GAC2010 — Groups, Actions and Computations

GAP session 2

Getting to know GAP

These tasks are meant to keep **everybody** busy for at least an hour. Do **not dispair** if you finish only part of them. There are hints on this sheet. We suggest that you first try to solve the exercises without using the hints, however, if you get **stuck** with one, then first read only the first hint and try again. If this does not help, try the second hint and so on. Finally, if nothing helps, **ask** someone.

1. Let G be the group generated by the following two permutations:

   $$(1, 10)(2, 3, 6, 9, 5, 8, 4, 11) \text{ and } (1, 2, 5, 9)(7, 10, 11, 8).$$

   We first want to analyse the action of this group on $M := \{1, 2, \ldots, 11\}$:
   Find, using GAP, the largest k such that G acts k-transitively on M.

   Hint 0: To enter the group, use the `Group` command ($\rightarrow$ `?Group`).

   Hint 1: Compute the orbit of 1 under G and decide, whether or not the action is transitive ($\rightarrow$ `?Orbit` and $\rightarrow$ `?IsTransitive`).

   Hint 2: Compute the stabiliser of 1 in G and apply the same method to it, of course using a different starting point ($\rightarrow$ `?Stabilizer`)

   Hint 3: Repeat.

2. For the k you found in 1: does G act **sharply** k-transitively?

   Hint 1: Look at the last stabiliser you computed.

3. Let's analyse the structure of this group a bit: Compute the group order ($\rightarrow$ `?Size`).

4. Compute the center of this group.

   Hint 1: $\rightarrow$ `?Center`

5. Compute the derived subgroup of this group.

   Hint 1: $\rightarrow$ `?DerivedSubgroup`

6. Check if this group is simple.

   Hint 1: $\rightarrow$ `?IsSimple`

7. Compute the 2-, 3-, 5- and 11-Sylow subgroups of G.

   Hint 1: $\rightarrow$ `?SylowSubgroup`

8. Compute the stabiliser of 1 in G and apply the above methods to it to find out something about its structure.

   Hint 1: $\rightarrow$ `?Stabiliser`

9. Let's study the derived subgroup D of $\text{Stab}_G(1)$: Confirm that it is a simple group of order 360.

10. We suspect that this might be isomorphic to the alternating group A_6 on 6 points. Verify this with GAP and compute an explicit isomorphism.
    Compute the images of the generators of D under this isomorphism and the preimages of the standard generators of A_6.

    Hint 1: $\rightarrow$ `?IsomorphismGroups`

    Hint 2: This gives you a GAP object representing an isomorphism. You can access the generators of a group with `GeneratorsOfGroup`. You can map elements using `ImageElm` and compute preimages with `PreImage`.

11. Find out what the following command does and why it does this (assuming that the above group D is stored in the variable `D`):

    ```
    List(GeneratorsOfGroup(D),x->ImageElm(iso,x));
    ```

    Hint 1: $\rightarrow$ `?List`

    Hint 2: $\rightarrow$ `?arrow notation`

12. We want to construct this isomorphism in another way. To this end, understand the following sequence of commands:

    ```
    c := ConjugacyClassesMaximalSubgroups(D);
    List(c,Size);
    r := List(c,Representative);
    List(r,Size);
    ```

13. The above computation gave you two subgroups of index 6. Compute the two actions of D on the right cosets of them.

    Hint 1: $\rightarrow$ `FactorCosetAction`

14. Derive explicitly an automorphism of D which does not come from conjugation in the symmetric group S_6.