Final Discussion Session: Theory

Abhay Ashtekar
Institute for Gravitation and the Cosmos, Penn State

Format

1. Opening Remarks

(Overview & A mathematical topic that was not covered)

2. Open Discussion

(Comments Questions addressed to all speakers & partipants)

Future of Gravitational Wave Astronomy; ICTS Bangalore; Aug 2019

1.A Overview

- Spectacular transition/transformation in the GRG community in India!

> *General theory of relativity is a theory of gravitation; and like the Newtonian theory of gravitation which it refines and broadens, its natural home is astronomy.*
>
> *S. Chandrasekhar (1980)*
>
> *Address given to the 17th General Assembly of the IAU*

Stan Whitcomb's Chandra Story: In spite of this comment, Chandra would have been quite surprised by the impact that detection has had on astronomy!

- Exciting upcoming decade for gravitational physics: 2020s
- Near future: Importance of getting LIGO-India online on schedule.

1.A Overview (contd)

- What about longer term future?

Enhancing the creation of human resources paramount for LIGO-India: I will focus on the theoretical aspects because Stan will cover the experimental aspects

1) Data Analysis and Source Modeling.
   Two aspects: Detection; and, Parameter Estimation.
   As emphasized by many speakers, importance of improving accuracy.

2) Tests of GR and alternate theories

3) Physics of Nuclear Matter in extreme conditions;

4) Applications to Astronomy and Astrophysics

5)Cosmology

This Discussion Meeting, "Future of Gravitational Wave Astronomy " covered all these areas! Illustrates the astuteness and breadth of vision of the organizers.

1.B A Mathematical Topic that was not covered

- Gravitational waves in exact GR: Confusion was fully resolved by the work of Bondi, Sachs, Penrose, et al.

But the framework came with a Surprising finding:
Even though space-time is asymptotically flat, if it admits gravitational waves, the asymptotic symmetry group is not the Poincaré group $\mathcal{P}$ but an infinite dimensional generalization $\mathfrak{B}$ thereof: The BMS group!

- Origin of this enlargement:

$g_{ab} \to \eta_{ab} + O(1/r)$ terms. But η_{ab} not unique even asymptotically.
$g_{ab} \to \eta'_{ab} + O(1/r)$ terms, where
the respective Cartesian coordinates x^a and $x'^{\,a}$ differ by an angle dependent translation; called supertranslation. Ex: $t' = t + s(\theta, \varphi)$; $\vec{x}' = \vec{x}$

The Poincaré groups of η_{ab} and η'_{ab} are distinct. $\mathfrak{B}$ can be thought of as a 'consistent union' of all these Poincaré groups.

- Just as translations lead to the notion of energy momentum; supertranslations lead to supermomentum which has infinitely many components, one for each supertransaltion $s(\theta, \phi)$ $\Rightarrow$ infinitely many balance laws:

$P_{(s)}[u_1] - P_{(s)}[u_2] = F_{(s)}[u_1, u_2]$,
Flux carried by gravitational waves between u_1 & u_2.

Testing Accuracy of Waveforms from CBC

• Idea: If we knew the exact GR wave-forms for CBC, we could take any waveform in the template bank and find out the exact error or approximation involved in the sue of that wave form. But of course we don't have the exact wave form. But whatever the exact waveform is, it must satisfy the infinitely many supermomentum balance laws. Deviation from the law for any given waveform is a measure of the error/departure from exact GR.

• For CBC we have boundary conditions in the asymptotic past (Asymptotic stationarity in a sense much weaker than used in the PN analysis) and asymptotic future (NR: approach to Kerr). The infinite number of supermomentum balance laws together with these asymptotic conditions lead to an infinite number of constraints on the wave-forms:

Waveform: $\sigma(u, \theta, \varphi) := (h_+ + ih_\times)(u, \theta, \varphi)$
suppose the final BH has a kick in the z direction with velocity v. Then:

$$GM_{\rm total} - \frac{GM_{\rm final}}{\gamma^3(1-\frac{v}{c}\cos\theta)^3} = \int {\rm d}u\ \Big(|\dot\sigma|^2 + {\rm Re}(\eth^2\dot{\bar\sigma})\Big)(u, \theta\varphi)$$

This provides one constraint on the waveform for each (θ, φ): So infinitely many constraints. Violation of any of these constraints is an absolute measure of the error in the waveform, relative to exact GR. Details: arXiv 1906.00913.

Supermomentum considerations

• There are also constraints for finite time intervals that can be used to check accuracy of PN or NR or EOB or Phenom pieces of waveforms. But they require knowing Ψ_2° at the end of these time intervals. It can be readily calculated from the asymptotic form of the PN metric and efforts are nearly complete to calculate it in NR simulations as well (Kidder). The constraints can be used to discriminate between various strategies e.g., for where and how to 'glue' the PN waveforms to NR; or resolve possible ambiguities in the choice of the EOB (or phenom) model in regions of the parameter space where NR simulations are sparse.

• The issue of angular momentum and supertransltions.

Since the Bondi news $\dot{\sigma} = \dot{h}_+ + i\dot{h}_\times$ dies-off at $u = \pm\infty$, one can select Poincaré groups $\mathcal{P}^\pm$ of $\mathfrak{B}$. However, if the total gravitational memory is non-zero, then $\mathcal{P}^+ \neq \mathcal{P}^-$. (Related to infra-red/soft sectors in the quantum theory).

$\vec{J}_{\rm total}$ refers to $\mathcal{P}^-$ while $\vec{S}_{\rm final}$ refers to $\mathcal{P}^+$. (Analogous to $L_z(t_1)$ and $(L_z + dP_y)(t_2$ in Newtonian systems.) Care is needed to calculate the angular momentum radiated away!

• Enlargement from the 4-d translation to infinite dimensional supertranslation group is forced by gravitational waves. Therefore, supertranslations provide a powerful tool for gravitational waves, but one that has been largely ignored so far.

1.A Overview (contd)

- What next? Enhancing the creation of human resources: I will focus on the theoretical aspects because Stan will cover the experimental aspects

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