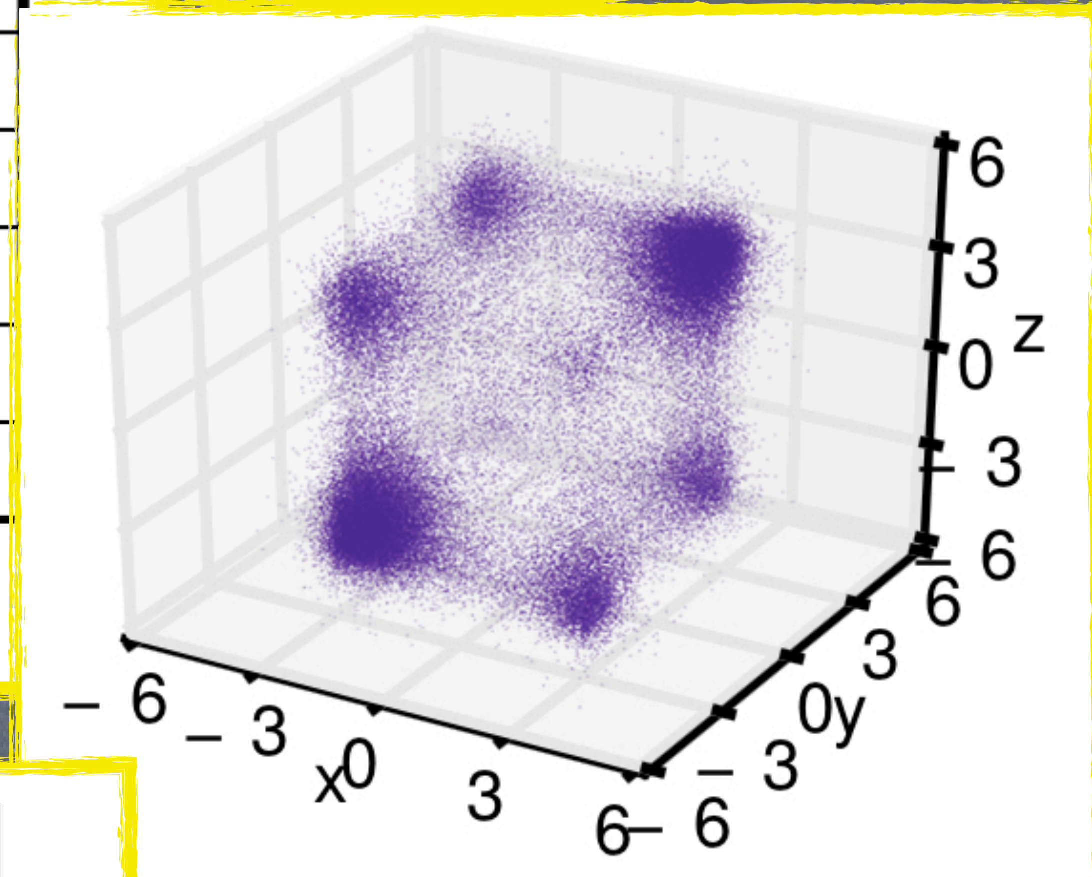
$$E^*(d) \equiv \frac{\sum_{i=1}^{N-d\tau} |x_{i+d\tau} - x_{n(i,d)+d\tau}|}{N - d\tau}; \; E2(d) \equiv \frac{E^*(d+1)}{E^*(d)}$$

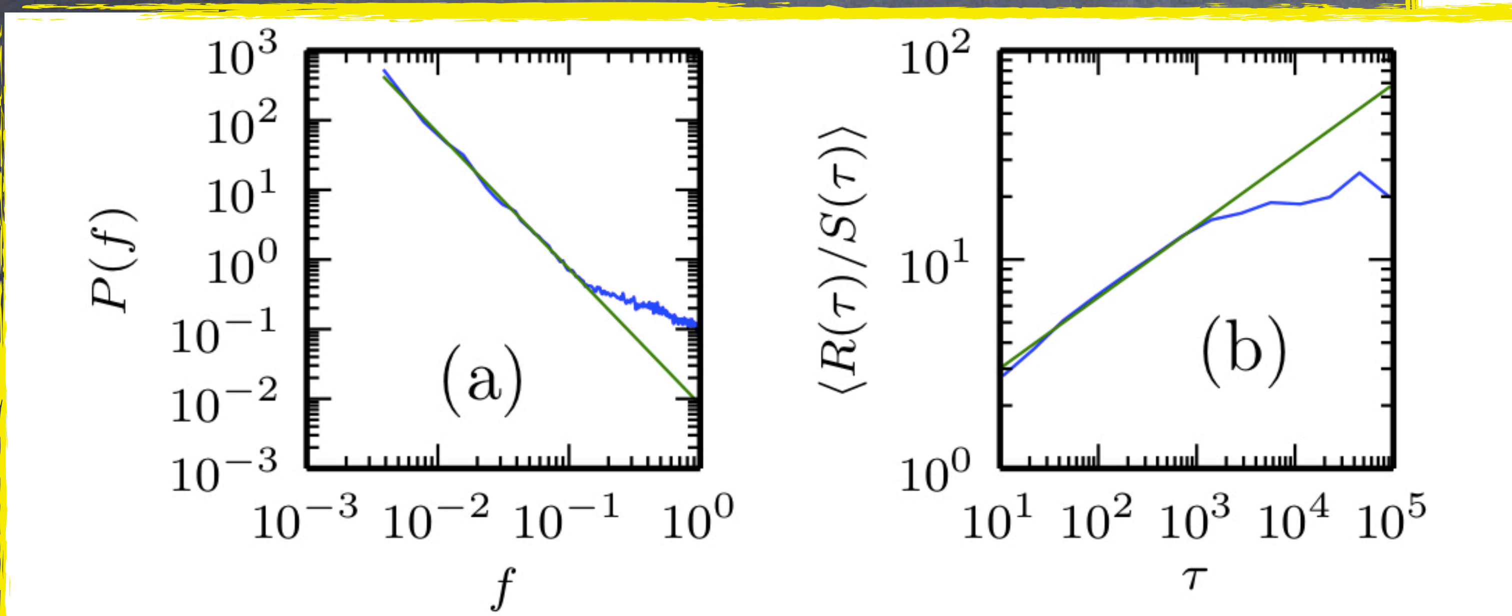


$$m = 4 \not> 2D_b$$

Reversal Time Series







(a) Power spectrum ($\beta = 1.95$) and (b) R/S analysis for real space velocity u_z ($H = 0.34$).

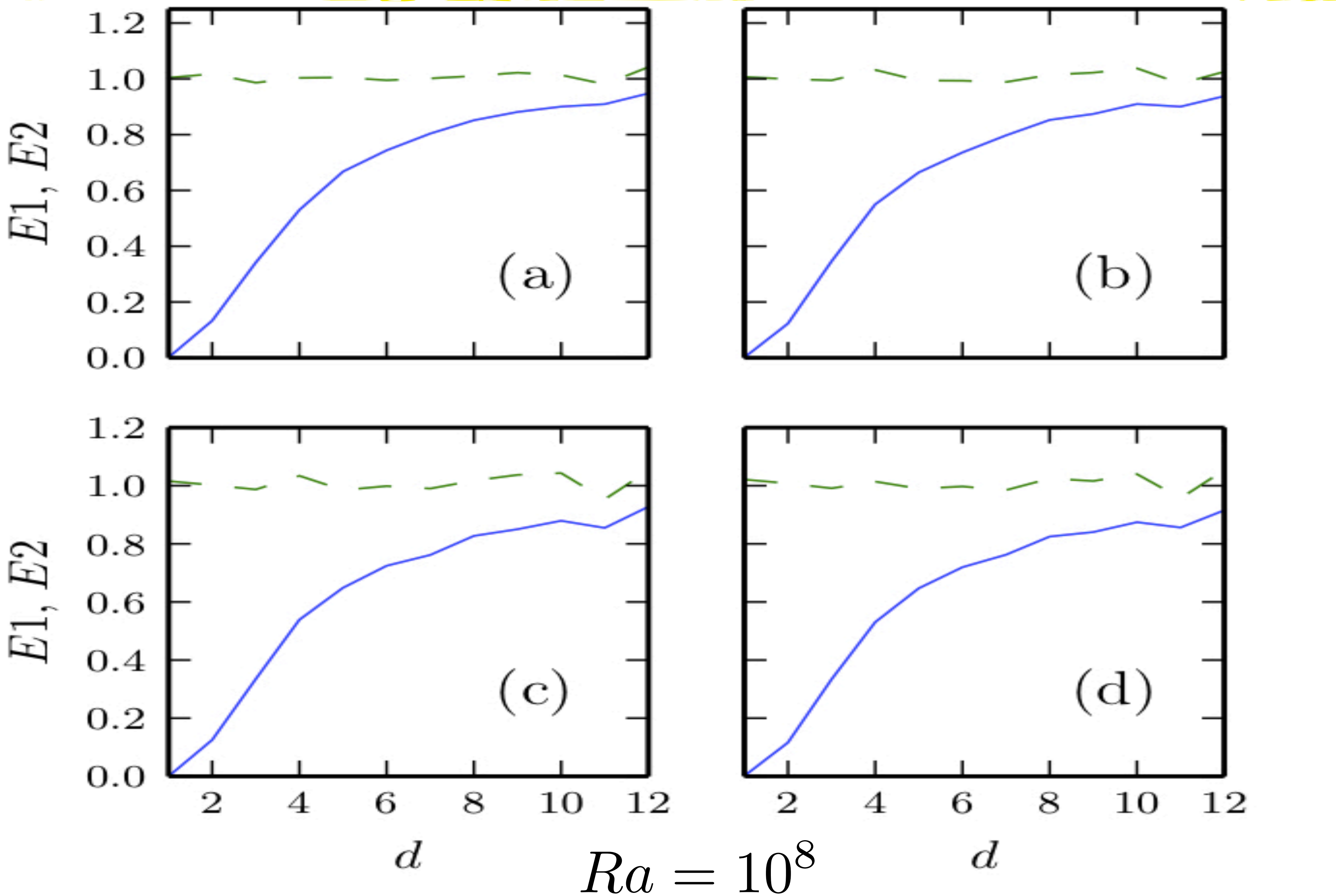
Time Series

is

Anti-Persistent

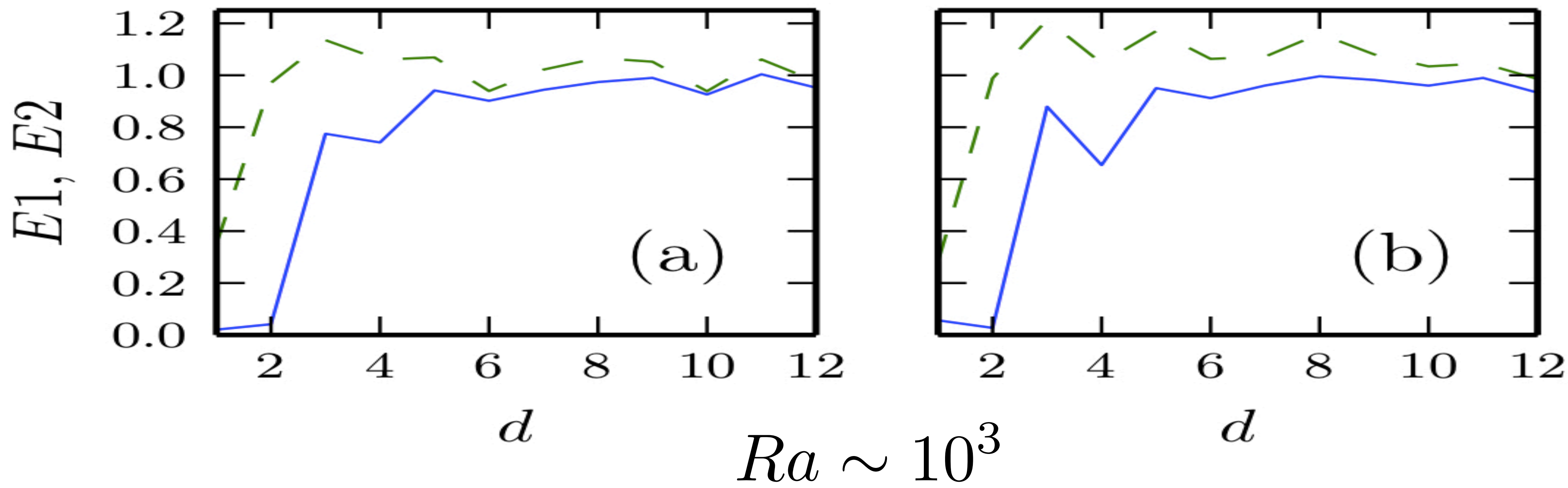
$$\because C(t) \sim 2^{2H-1} - 1$$

Reversal Time Series: AFNN



$E1$ and $E2$ curves. (a) u_z. (b) Fourier (1, 1) (c) Fourier (1, 3) (d) Fourier (1, 5).

Reversal Time Series: AFNN



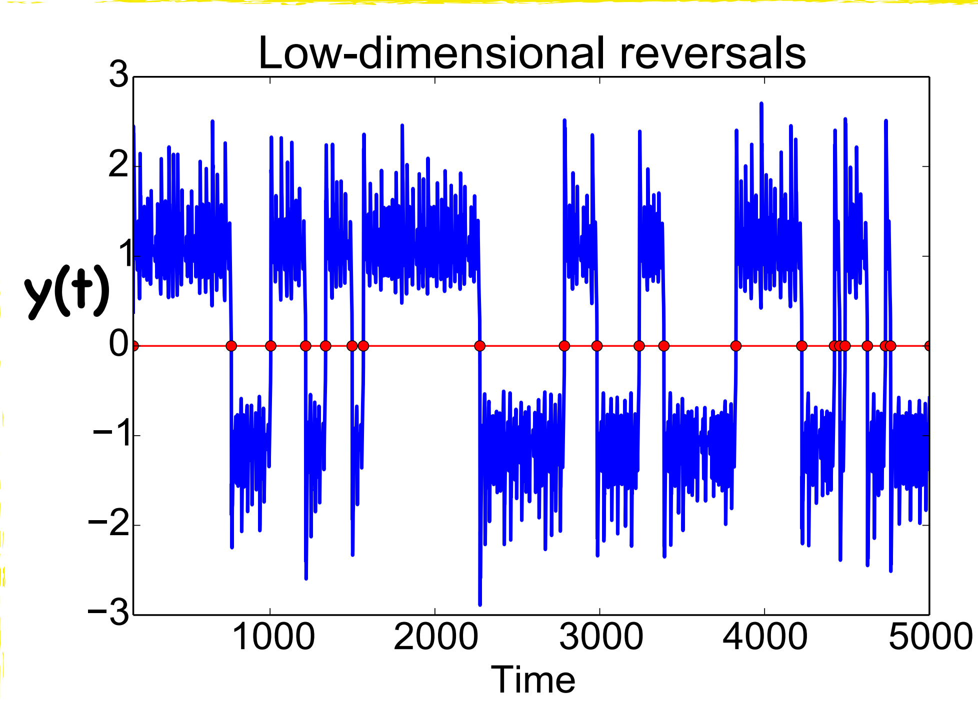
$E1$ and $E2$ curves. (a) u_z for Ra = 659.0 (b) u_z for Fourier mode (1,1) for Ra = 659.0

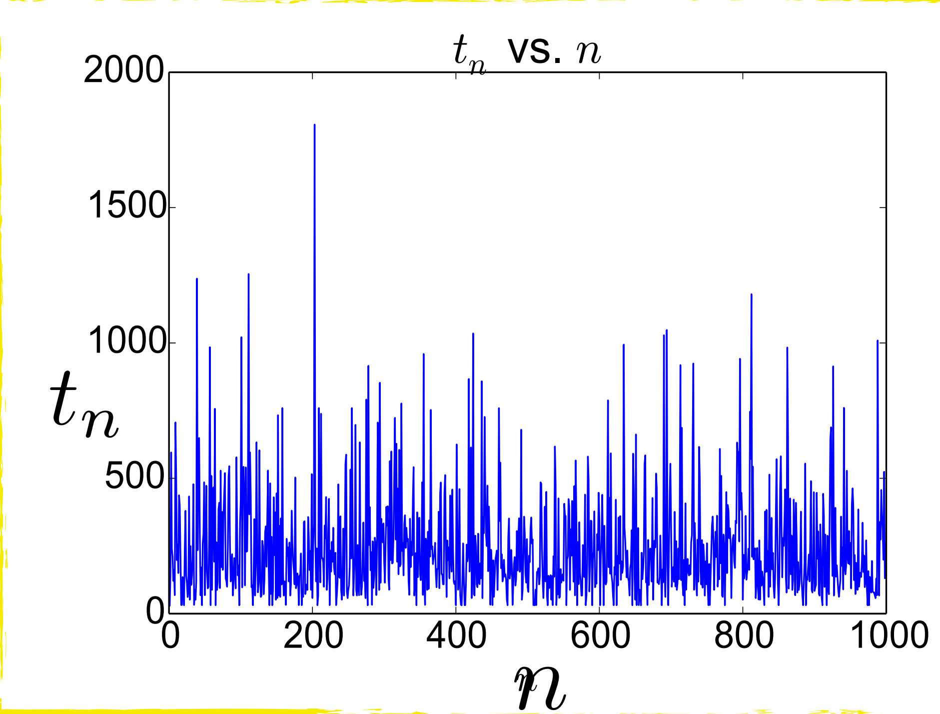
We can classify 'random' reversals in two classes:
(i) low-dimensional reversals, &
(ii) stochastic reversals.

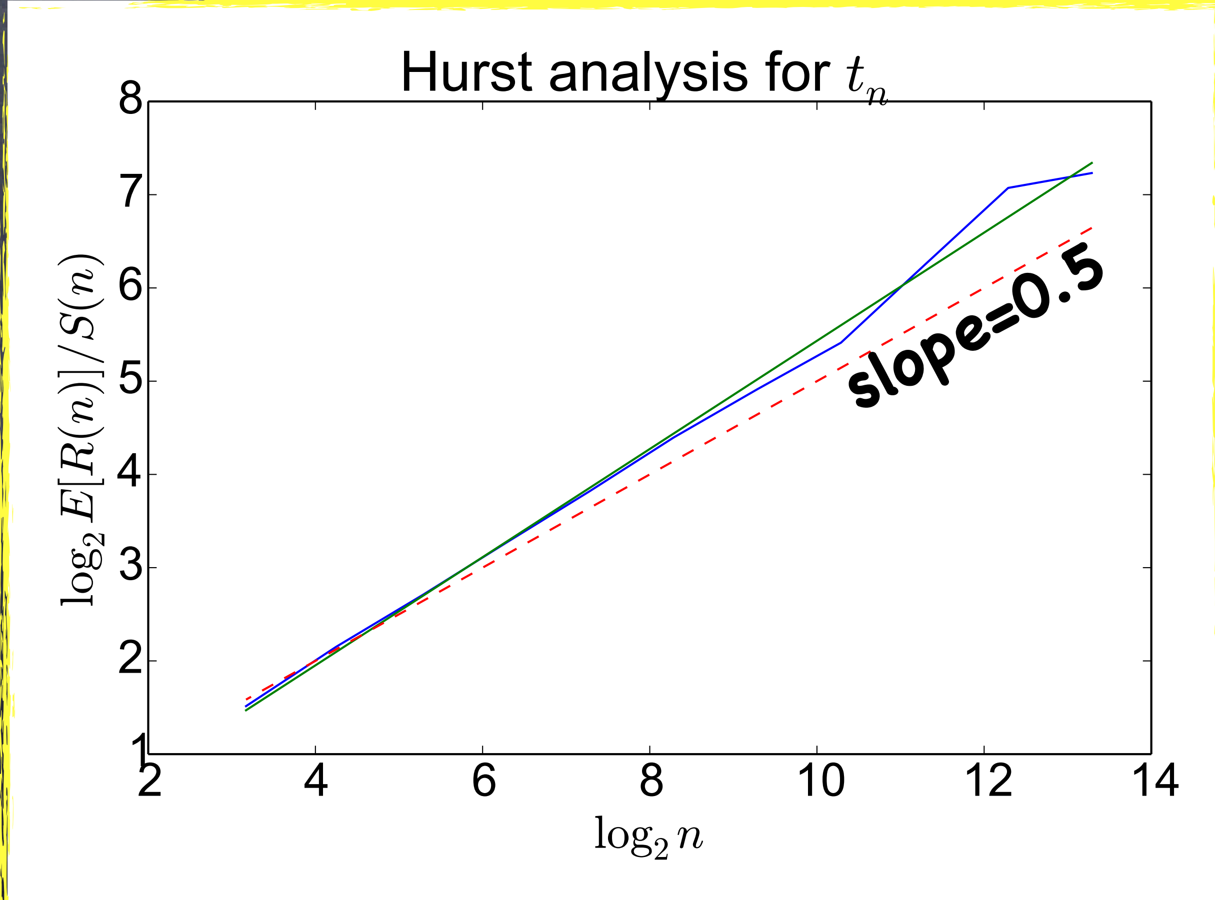
Seeking Low Dimensionality

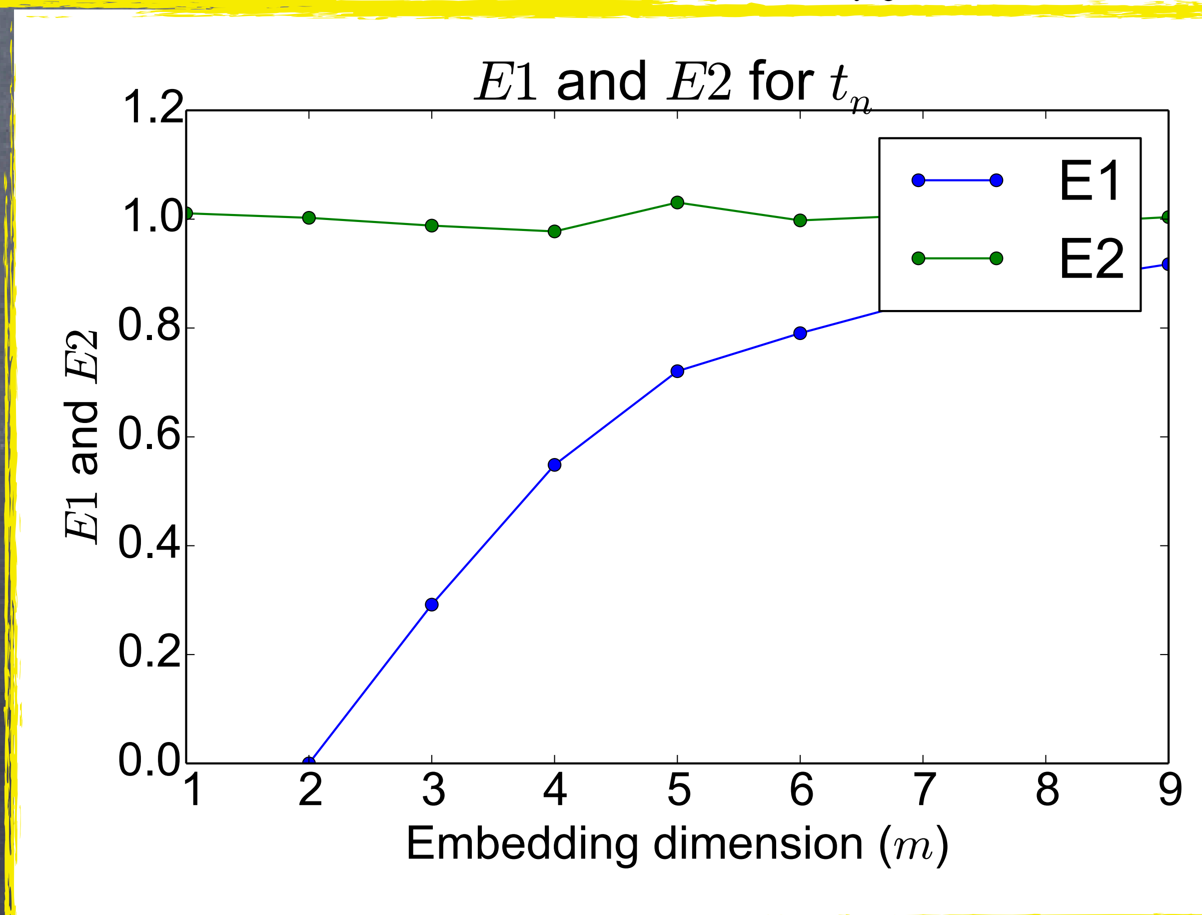
$$\dot{x} = \mu x - zy,$$
$$\dot{y} = -\nu x + zx,$$
$$\dot{z} = \Gamma - z + xy.$$

(Gissinger et al., 2010)









Conclusions

- There are two kinds of random reversals.
- Models should be constructed with Ra of convection in mind.
- Need to be careful with conclusions drawn from attractor reconstruction method.

...Thanks for your presence and patience!