

GAC2010 — Groups, Actions and Computations

GAP session 3

Using group libraries and conducting searches

1. How many non-abelian groups of order 24 are there? (Of course, with this formulation we always mean “up to isomorphism”.)

Hint : You want to use the **small groups library**, `→ ?NumberSmallGroups`, `→ ?AllSmallGroups`, `→ ?IsAbelian`

2. How many non-abelian groups of order 128 are there?

Compute the average of the sizes of their centres.

Hint 1: To automate the counting you can use `→ ?List` and `→ ?Collected` or use `→ ?Filtered` to get them all

Hint 2: For the centres the `→ ?arrow` notation could be convenient

3. What ID does the group generated by the three permutations

$(2, 4, 6, 8, 10), (1, 9)(2, 8)(3, 7)(4, 6)$ and $(1, 6)(2, 7)(3, 8)(4, 9)(5, 10)$

have?

Hint : `→ ?IdGroup`, `→ ?SmallGroup`

4. Find all composition series of all non-solvable groups of order 120.

Hint 1: You can avoid making all small groups of order 120 and then `Filtered` by using the more sophisticated syntax of `→ ?AllSmallGroups`

Hint 2: Simply compute all normal subgroups of them using `→ ?NormalSubgroups`

5. How many elements of order 3 do all groups of order 48 have together? (Of course, we mean to take one group of each isomorphism type.)

Hint 1: Fetch them all using `AllSmallGroups`, for each of them, ask for all elements (`→ ?Elements`) and let **GAP** count.

Hint 2: If you have a list `L` of groups, you can use a `→ ?for` loop to run through all of them like this:

```
for g in L do
  do stuff with g
```

```
od;
```

Hint 3: The same technique can be used to run through a list of elements. Use `→ ?if` to decide, whether or not an element has order 3 as in

```
if Order(x) = 3 then
  increase a counter
```

```
fi;
```

6. Show that the Sylow 2-subgroups of the Mathieu group M_{24} and the sporadic simple Held group He are isomorphic.

Hint 1: `→ ?MathieuGroup`

Hint 2: Use the **AtlasRep** package and `→ ?AtlasGroup` to fetch generators of He from the internet.

Hint 3: Use `→ ?SmallerDegreePermutationRepresentation` for the Sylow 2-subgroup of He

Hint 4: Use `→ ?IsomorphismGroups`

7. Find the group with the fewest elements that is non-abelian, has trivial center and contains an element a of order 2 and an element b of order 3 such that ab has order 5.