Shell model for dynamo for extreme Prandtl numbers

Mahendra K. Verma
Indian Institute of Technology Kanpur

Collaborators: Rohit Kumar (iitk)
Thanks to Abhishek Kumar (iitk)

mkv@Iitk.ac.in
http://turbulence.phy.iitk.ac.in

Mag energy grows with time at large length scales

Conditions for dynamo: Rm > 1

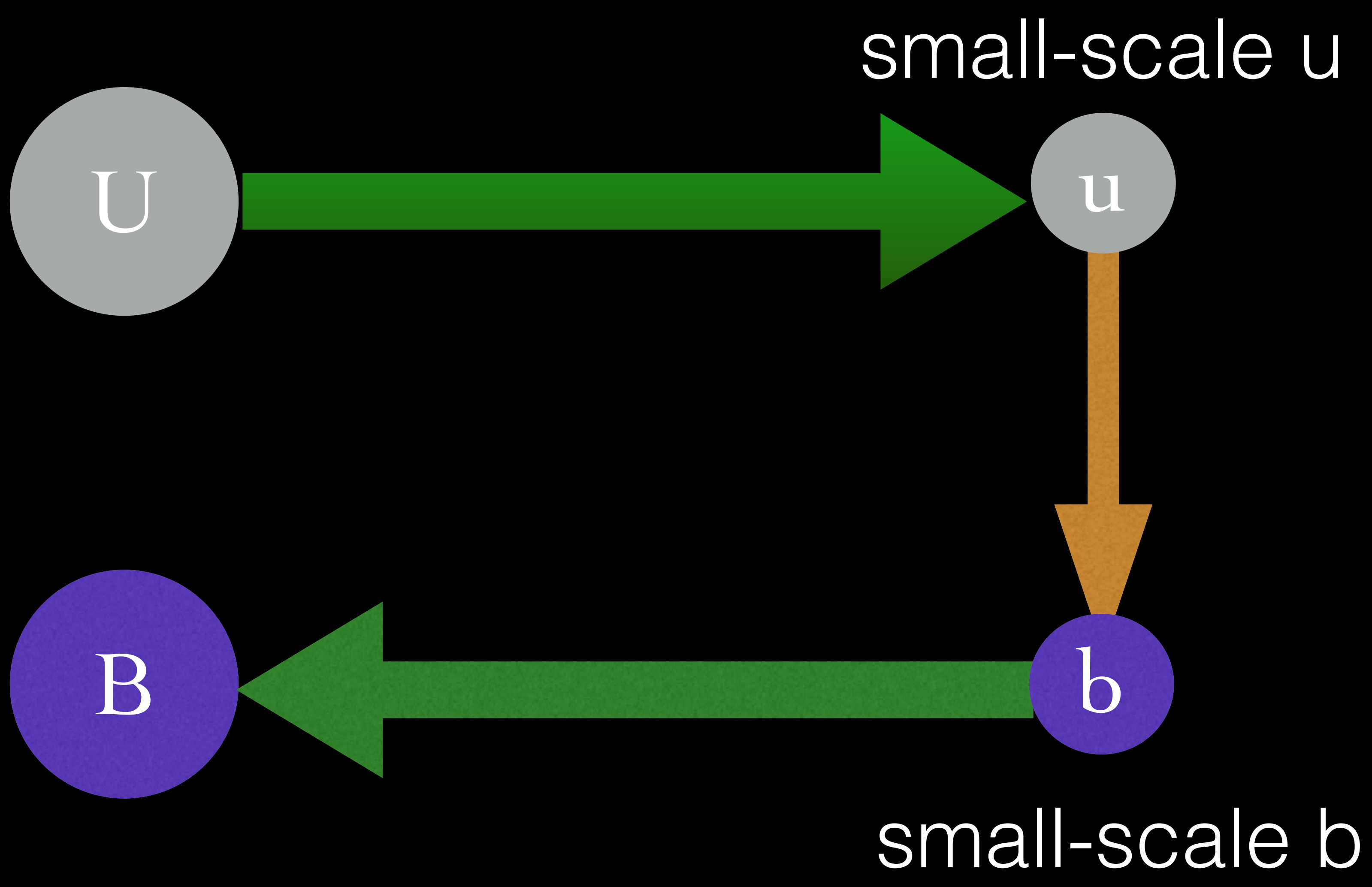
Schekochihin, Isakov, Proctor, Cowley, 2004-2007
Ponty et al., 2005, 2007

How does large-scale B field grow?

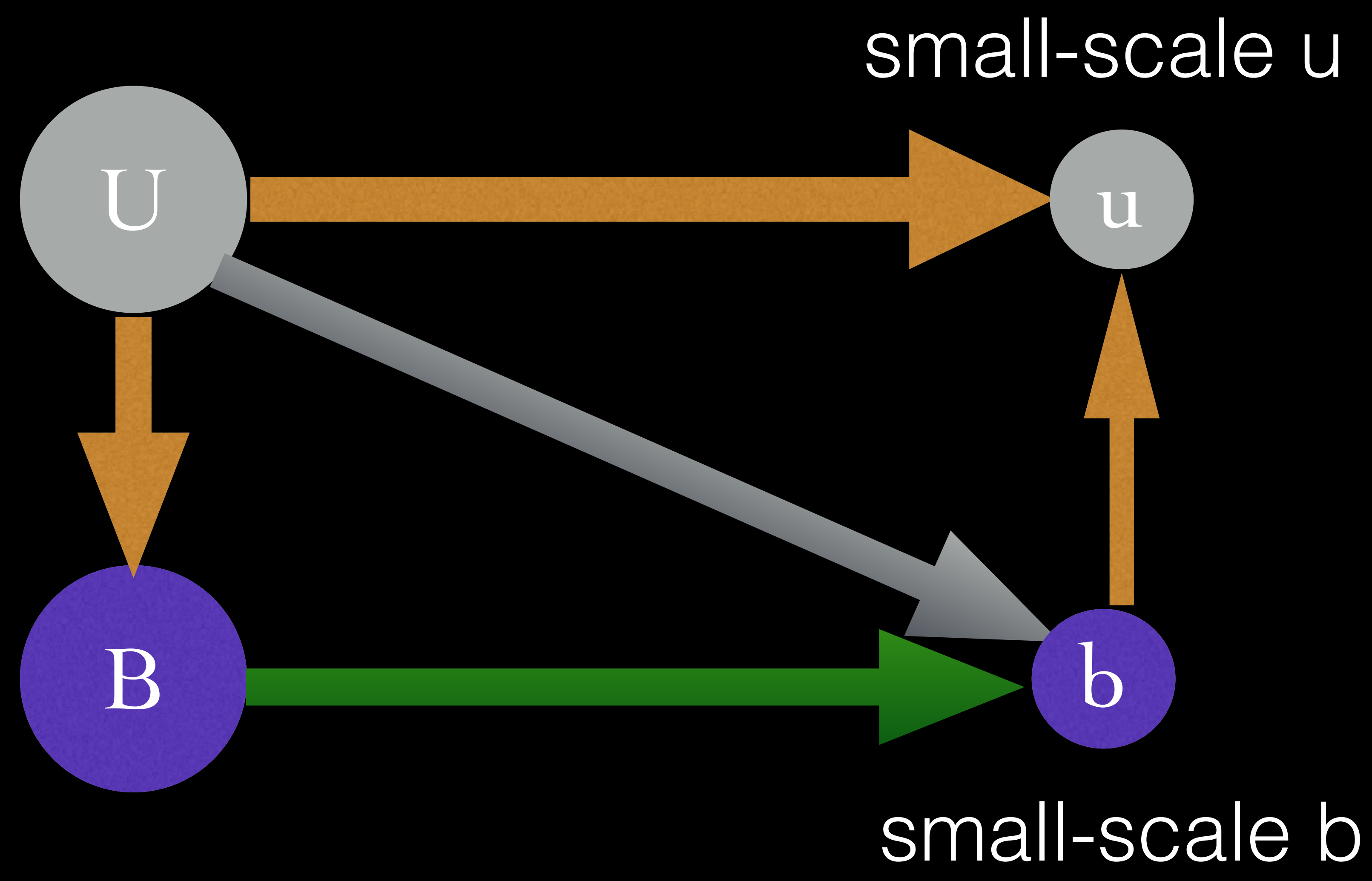
Inverse cascade of B?

Stapanov & Plunian, 2006, 2007, 2012

Energy Transfers in dynamo?

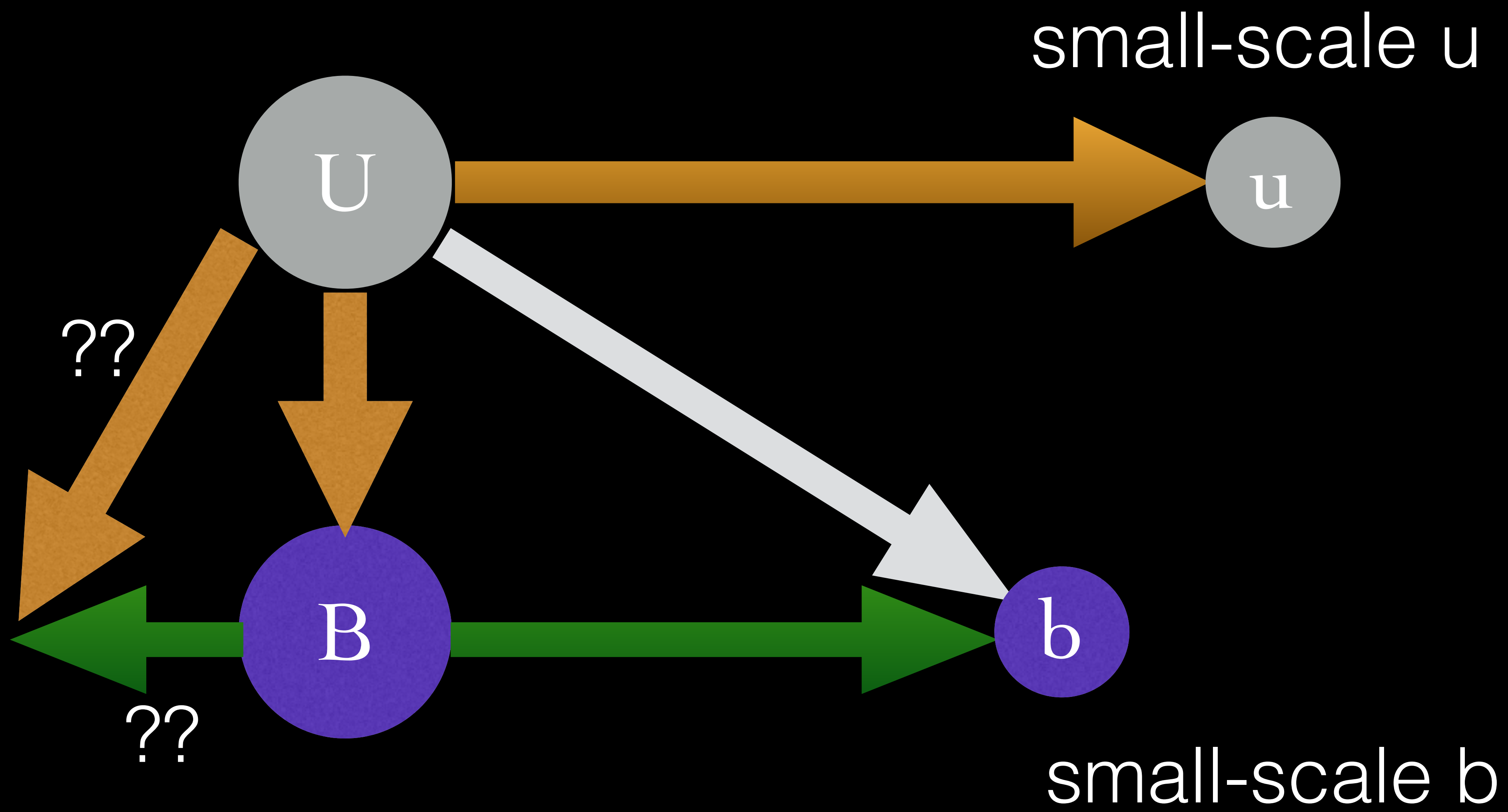


MHD turbulence (Pm=1)



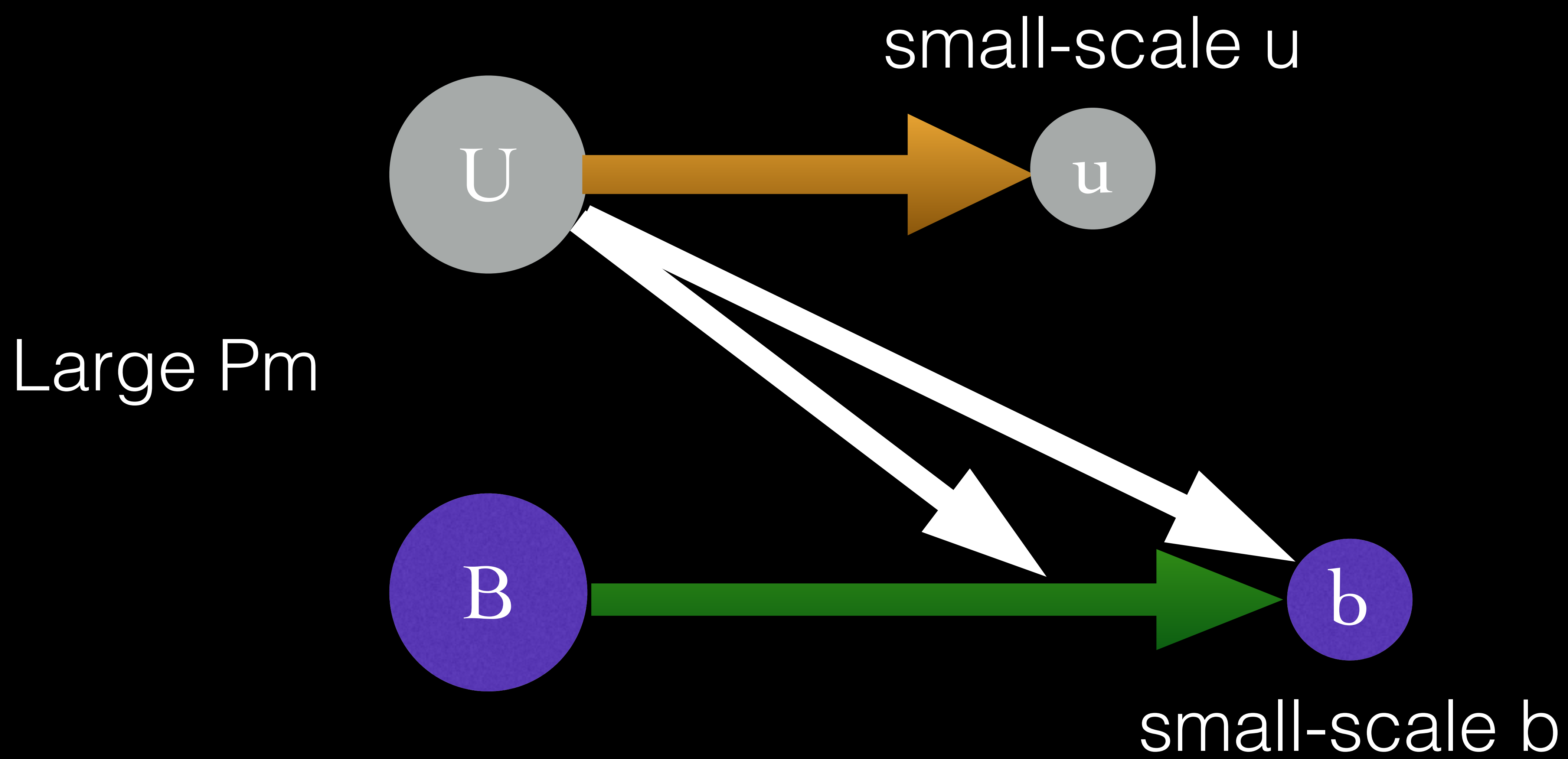
Dar et al. 2001; Verma 2004; Debliquy et al. 2005

Small Pm dynamo



Geodynamo

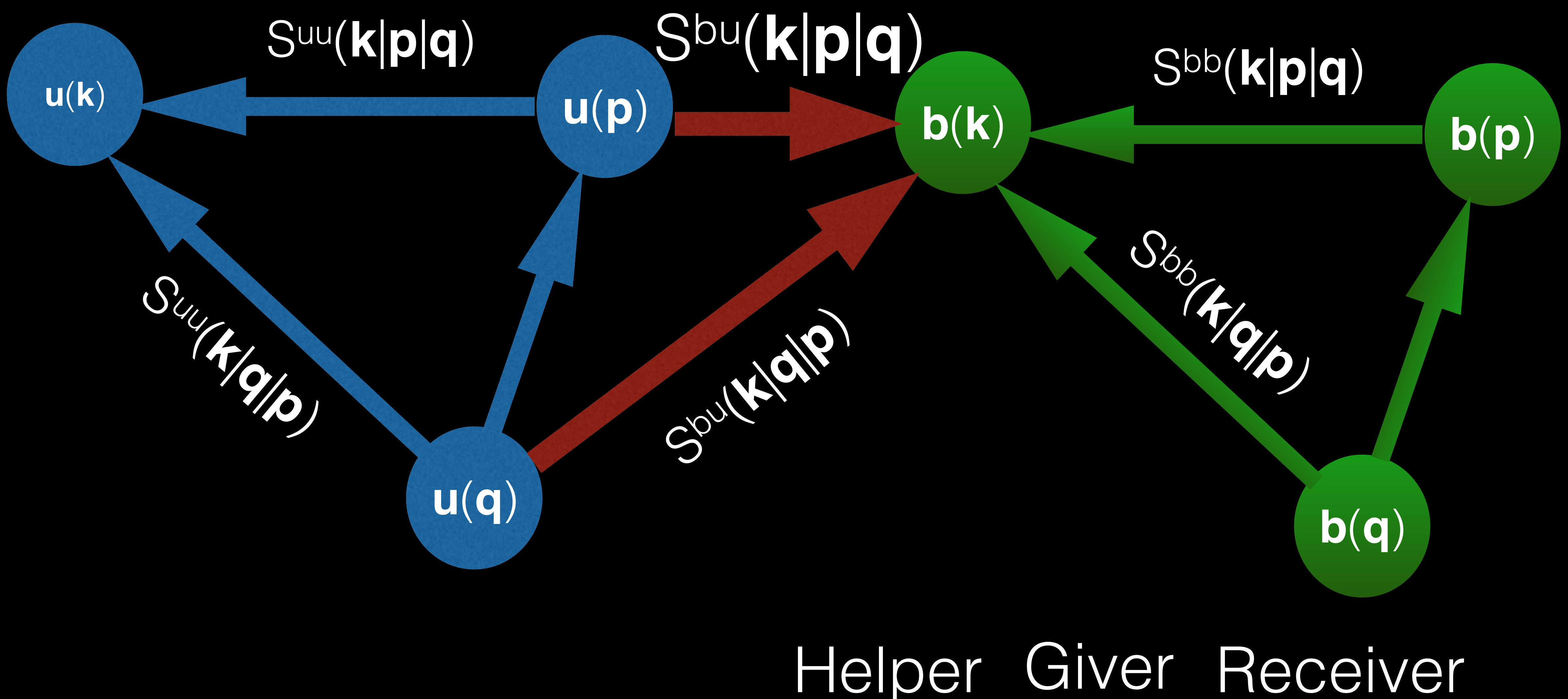
Large Pm dynamo



Galactic dynamo

Solar: both LSD & SSD

ET in MHD



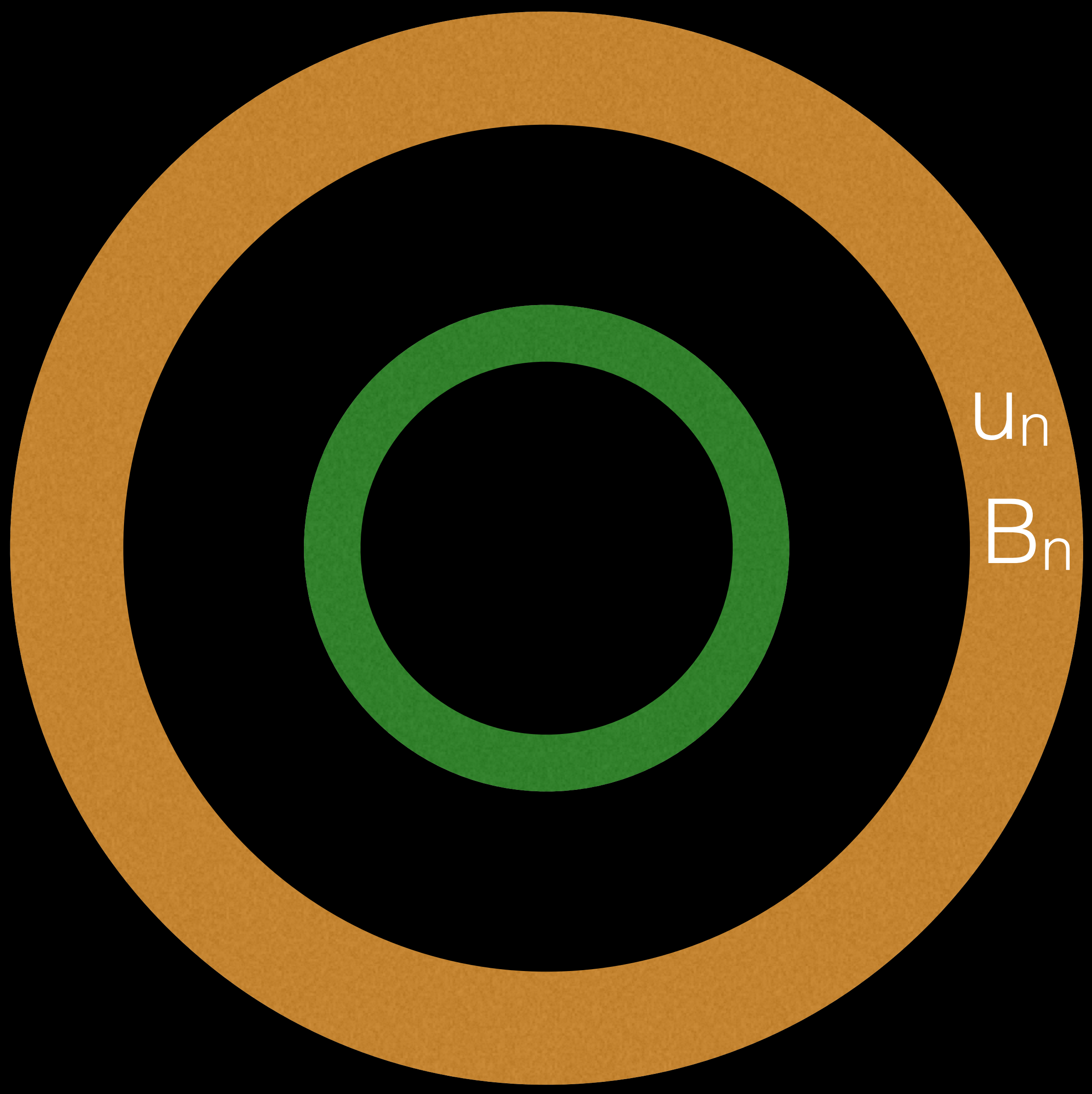
$$S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q}) = \mathrm{Im}\{[(\mathbf{k}\cdot\mathbf{u}(\mathbf{q})][(\mathbf{u}(\mathbf{p})\cdot\mathbf{u}^*(\mathbf{k})]\}$$

$$S^{bb}(\mathbf{k}|\mathbf{p}|\mathbf{q}) = \mathrm{Im}\{[(\mathbf{k}\cdot\mathbf{u}(\mathbf{q})][(\mathbf{b}(\mathbf{p})\cdot\mathbf{b}^*(\mathbf{k})]\}$$

$$S^{bu}(\mathbf{k}|\mathbf{p}|\mathbf{q}) = -\mathrm{Im}\{[(\mathbf{k}\cdot\mathbf{b}(\mathbf{q})][(\mathbf{u}(\mathbf{p})\cdot\mathbf{b}^*(\mathbf{k})]\}$$

GOY Shell model for MHD

Earlier models: Biskamp, Stepanov & Plunian, Pandit et al., Lessiness et al., Plunian et al. (2013)


u_n
B_n

$$\mathbf{u}\cdot\nabla\mathbf{u} \quad \mathbf{b}\cdot\nabla\mathbf{b}$$

$$\frac{dU_n}{dt} = N_n[U,U] + N_n[B,B] - \nu k_n^2 U_n + F_n$$

$$\frac{dB_n}{dt} = N_n[U,B] + N_n[B,U] - \eta k_n^2 B_n,$$

$$\mathbf{u}\cdot\nabla\mathbf{b} \quad \mathbf{b}\cdot\nabla\mathbf{u}$$

U2U channel: $\Re\left[U_n^* N_n(U,U)\right] = 0$

B2B channel: $\Re\left[B_n^* N_n(U,B)\right] = 0$

U2B+B2U channel: $\Re\left[U_n^* N_n(B,B) + B_n^* N_n(B,U)\right] = 0$

$$N_n[U,U] = -i(a_1 k_n U^*_{n+1} U^*_{n+2} + a_2 k_{n-1} U^*_{n+1} U^*_{n-1} + a_3 k_{n-2} U^*_{n-1} U^*_{n-2})$$

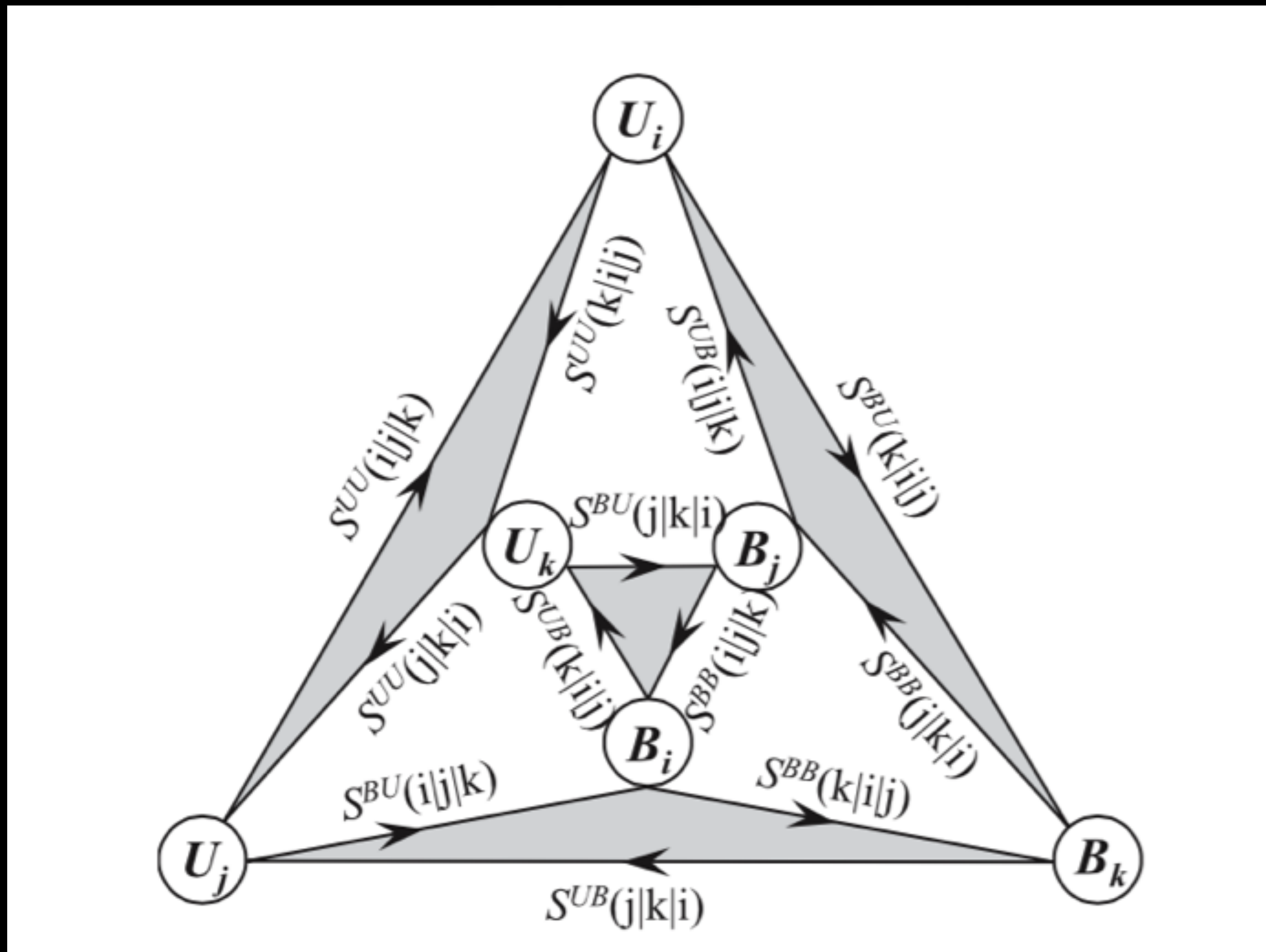
$$\begin{aligned} N_n[U,B] = -i[&k_n(d_1 U^*_{n+1} B^*_{n+2} + d_3 B^*_{n+1} U^*_{n+2}) \\ &+ k_{n-1}(-d_3 U^*_{n+1} B^*_{n-1} + d_2 B^*_{n+1} U^*_{n-1}) \\ &+ k_{n-2}(-d_1 U^*_{n-1} B^*_{n-2} - d_2 B^*_{n-1} U^*_{n-2})] \end{aligned}$$

$$\mathrm{N}_n[\mathrm{B},\mathrm{B}] = -2i(b_1 k_n B^*_{n+1} B^*_{n+2} + b_2 k_{n-1} B^*_{n+1} B^*_{n-1} + b_3 k_{n-2} B^*_{n-1} B^*_{n-2})$$

$$\begin{aligned} N_n[B,U] = i[&k_n(b_2 U^*_{n+1} B^*_{n+2} + b_3 B^*_{n+1} U^*_{n+2}) \\ &+ k_{n-1}(b_3 U^*_{n+1} B^*_{n-1} + b_1 B^*_{n+1} U^*_{n-1}) \\ &+ k_{n-2}(b_2 U^*_{n-1} B^*_{n-2} + b_1 B^*_{n-1} U^*_{n-2})] \end{aligned}$$

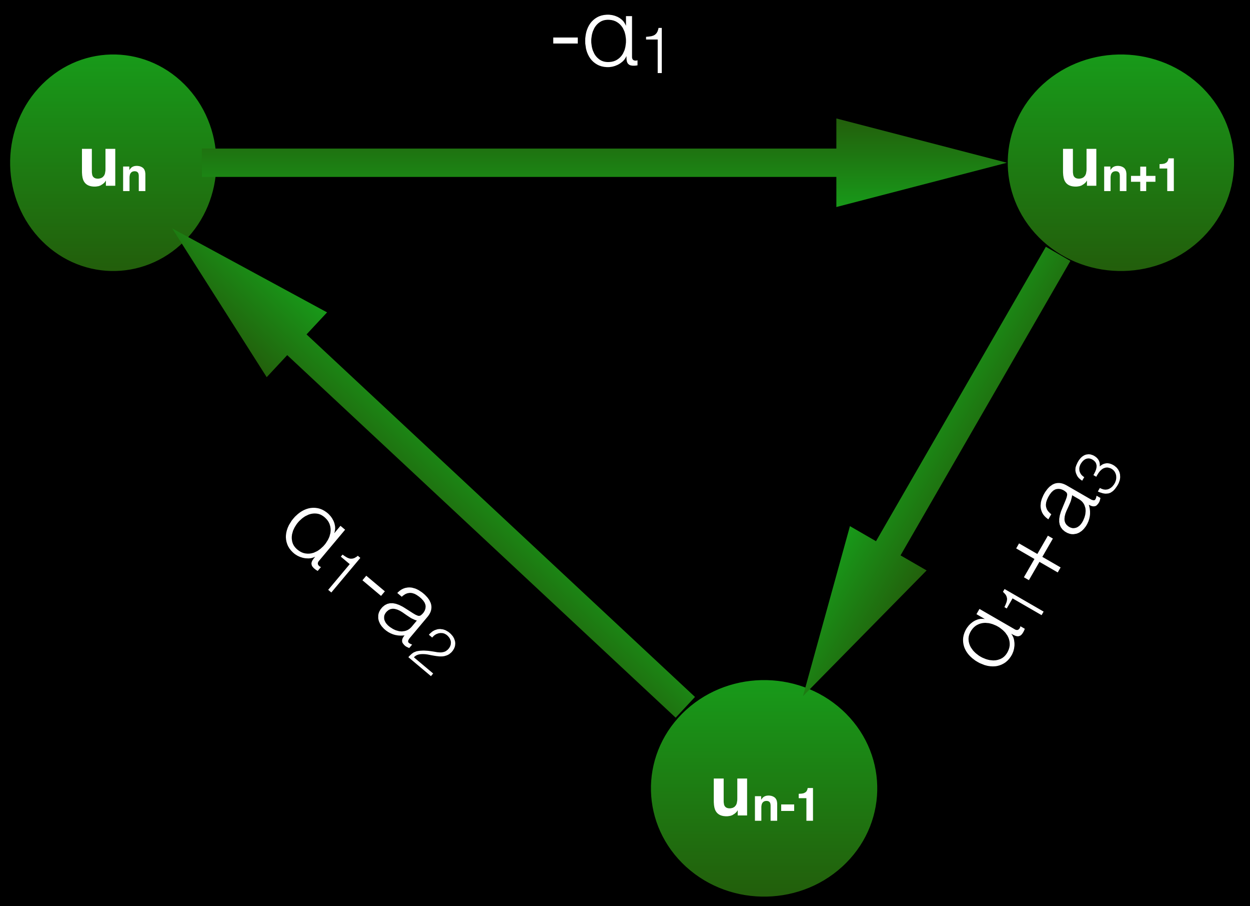
Coefficients determined by conservation laws, except one free parameter

Choice of coefficients a bit different from earlier models

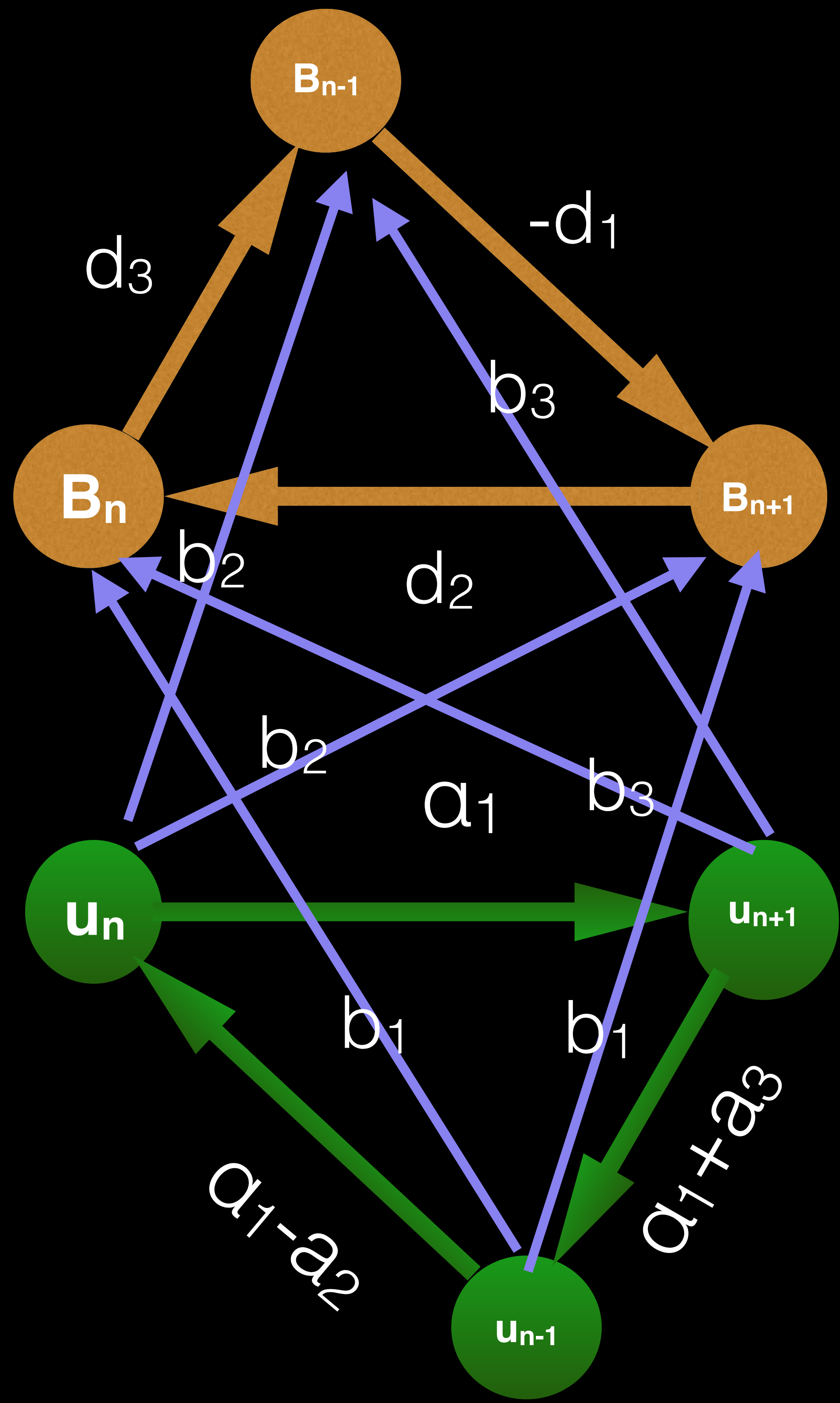


Plunian et al. (2013)

Shell2shell energy transfer



$$P_U^{UU} = -k_{n-1}\,\mathrm{Im}\{U^*_{n-1}U^*_n U^*_{n+1}\}$$

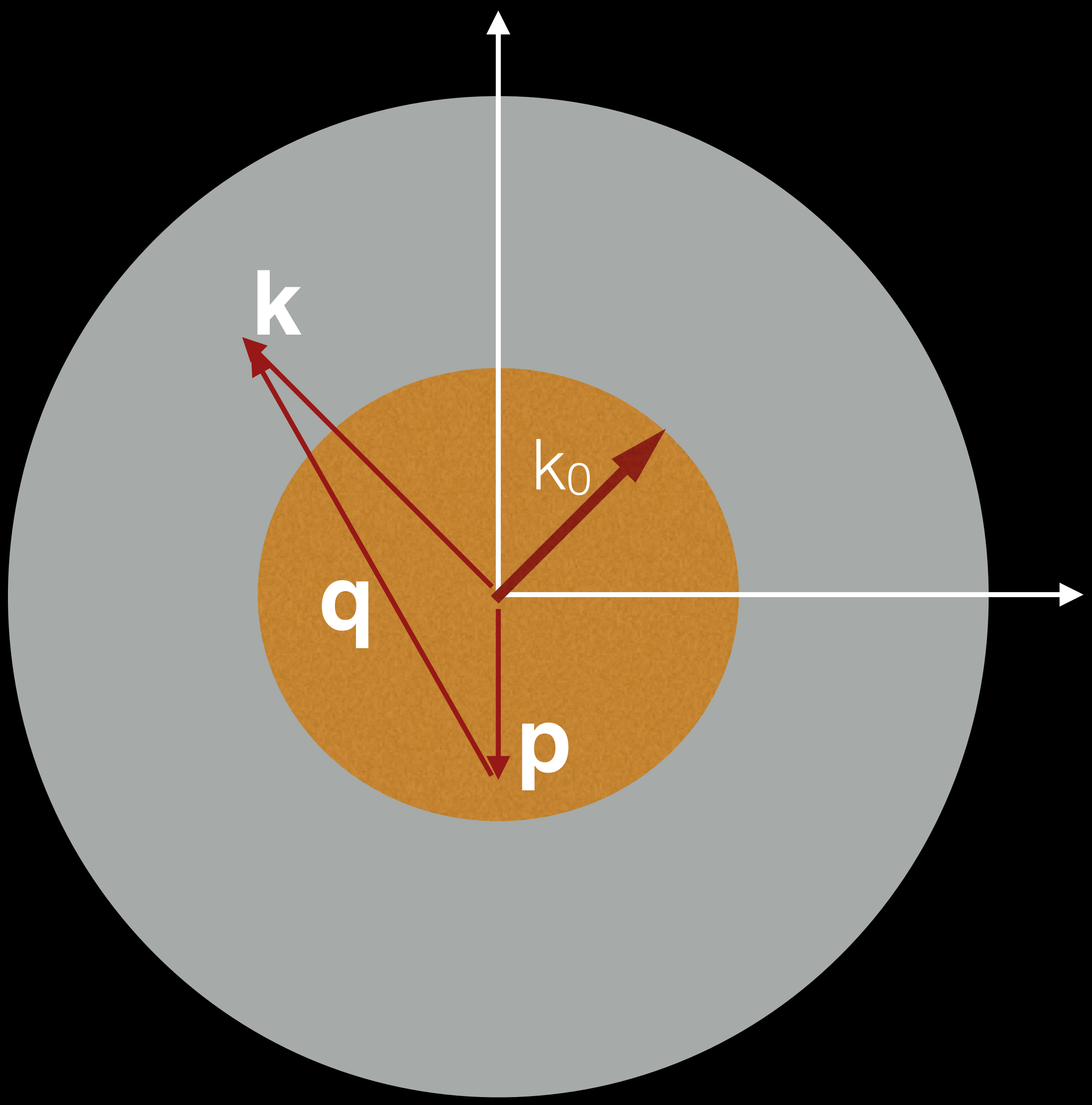


MHD ET

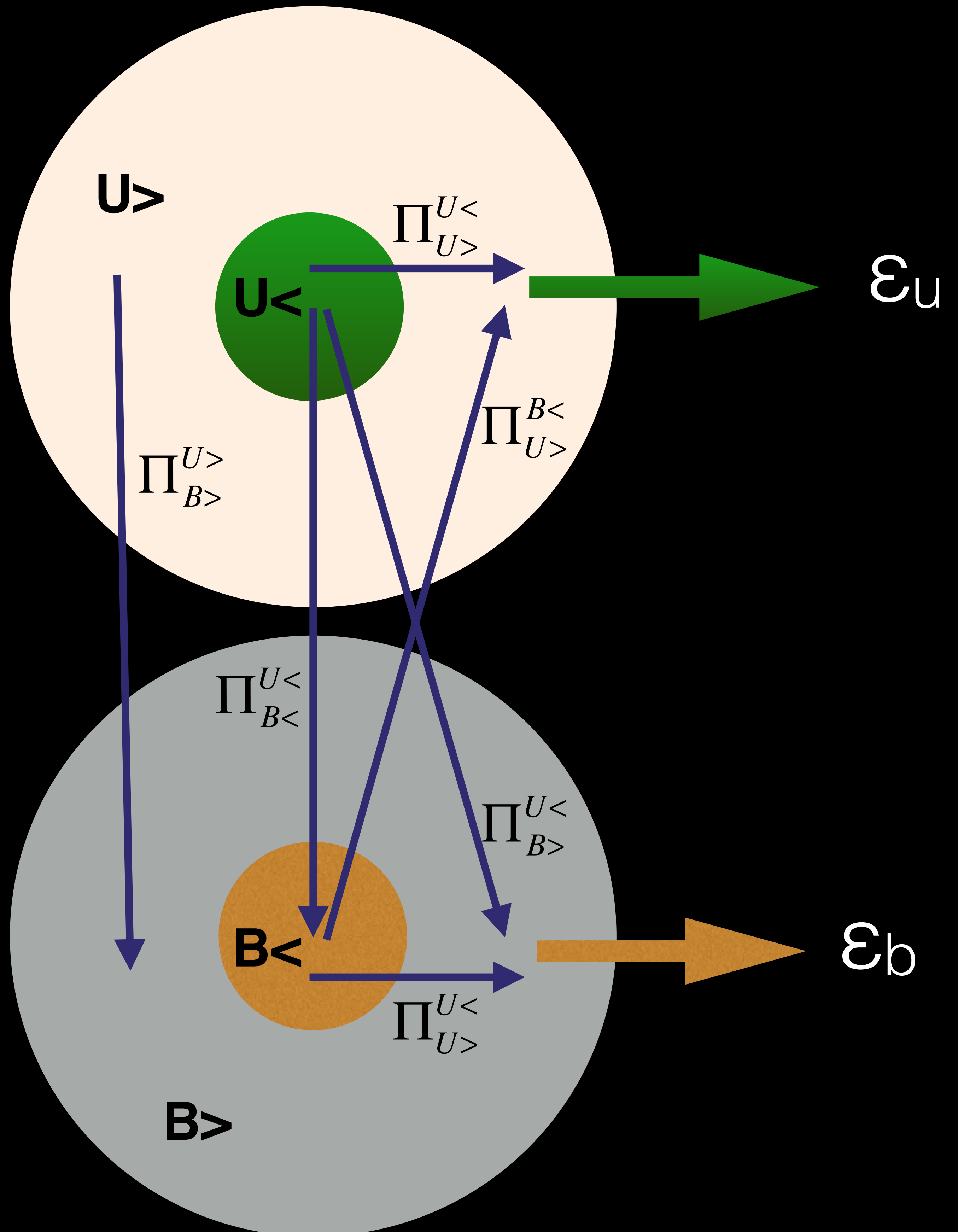
m: giver
n: receiver
p: helper

$$P_Z^{YX}(n\,|\,m\,|\,p) = -k_{\min}(n,m,p)\,\mathrm{Im}\{Y_n^* X_m^* Z_p^*\}$$

Flux in MHD



$$\Pi^{YX}(K) = \sum_{m \le K} \sum_{n > K} \sum_{p} S^{YX}(n \mid m \mid p)$$

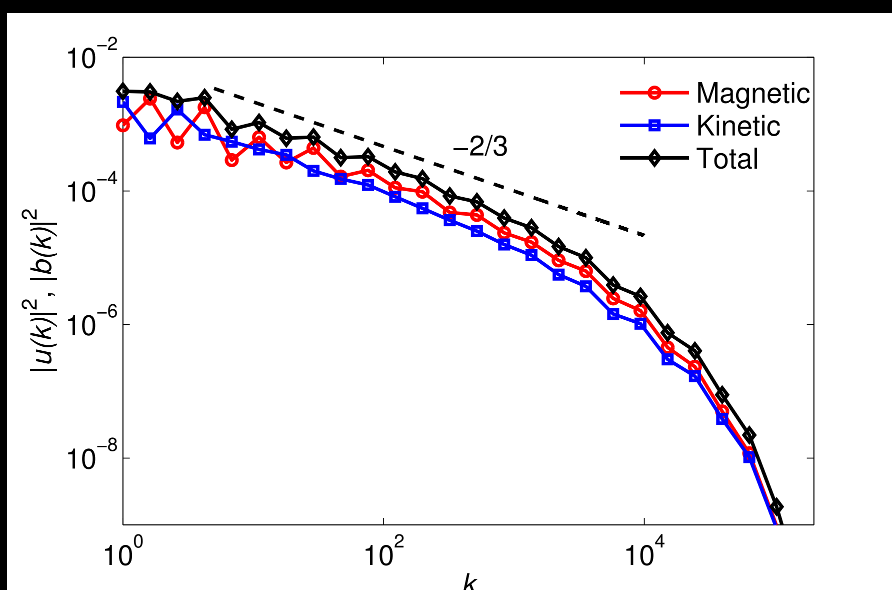

U>
U<
$\Pi_{U>}^{U<}$
ε_u
$\Pi_{U>}^{B<}$
$\Pi_{B>}^{U>}$
$\Pi_{B<}^{U<}$
$\Pi_{B>}^{U<}$
B<
ε_b
$\Pi_{U>}^{U<}$
B>

Simulations

- No of shells = 36
- Time stepping: RK4 method
- Both forcing & decaying simulations
- Random forcing at n=[3,4,5] [Stepanov & Plunian, 2006]

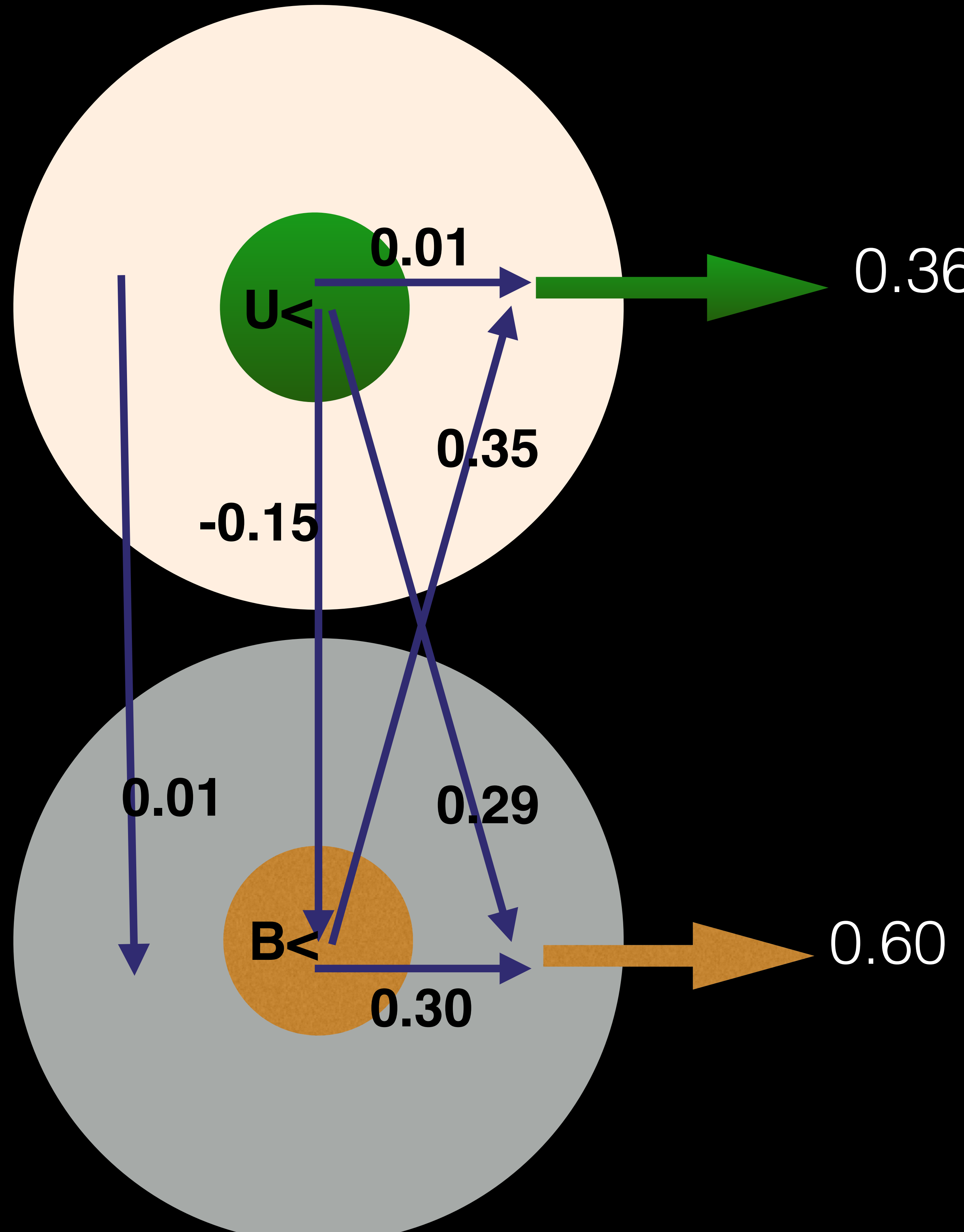
	Decaying	Forced 1	Forced 2	Forced 3
ν	10^{-6}	10^{-6}	10^{-9}	10^{-6}
η	10^{-6}	10^{-6}	10^{-6}	10^{-9}
Pm	1	1	10^{-3}	10^{3}
Re	3.2×10^{5}	9.3×10^{6}	9.2×10^{9}	9.7×10^{6}
Rm	3.2×10^{5}	9.3×10^{6}	9.2×10^{6}	9.7×10^{9}
$r_A = E_U/E_B$	0.5	1.5	1.47	1.58
$\varepsilon_U/\varepsilon_B$				

Decaying MHD (Pm=1)



Magnetic
Kinetic
Total
−2/3
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{0}
10^{2}
10^{4}
k
$|u(k)|^2, |b(k)|^2$

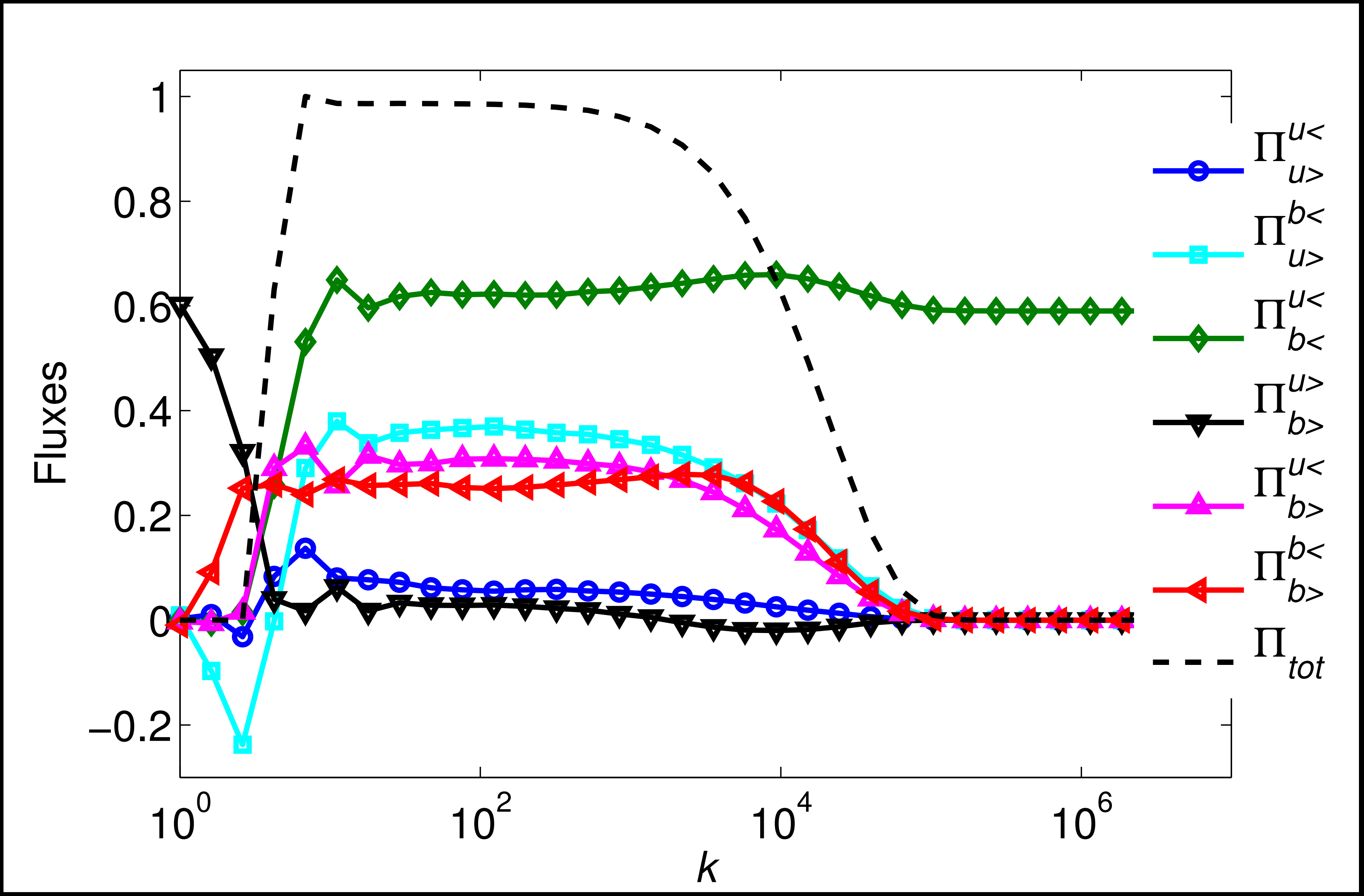
$r_A=0.5$



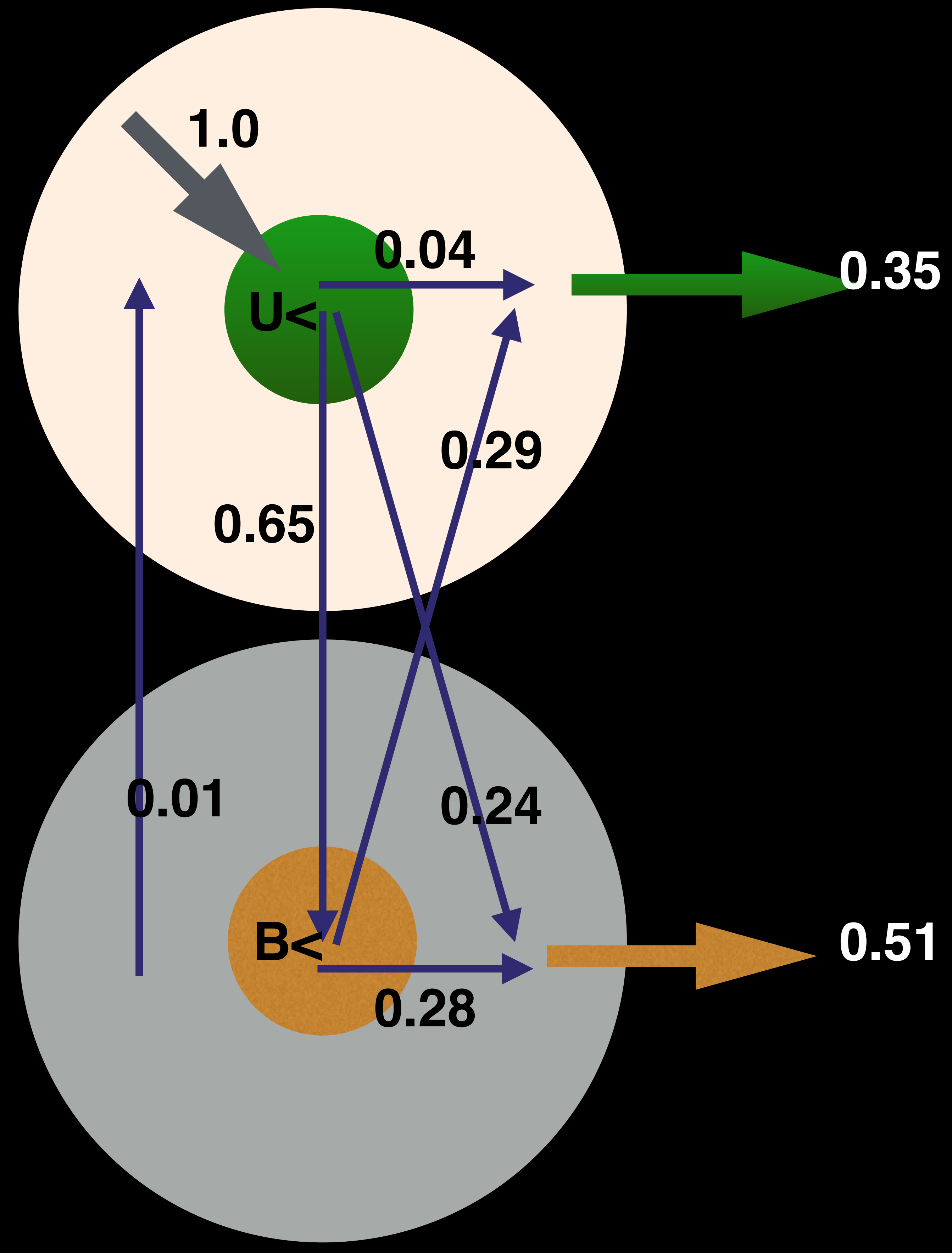
U<
0.01
0.36
0.35
-0.15
0.01
0.29
B<
0.60
0.30

Flux	$r_A = 0.6$ (Deb, DNS) (Decaying)	$r_A = 0.4$ (Deb, DNS) (Decaying)	$r_A = 0.5$ (shell model) (Decaying)	$r_A = 1.50$ (shell model) (Forced)
$\Pi^{u<}_{u>}$	0.073	0.066	0.01	0.04
$\Pi^{u<}_{b>}$	0.49	0.49	0.29	0.24
$\Pi^{b<}_{u>}$	0.13	0.13	0.35	0.29
$\Pi^{b<}_{b>}$	0.36	0.34	0.30	0.28
$\Pi^{u<}_{b<}$	−0.024	−0.12	−0.15	0.65
$\Pi^{u>}_{b>}$	0.22	0.22	0.01	−0.01
ϵ_ν	–	–	0.36	0.35
ϵ_η	–	–	0.60	0.51

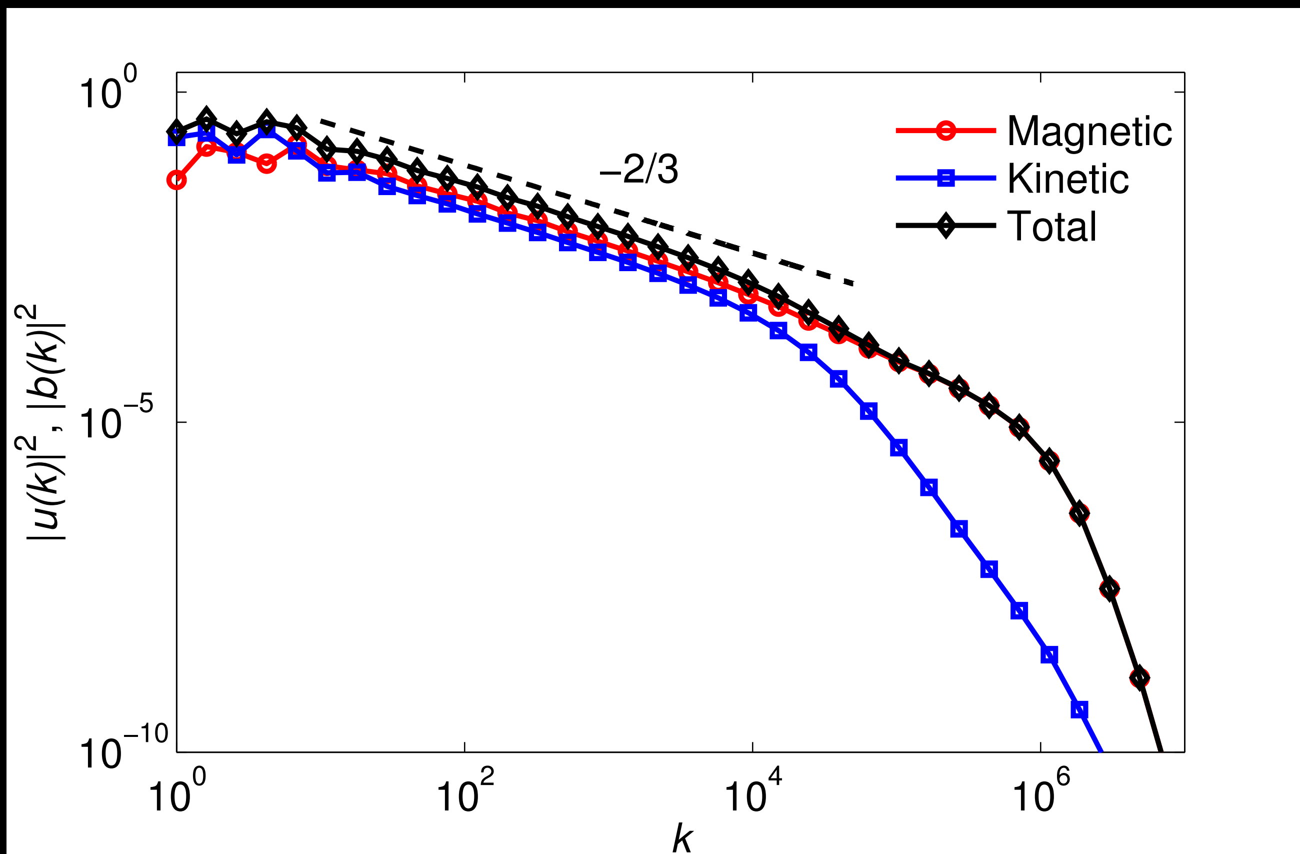
Forced MHD (Pm=1)



Fluxes
k
1
0.8
0.6
0.4
0.2
0
−0.2
10^0
10^2
10^4
10^6
$\Pi^{u<}_{u>}$
$\Pi^{b<}_{u>}$
$\Pi^{u<}_{b<}$
$\Pi^{u>}_{b>}$
$\Pi^{u<}_{b>}$
$\Pi^{b<}_{b>}$
Π_{tot}

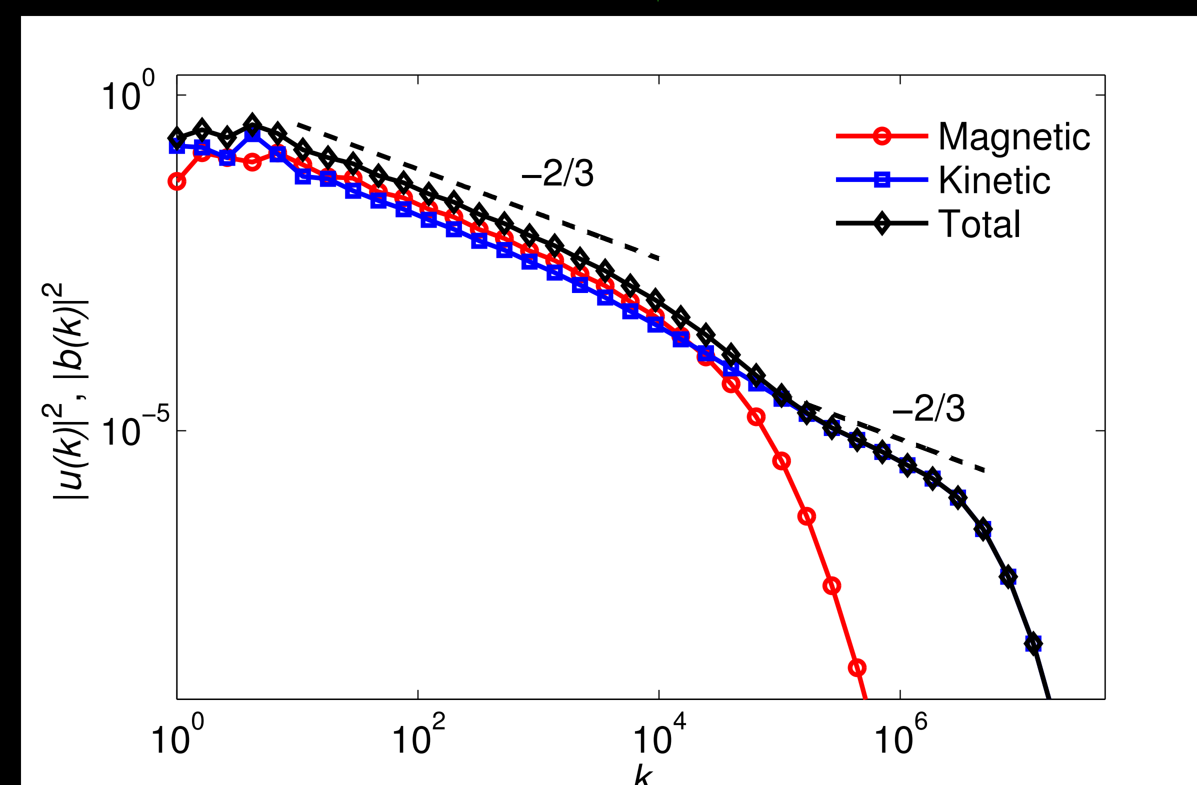

1.0
0.04
0.35
U<
0.29
0.65
0.01
0.24
B<
0.51
0.28

Forced MHD
($Pm=10^{-3},10^{3}$)



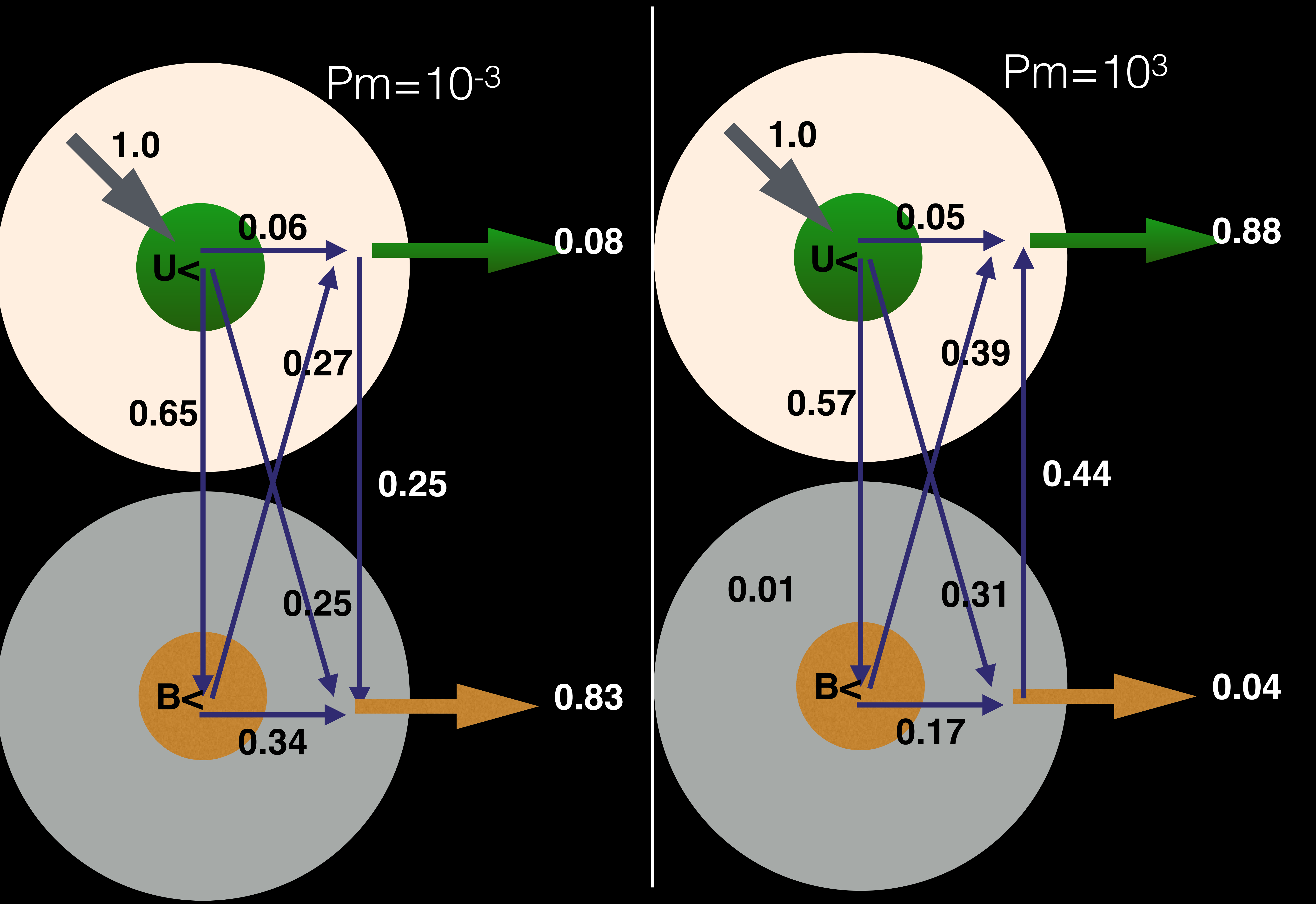
Magnetic
Kinetic
Total
−2/3
|u(k)|^2, |b(k)|^2
10^0
10^−5
10^−10
10^0
10^2
10^4
10^6
k

Pm = 10^{-3}



Magnetic
Kinetic
Total
−2/3
−2/3
|u(k)|^2, |b(k)|^2
10^0
10^−5
10^0
10^2
10^4
10^6
k

Pm = 10^3



Pm=10-3
1.0
0.06
U<
0.08
0.27
0.65
0.25
0.25
B<
0.83
0.34
Pm=103
1.0
0.05
U<
0.88
0.39
0.57
0.44
0.01
0.31
B<
0.04
0.17

Past work

- Lessiness, Carati, and Verma (2009): ET formalism developed.. but $\Pi_{B>}^{B<} < 0$
- Coefficients different since Hk conservation relaxed in our model.
- Extensive review by Plunian et al. (Phys Rep. 2013)…Shell2shell transfers.
- Sahoo et al. (2010): different coefficients

Conclusions

Our shell model reproduces approximately the fluxes of DNS (Pr=1).

Limitation: local energy transfers ONLY

U2U, B2B, and U2B energy transfers are forward

The asymptotic steady-state Eu/Eb is 1.5 for the forced MHD (Plunian et al. 2014)

For large Pm or large ν, $\varepsilon_U >> \varepsilon_B$

For small Pm or large η, $\varepsilon_U << \varepsilon_B$