GAC2010 — Groups, Actions and Computations

GAP session 6

p-group generation

1. Load the following packages:
```
LoadPackage("autpgrp");
LoadPackage("polycyclic");
LoadPackage("anupq");
```

2. Compute the p-cover of $G := C_2 \times C_2$ and look at its presentation.
Hint 1: Fetch this group from the `SmallGroups` library as a PC-group.
Hint 2: Use → `?PqPCover` from the anupq package.

3. Compute all immediate descendants of G.
Hint 1: Use → `?PqDescendants` from the anupq package.
Hint 2: The following function is useful for printing PC presentations:
```
PrintPC := function(g)
  PrintPcpPresentation(Image(IsomorphismPcpGroup(g)));
end;
```

4. Let H be the unique group among the immediate descendants of G of order 16 and nuclear rank 3. Compute all descendants of H up to p-class 4. How many have p-class 3 and how many have p-class 4?
Hint 1: Use → `?NuclearRank` to determine the nuclear rank.
Hint 2: Use the `ClassBound` option of the `PqDescendants` command. It is documented in section → `?ANUPQ options`.
Hint 3: To count the numbers you can use → `?Collected`.

5. In this exercise we want to find a 3-group with a small automorphism group. To this end, we start with $G := C_3 \times C_3$ and compute the immediate descendants. We take one with a relatively small automorphism group and compute its descendants and so on. In this way you can find a group with 243 elements whose automorphism group has order 486. What is its identication in the `SmallGroups` library?
Hint 1: Use → `?AutomorphismGroup` to compute the automorphism group.
Hint 2: Once you have it constructed, use `IdGroup` to find its ID.

6. Look at some examples for the anupq package by using the → `?PqExample` function.
Hint 1: Find out what examples there are using → `?AllPqExamples`.

7. Use anupq to find the largest 2-quotient of class 8 of the group
$$G := \left\langle a, b \mid a^4, b^4 \right\rangle$$
Hint 1: Use the → `?Pq` command with the options `Prime` and `ClassBound`.

8. Find the largest finite 3-group that has exponent 3 and can be generated by 4 elements.
Hint 1: Start with a free group of rank 4 and run the 3-quotient algorithm with the option `Identities`.
Hint 2: A convenient way to specify the exponent 3 identity is
```
x->x^3
```
Hint 3: → `?Pq` to learn that you have to give a list of functions for the `Identities` option.

9. Find the largest finite 2-generated 2-group of class 4 with the Engel-2 property (that is, for all elements a, b we have $[[a, b], b] = 1$).
Hint 1: Use the following function for `Identities`:
```
function(a,b) return Comm(Comm(a,b),b); end;
```