# Waves in the Earth's Core

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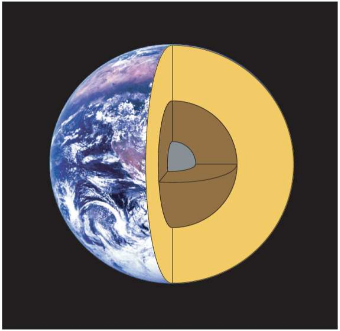
# Interior of the Earth



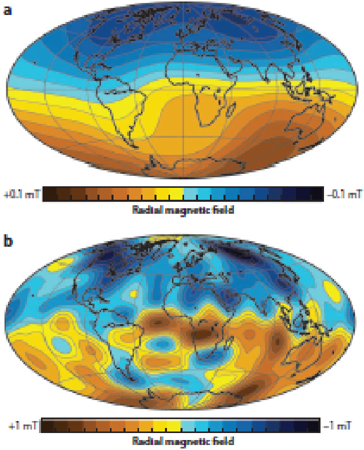
CMB = Core-Mantle Boundary

ICB = Inner Core Boundary

Tangent Cylinder = cylinder just enclosing the Inner core.
Fluid outer core is source of dynamo. Driven by convection arising from heat and light material released at inner core boundary.

Geomagnetic field is generated in liquid metal core, measured at Earth's surface and can be extended down to the CMB, assuming mantle is insulating. Limited to spherical harmonic degree $<\sim 14$.

# Secular Variation




The field has been mapped out in time over the last 300 years.
The time-derivative of the field is the secular variation.

# Why waves in the Core?

The magnetic field in the Earth's core varies on many timescales, but changes in the range 1-300 years can be observed in some detail, the secular variation.

By identifying waves in this signal, we can potentially discover the strength and form of the magnetic field inside the liquid metal core.

Waves on these timescales are affected by rotation, magnetic field and spherical geometry. Many types of wave are possible, but two are of particular interest as they have periods in the observable range.

Axisymmetric **Torsional Oscillations**, which have periods of about 6 years, the travel time of Alfvén waves. There is strong evidence these waves have been detected. A key question is how are they excited.

# Magnetic Rossby Waves

These are non-axisymmetric, and have slower periods, typically a few hundred years. The secular variation has nonaxisymmetric and axisymmetric components and varies on this timescale.

The difficulty in identifying these waves is that the core fluid speed is similar to the magnetic Rossby wave phase speed, so it is hard to separate wave motion from advection.

It has been controversial whether secular variation is due to wave or core flow. It is likely a combination of both.

The task is how can we use the improved data coming from satellites to separate the signal due to waves from the signal due to the flow? This requires an understanding of the dynamics of magnetic Rossby waves.

# Stably stratified core?

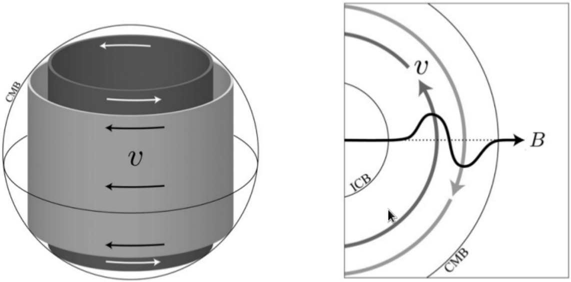
It is possible that the top few hundred kilometres of the Earth's core is stably stratified.

One reason is that conduction could possibly transport all the heat flux near the top of the core. We don't know the total heat going from the core into the mantle, but it is likely to be fairly close to the heat flux that can be conducted down the adiabat.

Another possibility is that light material (e.g. sulphur, oxygen) could separate out at the top of the core and form an 'inverted ocean'.

If the stable layer exists it will modify the existing waves and give rise to new internal gravity waves in which the buoyancy frequency is important. There has not been much exploration of these waves as yet.

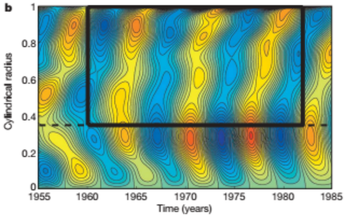
Sketches from Roberts and Aurnou 2012

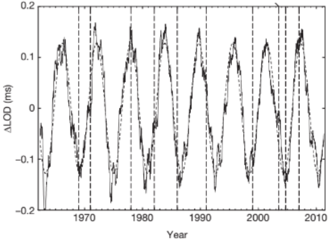



In spherical geometry, there must be no vortex stretching to obtain the pure Alfvén wave, so motion is independent of $z$ (geostrophic) in cylindrical coordinates $(s, \phi, z)$.
The magnetic field component $B_s$ provides the restoring force.

# Torsional oscillation observations

(a) Gillet et al 2010: (b) Holme & de Viron 2013




(a) The axisymmetric part of $u_\phi$ constructed from observations of the geomagnetic field.

(b) Length-of-day variations. Since total angular momentum is conserved, torsional waves change the mantle rotation rate.

An approximately six year period is seen in both data sets. Waves propagate out from the tangent cylinder, the cylinder surrounding the Earth's solid inner core.

# Dynamo model equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{Pm}{E}\left[\nabla p + 2\hat{\mathbf{z}} \times \mathbf{u} - (\nabla \times \mathbf{B}) \times \mathbf{B}\right]$$
$$+\frac{Pm^2 Ra}{Pr} T\mathbf{r} + Pm\nabla^2\mathbf{u}$$

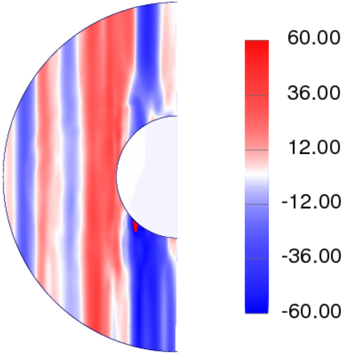
$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \frac{Pm}{Pr}\nabla^2 T + \epsilon, \quad \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u} \times \mathbf{B}) = \nabla^2 \mathbf{B}$$

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{B} = 0$$

$$E = \frac{\nu}{\Omega D^2}, \qquad Ra = \frac{g\alpha|\epsilon|D^5}{\nu\kappa\eta}, \quad Pr = \frac{\nu}{\kappa}, \qquad Pm = \frac{\nu}{\eta}$$

Boundary conditions used: no-slip, fixed flux; insulating magnetic. Zero compositional flux at CMB, uniform sink in interior. Parameters used: $E \in [10^{-6}, 10^{-4}]$, $Ra/Ra_c = 8.3$, $Pr = 1$, $Pm \in [1, 5]$, radius ratio $= 0.35$ These give 'Earth-like' dynamo models: basically dipolar but with an appropriate amount of reversed flux patches.

# Torsional Oscillations movie




Time-lapse movie of the axisymmetric part of $u_\phi$ on a meridional section. $E = 10^{-5}$.

The solid inner core of the Earth doesn't take part in the oscillations.

# Detecting Torsional Oscillations

Alfvén wave equation in spherical geometry is (Braginsky 1970)

$$\frac{\partial^2}{\partial t^2}\left(\frac{u_\phi}{s}\right) = \frac{1}{s^3 h}\frac{\partial}{\partial s}\left(s^3 h {U_A}^2 \frac{\partial}{\partial s}\left(\frac{u_\phi}{s}\right)\right)$$

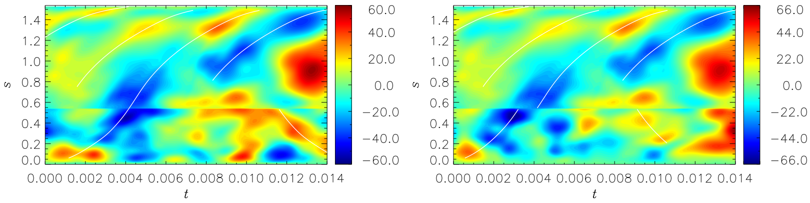
where ${U_A}^2 = \overline{\langle {B_s}^2/\mu\rho\rangle}$, the bar denoting $\phi$ average and angle brackets $z$-average. $h(s)$ is height of cylinder.

We run the dynamo simulation, and when initial transients have gone, we run for a further time $\tau$, and evaluate $U_A$ from $B_s$, the field averaged over time $\tau$.

We make plots in the $t$-$s$ plane of the fluctuating part of $u_\phi$, i.e. $u'_\phi = u_\phi - \widetilde{u_\phi}$ where $\widetilde{u_\phi}$ is the time-average over the whole $\tau$ run.

We then look for features in $u'_\phi$ propagating with the Alfvén speed.

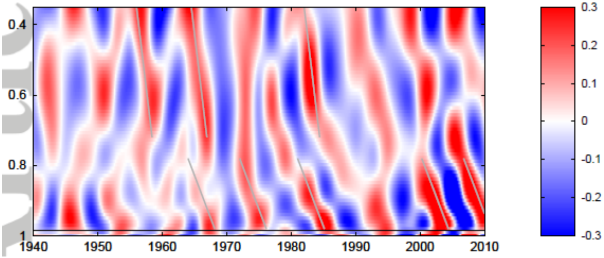
# Torsional Oscillations detected




Left: $u'_\phi$ inside Tangent Cylinder North. Right: $u'_\phi$ inside Tangent Cylinder South. $\langle\overline{u_\phi}\rangle'$, $E = 10^{-4}$, $Pm = 5$. White curves have gradient $U_A$. Similar pictures for $E = 10^{-5}$.

Plus points: torsional oscillations found in dynamo simulations. Mostly (but not exclusively) travel outwards. When field is scaled to observed $B_r$ at the CMB, travel time from tangent cylinder to equator is about 3 years, similar to Gillet et al. data.

Minus points: Origin inside the tangent cylinder rather than at the TC. Waves not as periodic as suggested by the Gillet et al. data.




Torsional waves reconstructed from secular variation data propagating from tangent cylinder (top) to equatorial region (bottom).

Note the slower speed near the equator, as seen in the simulations. Fast speed near tangent cylinder corresponds to approx $B_s$ =2mT, slower equatorial speed to $B_s$ =0.6mT.

Take the $\bar{\phi}$ and $\langle z \rangle$ average of the $\phi$-component of equation of motion so $\langle \bar{u}_\phi \rangle$ is the geostrophic part of the azimuthal flow

$$\left\langle \overline{\frac{\partial u_\phi}{\partial t}} \right\rangle = -\langle \overline{\hat{\phi} \cdot (\mathbf{u} \cdot \nabla)\mathbf{u}} \rangle + PmE^{-1} \langle \overline{\hat{\phi} \cdot ((\nabla \times \mathbf{B}) \times \mathbf{B})} \rangle + Pm \langle \overline{\hat{\phi} \cdot \nabla^2 \mathbf{u}} \rangle$$

Reynolds force, the Lorentz force and the viscous force

$$F_R + F_L + F_V.$$

Look at fluctuating part only

$$\left( \frac{\partial \langle \overline{u_\phi} \rangle}{\partial t} \right)' = F'_{LR} + F'_{LD} + F'_R + F'_V$$

In the core, we expect $F'_{LR}$ and $F'_{LD} \gg F'_R$ and $F'_v$.

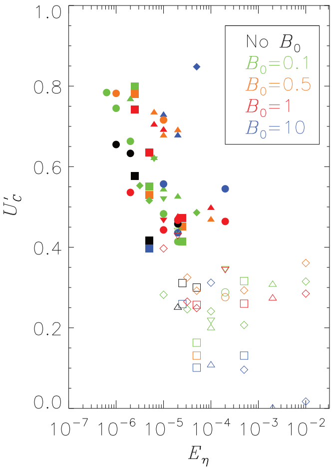
# Excitation of Torsional Oscillations 2.

But in dynamo simulations $F_R'$ is much overestimated: kinetic and magnetic energy is typically similar, whereas magnetic energy is much larger than kinetic energy in the core.

Also, the need to generate a magnetic field places severe restrictions on the accessible parameter space, e.g. very expensive to lower $Pm$.

So we used **Magnetoconvection** simulations. Same code, but change magnetic boundary condition: impose a dipole magnetic field at the core-mantle boundary.

Saves CPU time: don't have to wait for field to build up. Allows lower $E$, much lower $Pm$ and can get into dominant magnetic energy regime.



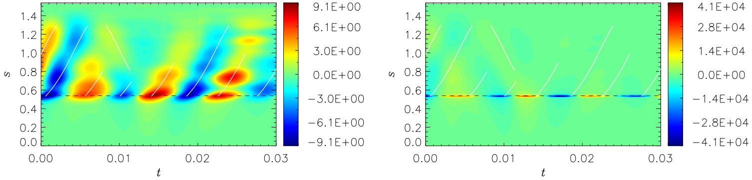

In magnetoconvection simulations, solutions exist where the torsional waves are a much larger part of the fluctuating signal.

We plot the geostrophy ratio,

$$U'_C = \sqrt{\frac{\|\langle \overline{u'_\phi} \rangle^2\|}{\|(\mathbf{u}')^2\|}}.$$

In Earth's core, value is about 0.45 (Gillet et al. 2015).

# Magnetoconvection results




In some magnetoconvection runs, the TO's are particularly clear.

$E = 5 \times 10^{-6}, Pm = 0.1, Ra = 5Ra_c, \Lambda \approx 100.$

Left: $u'_\phi$ contours. Right: $F'_L$ contours. $F'_R$ negligible.

In this strong field, low $E$ case, TO's are much more periodic, and originate from the tangent cylinder and propagate outwards. No significant reflection.

# Excitation in Lorentz force regime

We differentiate the equation of motion, and use the induction equation to replace the time derivatives of **B** in the Lorentz force term

$$\frac{\partial^2}{\partial t^2}\langle\overline{u_\phi}\rangle = \dot{F}_L = \frac{Pm}{E}\frac{1}{hs^2}\frac{\partial}{\partial s}s^2 h\overline{\langle sB_s(\mathbf{B}\cdot\nabla)\frac{u_\phi}{s}\rangle}$$

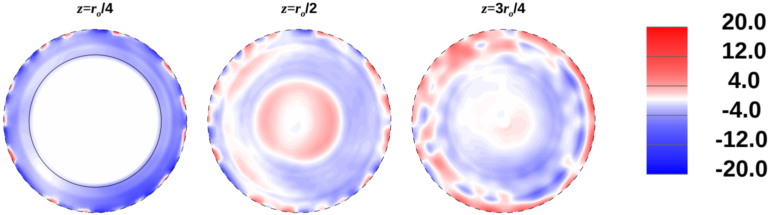
$+$ other less important terms.

$$\overline{\langle sB_s(\mathbf{B}\cdot\nabla)\frac{u_\phi}{s}\rangle} = s\langle\overline{B_s}\rangle^2\frac{\partial}{\partial s}\frac{\langle\overline{u_\phi}\rangle}{s} + s\langle\overline{B_s}\rangle\overline{\langle B_{sa}\frac{\partial}{\partial s}\frac{u_{\phi,a}}{s}\rangle}$$

First term on the right is the restoring force of the TO. The forcing is all magnetic, and located at the tangent cylinder. The second term is one of the forcing terms. The ageostrophic component of $u_\phi$ can combine with ageostrophic parts of the magnetic field to give a geostrophic forcing.

This is how ageostrophic convection, $u_{\phi,a}$ can drive a torsional oscillation, even though a TO itself cannot convect heat.

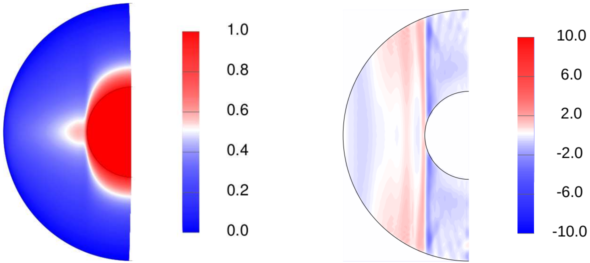
# $z$-sections of convective flow



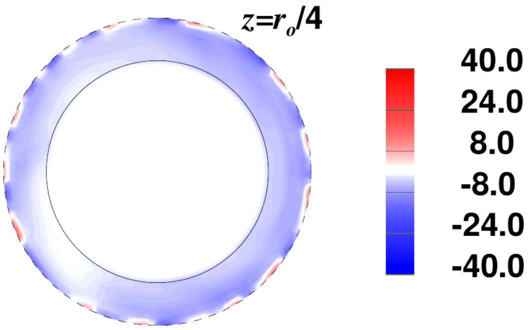

Many terms contribute to the forcing, but the dominant terms are those from the ageostrophic part of $u'_\phi$, $u'_{a\phi}$, combined with the ageostrophic part of $B_s^2$.

Left, middle and right plots are slices parallel to the equatorial plane showing this ageostrophic part, $u'_{a\phi}$.

This ageostrophic part is that part of the convective flow that has the torsional wave frequency. Corresponds to large azimuthal wavenumbers, $m \approx 20$.




Left: temperature in a meridional plane. Right: steady part of $u_\phi$. Buoyancy generated at ICB, and cannot easily cross the TC because of the Proudman-Taylor constraint.
Hotter inside TC, so a thermal wind is generated, giving a zonal flow with a shear layer at the TC. Time-dependent convection perturbs the shear layer, generating the strong $u'_{\phi,a}$ perturbations there.




Time-lapse movie of the axisymmetric part of $u_\phi$ on a constant $z = r_0/4$-section. $E = 5 \times 10^{-6}$.

Convection drives small-scale azimuthal flows with a non-zero $m = 0$ component. This forces the TO waves.

# How are the quasi-periodic waves produced?

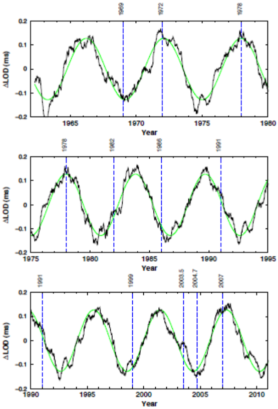
Take the unit of time to match the Alfvén speed at the CMB. Then ageostrophic velocity near the TC is $u'_{\phi,a} \sim 8.5 \times 10^{-4}$ m/sec.

Comparable to westward drift speeds.

Wavelength of the $m = 20$ convection $\sim 380$ km, so characteristic turnover time to travel half a wavelength $\sim 7$ years. There is significant power in the $m = 0$ mode at this frequency.

Surprisingly periodic temporal behaviour could be due to a periodic convective mode in the vicinity of the tangent cylinder.

# What is producing periodic behaviour?




The Holme and de Viron data suggest a periodic oscillator with fluctuating forcing.

Mound and Buffett (2006) suggested there might be gravitational coupling between the nonaxisymmetric mantle and the nonaxisymmetric inner core, giving a gravitational torsional oscillation with period 5.9 years. Davies et al. (2014) thought this period would be longer.

Could be that the convection gives an approximate 5.9 year forcing, which locks to the gravitational mode, or it could be that as in our model there is a particular magnetic mode with the right frequency.

# Summary on Torsional Oscillations

TOs observed in the core from secular variation and LOD signal.

Can be modelled using dynamo and magnetoconvection codes.

They originate from quite small scale ($\sim$ 400 km) convection, which has turnover time and period $\sim$ 6 years.

The convection is broad-band in azimuthal wavenumber and has an $m = 0$ component.

The convection disturbs the TC shear layer, and this forces the TOs which propagate outwards.

Still not clear where the period comes from: convection exciting particular wavemode close to the TC, or whether the gravitational Mound-Buffett mode plays a role.

With uniform magnetic field $\mathbf{B_0}$ and constant temperature gradient $T_0'$ we look for local wave-like solutions

$$\mathbf{u} = \mathbf{u_0} \exp i(\mathbf{k}\cdot\mathbf{x} - \omega t).$$

Linearised equations are

$$\frac{\partial \mathbf{u}}{\partial t} + 2\boldsymbol{\Omega}\times\mathbf{u} = -\nabla p' + g\alpha T'\hat{\mathbf{r}} + \frac{1}{\rho}\mathbf{j}\times\mathbf{B_0} + \nu\nabla^2\mathbf{u}$$

$$\frac{\partial \mathbf{b}}{\partial t} = (\mathbf{B_0}\cdot\nabla)\mathbf{u} + \eta\nabla^2\mathbf{b}, \qquad \mu\mathbf{j} = \nabla\times\mathbf{b},$$

$$\frac{\partial \mathbf{T}'}{\partial t} = -u_r T_0' + \kappa\nabla^2\mathbf{T}'$$

# Dispersion relation for rotating MHD waves

Ignoring diffusion and buoyancy, we get

$$4\omega^2(\mathbf{\Omega}\cdot\mathbf{k})^2 = \left(\frac{(\mathbf{B_0}\cdot\mathbf{k})^2}{\mu\rho} - \omega^2\right)^2 \mathbf{k}^2$$

$$\omega = \frac{(\mathbf{\Omega}\cdot\mathbf{k}) \pm \sqrt{(\mathbf{\Omega}\cdot\mathbf{k})^2 + \mathbf{k}^2(\mathbf{B_0}\cdot\mathbf{k})^2/\mu\rho}}{|k|}$$

Define

$$\omega_C = \frac{2(\mathbf{\Omega}\cdot\mathbf{k})}{|k|}, \quad \omega_M = \frac{(\mathbf{B_0}\cdot\mathbf{k})}{(\mu\rho)^{1/2}}$$

and note that in planets $\omega_C >> \omega_M$.

Taking plus sign, we get $\omega \approx \omega_C$, the fast inertial waves. taking minus sign, we get the slow wave

$$\omega_{MC} = \frac{\omega_M^2}{\omega_C}$$

These non-axisymmetric waves have frequency

$$\omega_{MC} = \frac{(\mathbf{B}\cdot\mathbf{k})^2|k|}{(\mathbf{\Omega}\cdot\mathbf{k})\mu\rho}$$

These slow MC are in magnetostrophic equilibrium;
time-dependence comes from induction equation, not equation of motion.

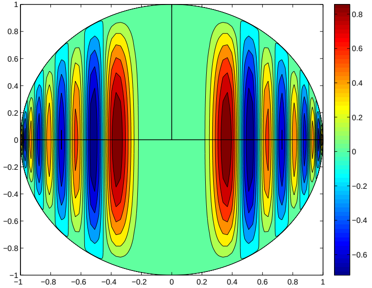
Large scale MC-waves typically have frequencies of thousands of years, too slow, so we focus on columnar waves that have small $(\mathbf{\Omega}\cdot\mathbf{k})$, the magnetic Rossby waves.

An exact solution is possible in spherical geometry for the special Malkus field,

$$\mathbf{B} = \frac{B_0 s \mathbf{e}_\phi}{a}$$

which has uniform current density in $z$ direction.

# Malkus model



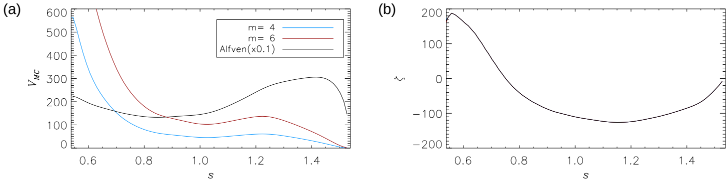

$u_r$ is shown for an eigenmode.

The solutions have 3 wavenumbers. An azimuthal wavenumber $m$, (picture has $m = 8$), a radial wavenumber in the $s$ direction (picture shows 8th radial mode), and a $z$-wavenumber.

The magnetic Rossby mode has the lowest possible $z$-wavenumber. It travels westward.

The next $z$-wavenumber has opposite equatorial parity and travels eastward, considerably slower.

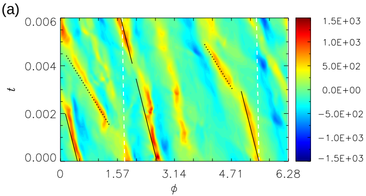
# Magnetic Rossby wave frequencies




The left plot is the wave speed and the right figure is the azimuthal flow speed from a dynamo simulation. If the field is azimuthal, the wave speed is ($k_s$ assumed small)

$$\frac{\omega}{m} = -\frac{m^2(r_0^2 - s^2)B_\phi^2}{2\Omega\mu\rho s^4}.$$

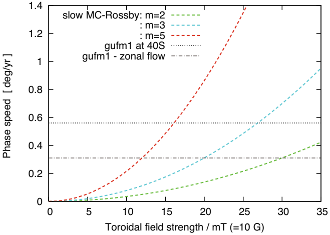
Magnetic Rossby waves go westward.
Both flow and wave speed are similar, but flow dominates near equator, waves dominate at higher latitudes. Higher latitudes more informative?




$E = 10^{-5}$, $Pm = 5$, $Ra = 8.32 Ra_c$.
Left: $\phi - t$ plot of $u_s$ at $s = 0.5 r_o$, latitude 60°.
Right: same at $s = 0.766 r_o$, latitude 40°.

White dashed: advection speed.

Black dashed: wave speed + advection speed for $m = 5$ and $m = 8$ in left, $m = 6$ in right.
At higher latitudes, waves + flow correlate well. At low latitudes, hard to distinguish.




At 40° S, black dotted is drift speed from observational model gufm1, black solid is drift speed with core flow removed (Hulot et al. 2002). Difference could be the wave speed relative to the flow.

The coloured dashed lines are the expected wave speeds from our model as a function of the *z*-averaged toroidal field.

Indicates that the azimuthal field is a lot stronger than the poloidal field of 3 mT if there is a wave component to the secular variation

# Conclusions on Magnetic Rossby waves

- Magnetic Rossby waves can be seen in dynamo simulations

- Waves are generally slow, so the wave drift will be dominated by advection at lower latitudes.

- Best chance of separating the waves from the advection velocity is at high latitudes, where the wave speed is significant compared to the flow speed.

- Need to analyze the data to see the wavespeeds of the different wavenumbers: dispersion relation depends on ratio of wave/advection speed.