Generalisations of Small Cancellation Theory

Max Neunhöffer

Van Kampen Diagrams
The Word Problem
Diagrams
Isoperimetric Functions

Small Cancellation Theory
Conditions
Theorems

Generalisations
Algorithmic approach
Forbidden Subdiagrams
Curvature redistribution
2-dimensional analysis
1-dimensional analysis
Changing the presentation

The Prototype

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University of St Andrews

Allahabad 11.9.2010

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Status of the Project

Joint work with:

Stephen Linton

Richard Parker

Colva Roney-Dougal

Richard already worked for several years on this.

The whole project is still in its early stages.

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Decide whether or not a word $w \in F(X)$ lies in N, that is, whether or not it represents the identity in G.

Of course:

$$w \in N \qquad \Longleftrightarrow \qquad w = \prod_{i=1}^{k} u_i r_i u_i^{-1} \quad \text{in } F(X)$$

for some $k \in \mathbb{N} \cup \{0\}$, $r_i \in R \cup R^{-1}$ and $u_i \in F(X)$.

Want: An algorithm to solve this Word Problem in G.

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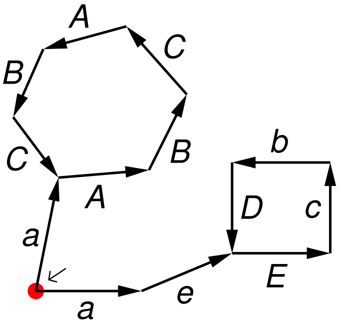

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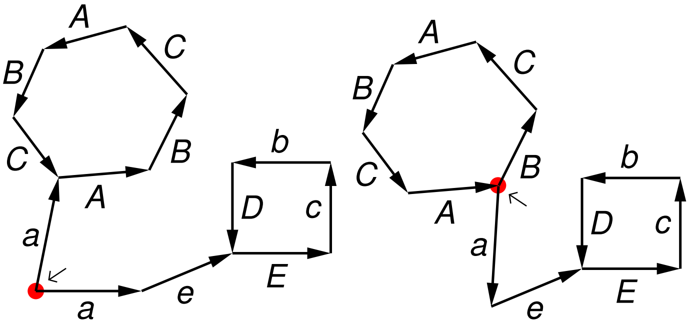

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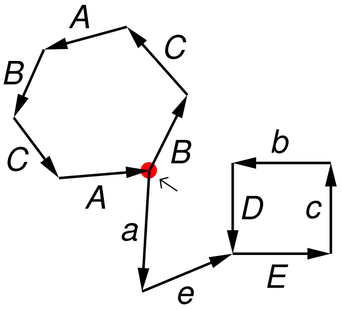

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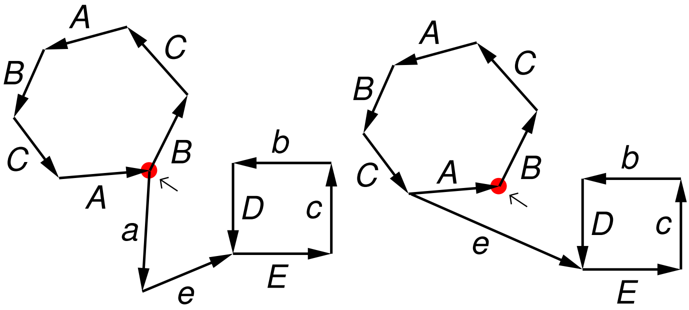

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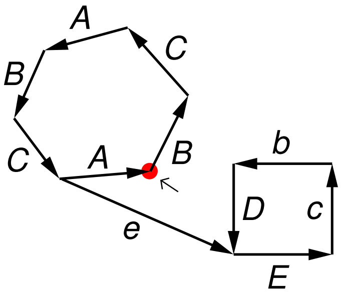

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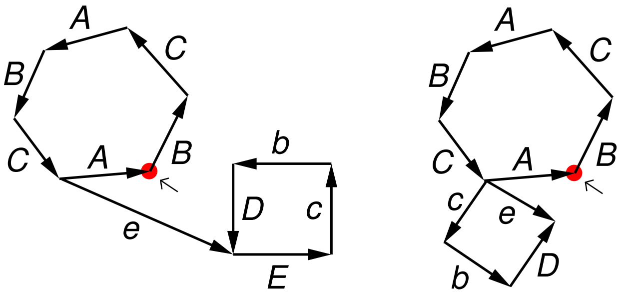

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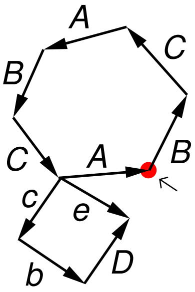

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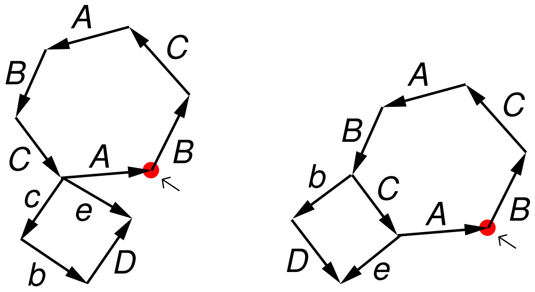

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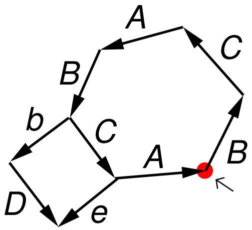

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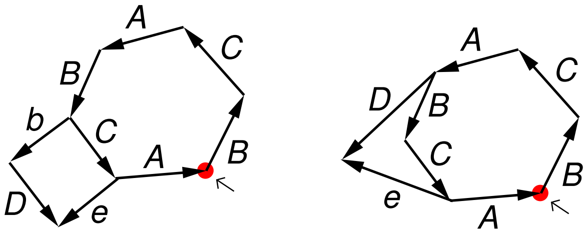

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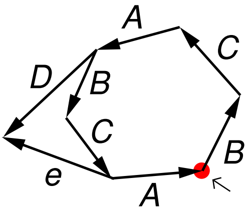

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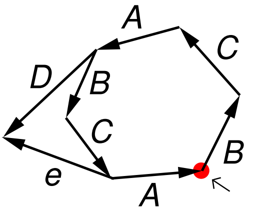

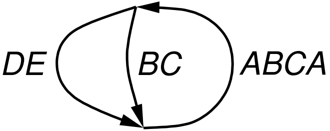

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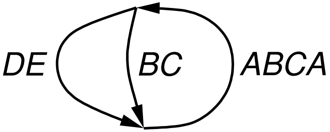

Definition

A van Kampen diagram is a connected, simply-connected 2-complex with oriented and labelled edges.

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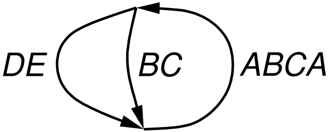

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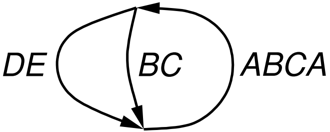

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It is a topological proof for a word to be in N.

It is a sphere if the boundary word is a relator.

Van Kampen Diagrams II

Theorem (Van Kampen, 1933)

Let $G = \langle X \mid R \rangle$ be a finitely presented group. Then a word $w \in F(X)$ is contained in the normal closure $N := \langle\langle R \rangle\rangle$ of R in $F(X)$ if and only if there is a van Kampen diagram with w as boundary word.

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Definition (Isoperimetric (or Dehn) Function)

A function $f : \mathbb{N} \to \mathbb{N}$ is called an isoperimetric function or Dehn function for $G = \langle X \mid R \rangle$ if the following holds:

for every reduced word w of length at most n representing 1 in G there is a van Kampen diagram proving this with at most $f(n)$ faces.

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Theorem (Gersten)

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Dehn's algorithm to solve the word problem

Whenever a cyclic word w contains more than half of a relator, replace this part by the inverse of the other, smaller part.

Conditions

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Let $\overline{R}$ be R together with all cyclic rotations and their inverses of all relators.

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Condition $T(k)$ for $k \geq 3$

If the presentation fulfills condition $T(k)$, then every internal vertex of a reduced van Kampen diagram has valency $\geq k$.

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Definition (Piece)

A word $1 \neq b \in F(X)$ is called a piece if there are relators $r_1, r_2 \in \overline{R}$ with $r_1 = bc_1$ and $r_2 = bc_2$ for $c_1, c_2 \in F(X)$ with $c_1 \neq c_2$.

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Condition $C'(\lambda)$ for $0 < \lambda < 1$

If $bc \in \overline{R}$ with a piece b, then $\ell(b) < \lambda \ell(bc)$.

Small Cancellation Theorems

Theorem (Small Cancellation)

If a presentation $G = \langle X \mid R \rangle$ fulfills

- $T(3)$ *and* $C'(1/6)$, *or*
- $T(4)$ *and* $C'(1/4)$,

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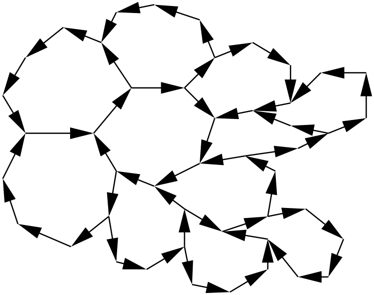

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Small Cancellation Theory is very much static.

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Idea 1: Analyse presentation dynamically

We want to write programs that analyse presentations and — if they succeed — come up with new, possibly more complicated local conditions which show that some word problem solving algorithm works.

This is a very dynamic and flexible approach.

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Already forbidden: two faces touching with inverse words.

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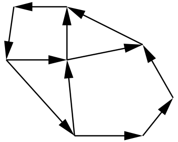

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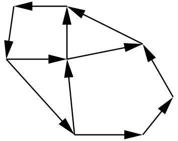

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$\Longrightarrow$ Can replace this subdiagram by one heptagon!

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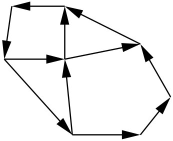

Assume this is a sphere, i.e. the boundary is a relator.

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Idea 2: Forbidden regions

We simply forbid certain subdiagrams (once we proved that they are not needed!) and check the local conditions only on the vK diagrams without forbidden regions.

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Curvature redistribution

Idea 3:

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In a planar graph, Euler's formula holds:

$$(\#\text{Vertices}) - (\#\text{Edges}) + (\#\text{Regions}) = 1$$

(not counting the "outside region").

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Combinatorial Curvature

If we endow every vertex with $+1$, every edge with -1 and every face with $+1$ units of "curvature", the total sum in a diagram is always equal to 1.

This way, curvature is a local phenomenon, about which we have global information.

Curvature redistribution

Idea 4: Move curvature around — Officers

We now start to move curvature around locally, e.g.:

- Distribute the curvature of each face to the adjacent edges according to their length (L).

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- If we can show that after the move, the curvature in the interior is always non-positive then

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Or any sequence of them, e.g. LE or LEA or LEALEA.

To which end?

- In the end, the sum is still 1.
- If we can show that after the move, the curvature in the interior is always non-positive then
  - there are no spheres, and

Curvature redistribution

Idea 4: Move curvature around — Officers

We now start to move curvature around locally, e.g.:

- Distribute the curvature of each face to the adjacent edges according to their length (L).
- Distribute the curvature of each edge equally to its adjacent vertices (E).
- Distribute the curvature of each vertex equally to its adjacent faces (A).

Or any sequence of them, e.g. LE or LEA or LEALEA.

To which end?

- In the end, the sum is still 1.
- If we can show that after the move, the curvature in the interior is always non-positive then
  - there are no spheres, and
  - in all vK diagrams, the curvature is at the boundary.

2-dimensional analysis by example

This analyses the interior of possible diagrams to conclude that an officer leaves no curvature in the interior.

Example: LE officer with $T(4)$ and $C'(1/4)$

Assume $T(4)$ and $C'(1/4)$ and run officer LE.

Curvature distribution (interior):

when	int. vertex	int. edge	face
initial	1	-1	1

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after E	< 0 $(**)$	0	0

$(*)$ since $-1/2 = -1 + 2 \cdot \frac{1}{4}$ using $C'(1/4)$.
$(**)$ since $0 = 1 + 4 \cdot \frac{-1/2}{2}$ using $T(4)$.

1-dimensional analysis

This analyses the boundary of possible diagrams to use positive curvature near the boundary to prove a word problem solver.

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Show that your favourite word problem solver can deal with all situations arising.

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Idea 5:

As part of our dynamic, algorithmic approach, we change the presentation (without changing the group!).

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Changing the presentation

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- Add generators and relations to change the geometry of diagrams (e.g. triangulation).
- Do all of these to make new local small cancellation conditions applicable.

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Generalisations of Small Cancellation Theory

Max Neunhöffer

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# gens	# random rels	time [seconds]
1 000	1 140	0.03
10 000	20 010	0.77
100 000	366 500	33
500 000	2 830 000	584
1 000 000	6 850 000	2145