New approaches to gravitational wave source modeling: From Particle Physics to General Relativity

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The Amplitude Program

Investigation in the theory of scattering amplitudes has thaught about

- structure of quantum field theory
- provided tools for phenomenological computations

Until recently, phenomenololy applied to high-energy colliders experiment results only, currently there is a widespread effort to connect the

amplitude program → GW astronomy

However not trivial: transient interaction vs. quasi-circular orbits ruled by conservative+dissipative interactions

Approximations to solve Einstein equations

Method	Abbrv.	Approximation parameter
post-Newtonian	PN	$v^2 \sim \frac{G_N M}{r}$
post-Minkowskian	PM	$G_N \, (\frac{G_N M}{r})$
gravitational self force	GSF	$\eta \equiv m_1 m_2 / M^2$

Output of these methods are *input* for EOB approach, which in turn provides a framework for phenomenological tests

Present status of 2 body problem conservative dynamics

PM expansion parameter is $G_N M/r$, vs PN expansion

$$\mathcal{L} = -Mc^2 + \frac{\mu v^2}{2} + \frac{GM\mu}{r} + \frac{1}{c^2}[\ldots] + \frac{1}{c^4}[\ldots]$$

Terms known so far

		N	1PN	2PN	3PN	4PN	5PN	6PN	...
0PM	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	v^{14}	...
1PM		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	v^{12}/r	...
2PM			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	v^{10}/r^2	...
3PM				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	v^8/r^3	...
4PM					$1/r^4$	v^2/r^4	v^4/r^4	v^6/r^4	...
5PM						$1/r^5$	v^2/r^5	v^4/r^5	...
6PM							$1/r^6$	v^2/r^6	...
...								...	...

3PM recently computed by Z. Bern et al. PRL (2019)
4PN G^6 by S. Foffa, P.Mastrolia, RS, C. Sturm, W. Torres Bobadilla PRL (2019)

Self force up to NLO

Spin Interactions (PN)

Conservative and dissipative

v order	v^3	v^4	v^5	v^6	v^7	v^8	...
$\vec{S}\times\hat{L}$	✓ ✓		✓ ✓	0 ✓	✓ ✓[1]	0 ✓[2]	...
$\vec{S}^2$		✓ ✓		✓ ✓	0 ×	✓[3] ×	...
$\vec{S}^3$			✓ ✓		×		...
$\vec{S}^4$				✓ ✓		×	...
...					...		...

with leading $S^n \sim v^{1+n/2}$, see Guevara et al. 1906.1007
For dissipative $\vec{S} = 0$ results: 3.5PN is known,
4PN should be produced shortly by the Blanchet's group.

[1] A. Behé et al., 1212.5520

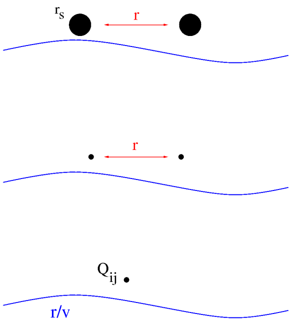
[2] Marsat & al. 1307.6793, Tagoshi et al. 9711072

[3] M. Levi & J. Steinhoff 1506.05794

The EFT point of view on the 2-body problem

Different scales in EFT (borrowing ideas from NRQCD):

- Very short distance $\lesssim r_s = G_N M$ negligible up to 5PN (effacement principle)
- Short distance: potential gravitons $H_{\mu\nu}$, $k_\mu \sim (v/r, 1/r)$ with $r \sim r_s/v^2$
- Long distance: GW's $k_\mu \sim (v/r, v/r)$ GWs $h^{GW}_{\mu\nu}$ coupled to point particles with moments



M. Beneke and V. A. Smirnov, NPB '98; I. Stewart and W. Manohar, PRD '07; W. Goldberger and I. Rothstein, PRD '06

EFT for compact binary systems: NRGR

Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}H(x)\exp\left[iS_{EH}(H+h_{GW}) + iS_{pp}(H+h_{GW})\right]$$

$$\begin{aligned} S_{pp} &= -m\int d\tau \\ S_{EH} &= \frac{1}{32\pi G_N}\int d^4x\left[(\partial h)^2 + h(\partial h)^2 + h^2(\partial h)^2 \ldots\right] \end{aligned}$$

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$$\begin{aligned} S_{pp} &= -m \int dt\, d^3x\, \delta(\vec{x} - \vec{x}_a(t)) \left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2 + h_{00}^2 \ldots\right) \\ S_{EH} &= \frac{1}{32\pi G_N} \int d^4x \left[(\partial_i h)^2 - (\partial_t h)^2 + h(\partial h)^2 + h^2(\partial h)^2 \ldots\right] \end{aligned}$$

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$$\begin{aligned} S_{pp} &= -m\int dt\, d^3x\, \delta(\vec{x}-\vec{x}_a(t))\left(\sqrt{G_N}\left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2\right) + G_N h_{00}^2 \ldots\right) \\ S_{EH} &= \frac{1}{2}\int d^4x\left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N}h(\partial h)^2 + G_N h^2(\partial h)^2 \ldots\right] \end{aligned}$$

EFT for compact binary systems: NRGR

Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

Effective

- Generic v-dependent potential terms
- Instantaneous couplings between world-line particles

$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}H(x)\exp\left[iS_{EH}(H+h_{GW}) + iS_{pp}(H+h_{GW})\right]$$

$$\begin{aligned} S_{pp} &= -m\int dt\, d^3x\,\delta(\vec{x}-\vec{x}_a(t))\left(\sqrt{G_N}\left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2\right) + G_N h_{00}^2 \ldots\right) \\ S_{EH} &= \frac{1}{2}\int d^4x\left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N}h(\partial h)^2 + G_N h^2(\partial h)^2 \ldots\right] \end{aligned}$$

$$S_{eff} = m_1\int dt\, d^3x\left[\frac{1}{2}v_1^2 + \frac{G_N m_2}{2r} + \frac{1}{8}v_1^4 + \frac{G_N^2 m_2}{2r^2}\left(\frac{G_N m_1}{2r} + v_1^2 + v_{1r}v_{2r}\right) + 1 \leftrightarrow 2\right]$$

3PN: Jaranowski, Schäfer, Damour; Blanchet, Faye; Itoh Futamase; Foffa & RS
4PN: Jaranowski, Schäfer, Damour; Blanchet et al, RS Foffa; RS Foffa, Mastrolia, Sturm

How to compute effective potential?

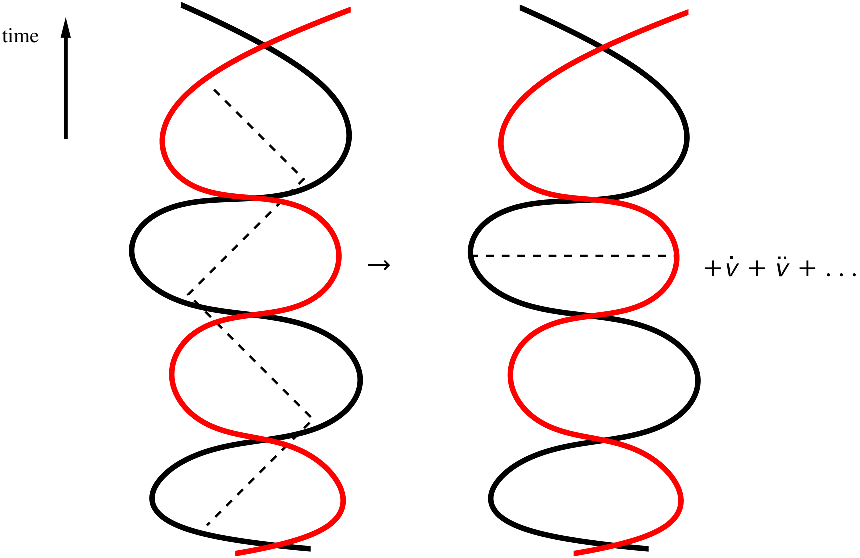
Iteratively solve Einstein equations for point particles ($J(x) \sim \delta^3(x-\bar{x})$):

$$
\begin{aligned}
-\int dt V_{1-2}(t,r) &\sim \int dt\, dt'\, d^3x\, d^3x' J(x) G(x-x') J(x') \\
&= 32\pi G_N m_1 m_2 \int dt\, dt' \frac{d^4k}{(2\pi)^4} \frac{e^{ik^\mu (x_1(t_1)-x_2(t_2))_\mu}}{k^2-\omega^2} \\
&\simeq G_N \int dt\, dt' \delta(t-t') \frac{m_1 m_2}{|x_1(t)-x_2(t')|}
\end{aligned}
$$

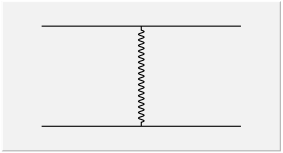
where k_0 has been neglected. Considering effects of v

$$
\begin{aligned}
V &\propto \int d\omega d^3k \frac{e^{-i\omega t_{12}+ikx_{12}}}{k^2-\omega^2} = \int d\omega d^3k \frac{e^{-i\omega t_{12}+ikx_{12}}}{k^2}\left(1+\frac{\omega^2}{k^2}+\ldots\right) \\
&= \delta(t_{12}) \int d^3k \frac{e^{ikx_{12}}}{k^2}\left(1+\frac{\partial_{t_1}\partial_{t_2}}{k^2}+\ldots\right) = \int d^3k \frac{e^{ikx_{12}}}{k^2}\left(1-\frac{k\cdot v_1 k\cdot v_2}{k^2}+\ldots\right)
\end{aligned}
$$

Trading knowledge over the full trajectory with knowledge of all derivatives of the trajectory at equal time

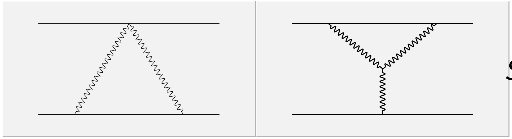


Newtonian potential:

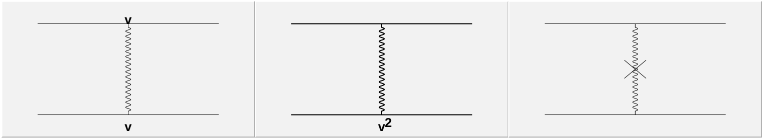


$$S_{eff} \sim \int dt \, \frac{G_N m_1 m_2}{r} \sim L$$

At 1PN:



$$S_{eff} = \int dt \frac{G_N^2 m_1^2 m_2}{r^2} \sim L v^2$$



$$V_{1PN} = -\frac{G_N m_1 m_2}{2r}\left[1 - \frac{G_N m_1}{2r} + \frac{3}{2}(v_1^2) - \frac{7}{2} v_1 v_2 - \frac{1}{2} v_1 \hat{r} v_2 \hat{r}\right] + 1 \leftrightarrow 2$$

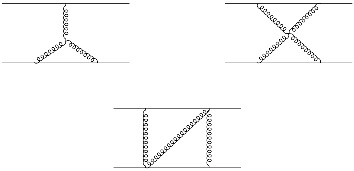
Method of regions, PN & PM

Internal graviton momentum can be expanded upon the following scaling:

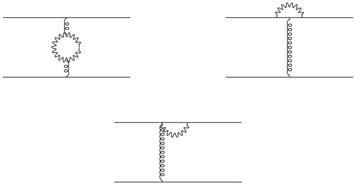
hard	(m, m)	quantum
soft	$(\vert\vec{q}\vert, \vert\vec{q}\vert)$	quantum
potential	$(v/r, 1/r)$	classical
radiation	$(v/r, v/r)$	classical

and then integrated over the full phase space
Only potential and radiation gravitons exchanged in classical processes: theory in terms of world lines selects diagrams that do not send source off-shell
Potential graviton → small change in energy wrt momentum, dominate classically

Ex. of classical connected diagrams



Ex. of quantum diagrams



From relativistic scattering amplitudes to 2-body potential

Equivalently, loop with massive particles contain a classical piece for $m \gg |\vec{q}|$:

$$V(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} A(\vec{q}, m_1, m_2, \vec{v}_1, \vec{v}_2, \ldots) e^{i\frac{\vec{q}\cdot\vec{r}}{\hbar}}$$

$$\supset \frac{1}{m^2 - \vec{k}^2} \quad \to \quad \underset{classical}{\frac{1}{m^2}} \quad + \quad \underset{quantum}{\frac{\vec{k}^2}{m^4}} \quad + \quad \ldots$$

E.g.: $\frac{G_N}{q^2}, \frac{G_N^2}{|q|}, G_N^3 \log|q| \ldots$ are classical contributions

PM and G_N expansions

In the binary scattering problem natural expansion in $G_N M/r$, $r \sim 1/(vq)$, $q \sim 1/b$, b, impact parameter, $J \sim \eta M v/q$. Expansion parameters

- G_N: $G_N Mq/v \sim G_N \eta M^2/J \equiv 1/j \to V_{PM} = \frac{V_{1PM}}{j} + \frac{V_{2PM}}{j^2} + \dots$
- v: non relativistic parameter

By treating sources relativistically:

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{16\pi G_N} R + \frac{1}{2}\sum_{a=1,2} D^\mu \phi_a D_\mu \phi_a - m_a \phi_a^2\right]$$

one can solve for the relativistic PM expansion, but isolate classical contributions. Technical helps in the task

- Double copy: gravity ~ gauge^2. Tree amplitudes through the Kawai-Lewellen-Tye (KLT) and Bern-Carrasco-Johansson (BCJ)
- Discard all quantum contributions
- Generalized unitarity used to glue tree level amplitudes into loop diagrams (helicity states compactify formulae)
- Various methods for integration: Differential equations, Mellin-Barnes, integration by parts

Z. Bern, C. Cheung, R. Roiban, C.-H. Shen, M. P. Solon, M. Zeng 1908.01493

More inputs from Amplitudes

Spin-2s particles with minimal coupling to gravity reproduce Kerr black-hole coupling at S^s order

Vaidya 1410.5384; Guevara, Ochirov, Vines 1906.10071

Scattering amplitudes ~ Full Gravitational Theory ~ 2-body potential
Quantum amplitudes contains information about both bound and unbound orbit altough in a complicated, involved way.
However an improved versions of the two-body dynamics from the (gauge-invariant) scattering function expressing the scattering angle χ as a function of E, J:

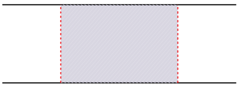
$$\frac{1}{2}\chi = \phi(e, j, \eta) = \frac{\chi_1}{j} + \frac{\chi_2}{j^2} + \ldots$$

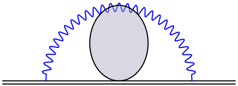
with $e = E/M$, $j \equiv J/(G_N m_1 m_2)$, reproducing the PM expansion

Damour 1609.00354, Damour 1710.10599

Mixing potential/radiation modes

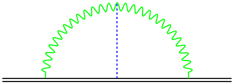
Within the PN approximation starting from 4PN (from G_N^4 in PM),
radiation emitted-scattered-reabosred alter the *conservative* dynamics of the system
UV div. from radiation m. kill IR div. from potential m. (zero bin subtruction) leaving UV divergencies in potential m. (treated with local counterterms)



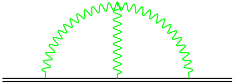


S. Foffa, R. Porto, RS, I. Rothstein 1903.05118
5PN in S.Foffa, RS 1907.02869

Tail (hereditary)



Memory (local-in-time)



Outlook

- Deep knowledge and large community of people involved in amplitude computations, their implications for 2-body bound orbit dynamics not fully explored
- Not only new (and old) methods are being applied to old problems, but an entire community is devoting its attention to gravitational sources
- In particular one can hope that some of these methods may shed light on non-perturbative aspects of the two-body problem.

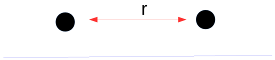
Recent results at 5PN

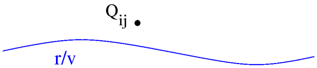
PN approximation to General Relativity

Small expansion parameter v, related to metric perturbation $v^2 \sim \frac{G_N M}{r}$

Near zone, $D \sim r$

Far zone, $D \gtrsim \lambda = r/v$





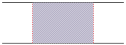
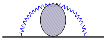
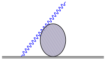
Describe conservative dynamics

conservative + dissipative

EFT framework pioneered by W. Goldberger and I. Rothstein, PRD '06

PN approximation for compact binary systems

	Near	Far
World-line	$-m_a \int d\tau = m_a \int dt \times$ $\left(\phi + A_i v^i + \sigma_{ij} v^i v^j + \ldots\right)$ $O_{ijk} E^{ij,k} + J_{ij} B_{ij} + \ldots$	$\int d^4x \Big(E h_{00} + \frac{1}{2} L_{\alpha\beta} \omega_\mu^{\alpha\beta}$ $+ Q_{ij} E^{ij} \ldots \Big)$
Bulk	$\frac{1}{16\pi G_N} \int d^4x \left[R - \frac{1}{2} \left(g^{\alpha\beta} \Gamma^\mu_{\alpha\beta} \right)^2 \right]$	
5PN	G_N, G_N^2, G_N^3 ✓ [4] G^4, G^5 × G^6 ✓ RS et al. PRL (2019)	✓ Foffa& RS 1907.02869

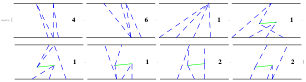
5PN is the lowest order finite size effect are not forbidden effacement principle, but expected at > 5PN order $(Love_{BH} = 0)$[5]

[4] PM: Duff ('73); Westpfahl & Goller, LNC ('79); Damour PRD ('18), Cheung et al. PRL ('18), Bern et al. PRL ('19)

[5] See Binnington & Poisson, Damour & Nagar PRD ('09); Kol & Smolkin JHEP ('12)

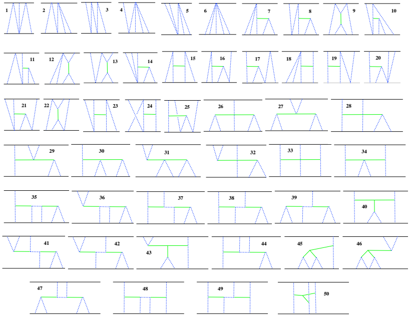
Near zone static sectors 3 and 4PN

3PN



All factorizable

4PN



25 factorizable + 25 prime

Factorization theorem in the static sector

Theorem

At $(2n+1)$-PN order all static graphs are factorizable

Proof.

$$V \propto G^{d_M-1} m_1^{d_{m_1}} m_2^{d_{m_2}} = G_N^{d_M-1} m_1^{d_M} \left(\frac{m_2}{m_1}\right)^{d_{m_2}}$$

Only world-line $m_i\phi^n$ and bulk $\phi^2\sigma^k$ vertices matter
Prime diagrams must have all $n=1 \implies d_M = 2m$ since all internal vertices have even number of ϕ. Then $V \propto G^{2m-1} \subset (2m-2)$-PN □

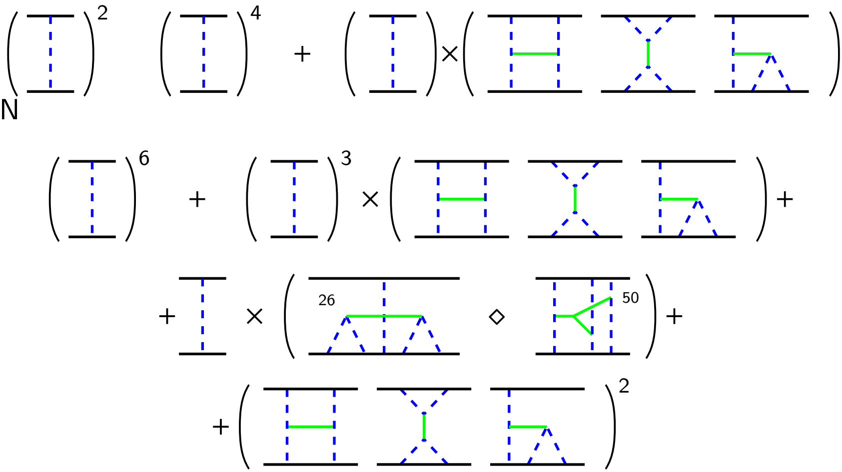
At $(2n+1)$-PN order no integration needed, just multiplications!

How it works in practice:

1PN

3PN

5PN

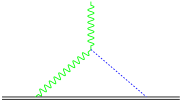
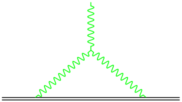
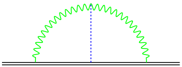
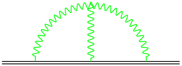


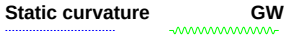
- Finite- and rational-ness inherited from 4PN (non trivially!)
- Schwarzschild limit OK ($m_2 \ll m_1$)
- Result confirmed by explicit, brute force calculation of ~ 100 diagrams in Blümlein et al arXiv:1902.11180
- Factorization does not hold for all diagrams at n-PN sectors $G^{1+n-j} v^{2j}$ $(0 < j \leq n)$ but for $\gtrsim 50\%$ of them.

$$V_{5PN\,static} = \frac{G_N^5 m_1^3 m_2}{r^5}\left(\frac{5}{16}m_1^2 + \frac{91}{6}m_1 m_2 + \frac{653}{6}m_2^2\right)$$

Hereditary terms

Wform depends on the history rather than source's state at retarded time (propagation inside the light-cone)

	Tail:	Memory:
1pt		
	hereditary Blanchet & Damour PRD (1988)	hereditary Christodoulou PRL (1991); Blanchet & Damour PRD (1992)
Self-energy	↓ 	↓ 
	hereditary Foffa & RS PRD (2013)	instantaneous Foffa & RS 1907.02869



Far zone Self Energy results at 5PN

Real part → conservative dynamics (to be added to near zone results, starting 4PN order)

Imaginary part matches into flux formula $F \propto \dddot{Q}_{ij}^2 + \dots$

Divergent graphs regularized in dim. reg.:

divergence (and coeff. of logarithmic term) linked to imaginary part

$$
\begin{aligned}
S_{5PN\,tail} &= G_N^2 M \int \frac{dk_0}{2\pi}\left[-\frac{1}{5}\left(\frac{1}{\epsilon}+\log\left(k_0^2/\bar{\mu}^2\right)-i\pi+\frac{41}{30}\right)|Q_{ij}|^2\right.\\
&\quad -\frac{1}{189}\left(\frac{1}{\epsilon}+\log\left(k_0^2/\bar{\mu}^2\right)-i\pi+\frac{163}{35}\right)|O_{ijk}|^2\\
&\quad \left.-\frac{16}{45}\left(\frac{1}{\epsilon}+\log\left(k_0^2/\bar{\mu}^2\right)-i\pi-\frac{127}{60}\right)|J_{ij}|^2\right]\\
S_{5PN\,Ltail} &= \frac{8}{15} G_N^2 \int dt\, \ddddot{Q}_{il}\dddot{Q}_{jl}\epsilon_{ijk}L_k\\
S_{5PN\,memory} &= G_N^2 \int dt\left[-\frac{11}{14}\ddddot{Q}_{il}\ddddot{Q}_{jl}Q_{ij}-\frac{1}{5}\dddot{Q}_{il}\dddot{Q}_{jl}\ddot{Q}_{ij}\right]
\end{aligned}
$$

- Imaginary part of self energy diagrams linked to n-PN flux formula (trivial) and to divergent (and log) part of real part
- Real part combines with near zone dynamics at $(n+4)$-PN, its divergence fixed by flux formula at n-PN
- Log-term is non-instantaneous (but causal) adding to the conservative dynamics
- Log-term becomes instantaneous on circular orbits, contribution to $E(x)$ agrees with 5PN log computed in Le Tiec et al. PRD (2012) and Bini & Damour PRD (2014)

In-in vs. in-out

- We work with Feynman propagator within the time symmetric in-out framework, which returns *conservative* dynamics and *averaged* emission.
- The in-in formalism is appropriate for time-asymmetric problems, needed to compute *back-reaction* force and *instantaneous* emission, see Galley & Tiglio PRD (2009) and Galley, Leibovich, Porto and Ross (2016)