Planetary Magnetism
3. Magnetic fields and simulations

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Magnetoconvection

How is the convection affected by the presence of an imposed magnetic field?

Linear theory: flow close to onset is slow, so the imposed field is not much changed. There is a first order perturbation to basic field.

The Lorentz force, $\mathbf{j} \times \mathbf{B}$, opposes the stretching of field lines: similar to vortex stretching.

Plane layer problem: magnetic field delays the onset of convection and can lead to oscillatory modes.

MHD equations

The MHD equations are

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \mu \mathbf{j}, \tag{1,2}$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \cdot \mathbf{E} = \frac{\rho_c}{\epsilon}. \tag{3,4}$$

Ohm's law relates current density to electric field. This depends on material, so empirical. In a material at rest

$$\mathbf{j} = \sigma \mathbf{E},$$

in a moving frame

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}). \tag{5}$$

In very intense fields, the Hall effect can change this and in some plasmas, ambipolar diffusion becomes significant.

Dividing Ohm's law (5) by σ and taking the curl and using (1)

$$\nabla \times (\frac{\mathbf{j}}{\sigma}) = \nabla \times \mathbf{E} + \nabla \times (\mathbf{u} \times \mathbf{B}) = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{u} \times \mathbf{B}),$$

and using (2) to eliminate **j**,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times \eta(\nabla \times \mathbf{B}), \tag{6}$$

where $\eta = 1/\mu\sigma$ is the magnetic diffusivity (units $\mathrm{m}^2\mathrm{s}^{-1}$). (6) is the induction equation.

Convection with Magnetic Field and Rotation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\mathbf{j} \times \mathbf{b} + g\alpha T' \mathbf{e_z} + \nu \nabla^2 \mathbf{u}.$$

Surprisingly, magnetic field can reduce the critical Rayleigh number in rotating convection. In rapidly rotating convection, viscosity breaks the Proudman-Taylor constraint: very thin columns.

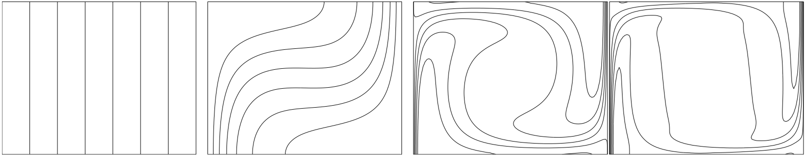
Magnetic field can also break the Proudman-Taylor constraint, and because magnetic field can act over long distances, **thick** columns are possible.

This means the Rayleigh number can be lower. There is an optimal magnetic field strength to minimise Ra.

$$\Lambda = \frac{B^2}{\mu \rho \Omega \eta}$$

is the Elsasser number, and minimum Ra occurs for $\Lambda \sim 1$.

Flux expulsion



An initially uniform field stirred by convection quickly expels the magnetic field out of the roll, so effective magnetic field inside the roll much weaker.

A Dynamic Elsasser number

$$\Lambda = \frac{B^2}{\mu \rho \Omega U_* \ell}$$

where U_* is the typical velocity and ℓ the roll-width, may be more appropriate.

Spherical geometry

Fields considered are azimuthal fields (Fearn, 1979, Jones et al. 2003) and axial fields (Sakuraba, 2002). Also studies in annulus and plane geometry.

Magnetic fields can reduce the critical *Ra* by the Lorentz force counteracting the PT constraint, just as in plane layers.

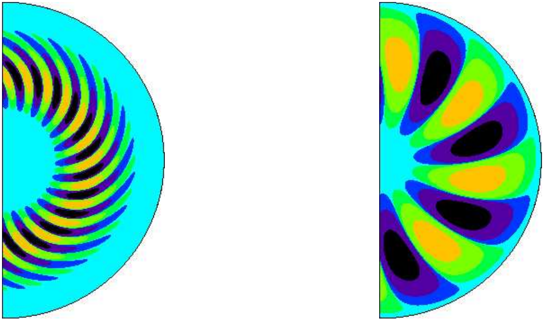
Magnetic fields reduce the critical value of m at onset. Important, as very thin columns won't give a dynamo.

Convenient azimuthal field is $B = B_0 s\ \hat{\phi}$ in cylindrical coordinates (s, ϕ, z).

This is called the Malkus field.

It has uniform current $\mathbf{j} = 2B_0\hat{\mathbf{z}}$.

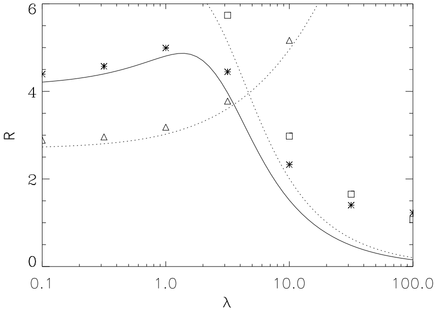
Magnetic fields reduce m



$\Lambda = 0.01,\ E = 10^{-6}$ $Pr = Pm = 1$

$\Lambda = 0.31,\ E = 10^{-6}$ $Pr = Pm = 1$

Onset of convection with magnetic field $B = B_0 s\ \hat{\phi}$.
Elsasser number $\Lambda = B_0^2/\rho\Omega\mu\eta$.
Note that even a modest azimuthal field greatly expands the columns. Axial field behaves similarly, but requires stronger field.

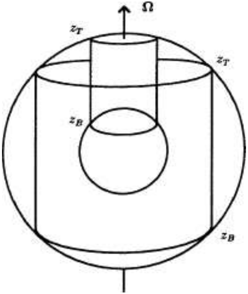


Solid line and *,
$Pr = 1$, $Pm = 0.5$

Dashed lines, triangles and squares, $Pr = 0.5$, $Pm = 1$.

$R = RaE^{-4/3}$, $E = 10^{-6}$ for points, $E \to 0$ for curves

Onset of convection with magnetic field $B = B_0 s\, \hat{\phi}$.
Elsasser number $\Lambda = \lambda/100 = B_0^2/\rho\Omega\mu\eta$.
Strong fields reduce the critical Rayleigh number, so enhance convection.



Coriolis term is zero, because no net flow across the cylinder. Reynolds stress small.

The last term comes from the viscous friction at the boundaries, which is produced by Ekman suction. It is small, because E is small.

Lorentz force term must be small, so positive/negative parts of integrand must cancel.

$$\frac{\partial}{\partial t}\int \rho u_\phi \, ds + \int 2\rho u_s \Omega \, ds = \int (\mathbf{j} \times \mathbf{B})_\phi \, ds - 2\pi s \frac{u_\phi (2E)^{1/2}}{(1-s^2)^{1/4}}$$

With uniform magnetic field $\mathbf{B_0}$ and constant temperature gradient T_0' we look for local wave-like solutions

$$\mathbf{u} = \mathbf{u_0} \exp i(\mathbf{k} \cdot \mathbf{x} - \omega t).$$

Linearised equations are

$$\frac{\partial \mathbf{u}}{\partial t} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\nabla p' + g\alpha T' \hat{\mathbf{r}} + \frac{1}{\rho} \mathbf{j} \times \mathbf{B_0} + \nu \nabla^2 \mathbf{u}$$

$$\frac{\partial \mathbf{b}}{\partial t} = (\mathbf{B_0} \cdot \nabla)\mathbf{u} + \eta \nabla^2 \mathbf{b}, \qquad \mu \mathbf{j} = \nabla \times \mathbf{b},$$

$$\frac{\partial \mathbf{T}'}{\partial t} = -u_r T_0' + \kappa \nabla^2 \mathbf{T}'$$

Dispersion relation for rotating MHD waves

Ignoring diffusion and buoyancy, we get

$$4\omega^2(\mathbf{\Omega}\cdot\mathbf{k})^2 = \left(\frac{(\mathbf{B_0}\cdot\mathbf{k})^2}{\mu\rho} - \omega^2\right)^2 \mathbf{k}^2$$

$$\omega = \frac{(\mathbf{\Omega}\cdot\mathbf{k}) \pm \sqrt{(\mathbf{\Omega}\cdot\mathbf{k})^2 + \mathbf{k}^2(\mathbf{B_0}\cdot\mathbf{k})^2/\mu\rho}}{|k|}$$

Define

$$\omega_C = \frac{2(\mathbf{\Omega}\cdot\mathbf{k})}{|k|}, \quad \omega_M = \frac{(\mathbf{B_0}\cdot\mathbf{k})}{(\mu\rho)^{1/2}}$$

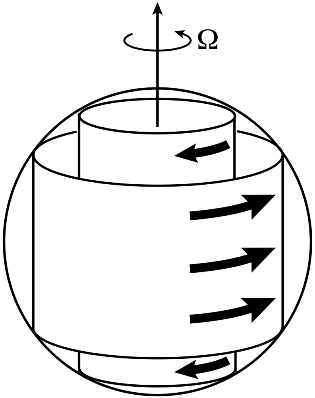
and note that in planets $\omega_C >> \omega_M$.

Taking plus sign, we get $\omega \approx \omega_C$, the fast inertial waves. taking minus sign, we get the slow wave

$$\omega_{MC} = \frac{\omega_M^2}{\omega_C}$$

Periods of rotating MHD waves

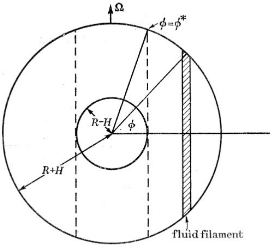
In the core, ω_M gives a period about 6 years, ω_C is typically hours or days, so the ω_{MC} waves usually have periods of thousands of years. These waves evolve through magnetostrophic equilibria, inertia being negligible.



However, if $\mathbf{\Omega} \cdot \mathbf{k} = 0$, i.e. the waves have no z variation, they have a frequency of ω_M, these are the torsional Alfvén waves, or torsional oscillations, with period about 6 years.

These TO's can be seen in the secular variation and the length of day signal.

Magnetic Rossby waves



From the dispersion relation

$$\omega_{MC} = \frac{\omega_M^2}{\omega_C}$$

we see that to get faster MC-waves, we need to reduce ω_C.

This can be done by considering Rossby waves, that is waves coming from vortex stretching in the interior of a sphere.

The slow Rossby waves are called magnetic Rossby waves.

Nonmagnetic Rossby waves travel eastwards in planetary cores, but magnetic Rossby waves go westwards, with periods of hundreds of years.

They may be connected with the westward drift of the secular variation.

Dynamo models

Solve induction equation and the Navier-Stokes equations simultaneously. Most work has been done on convection-driven dynamos.

Anti-dynamo theorems tell us dynamo process is three-dimensional, so expect 3D magnetic fields.

(i) No dynamo can be maintained by a planar flow

$$(u_x(x, y, z, t), u_y(x, y, z, t), 0).$$

No restriction is placed on whether the field is 2D or not in this theorem.

(ii) Cowling's theorem. An axisymmetric magnetic field vanishing at infinity cannot be maintained by dynamo action.

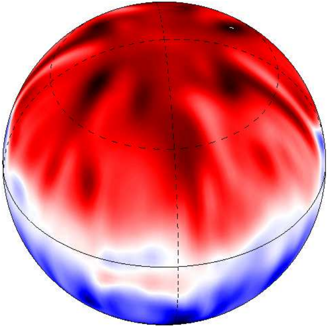
(iii) A purely toroidal flow, that is one with $\mathbf{u} = \nabla \times T\mathbf{r}$ cannot maintain a dynamo. Note that this means that there is no radial motion, $u_r = 0$.

In the Earth's core, the flow is clearly 3D, and rotating convection leads to 3D flows.

Simulations are therefore computationally demanding, and require clusters of processors for serious study. Only models close to critical can be studied with desktop machines.

Parallel programming (MPI) is required, and code construction is challenging.

Results from dynamo codes



$$Pr = Pm = 1, Ra = 750, E = 10^{-4}.$$

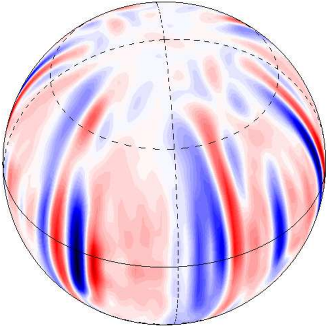
Radial magnetic field snapshot at the CMB

No internal heating. No-slip, fixed temperature, insulating boundaries.

Very dipolar, doesn't reverse. Field slightly weaker at the poles.

Intense flux patches at high latitudes.

Velocity field



$$Pr = Pm = 1, Ra = 750, E = 10^{-4}.$$

Radial velocity snapshot at

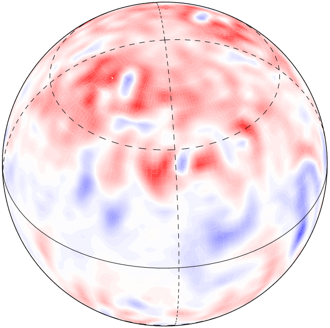
$r = 0.8r_{CMB}$.

Note the columnar nature of the convection rolls, local Rossby number small.

Intense flux patches at the top of these columnar rolls.

Pattern propagates westward.

Higher local Rossby number



$$Pr = Pm = 0.2, Ra = 750, E = 10^{-4}.$$

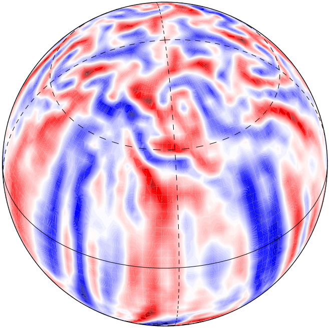
Radial magnetic field snapshot at the CMB

Much less dipolar. Field strength is weaker.

This type of dynamo can reverse.

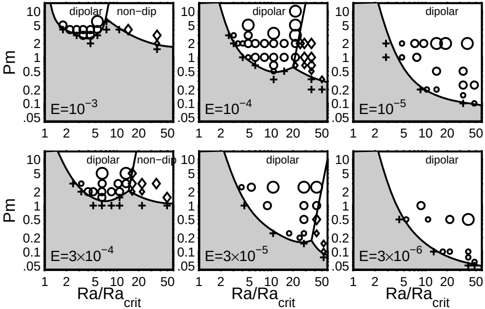
Rossby number is larger, and inertia is playing a significant role.

Flow at higher local Rossby number



$Pr = Pm = 0.2, Ra = 750, E = 10^{-4}.$

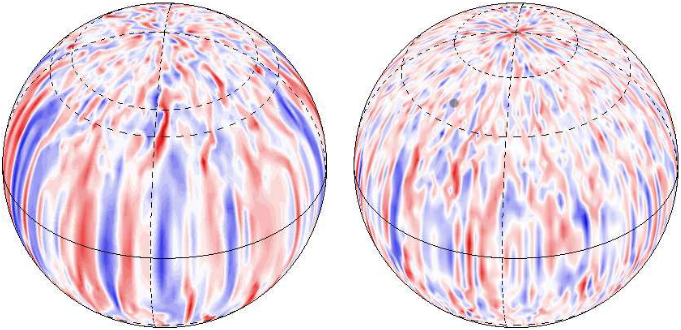
Radial velocity snapshot at
$r = 0.8 r_{CMB}$.
More activity near the poles,
less columnar convection rolls.
Between these patterns lies an Earth-like regime.



Christensen and Aubert 2006 summarise the dependence on Ekman number E and Pm.

At low E, low Pm dynamos are possible, provided Ra is large enough. Important, as liquid metals have low Pm.

Small E, low Pm dynamo, Flow pattern

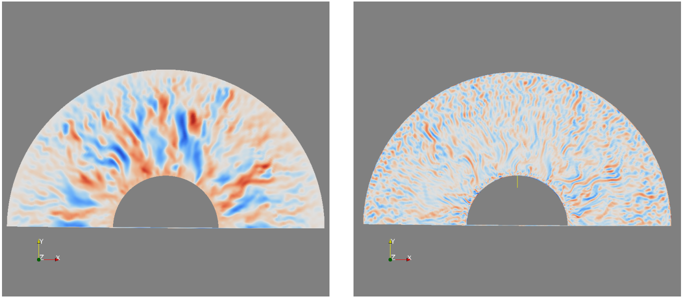


$$Pr = 1, Pm = 0.1, Ra = 50Ra_{crit}, E = 3 \times 10^{-6}.$$

Left: radial velocity at $r = 0.5r_0$. Right: radial velocity at $r = 0.8r_0$.

At lower E the convective columns are much thinner, particularly further out from the ICB.

Small E, low Pm dynamo, Scale separation



Left: radial magnetic field at $z = 0.2$. Right: vorticity at $z = 0.2$. Notice that the magnetic field is on a much larger scale in these low Pm calculations. Temperature fluctuations on same scale as magnetic field.

Scaling laws: magnetic field strength

In the Earth's core, magnetic energy is much greater than kinetic energy, so a simple balance as used in astrophysics won't work here.

We start with

Ohmic dissipation + Viscous dissipation = rate of working of buoyancy forces,

The rate of working work done by the buoyancy forces is

$$\int \rho g \alpha T' u_r \, dv$$

which can be written in terms of the heat flux.

Ignore the viscous dissipation and we obtain

$$\int \frac{g \alpha F_{conv}}{c_p} \, dv \sim \int \eta \mu \mathbf{j}^2 \, dv$$

Now need to relate magnetic energy to magnetic dissipation.

Christensen-Tilgner law

The dissipation time is the time taken for the magnetic energy to be dissipated through ohmic loss,

$$\tau_{diss} \int \eta\mu\mathbf{j}^2 \, dv = \int \mathbf{B}^2/2\mu \, dv$$

Equivalently, the magnetic dissipation length

$$\delta_B = \left(\frac{\tau_{diss}}{\eta}\right)^{1/2}.$$

Christensen and Tilgner (Nature, 2004) proposed that

$$\delta_B \sim d Rm^{-1/2}$$

mainly on the basis of simulations and laboratory experiments. It also has some theoretical support, because at high Rm flux ropes of this thickness are formed.

Predicted field strength

We now have

$$\eta\mu\mathbf{j}^2 \sim \eta\frac{(\nabla \times \mathbf{B})^2}{\mu} \sim \frac{\eta\mathbf{B}^2}{\mu\delta_B^2} \sim \frac{g\alpha F}{c_p},$$

giving

$$B_* \sim \left(\frac{g\alpha F_{conv}\mu d}{U_* c_p}\right)^{1/2}.$$

With the inertial theory scaling for U_* this gives

$$B_* \sim \mu^{1/2} d^{2/5} \rho^{1/5} \Omega^{1/10} \left(\frac{g\alpha F}{c_p}\right)^{3/10}.$$

Remarkable feature is weak dependence of B on Ω. We are assuming though that the planet is in the rapidly rotating low Ro regime.

$$\frac{D\mathbf{u}}{Dt} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\frac{1}{\rho}\nabla p' + \mathbf{g}\frac{\rho'}{\rho} + \frac{1}{\rho}\mathbf{j} \times \mathbf{B} + \mathbf{F_v} \qquad (1)$$

$$p = p_0(r) + p', \quad \rho = \rho_0(r) + \rho', \quad dp_0/dr = -g\rho_0.$$

Entropy $S = c_p\left(\frac{1}{\gamma}\ln p - \ln\rho\right)$, $\qquad p = \mathcal{R}\rho T$,

So provided $p'/p_0 \ll 1$, $\rho'/\rho_0 \ll 1$, $\quad S' = c_p\left(\frac{p'}{\gamma p_0} - \frac{\rho'}{\rho_0}\right)$.

$$-\frac{1}{\rho_0}\nabla p' + \mathbf{g}\frac{\rho'}{\rho_0} = -\nabla\left(\frac{p'}{\rho_0}\right) + \frac{p'}{\rho_0 c_p}\nabla S_0 - \frac{S'}{c_p}\mathbf{g} \qquad (2)$$

Anelastic approximation: entropy drop across layer is small,
$\Delta S/c_p = \epsilon \ll 1$.
All components of $\mathbf{u}$ of same order U_0. $\quad \partial/\partial t \sim U_0/d$.

$$(\rho_0 T_0)\left[\frac{DS'}{Dt} + u_r\frac{dS_0}{dr}\right] = \nabla\cdot\kappa\rho_0 T_0\nabla S' + Q_\nu + Q_j \qquad (3)$$

$u_r\frac{dS_0}{dr}$ is the source term for the entropy, so $S' \sim \epsilon c_p$.
Put this into the equation of motion, radial component gives $U_0^2/d \sim \epsilon g$, horizontal components give

$$p'/p_0 \sim \rho'/\rho_0 \sim T'/T_0 = O(\epsilon).$$

$U_0^2 \sim \epsilon g d, \qquad U_0^2/c^2 \sim \epsilon.$
In equation (2),

$$(p'/\rho_0 c_p)\nabla S_0 \sim \epsilon p'/\rho_0 d \sim \epsilon^2 g \ll (S'/c_p)g.$$

This term should therefore be omitted (Lantz/Braginsky & Roberts).
$\frac{\partial\rho'}{\partial t} \ll \nabla\cdot\rho_0\mathbf{u}$ so the continuity equation is

$$\nabla\cdot\rho_0\mathbf{u} = 0 \qquad (5)$$

Anelastic equations 3.

In dimensionless units,

$$\frac{E}{Pm}\frac{D\mathbf{u}}{Dt} + 2\hat{\mathbf{z}} \times \mathbf{u} = -\nabla\left(\frac{p'}{\rho_0}\right) + \frac{1}{\rho_0}(\nabla \times \mathbf{B}) \times \mathbf{B} + \mathbf{F}_\nu + \frac{RaEPmS\hat{\mathbf{r}}}{Pr\, r^2}$$

Toroidal/poloidal representation, $\rho_0\mathbf{u} = \nabla \times \rho_0\mathcal{T}\mathbf{r} + \nabla \times \nabla \times \rho_0\mathcal{P}\mathbf{r}$. The r-component of the curl and double curl of the equation of motion give the equations for $\mathcal{T}$ and $\mathcal{P}$. p' does not appear. Entropy equation takes form

$$\frac{DS}{Dt} = \frac{Pm}{Pr}\zeta^{-n-1}\nabla \cdot \zeta^{n+1}\nabla S + \frac{Di}{\zeta}\left[E^{-1}\zeta^{-n}(\nabla \times \mathbf{B})^2 + Q_\nu^*\right]$$

with $Di = \dfrac{c_1 Pr}{Pm\, Ra}$. Note that turbulent entropy diffusion assumed to dominate laminar temperature diffusion, because small-scale turbulence mixes entropy not temperature.

Difference between Jupiter and Geodynamo models

(i) Stress-free outer boundary $\rightarrow$ much stronger zonal flows, particularly near equator

(ii) Non-conducting outer region $\rightarrow$ magnetic field dissipates there

(iii) Uniform heating (entropy) source, rather than driving near inner core: enhances dynamo in outer regions, weaker near the core

(iv) Relatively smaller inner core

(v) Large density variations: similar effect to (iii)

Dipoles rare for Jupiter dynamo models: Why?

Zonal flow near equator $\rightarrow$ strong equatorial fields which migrate and disrupt the dipole

If Pr is large, zonal flow is weak, but thermal wind develops $\rightarrow$ hemispherical dynamos with field almost all in one hemisphere

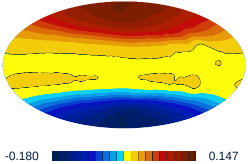
How to overcome these effects?

(i) Put driving near core, and keep Rm near critical. Not very Jupiter-like

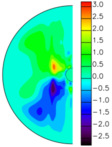
(ii) Go for strong field dynamo, magnetic braking strong enough to keep zonal flow small. Seems to work, but

(a) Needs large Ra, therefore very small E to maintain columnar convection. Computationally very demanding.

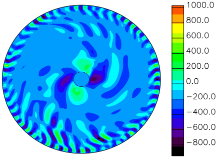
(b) Need Pr small to get strong convection everywhere (linear theory hint).



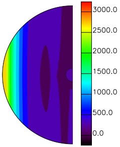
Left: Radial B_r at surface



Right: Axisymmetric part of B_ϕ in meridional plane



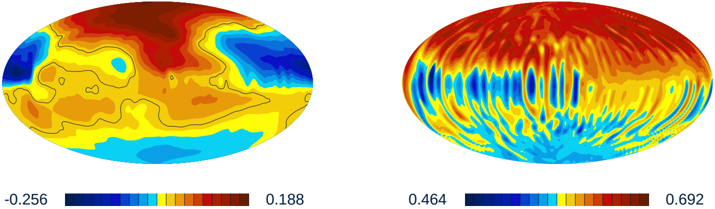
Left: Equatorial section u_r



Right: Axisymmetric part of u_ϕ in meridional plane

$E = 2 \times 10^{-5}$, $Ra = 2.4 \times 10^7$, $Pr = 0.25$, $Pm = 3$. Fixed inner core temperature, fixed flux outer boundary. Significant heat flux from inner core.

Hemispherical Dynamo: 1



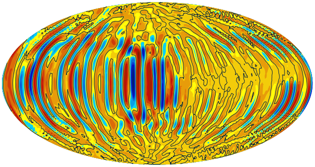
Left: Snapshot of B_r at surface   Right: Entropy at $r = 0.75R_J$

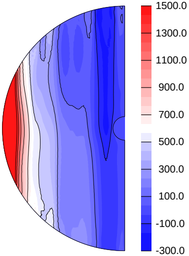
$Ra = 5e07$, $Pr = 1$, $Pm = 3$, $E = 2.5e - 05$. Fixed entropy bc, with uniform internal heating. Small heat flux going in from core, larger heat flux out through surface. Note N. Hemisphere hotter.

Hemispherical dynamo much more common in anelastic models than Boussinesq models, especially at $Pr = 1$.

Magnetic field and entropy asymmetry are linked.

Hemispherical Dynamo: 2



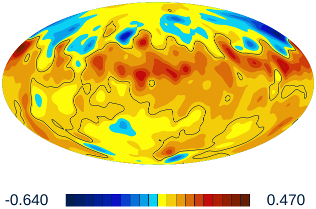


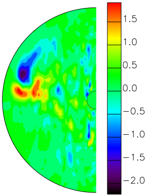
Left: u_r at $r = 0.75R_J$ Right: Axisymmetric meridional u_ϕ

Asymmetric entropy between the hemispheres drives a thermal wind: $2\Omega\partial u_\phi/\partial z = -(1/r)dT_0/dr\,\partial S/\partial\theta$ so the zonal flow is asymmetric. Dynamo waves propagating along the shear are therefore skewed: southern hemisphere waves propagate into insulating region and die?

Magnetic field in northern hemisphere enhances convection there, which leads to a higher entropy there?

Small scale dynamo (Multipolar)





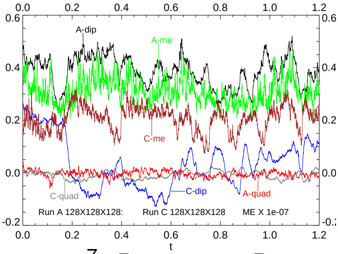
Left: B_r at surface Right: Axisymmetric meridional B_ϕ

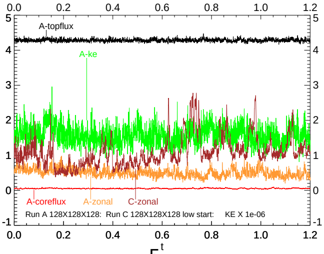
$E = 5 \times 10^{-5}$, $Ra = 1.5 \times 10^7$, $Pr = 0.25$, $Pm = 3$.

There is an active dynamo, but magnetic field is very messy. As with the hemispherical dynamo, B_ϕ is largest in the shear zone.

Larger E, so not rotationally dominated. Local Rossby number $U/\Omega\ell$, where ℓ is the dominant convective length scale, not sufficiently small. Inertial terms dominate over Coriolis terms.

Message is that small E is unfortunately essential for dipolar fields!





$Ra = 1.1 \times 10^7$, $Pr = 0.10$, $Pm = 3.0$, $E = 2.5 \times 10^{-5}$.

Run A has run for over two diffusion times, and is robust in that moderate changes of Ra and E still give a stable dipole. Small quadrupole component. Core flux small compared to top flux. Strong field controls zonal flow.

Bistable: run C starts from a small field, and is a dynamo, but not dipolar. Zonal flow stronger. At $E = 1.5 \times 10^{-5}$, $Ra = 2 \times 10^7$ dipole solution grows from small seed field.

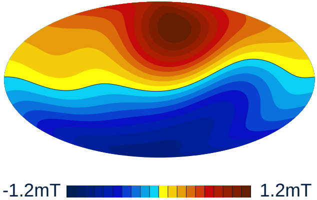
Why is the dipole maintained?

To maintain the dipole, need large *Ra* to promote vigorous convection. This generates strong magnetic field everywhere.

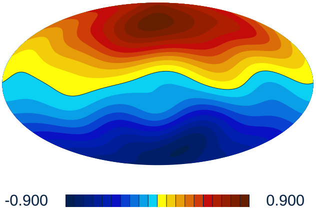
Why doesn't this give $\alpha - \omega$ behaviour? The field pushes the zonal flow into the non-conducting region (magnetic braking). Balance between zonal flow production and braking is crucial.

But *E* must be small enough for the local Rossby number $U/\Omega\ell$ to be small, where ℓ is the convective length scale. Also helps to have low *Pr*, as this gives more symmetic entropy and reduces thermal wind.

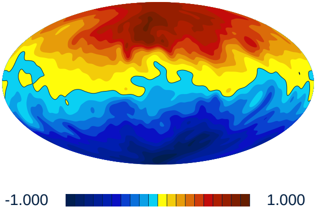
$Pr = 0.1,\ E = 2.5 \times 10^{-5}$ solution snapshot



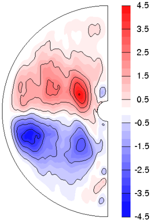
Jupiter surface radial B_r



Model radial B_r snapshot truncated at harmonic $\ell \leq 5$

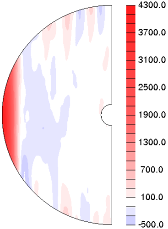


Model radial B_r (full resolution)

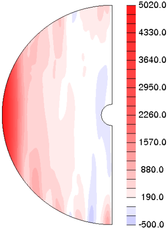


Axisymmetric meridional B_ϕ

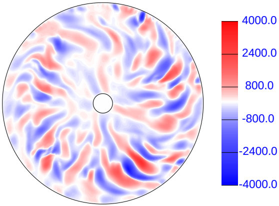
Flow for $Pr = 0.1$ solutions: units of Rm



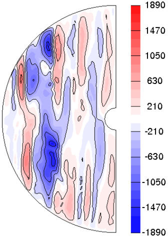
Axisymmetric part of u_ϕ run A



Axisymmetric part of u_ϕ run C

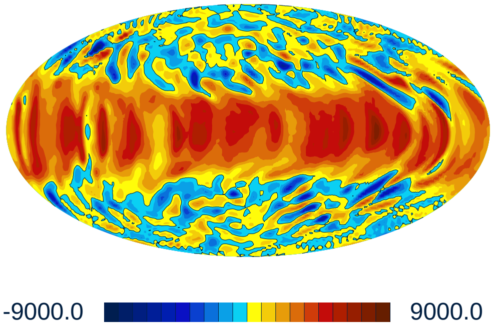


Equatorial section of u_r



Random meridional slice of u_r

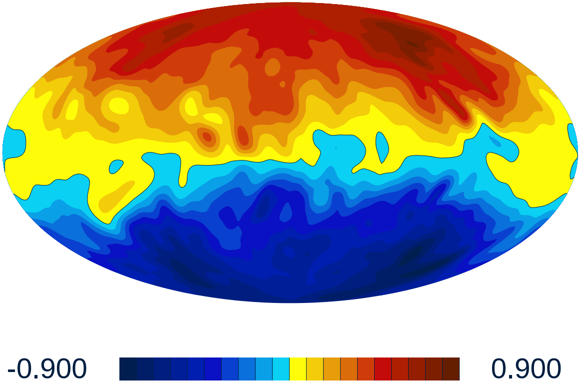
Surface Zonal Flow for $Pr = 0.1$ solution



Zonal flow at the cut-off surface: shows the relative size of the convective flow and the zonal flow.

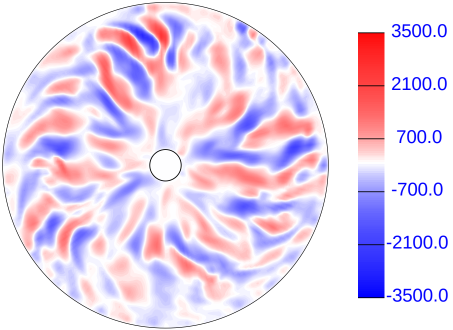
As usual in simulations, the ratio is small, but not the 10^{-4} smaller expected in Jupiter.

$Pr = 0.1$ Dynamo Movie

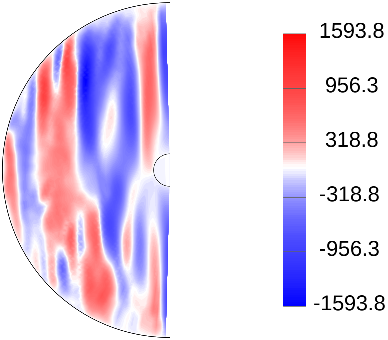


Time-lapse movie of B_r at the surface, for the $Pr = 0.1$ internal heated dynamo. Fixed entropy boundaries. $Ra = 1.2 \times 10^7$, $Pm = 3.0$, $E = 2.5 \times 10^{-5}$. Dynamo remains dipolar dominant.

Flow Movie for the $Pr = 0.1$ Dynamo



Time-lapse movie of u_r in the equatorial plane, for the $Pr = 0.1$ internal heated dynamo. Fixed entropy boundaries.
$Ra = 1.2 \times 10^7$, $Pm = 3.0$, $E = 2.5 \times 10^{-5}$. Note the difference between the magnetically locked interior and zonal flow dominated molecular region.



Time-lapse movie of u_r in a meridional plane, for the $Pr = 0.1$ internal heated dynamo. Fixed entropy boundaries. $Ra = 1.2 \times 10^7$, $Pm = 3.0$, $E = 2.5 \times 10^{-5}$. Note the columnar convection, though columns don't reach to boundaries.