GAC2010 — Groups, Actions and Computations

GAP session 5

Polycyclic Groups

1. Fetch $g :=$ `SmallGroup(120,33)` from the library and find out its polycyclic presentation.
   Hint 1: You have to get an `IsomorphismFpGroup`, take the `Image` and ask that for $\rightarrow$ `?RelatorsOfFpGroup`.
2. Generate a few random elements and multiply them to see the collection working.
   Hint 1: Use either `PseudoRandom` or `Random`.
   Hint 2: Observe how collections brings the products back into normal form.
3. Ask for a polycyclic generating sequence and determine the relative orders.
   Hint 1: $\rightarrow$ `?Pcgs` and $\rightarrow$ `?AsList`
   Hint 2: Check the order of the groups generated by the right subsequences of your polycyclic sequence ($\rightarrow$ `?Size`).
4. Type in the following polycyclic presentation as an FP group:

$$G := \mathrm{Pc}\left\langle x_1, x_2, x_3 \mid x_1^3 = x_3, x_2^2 = x_3, x_1^{-1}x_2x_1 = x_2x_3, x_1x_2x_1^{-1} = x_2x_3 \right\rangle$$

   and use $\rightarrow$ `?IsomorphismPcGroup` to compute a polycyclic presentation. Ask for a `Pcgs` and use this to determine the `RelativeOrders`.
5. Load the polycyclic and AtlasRep packages using

```
LoadPackage("polycyclic");
LoadPackage("atlasrep");
```

   These are needed for the following exercises.
6. Type in the following presentation (as an FP group) and use $\rightarrow$ `?IsomorphismPcGroup` to compute a polycyclic presentation (note that you first have to ask `IsPolycyclicGroup` for this to work):

$$G := \left\langle a, b \mid a^3b^{-3}, ababa^{-3}, [a,b]^2 \right\rangle$$

   Use then $\rightarrow$ `?IsomorphismPcpGroup` from the polycyclic package to find a PCP presentation, which you can then print using `PrintPcpPresentation`
7. Fetch `AtlasSubgroup("Fi22",11)` and call it S. Compute an isomorphism to a PC group and call the image P. Now compute a few things for both S and P (for example `SylowSubgroup(S,2)` and `SylowSubgroup(P,2)`). Compare the runtimes.
   Hint 1: This fetches a solvable subgroup of the sporadic simple Fischer group Fi_{22} using the AtlasRep package.
   Hint 2: To display the runtime a GAP command needed look at the `time` variable immediately after you executed the command. It shows the runtime in milliseconds.
8. Compute the derived subgroup S'' of the derived subgroup S' of S, compute a finite presentation for it and use $\rightarrow$ `?PQuotient` and $\rightarrow$ `?EpimorphismQuotientSystem` to find the largest 2-quotient of S''.
9. Download the file `permrep.g` from
   `http://tinyurl.com/33oxgkk`
   and read it into GAP. This defines a solvable permutation group G. Compute an isomorphism to a PC group and call the image P. Compare again some runtimes.