

# Probing neutron star core matter using EM observations

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# Measurement of neutron star parameters

## Neutron Star: Surface and Interior

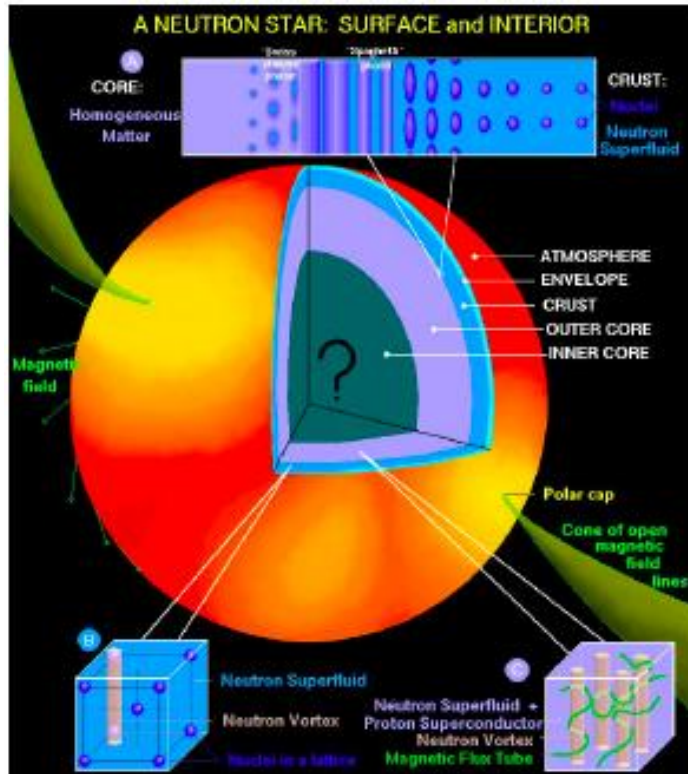
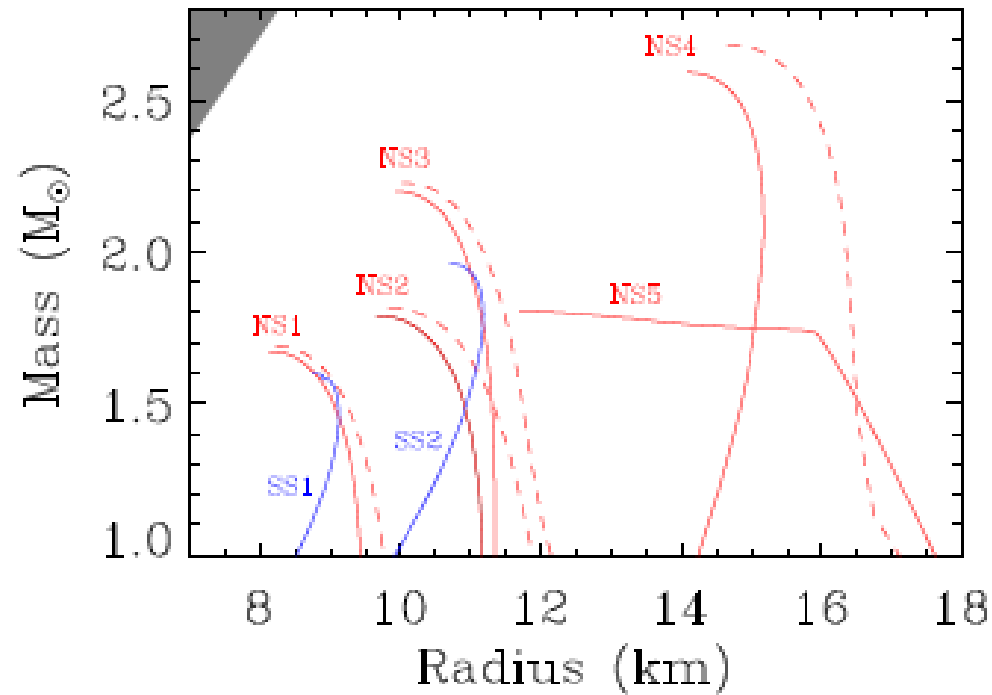


Figure courtesy: D. Page

## How to constrain EoS models?

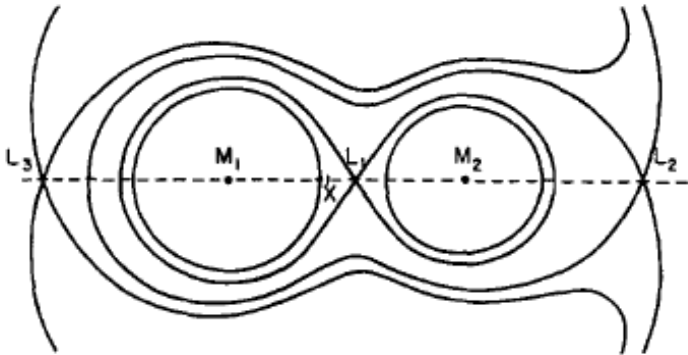


SB, AdSpR, 45, 949 (2010)

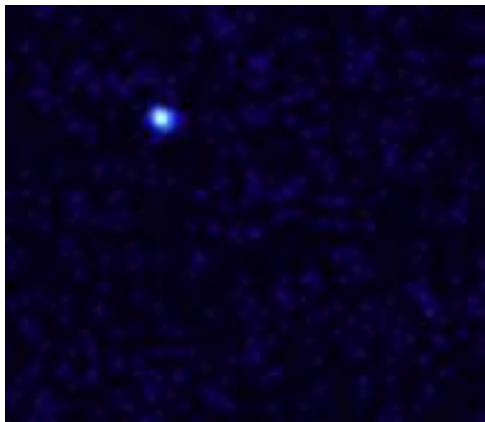
Mass, radius and spin frequency, or three independent structural parameters of the *same* neutron star are to be measured in order to constrain equation of state models.

# Low-mass X-ray binary (LMXB)

Equipotential surfaces in a binary system



Courtesy: Bhattacharya & van den Heuvel (1991)

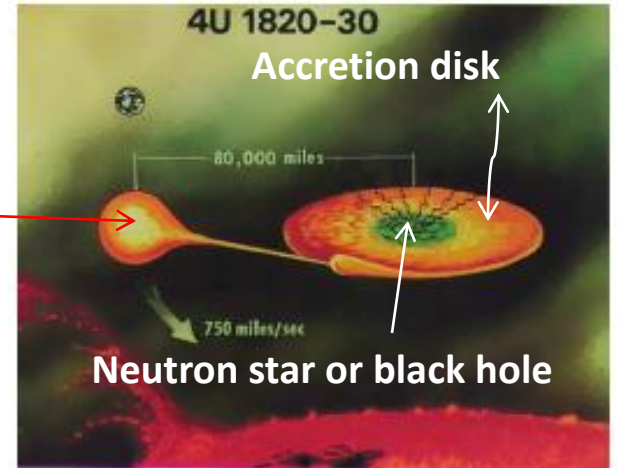


Chandra image of KS 1731-260  
Courtesy: NASA website

Angular size is so small  
that an LMXB cannot be  
spatially resolved.

Only spectral and timing  
methods are available to  
probe LMXBs.  
(Exception: some jets).

Low-mass X-ray binary (LMXB)  
(Artist's impression; NASA website)



Primarily emits X-rays. But also  
emits in other wavelengths.

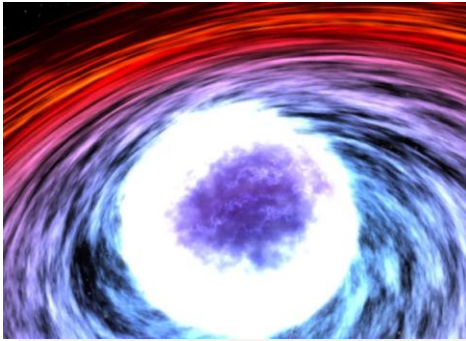
A reasonably clean accretion  
process with disk extended close to  
the compact object →  
Ideal to probe strong gravity and  
compact object properties.

# List of complementary methods to measure parameters of neutron stars in low-mass X-ray binaries

1. Thermonuclear X-ray burst: Continuum spectrum method
2. Thermonuclear X-ray burst: PRE burst method
3. Thermonuclear X-ray burst: Milli-Hz QPO method
4. Thermonuclear X-ray burst: Burst oscillation method
5. Accretion-powered pulsation method
6. Kilo-Hz QPO method
7. Broad relativistic iron line method
8. Quiescent emission method
9. Mass measurement: binary orbital motion method

Many complementary methods may be required to overcome the systematic effects.

# Thermonuclear X-ray Bursts



Accretion on neutron star

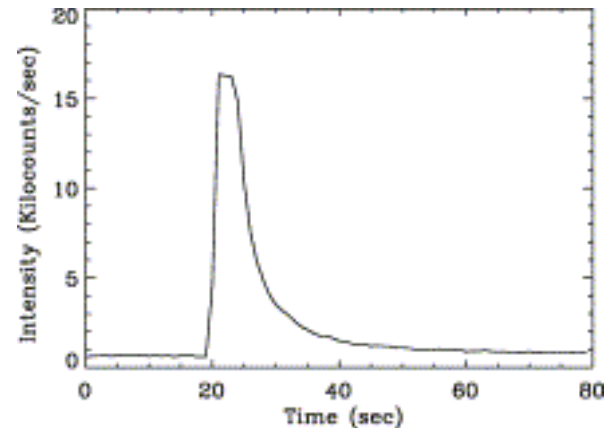
Unstable nuclear burning of accreted matter on the neutron star surface causes type I (thermonuclear) X-ray bursts.

Rise time  $\approx 0.5 - 5$  seconds  
Decay time  $\approx 10 - 100$  seconds  
Recurrence time  $\approx$  hours to day  
Energy release in 10 seconds  
 $\approx 10^{39}$  ergs



Sun takes more than a week to release this energy.

Burst light curve



Why is *unstable* burning needed?

Energy release:

Gravitational  $\approx 200$  MeV / nucleon

Nuclear  $\approx 5$  MeV / nucleon

**Accumulation of accreted matter for hours  $\rightarrow$  Unstable nuclear burning for seconds  $\Rightarrow$  Thermonuclear X-ray burst.**

# Thermonuclear X-ray Bursts

Bursts provide complementary methods to measure parameters of neutron stars in low-mass X-ray binaries

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4. Thermonuclear X-ray burst: Burst oscillation method

# 1. Continuum spectrum method

Burst spectra are normally well fit with a blackbody model.

In principle, neutron star radius can be measured from the observed bolometric flux ( $F_{\text{obs}}$ ) and blackbody temperature ( $T_{\text{obs}}$ ), and the known source distance ( $d$ ):

$$R_{\text{obs}} = d \cdot (F_{\text{obs}} / (\sigma T_{\text{obs}}^4))^{1/2}$$

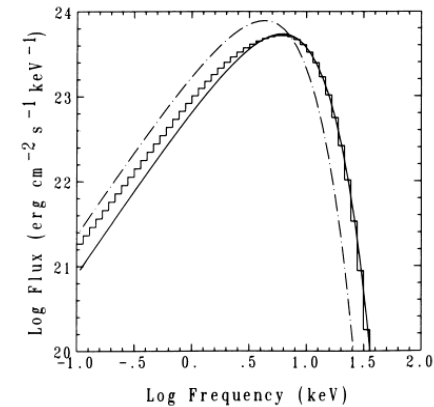
But there are systematic uncertainties:

- (1) unknown amount of spectral hardening;
- (2) effect of unknown gravitational redshift;
- (3) unknown distance;
- (4) if part of the surface emits.

$$\begin{aligned} T &= T_{\text{obs}} \cdot (1+z)/f \\ R &= R_{\text{obs}} \cdot f^2 / (1+z) \end{aligned} \quad \left\{ \begin{array}{l} z > 0; f \sim 1.0 - 2.0 \\ 1+z = [1 - (2GM/Rc^2)]^{-1/2} \end{array} \right.$$

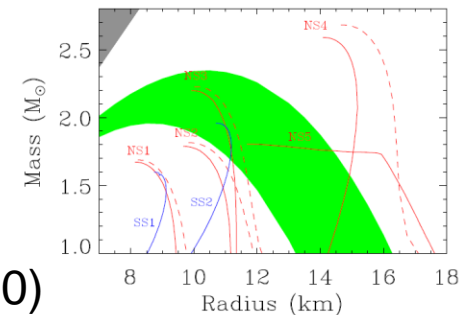
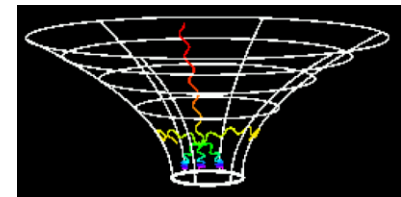
**Atmospheric chemical composition,  
surface gravity, temperature  $\Rightarrow f$   
(primary problem)**

Burst spectra



London, Taam & Howard (1986)

Gravitational redshift





# Thermonuclear X-ray Bursts

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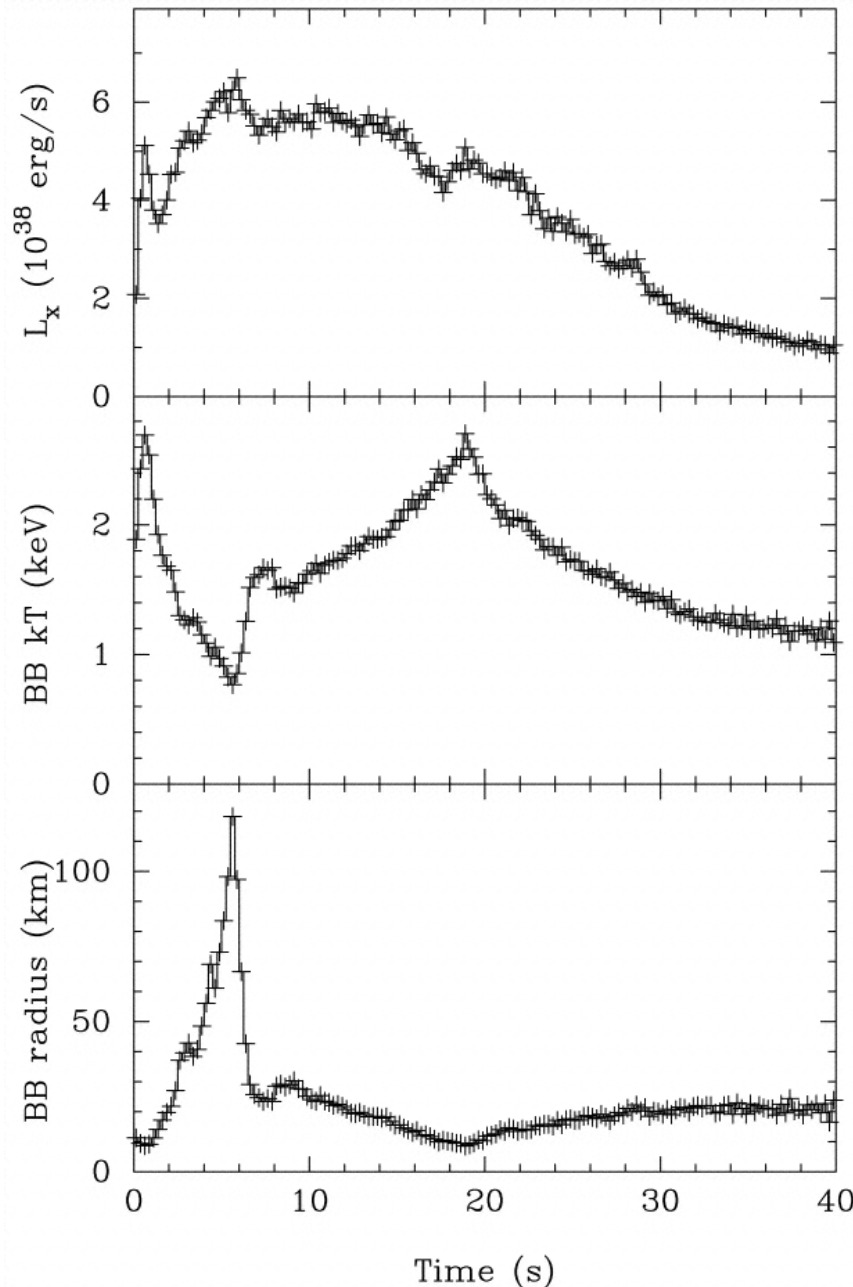
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## 2. PRE burst method



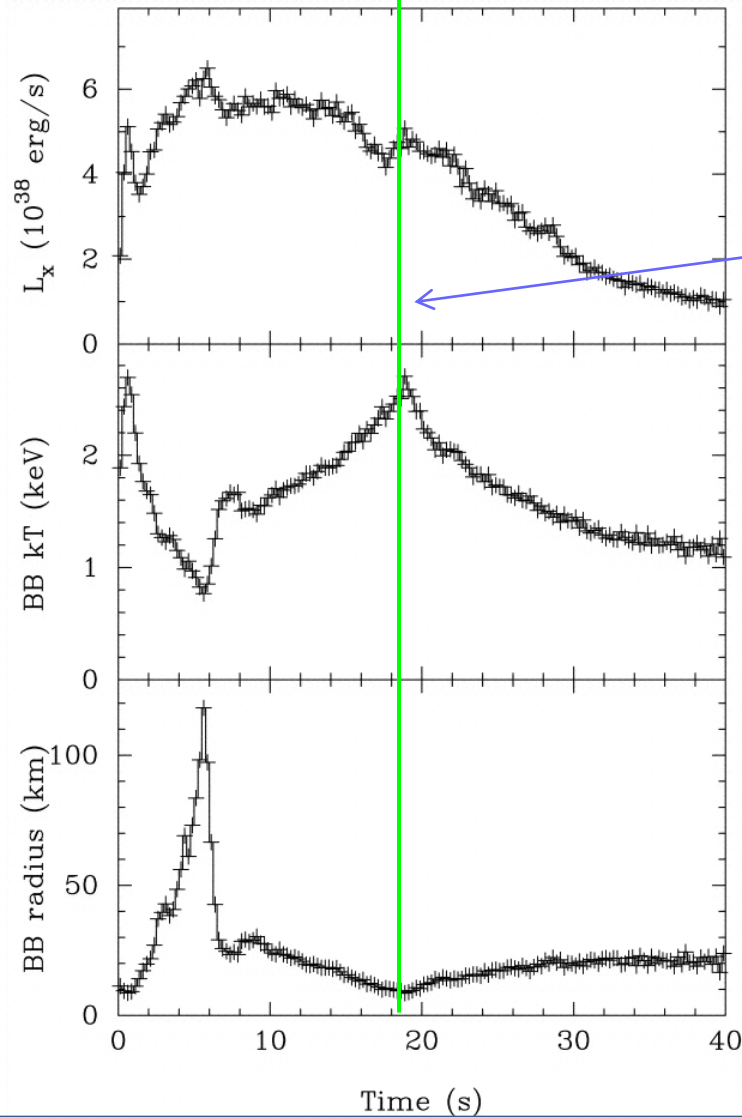
### Photospheric Radius Expansion (PRE) bursts

The burst is so strong that the radiative pressure pushes the photosphere or the neutron star atmosphere away from the stellar surface temporarily.

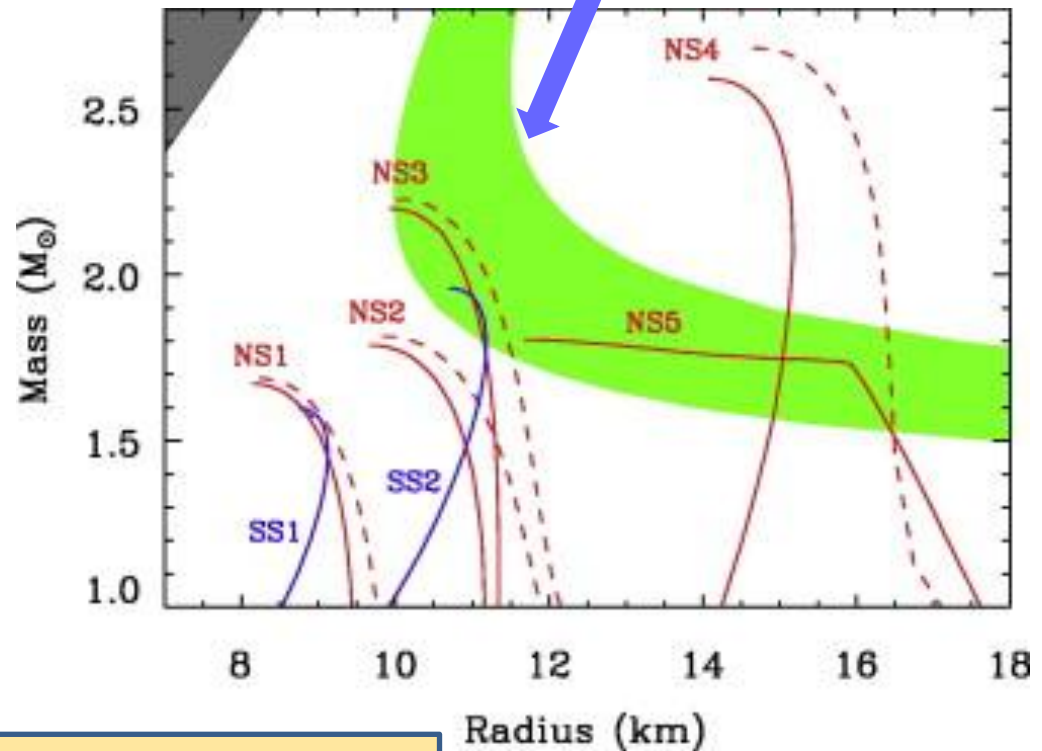
Smale (2001)

## 2. PRE burst method

Smale (2001)



**Eddington flux =**  
**Touch-down flux**



$$F_{\text{Edd}} = (1/4\pi d^2)(4\pi GMc/\kappa)(1-2GM/Rc^2)^{1/2}$$

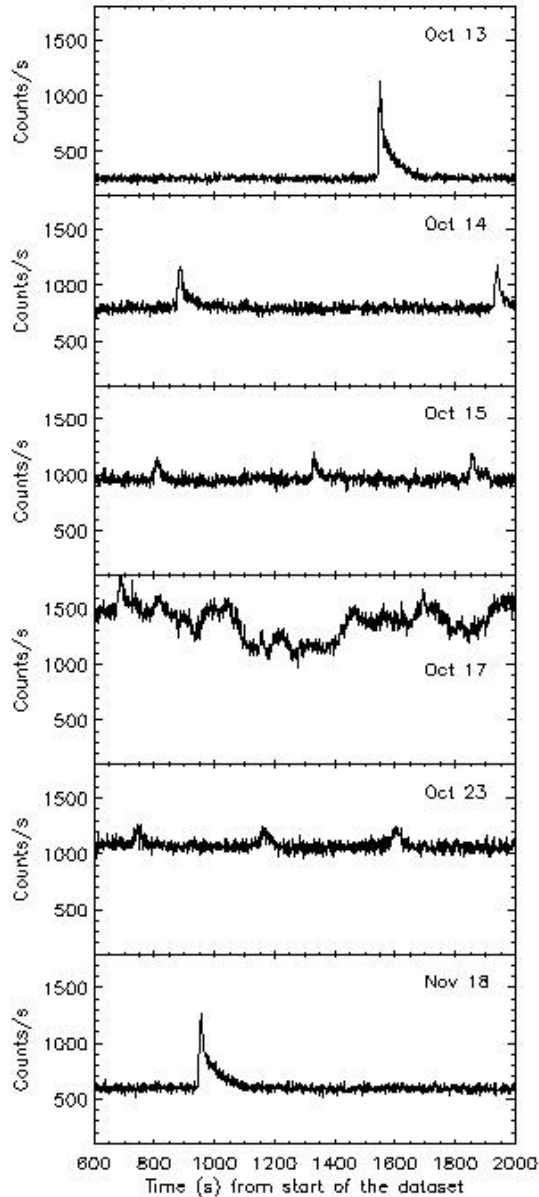
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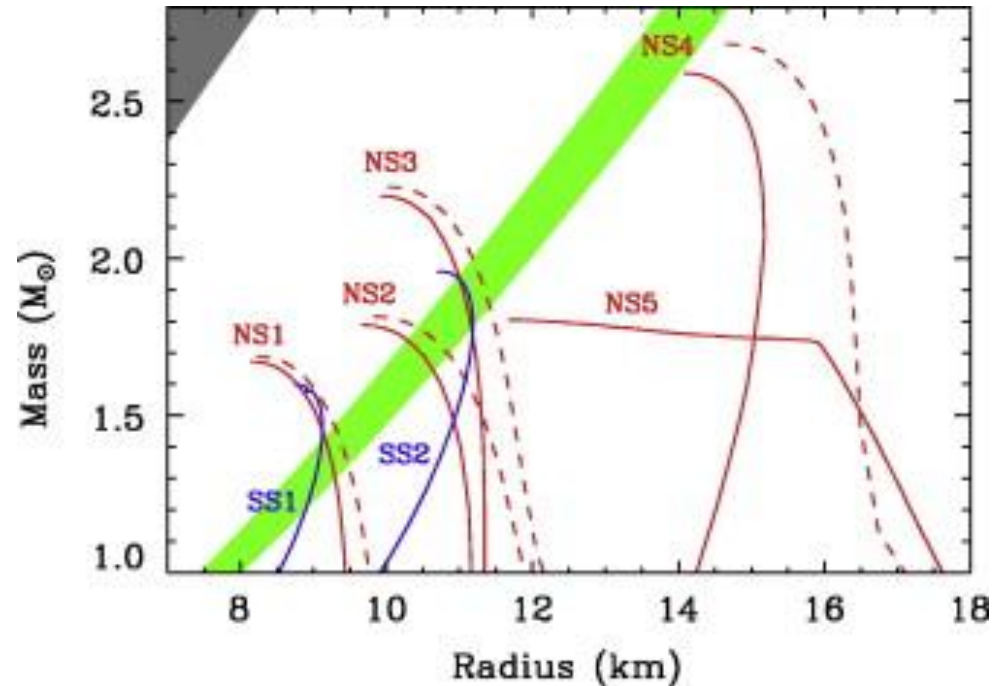
### 3. Milli-Hz QPO method

#### IGR J17480-2446



From the marginal burning stability condition, the oscillation frequency is a strong function of the surface gravity:

$$g \cong GM/R^2[1 - 2GM/RC^2]^{-1/2}$$



SB, AdSpR, 45, 949 (2010)

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# 4. Burst oscillation method

## Fast Timing Properties of X-ray Bursts (Burst Oscillations)

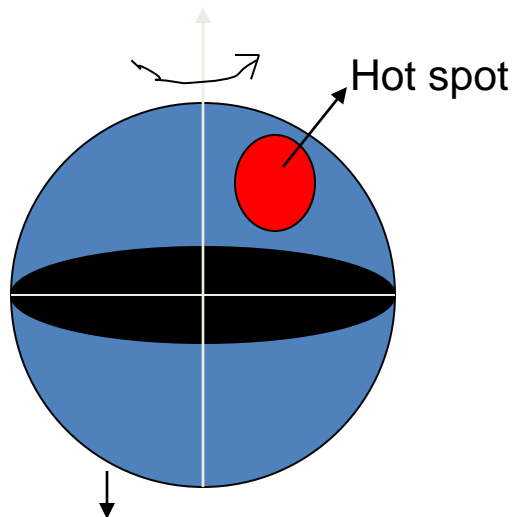
### What are burst oscillations?

These are millisecond period variations of observed intensity during thermonuclear X-ray bursts.

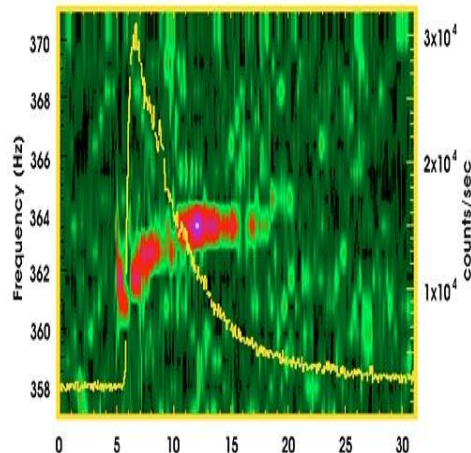
### What is their origin?

Asymmetric brightness pattern on the spinning neutron star surfaces.

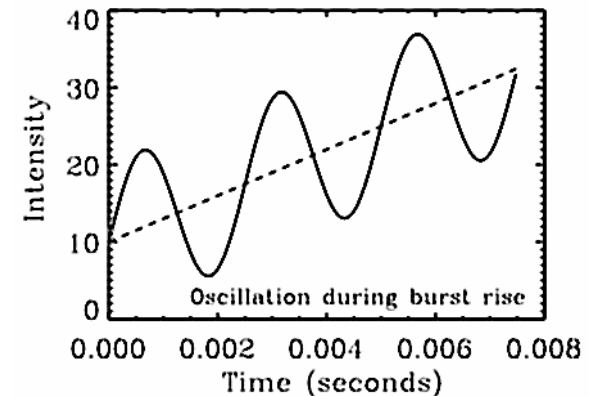
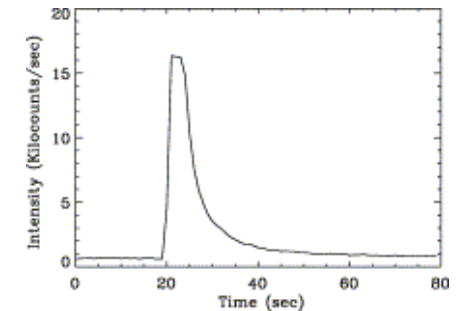
**Neutron star spin frequency  
= Burst oscillation frequency**



Spinning neutron star



Burst light curve



## 4. Burst oscillation method

### Burst oscillation method to measure neutron star parameters:

**SB**, Strohmayer, Miller, Markwardt, ApJ, 619, 483 (2005)

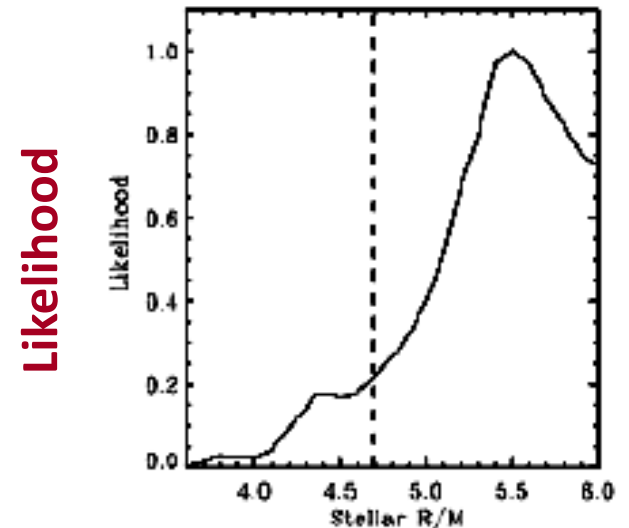
Physical effects included in the model:

(1) Doppler boosts; (2) special relativistic beaming; (3) gravitational red-shift; (4) light bending; (5) frame dragging.

Free parameters for a chosen EoS model:

- (1) NS radius-to-mass ratio:  $R/M$ ;
- (2)  $\theta$ -position of the centre of the hot spot:  $\theta_c$ ;
- (3) angular radius of the spot:  $\Delta\theta$ ;
- (4) observer's inclination angle w.r.t. NS spin axis:  $i$ ;
- (5) beaming (due to NS atmosphere) parameter:  $n$   
[specific intensity as a function of angle  $\psi$  (in emitter's frame) from surface normal is given by  $I(\psi) \propto \cos^n(\psi)$ ];
- (6) background.

XTE J1814-338  
(RXTE-PCA data)



**NS  $R/M$**

The vertical dashed line gives the lower limit of the stellar radius-to-mass ratio with 90% confidence.

$$q(\theta_k) = \int d\theta_1 \dots d\theta_{k-1} d\theta_{k+1} \dots d\theta_n p(\theta_1 \dots \theta_n). \quad (1)$$

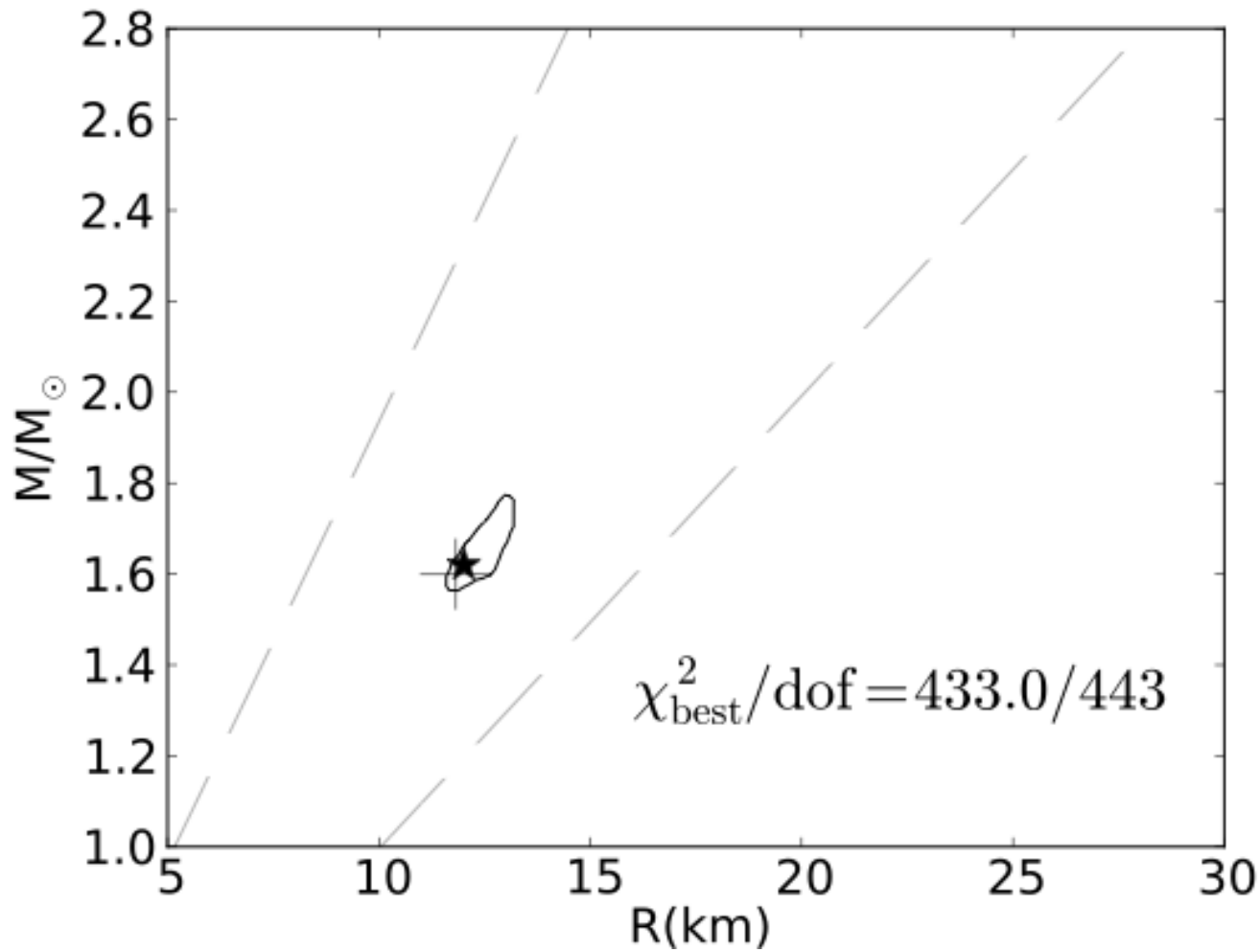
Integration over “nuisance” parameters.  
 $p \rightarrow$  posterior probability density



## 4. Burst oscillation method

Burst oscillation method to measure neutron star parameters:

### Simulated data fitting



A 10 square metres future timing X-ray instrument (originally done for **LOFT**) is considered for simulation.

Same method is being used for **NICER**, and may be used for other satellites (e.g., **eXTP**).

Ka Ho Lo, Cole Miller, **SB**, Fred Lamb, ApJ, 776, 19 (2013)

# List of complementary methods to measure parameters of neutron stars in low-mass X-ray binaries

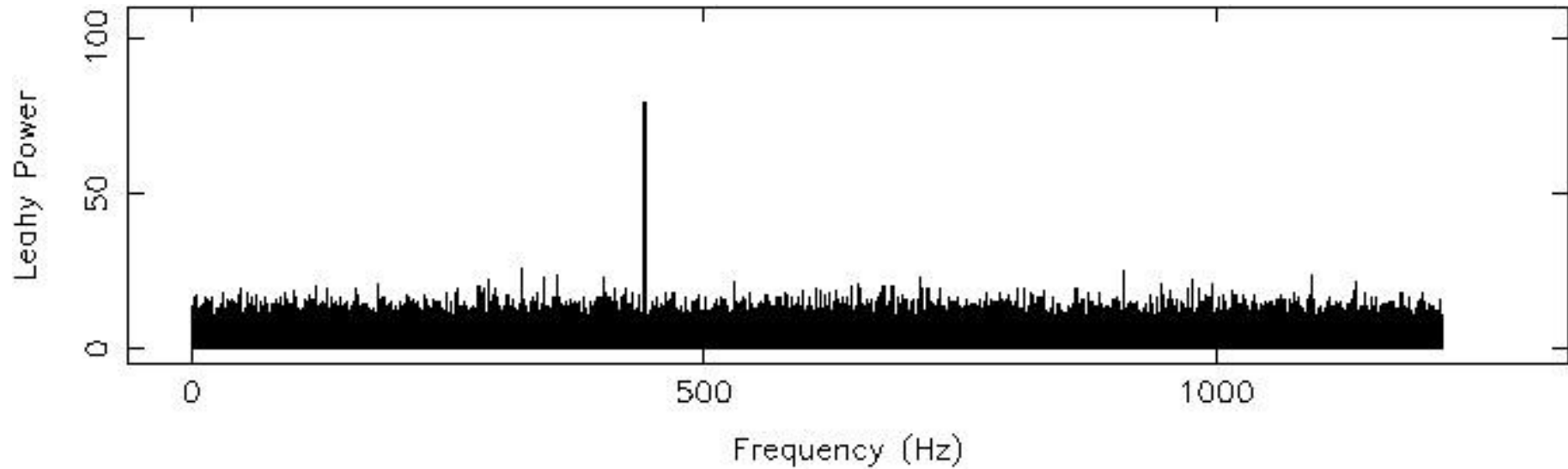
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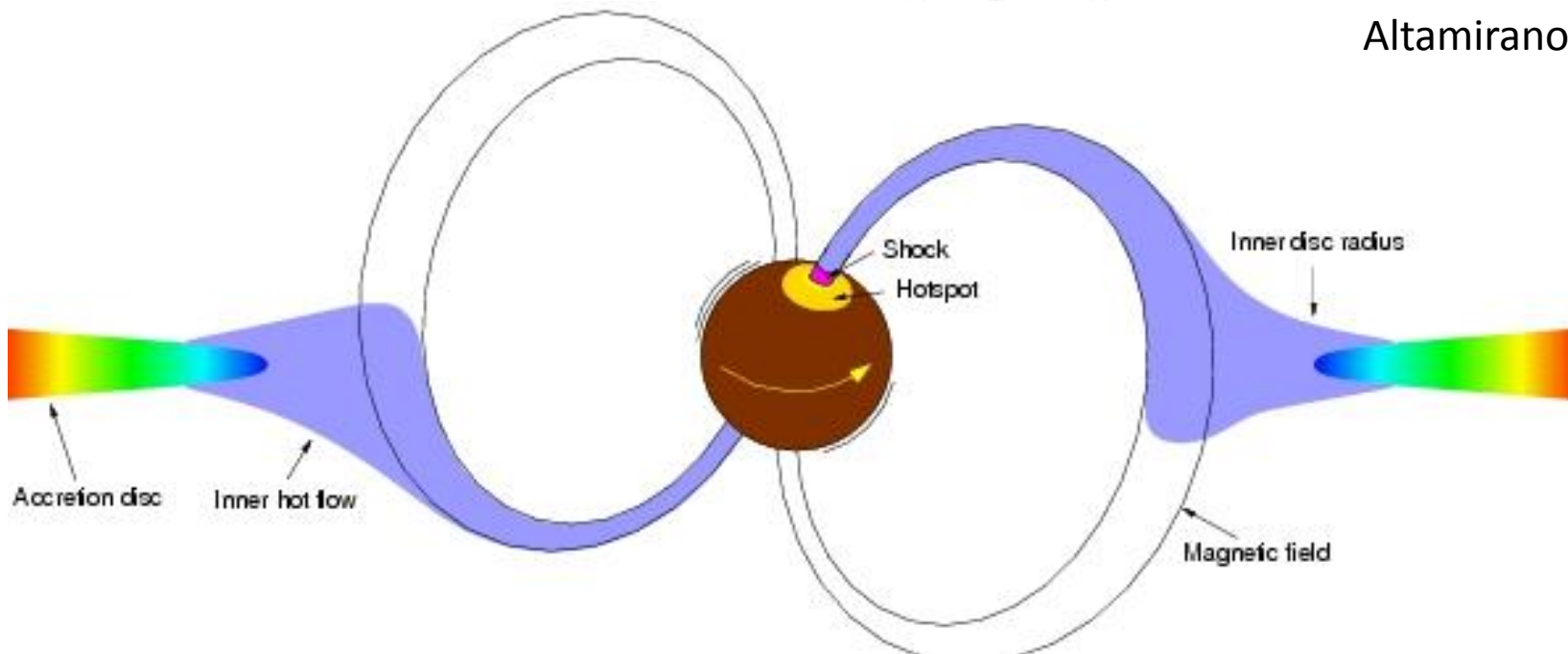
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# Accretion-powered pulsars



Altamirano et al. (2008)

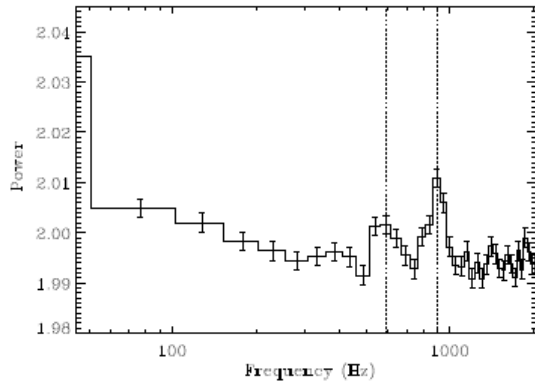


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# Kilohertz quasi-periodic oscillations (kHz QPOs)



**KHz QPOs often appear in a pair in the power spectrum, in the 200-1200 Hz frequency range.**

## Fourier Transform

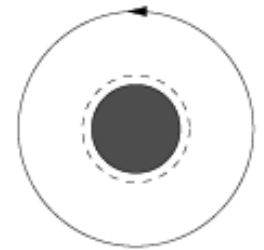
$$\hat{f}(\xi) := \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx.$$

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{2\pi i x \xi} d\xi$$

**Power is the Fourier transform of the spectrum intensity vs. time curve.**

**This observationally robust timing feature have been detected from many neutron star LMXBs.**

**According to almost all the models, the uniquely high kHz QPO frequencies are either the following accretion disk frequencies, or the beating or resonances among them, or with the neutron star spin frequency  $\nu_{\text{spin}}$ .**



$\nu_{\phi}$  : Orbital frequency

$\nu_r$  : Radial epicyclic frequency

$\nu_{\theta}$  : Vertical epicyclic frequency

$\nu_{\phi} - \nu_r$  : Periastron precession frequency

$\nu_{\phi} - \nu_{\theta}$  : Nodal or 'Lense-Thirring' precession frequency

# Kilohertz quasi-periodic oscillations (kHz QPOs)

## Accretion disk frequencies:

$$\nu_\phi = \frac{\sqrt{GM/r^3}/2\pi}{1 + j(r_g/r)^{3/2}} = \nu_K(1 + j(r_g/r)^{3/2})^{-1}$$

$$r_g \equiv GM/c^2$$

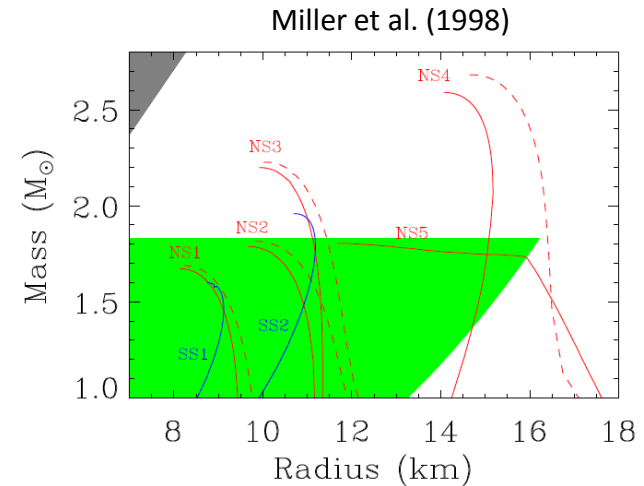
$$\nu_r = \nu_\phi \left(1 - 6(r_g/r) + 8j(r_g/r)^{3/2} - 3j^2(r_g/r)^2\right)^{1/2}$$

$$j \equiv Jc/GM^2$$

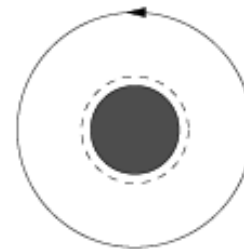
$$\nu_\theta = \nu_\phi \left(1 - 4j(r_g/r)^{3/2} + 3j^2(r_g/r)^2\right)^{1/2}$$

So when the correct model is identified, kHz QPOs can be used to measure the neutron star parameters **M** and **J**.

Apart from neutron star parameter measurement, kHz QPOs can be useful to study the matter flow in strong gravity region, and to test a law of gravitation.



Such a constraint on the neutron star mass-radius space is possible, if one of the kHz QPO frequencies is  $\nu_\phi$ , and the corresponding radial distance **r** is greater than or equal to the neutron star and the ISCO radius.



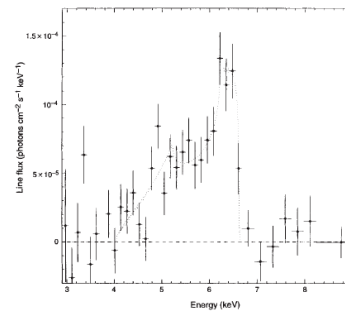
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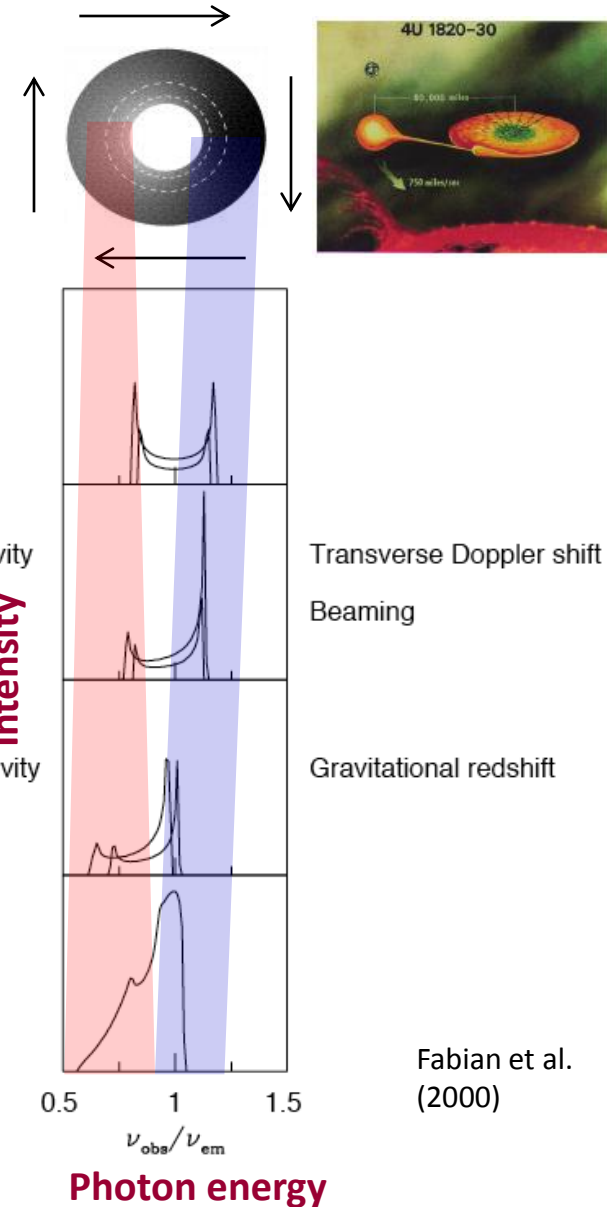
# Broad relativistic spectral line and its use

Broad asymmetric iron  $K\alpha$  emission lines are observed from accreting super-massive black hole (AGN) and stellar-mass black hole (black hole LMXB) systems. They originate from the inner part of the accretion disk.



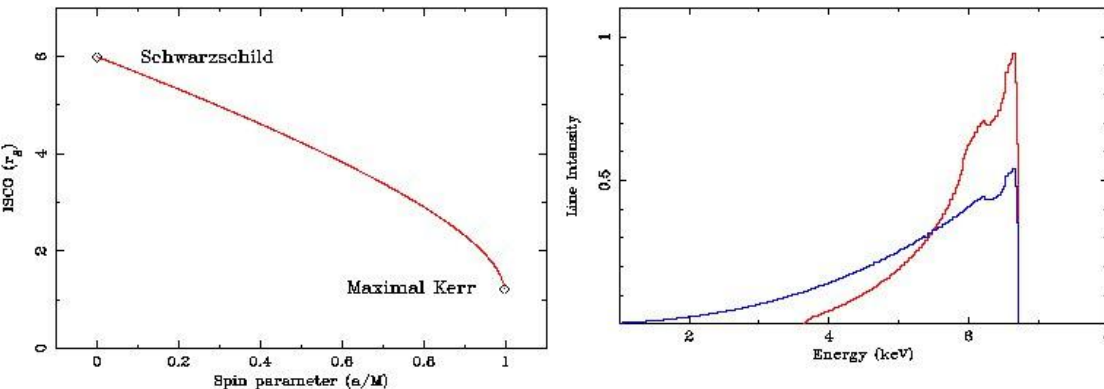
Tanaka et al. (1995)

Newtonian



Fabian et al. (2000)

They are a nature-given tool to measure the black hole spin and to probe the strong gravity regime.



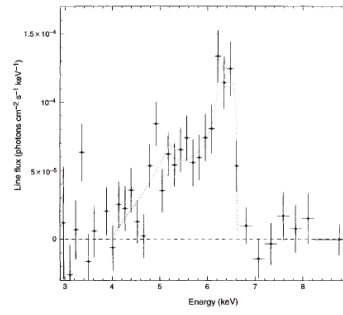
Theoretical models of broad line for two BH spin parameter ( $a/M \equiv Jc/GM^2$ ) values. (Miller 2007)



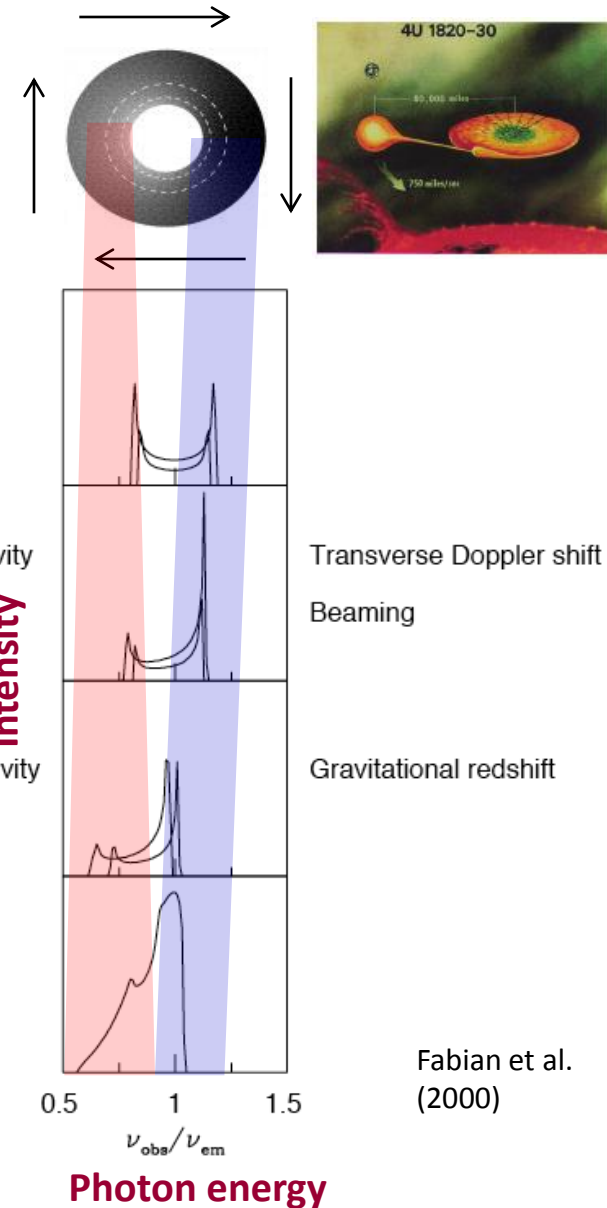
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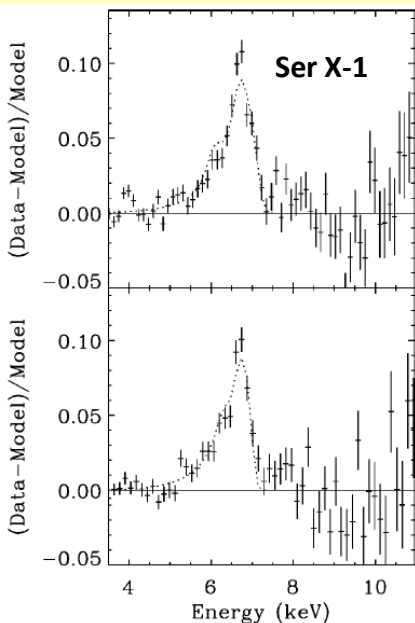
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Tanaka et al. (1995) Newtonian



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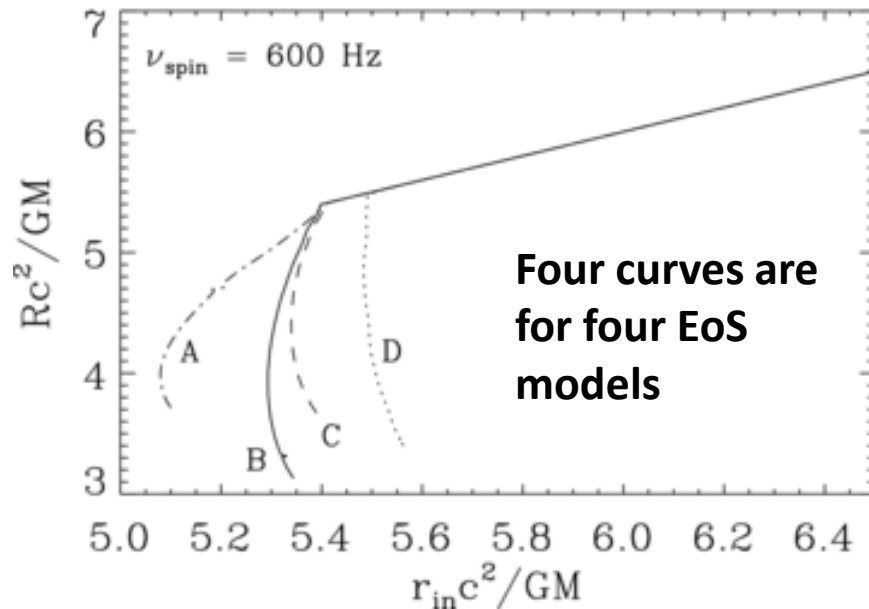


The first relativistic iron line from a neutron star LMXB was discovered by Bhattacharyya & Strohmayer, ApJ, 664, L103 (2007).

This opened up a new way to probe the strong gravity regime around neutron stars and to measure the stellar parameters.

# Broad relativistic spectral line and its use

**Rapidly spinning neutron star structures** have been calculated **with full GR effects** using the formalism of Cook, Shapiro & Teukolsky.



SB, MNRAS, 415, 3247 (2011)

**Conclusion:**  $r_{\text{in}} c^2 / GM$  is to be measured with better than an accuracy of **0.1** to effectively constrain EoS models.

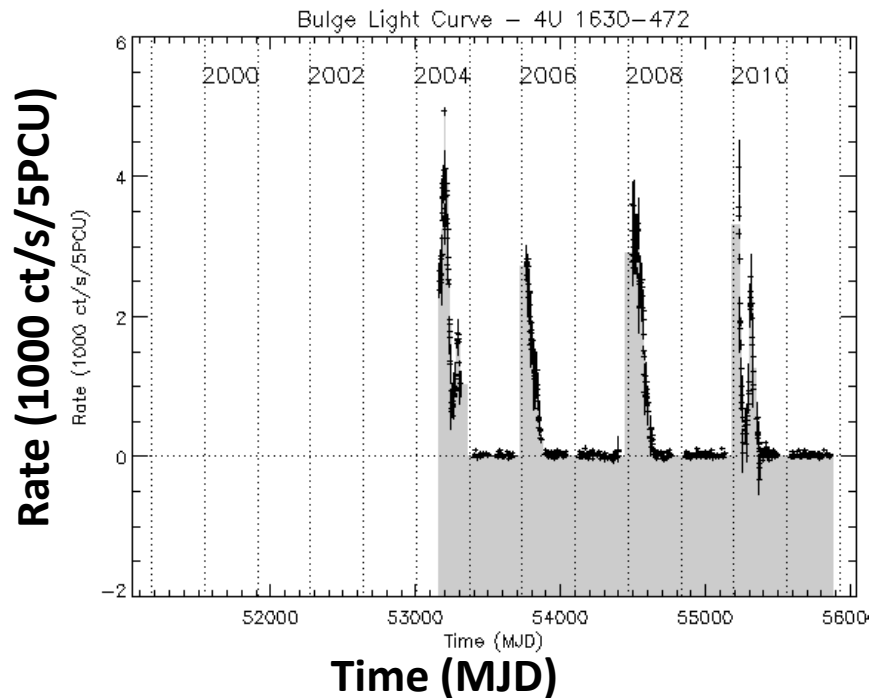
A section of the **White Paper** (Watts, Yu, Poutanen, Zhang, SB, et al., Science China PMA, 2019) of **eXTP's Science Working Group 1 on Dense Matter** was written based on this work.

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# Quiescent emission method



Transient sources show big outbursts, and quiescent periods between two subsequent outbursts.

RXTE/PCA light curve of 4U 1630-472 from Galactic bulge scan

Courtesy:

[http://asd.gsfc.nasa.gov/Craig.Markwardt/gal\\_scan/](http://asd.gsfc.nasa.gov/Craig.Markwardt/gal_scan/)

In the quiescent period, the neutron star surface emits almost like a blackbody.

So, in principle, neutron star radius can be measured from the observed bolometric flux ( $F_{\text{obs}}$ ) and blackbody temperature ( $T_{\text{obs}}$ ), and the known source distance ( $d$ ):

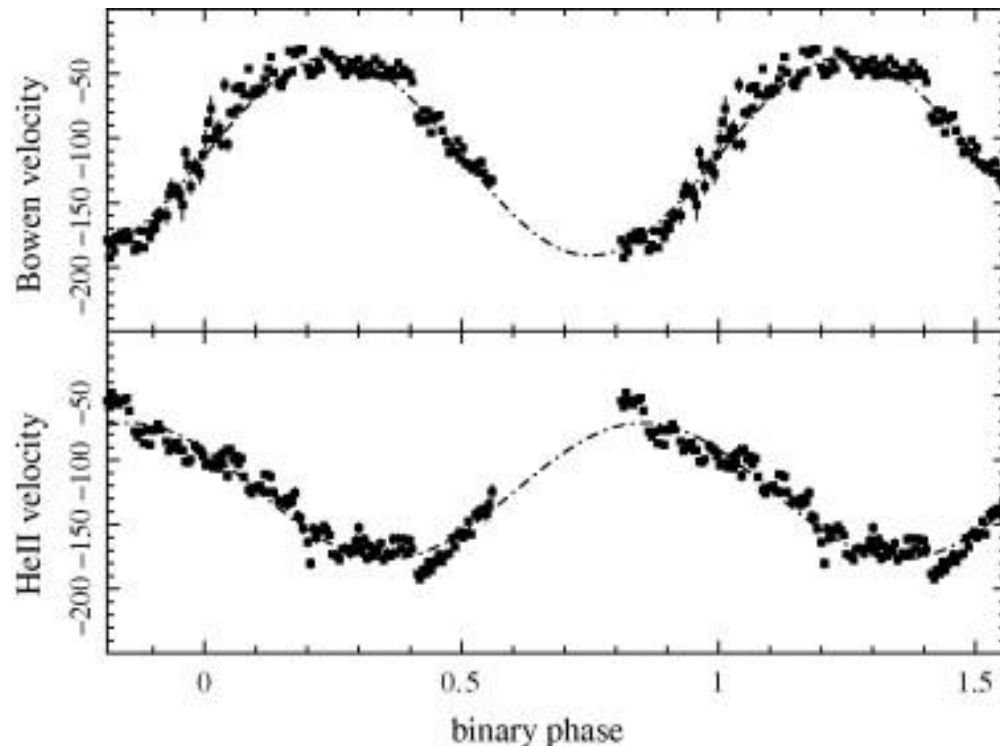
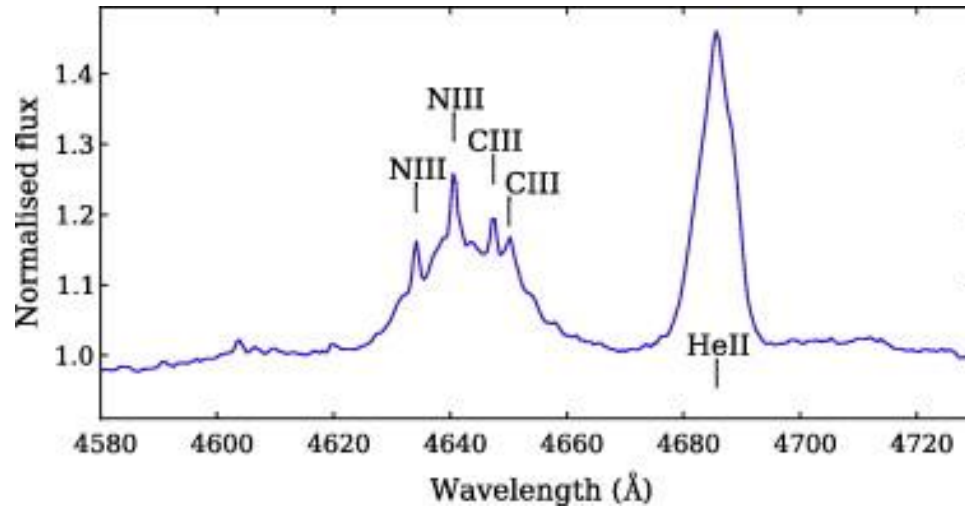
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# Mass measurement: binary orbital motion method



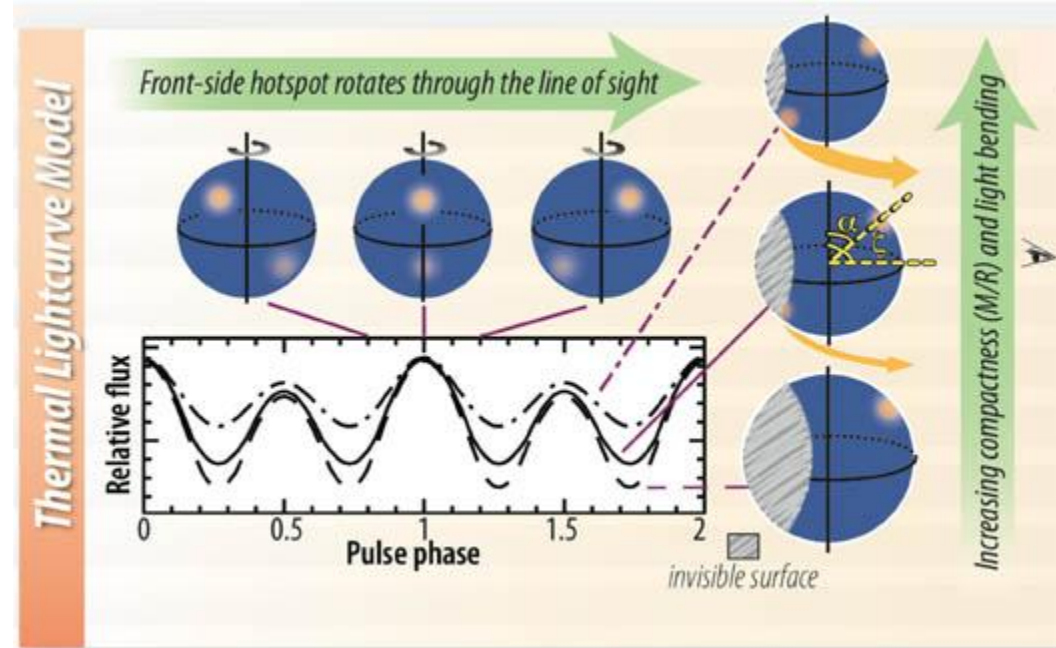
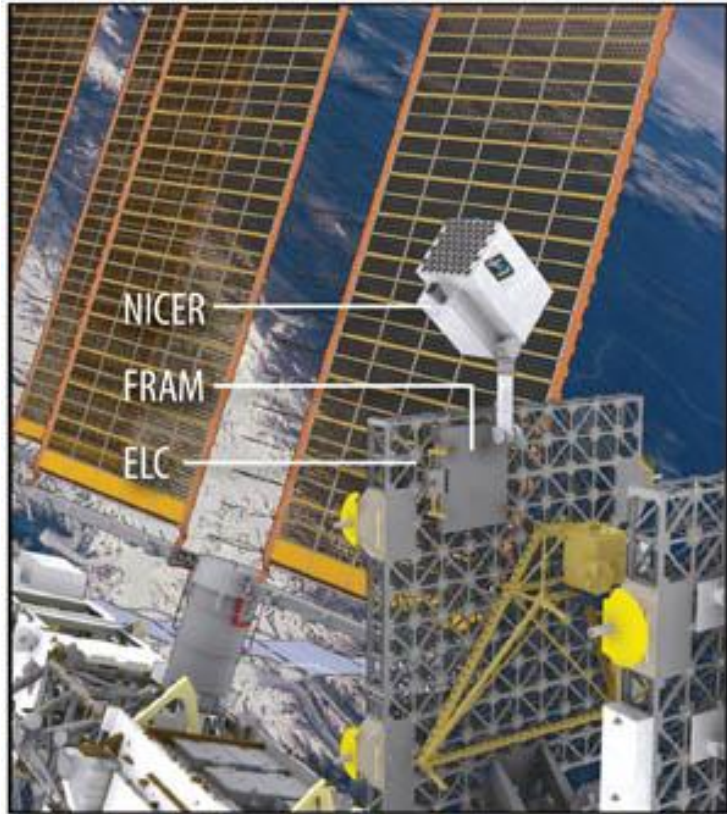
Velocity curves can be measured from the Doppler shifts of the spectral lines.

Mass can be estimated from the velocity curves and Kepler's 3<sup>rd</sup> law.

Steeghs and Casares (2002)

SB, AdSpR, 45, 949 (2010)

# Neutron star parameters from the measurement of spin-phase folded X-ray light curves of millisecond radio pulsars



[https://heasarc.gsfc.nasa.gov/docs/nicer/nicer\\_about\\_merged.html](https://heasarc.gsfc.nasa.gov/docs/nicer/nicer_about_merged.html)

**NICER satellite**



# Millisecond pulsars: Equation of state

**NICER** should tightly constrain the neutron star mass-radius space for a few millisecond pulsars.

For many equation of state (EoS) models, we numerically compute the rapidly spinning neutron star structures (considering the full effect of GR) for the known stellar spin frequencies, and we will try to constrain the EoS models from NICER results.

We particularly consider the possibility of transition from a neutron star to a strange (or to a hybrid) star.

**SB**, Bombaci, Logoteta, Thampan, ApJ, 848, 65 (2017)

